Economics 451   Exam 1   Fall 2008   Prof Montgomery

Answer all questions. 100 points possible.

1. [30 points] Consider the model of the division of labor in the household offered by Becker in Chapter 2 of his *Treatise on the Family*. In particular, consider the simplest case we studied in lecture: the household is composed of a husband and wife; both the market and household good are normal goods in the household's utility function; each household member allocates time between the market and household sectors; each household member has fixed coefficients of market and household production (and hence constant returns in both sectors); the husband has the comparative advantage at market production, while the wife has the comparative advantage at household production.

a) Assuming that the household is efficient, what divisions of labor (i.e., time allocations by the husband and wife) might we observe in this household?

b) Now suppose that the government starts a new program that provides free childcare to this household. (For our purposes, you should conceptualize this increase in childcare as an increase in the amount of the household good received by the household.)

i) How would this government program affect the household's overall production possibilities curve? Draw a PPC diagram to illustrate.

ii) How might the government program alter the household's level of consumption of the market and household goods? In particular, will the household necessarily consume more of the household good? Will the household necessarily consume more of the market good? Explain.

iii) How might the government program affect the optimal allocation of time by the members of the household? In particular, will both individuals necessarily decrease time spent working in the home? Will both individuals necessarily increase time working in the market? Explain.

iv) Does the government program alter your answer to part (a)? Explain.

2. [10 points] In his discussion of the division of labor in households, Becker suggests that smaller households should combine into larger households to capture the maximal benefit of the division of labor. But toward the end of Chapter 2, Becker suggests one source of diseconomies of scale that might limit household size. What is this source of diseconomies of scale? According to Becker, why do business establishments tend to be larger than households?

3. [40 points] Consider a marriage market with 3 males and 3 females. Utilities generated by each possible match are given by the following matrix: entry (i,j) gives the payoffs (m_{ij}, f_{ji}) that would be received by male i and female j if they marry. Assume that every individual would receive a payoff equal to zero if he/she remains single.

$$\begin{array}{cc} & \text{females} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{males} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} (1,3) & (4,2) & (6,1) \\ (3,7) & (5,6) & (4,5) \\ (2,5) & (9,1) & (3,2) \end{bmatrix} \end{array}$$

a) Assuming that utility is NOT transferable, find an equilibrium match structure. Is there more than one equilibrium match structure? If so, report a second match structure. If not, explain how you know there is only one equilibrium match structure.

b) If we now assume that individuals CAN transfer utility to their match partners, what is the equilibrium match structure? Explain, giving the relevant computations. Would this match structure be supported by an equal sharing rule (i.e., if the surplus was split equally between each matched pair)? If so, explain why, giving the relevant computations. If not, explain why not, and then find a different (vector of) shares that would support the equilibrium match structure. [HINT: You would merely need to find one possible solution, not derive every possible solution.]

4. [20 points] Consider a single mother deciding on the optimal number of children to have (or adopt). She recognizes that there is a direct monetary cost p_n and a time cost t_n associated with each child. (For simplicity, assume that any time not spent with her children is spent in market work.) Formally, the mother faces a budget constraint

$$p_n\, n + p_z\, z \;=\; w\,(T - t_n\, n)$$

where p_n = price per child,
n = number of children,
p_z = price of composite good (i.e., everything other than children),
z = amount of composite good,
w = wage rate,
T = total time available,
t_n = time required per child.

Finally, assume that the mother has a utility function $U(n,z)$ where both n and z are normal goods. If the mother's wage increases, how would this affect the number of children that she would optimally choose? Explain, using an indifference-curve / budget-constraint diagram.

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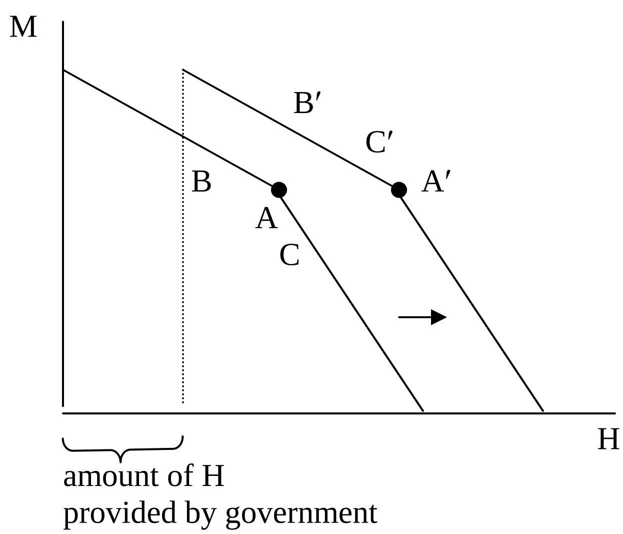
1a) [5 pts] Consider every point along the household's PPC. We might observe: both husband and wife completely specialized in market production; the husband completely in the market and the wife split between sectors; the husband specialized completely in the market and the wife specialized completely in the household; the husband split between sectors and the wife completely in the household; both completely specialized in household production.

bi) [5 pts] Free childcare from the government will shift the household's PPC:



ii) [7 pts] Given that M and H are normal goods, the household would necessarily increase H, and might either increase M or maintain constant M depending on the household's indifference map. For instance, if the household was initially at the corner solution A, the shift in the PPC might cause it to switch to the corner solution A′, so that H rises while M remains constant. But if the household was not initially at a corner solution, both M and H must rise. (Any possible counterexample would require an indifference map inconsistent with the assumption that M and H are normal goods.)

iii) [7 pts] It is *not* necessary that both individuals would decrease time in the household (or increase time in the market). For instance, if the household moved from A to A′, the time allocations of the husband and wife are not affected at all by the shift in the PPC. However, it is *possible* that one member of the household would increase market hours (consider the switch from B to B′; the wife increases her market hours), and even possible for *both* members to increase market hours (consider the switch from C to C′).

iv) [6 pts] No, the answer to part (a) still holds. The government's program merely shifted the PPC rightward. Moving along the PPC, the same divisions of labor remain possible.

2) [10 pts] See Becker, pp 48-53. Becker argues that household size is limited by potential shirking problems and the demand for privacy. He argues that business establishments are larger than households because they are more capital intensive. Business establishments may also be better able to monitor shirking, though they can't rely on altruism like households might.

3a) [16 pts] Using the Gale-Shapley algorithm with men making the offers, the resulting equilibrium match structure is {M1-F1, M2-F2, M3-F3}. This same equilibrium match structure results when women make the offers. The Gale-Shapley Theorem tells us that the match structure that results when men (women) make the offers is the best (worst) from the mens' perspective. Given that the match structure{M1-F1, M2-F2, M3-F3} is both the best and the worst for men, it must be the unique match structure.

b) [24 pts] Given transferable utility, the equilibrium match structure must maximize aggregate surplus. Given that all matches generate positive surplus, we need to consider only the six possible match structures in which all individuals are married:

{M1-F1, M2-F2, M3-F3} generates aggregate surplus 4 + 11 + 5 = 20;
{M1-F1, M2-F3, M3-F2} generates aggregate surplus 4 + 9 + 10 = 23;
{M1-F2, M2-F1, M3-F3} generates aggregate surplus 6 + 10 + 5 = 21;
{M1-F2, M2-F3, M3-F1} generates aggregate surplus 6 + 9 + 7 = 22;
{M1-F3, M2-F1, M3-F2} generates aggregate surplus 7 + 10 + 10 = 27;
{M1-F3, M2-F2, M3-F1} generates aggregate surplus 7 + 11 + 7 = 25.

Thus, the equilibrium match structure is {M1-F3, M2-F1, M3-F2}. This match structure would *not* be supported by an equal sharing rule (i.e., $\theta_{13}^* = \theta_{21}^* = \theta_{32}^* = ½$). Note that M2 and F2 would leave their current partners for each other because their joint surplus, $s_{22} = 11$, would be bigger than the sum of their present payoffs, $\theta_{21}^* s_{21} + (1-\theta_{32}^*)s_{32} = (1/2)(10) + (1/2)(10) = 10$. [Further note that, given equal shares, M2 and F3 would leave their current partners for each other because their joint surplus, $s_{23} = 9$, would be bigger than the sum of their present payoffs, $\theta_{21}^* s_{21} + (1-\theta_{13}^*)s_{13} = (1/2)(10) + (1/2)(7) = 8.5$.] To support the equilibrium match structure, we need to find a vector $(\theta_{13}^*, \theta_{21}^*, \theta_{32}^*)$ that satisfies the conditions

$$4 < \theta_{13}^*\, 7 + (1-\theta_{21}^*)\, 10$$
$$6 < \theta_{13}^*\, 7 + (1-\theta_{32}^*)\, 10$$
$$11 < \theta_{21}^*\, 10 + (1-\theta_{32}^*)\, 10$$
$$9 < \theta_{21}^*\, 10 + (1-\theta_{13}^*)\, 7$$
$$7 < \theta_{32}^*\, 10 + (1-\theta_{21}^*)\, 10$$
$$5 < \theta_{32}^*\, 10 + (1-\theta_{13}^*)\, 7$$

Many vectors would work. For instance, $(\theta_{13}^* = 1/2, \theta_{21}^* = 3/5, \theta_{32}^* = 2/5)$. Intuitively, because M2 and F2 could form a match with a lot of surplus, they need to be given larger shares within their current marriages.

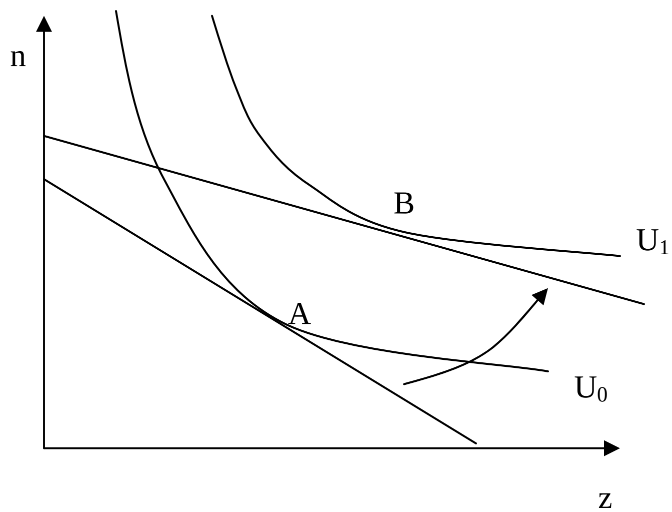
4) [20 pts] Rewriting the budget constraint, we obtain

$$(p_n + w\, t_n)\, n + p_z z = wT.$$

The term $(p_n + w\, t_n)$ can be interpreted as the "effective price" of children while the right-hand side of the equation (wT) can be interpreted as "potential" income. Thus, an increase in the wage (w) causes both the effective price of children and potential income to rise. To plot the budget constraint (placing z on the horizontal axis and n on the vertical axis), we may rewrite the budget constraint as

$$n = [wT / (p_n + w\, t_n)] - [p_z / ((p_n + w\, t_n)]\, z$$

(Note that, because the parameters w, T, p_n, t_n, and p_z are all constants, this budget constraint is linear.) Thus, as the mother's wage w increases, her budget constraint shifts upward (the intercept is higher) and becomes flatter (the slope is smaller). Grapically,



On my graph, the shift/rotation of the budget constraint moves the mother from point A to point B, so that consumption of both n and z increases. But more generally, it is important to recognize that income and substitution effects go in the opposite direction. The increase in the mother's potential income would cause her to have more children (since children are a normal good); the increase in the effective price of children would cause her to have fewer children; the overall effect is ambiguous.

Economics 451 Exam 2 Fall 2008 Prof Montgomery

Answer all 4 questions. 100 points possible.

1. [28 pts] A husband and wife are having trouble agreeing how much of their household income to spend on their child's consumption (versus their own consumption). The household budget constraint is

$$Z_P + Z_C = I$$

where I is total household income
Z_P is the parents' consumption level (which must satisfy $0 \leq Z_P \leq I$)
Z_C is the child's consumption level (which must satisfy $0 \leq Z_C \leq I$)

The husband's utility function is $U_H = Z_P + (2/3)\, Z_C$
The wife's utility function is $U_W = Z_P + (5/3)\, Z_C$

a) If the husband could allocate income unilaterally (using his own utility function), what consumption levels Z_P and Z_C would he choose? If the wife could make this decision unilaterally (using her own utility function), what levels Z_P and Z_C would she choose? Draw budget-constraint / indifference-curve diagrams to illustrate your answers. [NOTE: Your diagrams don't need to be perfect, but should be qualitatively accurate, reflecting the correct shapes of the relevant curves.]

b) Suppose the outcome negotiated by the husband and wife is given by the Nash bargaining solution. Letting T_H denote the husband's threat point, and letting T_W denote the wife's threat point, derive equations for the following outcomes of the negotiation:

(i) the utility level of the wife
(ii) the utility level of the husband
(iii) the consumption level of the child

[HINT: You should use calculus to determine the answers (not worrying about potential corner solutions). Each of the 3 outcomes is a function of the parameters I, T_H, and T_W.]

2. [12 pts] State Arrow's Impossibility Theorem. Be sure to give a brief verbal description for each of the conditions mentioned in the theorem. Why is this an important theorem?

3. [30 pts] Given 6 social outcomes (u, v, w, x, y, z), suppose that the Condorcet procedure was used to determine social preferences. More specifically, suppose that majority voting between each pair of outcomes yielded the social preferences

uPx, vPu, wPu, wPv, wPx, wPy, xPv, yPu, yPv, yPx, yPz, zPu, zPv, zPw, zPx,

where iPj indicates that outcome i is strictly preferred to outcome j. [HINT: It's not required, but you might want to draw a graph (with nodes and directed edges) to keep track of these social preferences.]

a) Which outcomes are contained in the Condorcet set (i.e., the "top cycle")?

b) Consider the following agenda (using "amendment procedure"): u is compared to v; the winner is compared to w; the winner is compared to x; the winner is compared to y; the winner is compared to z. If voting is *sincere*, which outcome is wins each round of the agenda? Which outcome is the ultimate winner?

c) Suppose that you're put in charge of designing the voting agenda. If voting is *sincere*, is it possible to construct an agenda (using the amendment procedure) so that

i) outcome x is the ultimate winner?
ii) outcome y is the ultimate winner?

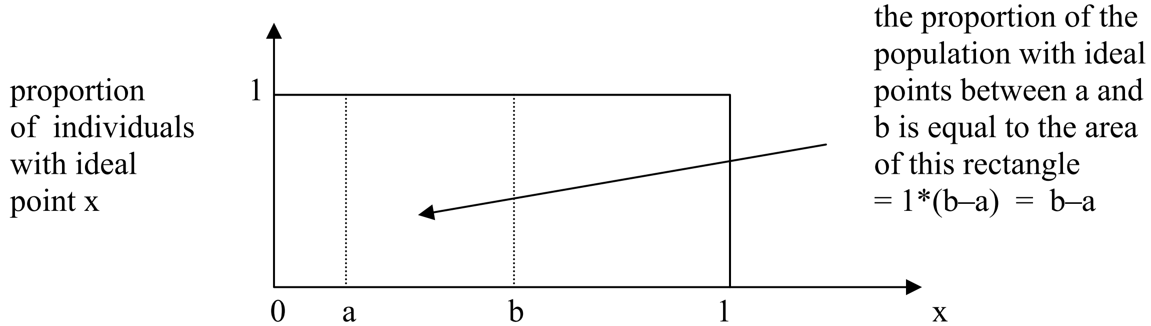
For each case, you should either give an agenda or explain why it is not possible.

d) Given the agenda described in part (b), which outcome is the ultimate winner when voting is *sophisticated*? Explain why, using the appropriate diagram. [HINT: Start your analysis at the fourth round of the agenda, in which y is compared to the winner from the third round (= u or v or w or x). Does it matter which outcome won the third round?]

4. [30 pts] Consider a one-dimensional spatial voting model in which candidates simultaneously commit to policy positions between 0 and 1. Individual i's utility for a candidate who chooses location x is given by

$$U_i(x) = 1/3 - |x_i - x|$$

where x_i is the ideal point for individual i. Further suppose a very large population of individuals, with the ideal points for the population distributed "uniformly" between 0 and 1. As illustrated below, this means that the probability density function is a rectangle with height 1. Consequently, for any locations a and b (where $0 \leq a \leq b \leq 1$), the proportion of individuals with ideal points between a and b is equal to $b - a$.



In contrast to the standard Hotelling model, suppose that an individual **does not vote** if both candidates are too far from the individual's ideal point. More precisely, suppose that individual i votes for the nearest candidate if this candidate's location implies $U_i \geq 0$, but that individual i **does not vote** if $U_i < 0$ for both candidates.

a) Suppose that candidate 1 is the only candidate running for office, and that this candidate chooses location $x_1 = 3/5$. What is the range of ideal points for individuals voting for this candidate? What proportion of the population does **not** vote?

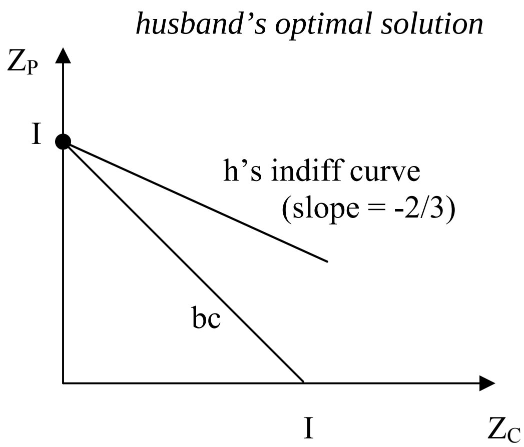
b) Suppose that candidate 1 remains at location $x_1 = 3/5$, but there is now a second candidate who chooses location $x_2 = 1/2$. What range of individuals vote for each candidate? What proportion of the population votes for each candidate? What proportion does **not** vote?

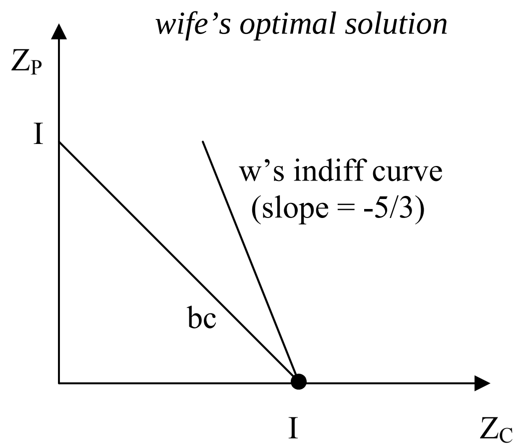
c) Suppose that each candidate wants to maximize the proportion of the population that votes for him/her. In part (b), is candidate 2 making a best response to candidate 1's position (at $x_1 = 3/5$)? If so, is the pair of actions ($x_1 = 3/5$, $x_2 = 1/2$) a Nash equilibrium? If not, what is candidate 2's best response to candidate 1's choice of $x_1 = 3/5$?

d) Given two candidates, is ($x_1 = 1/2$, $x_2 = 1/2$) a Nash equilibrium? If so, explain why. If not, what is the Nash equilibrium?

Economics 451 Exam 2 Fall 2008 Solutions

1a) [10 pts] Given the linearity of the utility functions (as well as the linearity of the household budget constraint), both the husband and the wife are at **corner solutions**. The husband would prefer to set ($Z_C = 0$, $Z_P = I$); the wife would prefer to set ($Z_C = I$, $Z_P = 0$). Graphically,





b) [18 pts] Substituting the household budget constraint into each utility function, we obtain each spouse's utility as a function of Z_C:

$$U_H = Z_P + (2/3)\,Z_C = I - Z_C + (2/3)\,Z_C = I - (1/3)\,Z_C$$
$$U_W = Z_P + (5/3)\,Z_C = I - Z_C + (5/3)\,Z_C = I + (2/3)\,Z_C$$

To obtain the efficient frontier (U_W as a function of U_H), we rewrite the husband's equation as $Z_C = 3(I–U_H)$, and substitute this into the wife's equation:

$$U_W = I + (2/3)\,3(I–U_H) = 3I - 2U_H$$

The Nash bargaining solution maximizes the product $(U_H - T_H)(U_W - T_W)$

Substituting in the efficient frontier, this becomes $(U_H\text{-}T_H)(3I - 2U_H - T_W)$

Differentiating with respect to U_H, we obtain $(1)(3I - 2U_H - T_W) + (-2)(U_H\text{-}T_H) = 0$

which yields $U_H = (3/4)I + (1/2)T_H - (1/4)T_W$

and thus $U_W = 3I - 2\,[(3/4)I + (1/2)T_H - (1/4)T_W] = (3/2)I + (1/2)T_W - T_H$

$Z_C = 3(I - [(3/4)I + (1/2)T_H - (1/4)T_W]) = (3/4)I - (3/2)T_H + (3/4)T_W$

[Note that you could also have obtained these answers by substituting $U_H = I - (1/3)\,Z_C$ and $U_W = I + (2/3)\,Z_C$ into $(U_H - T_H)(U_W - T_W)$, and then maximized with respect to Z_C.]

2) [12 pts] Arrow's Theorem states there is no social welfare function (mapping individual preference orders into a complete, transitive social preference order) that simultaneously satisfies the conditions U (universal admissibility), P (Pareto optimality), D (non-dictatorship), and I (independence from irrelevant alternatives). The theorem shows that there is no problem-free institutional arrangement for aggregating individual preference orders into a collective preference order. Any voting method that avoids the Condorcet paradox (i.e., generates a social preference order without voting cycles) must violate one or more of the conditions.

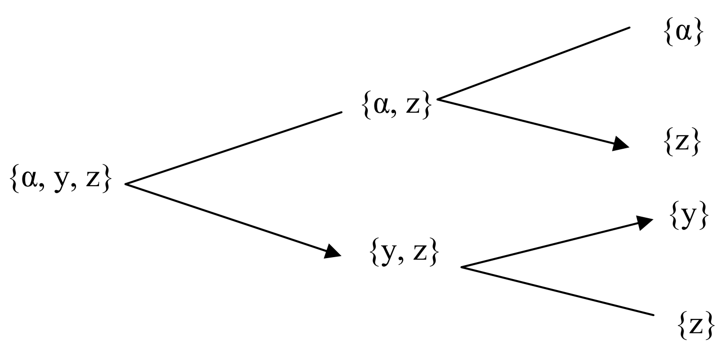
3a) [6 pts] The Condorcet set is {w, y, z}.

b) [5 pts] The winners are: v, then w, then w, then w, then z (the ultimate winner)

ci) [5 pts] not possible because x is not in the Condorcet set

cii) [5 pts] Many answers are possible. One agenda compares u to v; the winner is compared to x; the winner is compared to w; the winner is compared to z; the winner is compared to y.

d) [9 pts] Suppose the winner of round 3 is outcome α (which could be u, v, w or x). The remainder of the game is described by the decision-tree diagram below. (The sets indicate the "live options" remaining at each stage.)



Given sophisticated voting, we apply backward induction – i.e., we start with the final (5^{th}) round, and then work backwards. Suppose the final round is between α and z. Given the social preferences, z would beat α (regardless of whether α is u, v, w, or x), as indicated by the arrow on the decision-tree diagram. Alternatively, suppose the final round is between y and z. Given the social preferences, y would beat z.

Moving back to the previous (4^{th}) round, sophisticated voters recognize that this round is "really" between z (not α) and y. Thus, y wins the 4^{th} round, and then goes on to become the ultimate winner.

4a) [6 pts] Individuals will vote for candidate 1 if their ideals points are within 1/3 of candidate 1's position. Thus, the range of ideal points is [4/15, 14/15]. This implies that 2/3 of the population votes for candidate 1, while the remaining 1/3 does not vote.

b) [10 pts] Voter i with ideal point $x_i = 11/20$ (which is equal to the mean of $x_1 = 3/5$ and $x_2 = 1/2$) is indifferent between the candidates, and receives positive utility from both candidates (i.e., $U_i(1/2) = U_i(3/5) > 0$). Thus, the range of individuals voting for candidate 2 is [1/6, 11/20]; the range of individuals voting for candidate 1 is [11/20, 14/15]; voters with ideal points outside these ranges do not vote. Thus, each candidate receives votes from 23/60 of the population, while 7/30 of the population does not vote.

c) [7 pts] No, candidate 2 is not making a best response. As candidate 2 begins to move a little toward the left, she will pick up more votes (from the "far left") than she loses (from "centrist" voters). But candidate 2 won't move further left than 1/3, since there are no additional voters on the "far left" to be gained. Thus, if $x_1 = 3/5$, candidate 2's best response is to choose $x_2 = 1/3$. Note that both candidates now receive votes from 7/15 (which is greater than 23/60) of the population.

d) [7 pts] No, ($x_1 = 1/2$, $x_2 = 1/2$) is not a Nash equilibrium. If both candidates locate at the median, they each receive votes from 1/3 of the population. Given the intuition from part (iii), candidate 2 would want to move to 1/3, while candidate 1 would want to move to 2/3. This outcome (in which each candidate receives votes from 1/2 of the population) is a Nash equilibrium, since both candidates are making a best response to the other.

Economics 451 **Exam 3** **Fall 2008** **Prof Montgomery**

Answer all questions. 100 points possible. Note that there are ***3 pages*** *of questions.*

1) [15 points] Consider a version of Pascal's Wager with multiple types of gods and multiple types of religious participation. As summarized in the table below, the type 1 god would reward anyone who joins any religion but punishes non-joiners; the type 2 god would reward those who join religion 2 but punish everyone else; the type 3 god would punish everyone regardless of their action. The focal individual must choose one of the four actions, and holds subjective beliefs reflected by the probabilities q_1, q_2, and q_3 given in the table. Note that the rewards (R) and punishments (P) and costs of belief (C) do not vary across gods (i.e., $R_1 = R_2 = R$, $P_1 = P_2 = P_3 = P$, $C_1 = C_2 = C_3 = C$). Further assume $R > 0$, $P > 0$, and $C > 0$, which implies $–P < 0$ and $–C < 0$.

	possible outcomes			
actions	type 1 exists (prob q_1)	type 2 exists (prob q_2)	type 3 exists (prob q_3)	no god exists (prob $1–q_1–q_2–q_3$)
join religion 1	R–C	–P–C	–P–C	–C
join religion 2	R–C	R–C	–P–C	–C
join religion 3	R–C	–P–C	–P–C	–C
join no religion	–P	–P	–P	0

a) Compute the expected value of each action (simplifying as much as possible).

b) If the focal individual maximizes expected utility, are there some actions that she would *never* choose (regardless of the subjective probabilities she assigns to each possible outcome)? Briefly explain. Then give a (mathematical) condition to describe which actions would be (optimally) chosen depending on her subjective probabilities.

2) [10 points] Empirically, comparing different Protestant denominations, is there a positive or a negative relationship between denominational strictness and the average income of members? How does Iannaccone explain this fact in his "Church and Sect" paper (*American Journal of Sociology* 1988)?

3) [10 points] Finke and Stark (in their book *The Churching of America*) argue that Protestant denominations in the US have tended to become less strict over time. Briefly sketch the "intergenerational social mobility" explanation for this phenomenon (formalized by Montgomery, *Rationality & Society* 1996). According to this model, why don't low-tension denominations tend to become more strict over time?

4) [10 pts] According to Iannaccone, Finke, and Stark (*Economic Inquiry* 1997), why is religious participation relatively low in Sweden? Why is it relatively high in the US?

5) [30 points] Consider an individual with a utility function

$$U(R, Z) = \theta \ln(R) + (1-\theta) \ln(Z)$$

where R is religious consumption, Z is secular consumption, θ is a preference parameter between 0 and 1, and ln denotes natural logarithm. Further assume

$$R = S\, t_R \qquad Z = t_Z \qquad t_R + t_Z = T$$

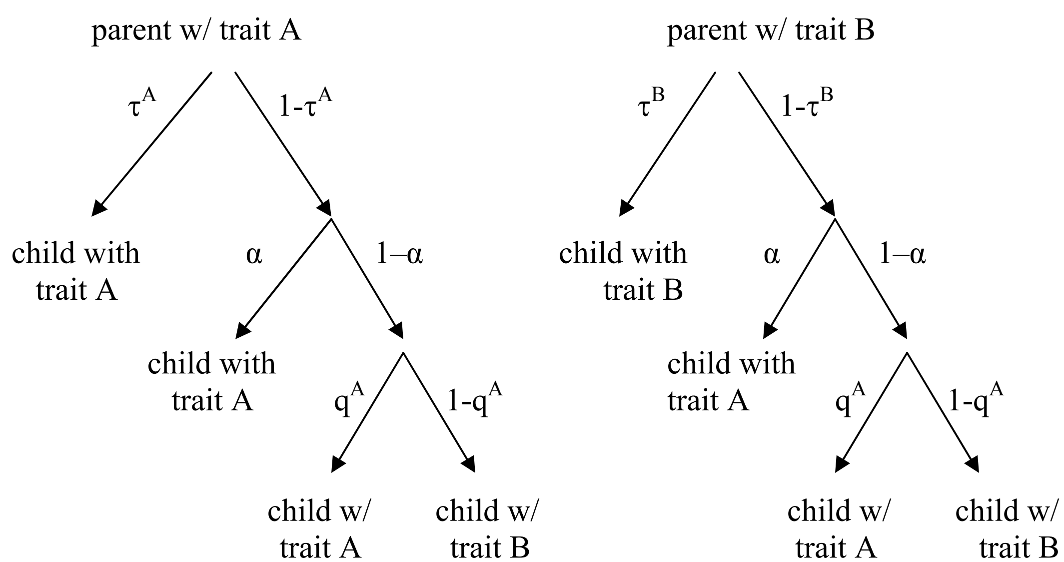
where S is the individual's stock of religious capital, t_R is time spent in religious participation, t_Z is time spent in secular (non-religious) participation, and T is total time available.

a) Taking the parameters S, T, and θ as exogenously given, find the individual's optimal levels of religious participation (t_R*) and secular participation (t_Z*). Then substitute these optimal choices into the utility function to obtain the individual's indirect utility function.

b) Now suppose that religious capital (S) is chosen by parents for their children (who then face the utility maximization problem just solved in part a). If parents wished to increase their child's *religious participation*, should they choose a higher or lower level of S? If parents wished to increase their child's *utility*, should they choose a higher or lower level of S?

c) Use an indifference curve diagram (with Z on the horizontal axis and R on the vertical axis) to illustrate how the child's optimal choice changes as S rises. How does the increase in S affect the budget constraint? As S increases, does the substitution effect cause the child's consumption of Z to rise or fall? Does the income effect cause the child's consumption of Z to rise or fall? What is the overall effect?

6) [25 points] Consider a variation on the model of intergenerational transmission developed by Bisin and Verdier (*Quarterly Journal of Economics* 2000). Individuals acquire either trait A or trait B. Let q^i denote the proportion of the population with trait i (= A or B) so that that $q^A + q^B = 1$. Extending the model developed in class, suppose that the socialization process now has three stages. In the first stage, parents attempt to directly socialize children to the parent's own type. Suppose parents with trait A succeed with probability τ^A, while parents with trait B succeed with probability τ^B. In the second stage (assuming that the parent did not succeed in the first stage), the public school system attempts to socialize children to trait A. Suppose that the public schools succeed with probability α. In the third stage (assuming that schools did not succeed in the second stage), the child is socialized by a "cultural parent" drawn randomly from the population. Thus, the socialization process is described by the following probability trees:



a) For parents with trait A, what is the probability that their child adopts trait A (i.e., $\Pr(A\rightarrow A)$)? What is the probability that their child adopts trait B (i.e., $\Pr(A\rightarrow B)$)? For parents with trait B, what is the probability that their child adopts trait A (i.e., $\Pr(B\rightarrow A)$)? What is the probability that their child adopts trait B (i.e., $\Pr(B\rightarrow B)$)? [HINT: For $\alpha = 0$, the model reduces to the model considered in class. Furthermore, if your answers are correct, you should have $P(A\rightarrow A) + P(A\rightarrow B) = 1$ and $P(B\rightarrow A) + P(B\rightarrow B) = 1$.]

b) Now suppose that each parent with trait i (= A or B) chooses τ^i to maximize her expected utility. Following Bisin and Verdier, suppose each parent with trait i receives utility level V^{iA} if her child acquires trait A, and receives V^{iB} if her child acquires trait B. Further suppose the parent's cost of direct socialization is given by $(1/2)(\tau^i)^2$. Write the expected utility function for each type of parent, and then solve these utility maximization problems to determine the optimal socialization choices (τ^{A*} and τ^{B*}).

c) Suppose that the schools become more successful at socializing children (i.e., α rises). How does this affect the optimal choice made by each type of parent? Briefly explain.

Econ 451 Exam 3 Fall 2008 Solutions

1a) [6 pts] EV1 = expected value of joining religion 1 = $q_1R - (q_2+q_3) P - C$
EV2 = expected value of joining religion 2 = $(q_1+q_2)R - q_3P - C$
EV3 = expected value of joining religion 3 = $q_1R - (q_2+q_3) P - C$
EVN = expected value of joining no religion = $-P(q_1+q_2+q_3)$

b) [9 pts] Given $R > 0$, $P > 0$, and $C > 0$, individuals would never choose to join religions 1 or 3. Formally, EV2 – EV1 = EV2 – EV3 = $q_2(R+P) > 0$. Intuitively, the type 1 and 3 gods don't distinguish between the different types of joiners, while the type 2 god only rewards type 2 joiners. So if you're going to join any religion, it should be religion 2.

Eliminating religions 1 and 3, the two remaining options are religion 2 and no religion. The individual chooses religion 2 when EV2 > EVN, which can be written as $(R+P)(q_1+q_2) > C$ or as $(q_1+q_2) > C/(R+P)$. Intuitively, you should join religion 2 when the probability of a type 1 or 2 god is high, and the ratio of costs to benefits is low.

2) [10 pts] As shown in the scatterplot in Iannaccone's 1994 *AJS* paper, there is a *negative* correlation between denominational strictness and average income of members.

Applying Iannaccone's 1988 "Church and Sect" model, each individual makes a conduct choice (C) that generates utility from social approval by their religious group (R(C)) and social approval by the society (Z(C)). The conduct level most approved by church groups (C_R) is not much different than the conduct most approved by society (C_Z). Thus, church members set a conduct level somewhere between C_Z and C_R, receiving moderate levels of both Z and R. Because sects require a high level of conduct that generates low Z, sects must compensate by offering higher R. Thus, sect members set a conduct level near C_R, receiving high R and low Z.

Suppose now that individuals differ in terms of their non-religious opportunities, so that $Z(C_Z)$ is larger for those with better opportunities and smaller for those with worse opportunities. Sect membership will continue to generate high R and low Z for both types of individuals. But now, the utility from church membership depends on non-religious opportunities. Individuals with high opportunities will be able to obtain high Z and moderate R. Individuals with low opportunities will be able to attain low Z and moderate R. To summarize the benefits across types of individuals:

	join church	join sect
high opportunities	high Z, moderate R	low Z, high R
low opportunities	low Z, moderate R	low Z, high R

Thus, individuals with low opportunities will tend to join sects, while individuals with high opportunities (at least those who care about Z) will tend to join churches. [See Figure 7 in Iannaccone 1988 for the relevant PPC and indifference curve diagram.]

3) [10 pts] Everything else equal, individuals with lower incomes prefer to join stricter denominations (perhaps for the reasons given in question 2 above). Because they self-select into the sect, the first-generation members of a new sect will tend to have low income. However, the income distribution within the sect may change over time as the initial members have children. If there is intergenerational social mobility (so that some children have higher income than their parents), and if children develop "loyalty" to their parents' denomination (denomination-specific religious capital), the average income of sect members will tend to rise over time. After several generations, the more affluent members (who would prefer lower strictness) may outnumber the less affluent members (who want to maintain higher strictness), causing the denomination to become less strict through a "voice" mechanism.

Given this intergenerational-social-mobility mechanism, we might also expect churches to become stricter over time (as the average income of church members falls over generations to the population average). However, these church-to-sect transitions would not occur if low-strictness denominations induce less loyalty (lower religious capital) so that less affluent children would simply exit their parents' church rather than staying and exercising "voice" to increase the strictness of this denomination.

4) [10 pts] Iannaccone, Finke, and Stark argue that cross-country differences in religious participation are driven by "supply side" rather than "demand side" forces. In particular, they maintain that government regulation of religious producers accounts for the differences between the US and Western Europe. In their paper, they argue that low religious participation is Sweden occurs because the state-supported church has little incentive to produce a "religious good" valued by consumers. In contrast, they argue that the lack of regulation in the US encourages a wide variety of producers (so that there are many different types of religion offered to consumers) and that the lack of government subsidies forces these producers to try harder to please their customers.

5a) [14 pts] $U(R,Z) = \theta \ln(R) + (1-\theta) \ln(Z) = \theta \ln(S\, t_R) + (1-\theta) \ln(T-t_R)$

To determine the optimal time spent in religious participation, differentiate with respect to t_R, set this derivative equal to zero, and then simplify.

$$dU/dt_R = \theta\, (1/(S\, t_R))(S) + (1-\theta)\, (1/(T-t_R))(-1) = 0$$
$$\rightarrow \quad \theta\, (T-t_R) = (1-\theta)\, t_R$$
$$\rightarrow \quad t_R^* = \theta T$$

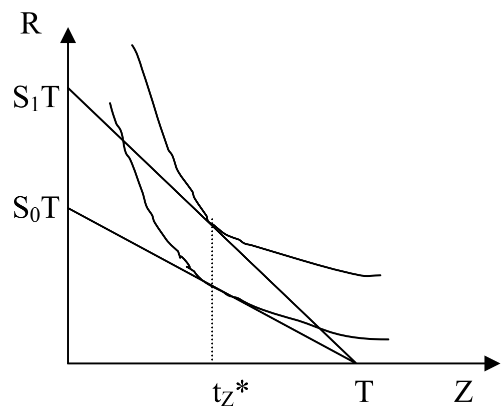
Thus, given the assumed utility function, t_R^* is a constant proportion (θ) of total time. The optimal time spent in secular participation and the indirect utility function are

$$t_Z^* = T - t_R^* = T - \theta T = (1-\theta)T$$

$$V(R^*,Z^*) = \theta \ln(S\theta T) + (1-\theta) \ln((1-\theta)T)$$

5b) [6 pts] The calculations in part (a) show that t_R* does not depend on S. Thus, the level of religious capital set by the parents does not influence the child's religious participation. However, looking at the indirect utility function, we see that the parents should increase S in order to increase the child's utility.

c) [10 pts] The individual's constraint is given by $t_R + t_Z = T$ which implies $R/S + Z = T$ and hence $R = S(T-Z)$. Thus, an increase in S causes the budget constraint to rotate outwards as S rises. Essentially, an increase in religious capital (from S_0 to S_1) decreases the relative price of the religious good.



The substitution effect (induced by the decline in the relative price of R) causes the individual to decrease Z. The income effect causes the individual to increase Z (given that Z is a normal good). Given the utility function in part (a), these effects precisely cancel each other out, so that the overall effect is zero. More generally (for other utility functions), the overall effect would be ambiguous.

6a) [8 pts]

$$P(A\rightarrow A) = \tau^A + (1-\tau^A)\alpha + (1-\tau^A)(1-\alpha)q^A$$
$$P(A\rightarrow B) = (1-\tau^A)(1-\alpha)(1-q^A)$$
$$P(B\rightarrow A) = (1-\tau^B)\alpha + (1-\tau^B)(1\text{-}\alpha)q^A$$
$$P(B\rightarrow B) = \tau^B + (1\text{-}\tau^B)(1-\alpha)(1-q^A)$$

b) [12 pts] The expected utility functions are

$$P(A\rightarrow A)V^{AA} + P(A\rightarrow B)V^{AB} - (1/2)(\tau^A)^2 \quad \text{for a parent with trait A}$$
$$P(B\rightarrow A)V^{BA} + P(B\rightarrow B)V^{BB} - (1/2)(\tau^B)^2 \quad \text{for a parent with trait B}$$

Differentiating with respect to τ^A (for trait-A parents) or τ^B (for trait-B parents), we obtain the optimal solutions

$$\tau^{A*} = (1-\alpha)(1-q^A)(V^{AA}-V^{AB}) \quad \text{and} \quad \tau^{B*} = [\alpha + (1-\alpha)q^A](V^{BB}-V^{BA})$$

c) [5 pts] From the optimal solutions, we see that an increase in α causes τ^{A*} to fall and τ^{B*} to rise. Intuitively, because the schools attempt to instill trait A in children, trait-A parents don't need to work as hard to socialize their kids, while trait-B parents try harder.