Place-Based Redistribution in Location Choice Models[⋆]

Morris A. Davis[a,*], Jesse Gregory[b]

[a]*Rutgers Business School, 1 Washington Park Room 627, Newark, NJ 07102*
[b]*University of Wisconsin-Madison, 1180 Observatory Drive, Madison, WI 53706-1393*

Abstract

In recent location choice models, households randomly vary with respect to their utility of living in a location. We demonstrate the distribution generating this randomness is not identifiable from location choice data and the optimal allocation chosen by a social planner is not identified. We propose an algorithm for setting the distribution generating the random utility across locations that implies a planner will optimally choose no redistribution in the absence of externalities or equity motives between different types of people. Our algorithm preserves a planner's motives to redistribute due to equity considerations between types of people and efficiency in production.

⋆We thank participants at The Future of Cities conference at Northwestern University as well as David Albouy, Cecile Gaubert, Jonathan Dingel, Jan Eeckhout, Andra Ghent, John Kennan, Patrick Kline, Fabrizio Perri, Esteban Rossi-Hansberg and Chris Taber for helpful conversations.

*Corresponding author

Email addresses: mdavis@business.rutgers.edu (Morris A. Davis), jmgregory@ssc.wisc.edu (Jesse Gregory)

1. Introduction

For decades, federal, state and local governments have directly or indirectly redistributed income across locations. This redistribution can take many forms: It can be a subsidy for development of new low-income housing (Davis et al., 2019); a subsidy to local businesses operating in low-income areas such as Empowerment Zones (Busso et al., 2013) or Opportunity Zones (Arefeva et al., 2021); a large-scale government works projects (Kline and Moretti, 2014); or other forms. Thus, a central area of investigation in economics is to understand the context in which redistribution across locations improves welfare.

Recent papers by Fajgelbaum and Gaubert (2019), Rossi-Hansberg et al. (2020) and Gaubert et al. (2020) extend this tradition by studying optimal transfers of income across households and locations using sophisticated equilibrium location choice models. The models include well-documented externalities in production and multiple types of households, for example low- and high-skill. Using calibrated models, these papers quantify transfers across people and locations that improve expected utility for reasons of both efficiency and equity.

We show that in location choice models a planner will have three motives to redistribute resources across locations and people relative to an environment in which households consume the income they generate and do not receive (or pay) transfers. The first, which we call "across-type equity," is to narrow inequality in consumption across different types of households, for example low-skill and high-skill. The second, which we call "efficiency," arises from externalities and spillovers across types in production; the planner will transfer resources to provide incentives for households to internalize the external impacts of their decisions. Understanding motives for redistribution arising from these first two reasons has been the focus of recent studies.

We show that a planner has a third reason to redistribute in these models: To equate the average marginal utility of consumption of otherwise identical households that make different location choices. A typical prediction is that a planner will redistribute resources from ex-ante identical households choosing to live and work in high-income locations to households choosing low-income locations. We call this third motive "within-type transfers."

To understand why a planner may wish to make within-type transfers in location choice models, we need to provide some background. For all locations to be occupied in models with ex-ante identical households, some households must choose to live in low-income locations. In an older literature that relies on the Rosen-Roback model (Roback, 1982) to describe the economic environment, utility in every location is assumed identical and each household is indifferent as to its location. The Rosen-Roback model implies that population elasticities are infinite with respect to a small change in location attributes such as consumption or amenities holding all else fixed.

This infinite elasticity is not realistic and many researchers now use a different framework where utility in every location is not assumed to be equal. Instead, households receive "location attachment" draws that affect the utility of living in each location. From the perspective of the researcher, these draws vary randomly across locations and households. The inclusion of these draws imply households are not indifferent as to where they live and some households will not leave their chosen location in response to marginal changes in utility. Researchers can calibrate the distribution of the draws to match empirical population elasticities with respect to changes in wages, amenities, or other location characteristics. The fact that some households are sticky with respect to location choice raises the possibility of welfare-improving place-based policies. The calibration of the distribution of the location attachment draws that generates this stickiness enables accurate predictions about behavioral responses to policy.

We document that the exact distribution of these location attachment draws is fundamentally not identifiable from location choice data. This implies the size and direction of optimal within-type transfers are not identified, even when a model includes all three motives for redistribution. Different, untestable assumptions about the distribution of location attachment draws can lead to large swings in predicted optimal within-type transfers. The uncertainty this creates potentially swamps predicted redistribution arising from the motives of across-type equity or efficiency. When researchers compare policies across a number of scenarios, we often do not know the role played by within-type transfers in generating changes to policy. We propose an adjustment to the standard planning problem that eliminates within-type transfers, while preserving motives for redistribution due to reasons of across-type equity or efficiency.

2. Proving Lack of Identification of Within-Type Transfers

2.1. A Common Model

We start by considering the predictions of a simple location choice model with no externalities and one type of household that is at the core of some more complicated models. The economy consists of a measure 1 of ex-ante identical households and each household must choose where to live from one of $n = 1, \ldots, N$ discrete locations. Households value consumption, which is produced and transferrable across locations. Each household living in location n produces z_n units of output. L_n is the measure of households living and working in n.

Denote c_n as consumption enjoyed by each household living in location n, not necessarily equal to z_n. The utility of household i choosing to live in location n is

$$u_{ni} = A_n c_n e_{ni}$$

A_n are amenities freely enjoyed by all households living in location n. e_{ni} is a level of attachment to location n by household i that varies across locations and households. Each household observes e_{ni} for $n = 1, \ldots, N$ before making a location choice. Households differ *only* with respect to e_{ni}. We assume, as is common, that the e_{ni} are drawn iid across locations for each household and iid across all households from the Fréchet distribution with shape parameter ν.

Consider a planner with the objective to maximize expected utility subject to satisfying aggregate feasibility, $\sum_n z_n L_n = \sum_n c_n L_n$, population feasibility, $1 = \sum_n L_n$, and respects that households choose the location offering the maximum value of u_{ni}, i.e. household i chooses n_i^* when $n_i^* = \text{argmax}\,\{u_{ni}\}_{n=1}^{N}$. We show in Appendix A that a planner that maximizes expected utility will relate the per-household consumption differential between any two locations n and n' to the per-household income differential of those locations as follows

$$c_n - c_{n'} = \left(\frac{\nu}{1+\nu}\right)(z_n - z_{n'}) \tag{1}$$

Equation (1) illustrates what we call within-type transfers, as the planner optimally redistributes consumption from households living in high income locations to those living in low income locations. Households choosing to work and live in low income locations receive subsidies that are funded by otherwise identical households choosing to work and live in high income locations. A typical calibration sets $\nu = 2$ (Rossi-Hansberg et al., 2020).

2.2. Economics of Within-Type Transfers

So why does the planner wish to redistribute income? After all, there are no externalities and all households have the ability to choose any location in which to live and earn the income of that location. We use intuition from the literature on optimal unemployment insurance to show why a planner makes within-type transfers. Consider a simple setup with only 2 locations where locations differ in their income per household, denoted w_1 and w_2. Assume residents of location 2 pay a tax t to finance a subsidy b paid to residents of location 1. For example, when $z_1 = w_1$ and $z_2 = w_2$ it can be shown equation (1) implies

$$b \;=\; \frac{(1 - L_1)\,(z_2 - z_1)}{1 + \nu} \quad \text{and} \quad t \;=\; \frac{L_1\,(z_2 - z_1)}{1 + \nu}$$

Each household draws idiosyncratic preferences for locations we label as ε_1 and ε_2 and chooses the location that provides the highest utility. Utility is derived from consumption bundled with each individual's draws of ε_1 and ε_2. Expected utility is

$$V \;=\; E_{\varepsilon_1,\varepsilon_2} \max\Big(u\big(w_1 + b, \varepsilon_1\big), u\big(w_2 - t(b), \varepsilon_2\big)\Big) \quad \text{with} \quad t(b) = \frac{L_1}{1 - L_1} b$$

The second expression is the government balanced budget condition assuming there is a measure 1 of households in the economy.

The optimal subsidy is the value of b at which $\dfrac{dV}{db} = 0$. To characterize this optimal subsidy, begin with the expression,

$$\frac{dV}{db} \;=\; \frac{\partial V}{\partial b} + \frac{dt}{db}\frac{\partial V}{\partial t} + \Big(\frac{\partial L_1}{\partial b} + \frac{dt}{db}\frac{\partial L_1}{\partial t}\Big)\underbrace{\frac{\partial V}{\partial L_1}}_{=0}$$

The third term is equal to zero because individuals who switch locations in response to a marginal policy change receive the same utility in both locations.[1]

The partial effects of changing b and t depend only on the populations of the two locations and the average marginal utility of consumption of residents in each location, denoted μ_1 and μ_2:

$$\begin{aligned}
\frac{\partial V}{\partial b} &= L_1 \; \overbrace{E\left[\frac{\partial u\big(w_1+b,\varepsilon_1\big)}{\partial w_1}\middle| u\big(w_1+b,\varepsilon_1\big) > u\big(w_2-t,\varepsilon_2\big)\right]}^{\equiv\mu_1} \\
\frac{\partial V}{\partial t} &= -(1-L_1)\, \underbrace{E\left[\frac{\partial u\big(w_2-t,\varepsilon_1\big)}{\partial w_2}\middle| u\big(w_1+b,\varepsilon_1\big) < u\big(w_2-t,\varepsilon_2\big)\right]}_{\equiv\mu_2}
\end{aligned}$$

The government balanced budget condition implies that

$$\frac{dt}{db} = \frac{L_1}{1-L_1}\left(1+\frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1}\right)$$

The first term is the mechanical change in the tax is necessary to finance the change in transfers in the absence of any behavioral responses. The second term captures the fact that an additional tax increase is necessary to offset the population response to the change in taxes/transfers. We can now write

$$\begin{aligned}
\frac{dV}{db} &= L_1\mu_1 - \frac{L_1}{1-L_1}\left(1+\frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1}\right)(1-L_1)\mu_2 \\
&= L_1\left[\mu_1-\mu_2\left(1+\frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1}\right)\right]
\end{aligned}$$

In the case when the current transfer is $b=0$, the planner wishes to transfer consumption to the location with the higher marginal utility of consumption. At the optimal transfer b (satisfying $dV/db=0$), we have

$$\frac{\mu_1-\mu_2}{\mu_2} = \frac{\frac{dL_1}{db}\frac{b}{L_1}}{1-L_1} \qquad (2)$$

[1] dt/db is not a partial derivative both because people move as a result of the policy and because of the need to balance the budget.

This result is exactly analogous to the Baily-Chetty formula of Chetty (2006) characterizing the optimal generosity of unemployment insurance. The left-hand side is the insurance benefit of moving 1 dollar (in total) from location 2 to location 1, which, for most commonly considered utility functions, is decreasing in b. The right-hand side describes the marginal cost of raising one dollar in total from location 2, which captures the fact that as benefits rise, the population of location 1 also increases which causes an excess burden of transferring one additional dollar.

2.3. Location Choice and Location Specific Preference Draws

In a two location model where e_{ni} are iid drawn from the Fréchet, we can derive the left-hand side of equation (2). A household chooses location 1 whenever $e_1 > e_2 t$ where $t = A_2 c_2 / (A_1 c_1)$. This implies the following expected values

$$\begin{aligned} E\left[e_1 \mid e_1 > e_2 t\right] &= (1+t^\nu)^{1/\nu}\,\Gamma\left(1-\frac{1}{\nu}\right) \\ E\left[e_2 \mid e_2 > e_1/t\right] &= (1+(1/t)^\nu)^{1/\nu}\,\Gamma\left(1-\frac{1}{\nu}\right) \end{aligned}$$

where Γ is the gamma function. The average marginal utility of consumption of households living in locations 1 and 2 is equal to the appropriate expression above multiplied by A_1 for location 1 and A_2 for location 2. After cancelling redundant terms, the left-hand side of equation (2) is equal to[2]

$$\begin{aligned} \left(\frac{A_1}{A_2}\right)\left[\frac{1+t^\nu}{1+(1/t)^\nu}\right]^{1/\nu} - 1 &= \left(\frac{A_1}{A_2}\right)\left[\frac{\frac{(A_1c_1)^\nu+(A_2c_2)^\nu}{(A_1c_1)^\nu}}{\frac{(A_2c_2)^\nu+(A_1c_1)^\nu}{(A_2c_2)^\nu}}\right]^{1/\nu} - 1 \\ &= \frac{1/c_1 - 1/c_2}{1/c_2} \end{aligned}$$

We now write down a transformation of the location attachment draws that yields exactly the same probability distribution over all location choices but different values for the left-

[2]Note that the Fréchet shape parameter ν only determines the marginal deadweight loss from increasing transfers, the right-hand side of equation (2). This expression shows ν does not determine any benefits, the left-hand side of equation (2).

hand side of equation (2) and therefore different optimal within-type transfers. Note that the optimal location choice for household i, call it n_i^*, satisfies

$$n_i^* = \text{argmax}\left[A_1 c_1 e_{1i},\ A_2 c_2 e_{2i},\ \ldots,\ A_N c_N e_{Ni}\right]$$

Suppose a researcher had considered a different distribution for the location attachment draws, $\widetilde{e}_{ni}$, such that the optimal location choice for household i resulting from this distribution, call it $\widetilde{n}_i^*$, satisfies

$$\widetilde{n}_i^* = \text{argmax}\left[A_1 c_1 \widetilde{e}_{1i},\ A_2 c_2 \widetilde{e}_{2i},\ \ldots,\ A_N c_N \widetilde{e}_{Ni}\right]$$

When $\widetilde{e}_{ni} = D_i e_{ni}$, with D_i random but taking on a single realized value for each household i, optimal location choices for every household are identical to those when household utility is $A_n c_n e_{ni}$:

$$\begin{aligned}
\widetilde{n}_i^* &= \text{argmax}\left[A_1 c_1 \widetilde{e}_{1i},\ A_2 c_2 \widetilde{e}_{2i},\ \ldots,\ A_N c_N \widetilde{e}_{Ni}\right] \\
&= \text{argmax}\left[A_1 c_1 D_i e_{1i},\ A_2 c_2 D_i e_{2i},\ \ldots,\ A_N c_N D_i e_{Ni}\right] \\
&= \text{argmax}\left[A_1 c_1 e_{1i},\ A_2 c_2 e_{2i},\ \ldots,\ A_N c_N e_{Ni}\right] \\
&= n_i^*
\end{aligned}$$

D_i is fundamentally not identifiable from location choice data: Optimal choices from $\widetilde{e}_{ni}$ are exactly the same as e_{ni}.[3]

[3] We can use the results of Matzkin (1993) to formally state what is identified in this model. Define utility in location n for household i as $A_n c_n D_i e_{ni}$. For arbitrary location m, the probability a household chooses to live in m, call it ρ_m, is

$$\begin{aligned}
& \rho_m\left(\{A_n\}_{n=1}^N, \{c_n\}_{n=1}^N\right) \\
= & \text{Prob}\{\log A_m + \log c_m + \log D_i + \log e_{mi} > \log A_n + \log c_n + \log D_i + \log e_{ni}\} \quad \text{for } n \neq m \\
= & \text{Prob}\{\log e_{mi} - \log e_{ni} > \log A_n - \log A_m + \log c_n - \log c_m\} \quad \text{for } n \neq m
\end{aligned}$$

The joint CDF of the N−1 terms $(\log e_{mi} - \log e_{ni})$ is all that is nonparametrically identified, assuming sufficient continuous variation in consumption or amenities. The D_i terms do not appear and therefore they are not identifiable.

2.4. Lack of Identification of Optimal Transfers

For predicting population responses to various changes in location attributes such as consumption and amenities, setting $D_i = 1$ for all households is harmless as location choice predictions do not depend on D_i. For the purposes of deriving optimal within-type transfers, setting $D_i = 1$ for all households is an arbitrary assumption with significant consequences as any correlation of D_i with one or more of the draws of e_{ni} for $n = 1, \ldots, N$ can change predicted optimal transfers.

To see this, define utility in location n for household i as

$$A_n c_n \widetilde{e}_{ni} \quad \text{with} \quad \widetilde{e}_{ni} \;=\; D_i e_{ni}$$

where e_{ni} are drawn iid from the Fréchet and D_i is defined as:

$$D_i \;=\; \left[\prod_{n=1}^{N} e_{ni}^{\phi_n}\right]^{-1}$$

The parameters $\phi_1, \phi_2, \ldots, \phi_N$ govern the correlation of D_i and each e_{ni}; for now, we assume these parameters are the same for all households. $\phi_1, \phi_2, \ldots, \phi_N$ are nuisance parameters since they are not identifiable from location choice data. Now consider three cases of (ϕ_1, ϕ_2) for the two location model with $A_1 = A_2 = 1$ and $z_1 = z_2$.

- *Case 1:* $\phi_1 = \phi_2 = 0$

$$\text{Utility in location 1} \;=\; c_1 e_{1i} \quad \text{and} \quad \text{Utility in location 2} \;=\; c_2 e_{2i}$$

  Household i chooses location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$. Given the e_{ni} are drawn iid from the Fréchet distribution, optimal transfers are characterized by equation (1). Since $z_1 = z_2$ the planner optimally sets $c_1 = c_2$ and no resources are transferred across locations.

- *Case 2:* $\phi_1 = 0$ and $\phi_2 = 1$

$$\text{Utility in location 1} \;=\; c_1 \left(\frac{e_{1i}}{e_{2i}}\right) \quad \text{and} \quad \text{Utility in location 2} \;=\; c_2$$

Household i chooses location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$. For any given values of e_{1i} and e_{2i}, households choose exactly the same locations as in case 1.

The planner will *not* optimally choose to set $c_1 = c_2$. Suppose that $c_1 = c_2$ and households choose location 1 whenever $e_{1i}/e_{2i} > 1$. The marginal utility of consumption for all households living in location 2 is always 1. The average marginal utility of consumption for all households choosing to live in location 1 at at this allocation is

$$E\left[\frac{e_{1i}}{e_{2i}} \mid \frac{e_{1i}}{e_{2i}} > 1\right] \quad > \quad 1$$

When $c_1 = c_2$, the average marginal utility of consumption of residents optimally choosing to live in location 1 is strictly larger than the average marginal utility of consumption of residents choosing to live in location 2. Therefore, the planner will transfer some consumption from location 2 to location 1 and $c_1 > c_2$.

- *Case 3:* $\phi_1 = 1$ and $\phi_2 = 0$ such that

$$\text{Utility in location 1} \quad = \quad c_1 \quad \text{and} \quad \text{Utility in location 2} \quad = \quad c_2\left(\frac{e_{2i}}{e_{1i}}\right)$$

Household i chooses location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$ and for any given values of e_{1i} and e_{2i}, households optimally choose exactly the same locations as in cases 1 and 2. Now consider the allocation $c_1 = c_2$, such that households choose to live in location 2 whenever $e_{2i}/e_{1i} > 1$. The marginal utility of consumption for all households living in location 1 is 1. The average marginal utility of consumption for all households choosing to live in location 2 is

$$E\left[\frac{e_{2i}}{e_{1i}} \mid \frac{e_{2i}}{e_{1i}} > 1\right] \quad > \quad 1$$

At the allocation $c_1 = c_2$, the average marginal utility of consumption of households optimally choosing to live in location 2 is strictly larger than the average marginal utility of consumption of households choosing to live in location 1. The planner will transfer some consumption from location 1 to location 2 and $c_2 > c_1$, exactly the opposite result as in case 2.

In each of cases 1-3, households choose to live in location 1 as long as $e_{1i}/e_{2i} \geq c_2/c_1$ and this choice is completely independent of the values of ϕ_1 and ϕ_2. Yet in case 1 the planner chooses no transfers, in case 2 the planner transfers consumption from location 2 to location 1, and in case 3 the planner transfers consumption from location 1 to location 2. The size and direction of the transfers is determined by the nuisance parameters ϕ_1 and ϕ_2.

This example is sufficient for the general point we wish to make: Since $\phi_1, \phi_2, \ldots, \phi_N$ are not identified from location choice data, optimal within-type transfers across locations are also not identified.

2.5. Numerical Examples

To illustrate the potential quantitative significance of this problem, we simulate the planning solution to a two location version of the model when utility for household i in location n is

$$\begin{aligned} u_{ni} &= A_n c_n \widetilde{e}_{ni} \\ \text{with} \quad \widetilde{e}_{ni} &= D_i e_{ni} \quad \text{and} \quad D_i = \left[\prod_{n=1}^{N} e_{ni}^{\phi_n} \right]^{-1} \end{aligned}$$

where e_{ni} is drawn iid from the Fréchet distribution with shape parameter $\nu = 2$. In simulations we consider values of $\phi_1 \in \{0.0, 0.5, 1.0\}$ and $\phi_2 \in \{0.0, 0.5, 1.0\}$. For all combinations of ϕ_1 and ϕ_2, we consider the case of equally productive locations, $z_1 = z_2 = 1.0$, and location 1 more productive, $z_1 = 4/3$ and $z_2 = 2/3$. We set $A_1 = A_2 = 1.0$ in all simulations. Given the draws of $\widetilde{e}_{ni}$, we assume each household chooses the location that provides the highest level of utility and then determine the allocation of consumption to residents of each location that maximizes overall average utility in the economy, subject to the resource constraint $\sum_n L_n (z_n - c_n) = 0$ and population constraint $\sum_n L_n = 1$.

The top panel of Figure 1 shows results for the case in which residents of both locations are equally productive and the bottom panel shows results when residents of location 1 are more productive. The y-axis shows transfers per person from location 2 to location 1, $(c_1 - c_2) - (z_1 - z_2)$; the x-axis marks the value of ϕ_2; and the different lines show results for the three values of ϕ_1 we consider. The dashed black lines mark allocations where consumption in each location equals production in that location and no within-type transfers occur.

Consider first the optimal allocation that arises when $\phi_1 = 0.0$ and $\phi_2 = 0.0$, a standard parameterization. When the two locations are equally productive, there are no transfers (top panel); and when households in location 1 are more productive than in location 2, the planner redistributes from location 1 to location 2, as the planner sets the difference in consumption of the two locations equal to $\nu/(1+\nu) = 2/3$ of the difference in TFP. Once we consider other values for ϕ_1 and ϕ_2, both panels of figure 1 make clear that optimal transfers can vary in both sign and magnitude depending on the values of these nuisance parameters. For any value of ϕ_1, increasing ϕ_2 – a movement from left to right along any given line – increases consumption allocated to households living in location 1 relative to those living in location 2. For any given value of ϕ_2, increasing ϕ_1 – moving down from a higher line to a lower line holding ϕ_2 fixed – increases allocations of consumption to households living in location 2 relative to those in location 1. These patterns are consistent with the intuition of the three cases discussed in section 2.4.

3. Method for Eliminating Motives for Within-Type Transfers

3.1. The Method

Since location choice data do not identify optimal within-type transfers, we advocate setting the distribution of the location attachment draws such that the planner optimally chooses no within-type transfers in simple models with one type of household and no externalities. Below, we propose a 5-step algorithm to find a distribution of location attachment draws that accomplishes this objective and does not change any household's optimal location choice. We describe this algorithm as implementing an adjustment to the planning problem:

1. Guess a variable $\omega_i = 1.0$ for all households in the simulation
2. Multiply the utility function by ω_i. Find the allocation of c_n for all $n = 1, \ldots, N$ that is feasible and maximizes the planner's objectives at the current guess for ω_i.
3. Given each household's optimal choice at this allocation, $\widehat{n}_i$, compute $\widehat{\omega}_i$ as the inverse of the marginal utility of consumption for that household at the optimally chosen loca-

tion. Using the framework described in section 2.5, if we define $\widehat{n}_i$ as the optimally chosen location for household i at the current guess of c_n then we set $\widehat{\omega}_i = (A_{\widehat{n}_i}\ D_i\ e_{\widehat{n}_i})^{-1}$.

4. Compute $\omega_i' = \omega_i + d \cdot (\widehat{\omega}_i - \omega_i)$, where $d \in (0, 1]$ is a dampening factor.
5. Update $\omega_i = \omega_i'$ and repeat steps 2-5 until c_n has converged.

This algorithm finds the solution the planner's problem that is consistent with the marginal utility of consumption equal to 1 for all households at the optimal allocation, thus setting the left-hand side of equation (2) to zero.

3.2. The Adjustment Applied to the Simple Model

Denote the planner's objective as $\mathcal{O}$. In the model we have analyzed so far, our adjustment normalizes the location attachment draws as follows

$$\mathcal{O} \;=\; E_i\left[\max_n \{A_n c_n \widehat{e}_{ni}\}_{n=1}^{N}\right] \quad \text{where} \;\; \widehat{e}_{ni} \;=\; \omega_i \widetilde{e}_{ni} \;\; \text{and} \;\; \widetilde{e}_{ni} \;=\; D_i e_{ni} \tag{3}$$

In the above, $\widehat{e}_{ni}$ are the normalized draws and ω_i is set to the inverse of the marginal utility of consumption for household i at the planner's optimal allocation. Explaining, if household i optimally chooses location n_i^* given the planner's allocation $c_1^*, c_2^*, \ldots, c_N^*$, then $\omega_i = \left(A_{n_i^*}\ D_i\ e_{n_i^* i}\right)^{-1}$. Since ω_i is fixed across locations for any given household it does not affect any location choices of households; additionally, since ω_i rescales the draws such that all households have the same marginal utility of consumption of 1 at the optimal allocation, the planner has no motives for within-type transfers. Given any initial researcher-chosen distribution of location attachment draws $\widetilde{e}_{ni}$, $\widehat{e}_{ni}$ implies exactly the same optimal location choices but removes motives for the planner to implement within-type transfers.

Referring again to Figure 1, when we set ω_i in this way, the planner optimally chooses the dashed line at 0.0 in both panels ($c_1 - c_2 = z_1 - z_2$) for all values of $z_1 - z_2$ and for any combination of the nuisance parameters ϕ_1 and ϕ_2.

3.3. More Complicated Models

3.3.1. Theory

In our introduction, we describe three possible motives for a planner to redistribute resources across people and locations: Across-type equity, efficiency, and within-type transfers.

So far, we have analyzed a simple model where a planner has no motives for redistribution due to across-type equity, as there is only one type of household, or efficiency, as there are no externalities. We now show that in a more complicated model where all three motives may be present, our adjustment removes motives for within-type redistribution but motives for redistribution due to across-type equity and efficiency remain.

Consider an environment in which there are are $n = 1, \ldots, N$ discrete locations, $\tau = 1, \ldots, T$ distinct types of people, and possible externalities and complementarities across types in production. We assume a planner can choose any level of consumption for any type of household in any location, as long as the overall allocation satisfies aggregate feasibility conditions and respects individual optimization, i.e. households are assumed to optimally choose locations given their location attachment draws and given the allocation of consumption across locations.[4]

The planner chooses consumption for each type in each location to maximize the social welfare function

$$\sum_{\tau} \Pi^{\tau} L^{\tau} U\left(V^{\tau}\right) \tag{4}$$

where Π^{τ} is the planner's Pareto weight on type τ households in the economy, L^{τ} is the total population of type τ, V^{τ} is the expected utility associated with a type τ household and U is a concave function. The constraints on the problem are listed below, with Lagrange multipliers placed to the left of the brackets:

Expected Utility, by type: $\tau = 1, \ldots, T$	$\lambda^{\tau}\left[E_{e_{ni}}\left(\max_{n'} u^{\tau}_{n'i}\right) - V^{\tau}\right]$	$=$	0
Resource constraint:	$P\left[\sum_{n}\sum_{\tau} t^{\tau}_{n} L^{\tau}_{n}\right]$	$=$	0
Population, by type: $\tau = 1, \ldots, T$	$\gamma^{\tau}\left[L^{\tau} - \sum_{n} L^{\tau}_{n}\right]$	$=$	0
Optimization, by type and location: $\tau = 1, \ldots, T$ and $n = 1, \ldots, N$	$W^{\tau}_{n}\left[\rho^{\tau}_{n} L^{\tau} - L^{\tau}_{n}\right]$	$=$	0

[4] In this framework, the planner does not need to and will not want to implement across-location transfers as a means to implement across-type transfers. We can modify the environment to restrict transfers across locations to be identical across types. If different types tend to occupy different locations, then across-location transfers accomplish some across-type redistribution (Gaubert et al., 2020). This changes details of the solution but does not affect our general conclusions.

L_n^τ is the population of type τ in location n, u_{ni}^τ for agent i of type τ is the function $u_n(c_n, D_i, e_{ni})$ with $c_n^\tau = z_n^\tau - t_n^\tau$ where z_n^τ is income generated by one type τ worker in location n.[5] ρ_n^τ is the probability that $n = \text{argmax}_{n'} u_{n'i}^\tau$ for $n' = 1, \ldots, N$.

In Appendix B.1 we derive the solution to this problem; below we copy the equation from that Appendix that characterizes optimal location- and type-specific transfers for type τ in location n

$$\underbrace{\frac{\kappa\mathcal{U}^\tau\Pi^\tau\mu_n^\tau - \overline{\mathcal{U}\Pi}\bar{\mu}}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(1)} - \underbrace{\frac{\kappa\epsilon_n^\tau}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(2)} = \underbrace{\left(\frac{1}{L_n^\tau}\right)\sum_m t_m^\tau L^\tau\left(\frac{\partial\rho_m^\tau}{\partial t_n^\tau}\right)}_{(3)}$$

This solution is similar to that of the simple model, but modified to allow for multiple types of people in the economy and the possibility of complementarities across types and externalities in production. For a given type τ in location n, the first term on the left-hand side captures the difference in the Pareto-weighted ($\kappa\mathcal{U}^\tau\Pi^\tau$) marginal utility of consumption of that type in that location (μ_n^τ) from the economywide-average marginal utility of consumption ($\overline{\mathcal{U}\Pi}\bar{\mu}$) and the second term captures the economy-wide utility net benefit of production spillovers generated by that type in that location ($\kappa\epsilon_n^\tau$).[6] The difference of these two terms is equated to the marginal deadweight loss from increasing transfers, the third term.

In Appendix B.2, we derive the impact of our 5-step procedure on the solution for the optimal transfer to type τ in location n, which we copy below

$$\left(\kappa\mathcal{U}^\tau\Pi^\tau - \overline{\mathcal{U}\Pi}\right) - \kappa\epsilon_n^\tau = \left(\frac{1}{L_n^\tau}\right)\sum_m t_m^\tau L^\tau\left(\frac{\partial\rho_m^\tau}{\partial t_n^\tau}\right) \tag{5}$$

After the adjustment, the planner continues to have motives to transfer resources across types and locations. The right-hand side of equation (5), the marginal deadweight loss from increasing transfers, does not change. The term in parentheses on the left-hand side,

[5] This can be a function of $L_n^{\tau'}$ for $\tau' = 1, \ldots, T$, for example $z_n^\tau = z\left(z_n, L_n^1, L_n^2, \ldots, L_n^T\right)$ where z_n is TFP for location N.

[6] The $\mathcal{U}^\tau$ term is the derivative of the U function in the planner's objective function for type τ. κ is a scalar related to economy-wide fiscal externalities, marginal utilities of consumption and average production externalities and spillovers.

$\kappa\mathcal{U}^\tau\Pi^\tau - \overline{\mathcal{U}\Pi}$, is constant across locations for any given type of household, but this term allows for transfers across different types of households arising from motives of across-type equity based on differences in Pareto weights Π^τ and the slope of the concave function U evaluated at the optimal policy.[7] The term $\kappa\epsilon_n^\tau$ measures the impact of spillovers and externalities in production. Within-type variation in this term determines across-location, within-type transfers.

3.3.2. Numerical Examples

We now demonstrate that optimal transfers across locations are unidentified in more complicated models by examining these transfers in an environment with one type of household (as before), but with a large number of possible locations in which to live and produce and an externality in production. We set output per person in location 1 equal to $z_1 \left(L_1/L^*\right)^\delta$, where L_1 is the population in location 1 and for all locations $n > 1$ we set output per person equal to z_n. We consider two cases: $\delta = 0.0$, no externalities, and $\delta = 0.15$, a large population externality in city 1.

The planner chooses c_m for all locations m to maximize expected utility, $\sum\limits_m L_m V_m$, where V_m is the expected utility of households optimally choosing to live in location m. Utility of household i choosing to live in location m is

$$
\begin{aligned}
& A_m c_m \widetilde{e}_{mi} \\
& \text{where } \widetilde{e}_{mi} \;=\; D_i e_{mi} \text{ and } D_i \;=\; \left[\prod_m e_{mi}^{\phi_m}\right]^{-1}
\end{aligned}
$$

implying $V_m \;=\; E_i\left[A_m c_m D_i e_{mi} \mid m \text{ chosen by } i\right]$. In all simulations, we draw e_{mi} iid from the Fréchet distribution with shape parameter $\nu = 2$ and we set $A_m = 1$ for all m.

In our simulations, we assume 26 total locations in the economy and set $z_1 = 4/3$ in location 1 and $z_n = 2/3$ in the other 25 locations. As before, we consider values $\phi_1 \in$

[7] Exactly at the optimal solution, this adjustment causes the marginal utility of consumption of all households of all types to be identical. This means that the planner will not redistribute from high-income types to low-income types unless the Pareto weights for low-income types are higher than for high-income types. Researchers can pick Pareto weights to replicate optimal across-type transfers that are the solution to the planner's problem prior to applying our adjustment, if desired.

$\{0.0, 0.5, 1.0\}$ and $\phi_n \in \{0.0, 0.5, 1.0\}$. In each simulation, we fix ϕ_n to be the same for all $n > 1$ but allow ϕ_n to be different from ϕ_1. Since all locations $n > 1$ have identical TFP and identical values of ϕ_n, a planner will set c_n to be identical in each location $n > 1$. We will study the planner's redistribution from c_n to c_1.

The top panel of Figure 2 shows optimal within-type redistribution for the case of $\delta = 0.0$, $(c_1 - c_n) - (z_1 - z_n)$. As before, the sign and magnitude of optimal transfers is not identified. By comparing the bottom panel of Figure 1 to the top panel of Figure 2 we can see the impact on optimal policy from expanding the model from 2 locations to 26 locations. For the standard case of $\phi_1 = \phi_n = 0$, the blue circles, the number of locations has no impact on optimal transfers. For other values of ϕ_1 and ϕ_n, the size of the optimal transfer depends on the total number of locations in the environment. In a many-location model, the planner can heavily subsidize one location by transferring a small amount of resources from multiple locations. This has the potential to increase total optimal transfers relative to a model with only a few locations, where large transfers may be more distortionary.

The bottom panel of Figure 2 shows optimal within-type redistribution for the 26-location economy when $\delta = 0.15$ and production in location 1 is subject to increasing returns. The y-axis of this panel is $(c_1 - c_n) - \left(z_1 \left(\frac{L_1}{L^*}\right)^\delta - z_n\right)$, which are optimal transfers from any location $n > 1$ to location 1 given TFP in location 1 of $z_1 \left(\frac{L_1}{L^*}\right)^\delta$. We set $L^* = 4/29$. When $L_1 = L^* = 4/29$, the ratio of both consumption and TFP of location 1 to location $n > 1$ is $(4/3) / (2/3) = 2$ and there are no transfers.[8]

By comparing the results in the bottom panel to those in the top panel, we can see the impact of the externality on redistribution. For any given value of ϕ_1 and ϕ_n, when the externality is present the planner wants to redistribute more to location 1 from locations $n > 1$. In Appendix C, we show this directly by deriving optimal transfers for the standard

[8]When $\nu = 2$, $L_1/L_n = (c_1/c_n)^\nu = [(4/3)/(2/3)]^\nu = 4$. Given $L_n = (1 - L_1)/25$, this gives $L_1/(1 - L_1) = 4/25$ and therefore $L_1 = 4/29$.

case of $\phi_1 = \phi_n = 0$:

$$c_1 - c_n \;=\; \left(\frac{\nu}{1+\nu}\right)\;\Bigg[\;\underbrace{z_1\left(\frac{L_1}{L^*}\right)^{\delta} - z_n}_{A} \;+\; \underbrace{\delta z_1\left(\frac{L_1}{L^*}\right)^{\delta}}_{B}\;\Bigg]$$

A is the TFP differential of locations 1 and n at the optimal allocation and B is the impact on output of existing residents at location 1 from a marginal increase in the population in location 1 due to the externality. As the bottom panel of Figure 2 illustrates, even with a large externality and a motive to transfer additional resources to location 1, when ϕ_1 and ϕ_n are unknown, both the sign and magnitude of optimal transfers are not identified.

This example illustrates the quantitative importance of within-type transfers on optimal policy relative to the importance of a large externality. Comparing the top and bottom panels of Figure 2, it is obvious that the large externality in location 1 shifts optimal policy, as all lines on the bottom panel are about 0.2 higher than all lines in the top panel. That said, the range of optimal policies arising from within-type redistribution due to the lack of identification of ϕ_1 and ϕ_n is substantially larger than the change in optimal policy at any ϕ_1 and ϕ_n once the externality is introduced.

To implement our adjustment, we redefine the location attachment draws for household i in location m such that utility is

$$A_m c_m \widehat{e}_{mi} \tag{6}$$

$$\text{where } \widehat{e}_{mi} \;=\; \omega_i \widetilde{e}_{mi},\;\; \widetilde{e}_{mi} \;=\; D_i e_{mi},\;\; D_i \;=\; \left[e_{1i}^{\phi_1^{\tau}} \prod_{n>1} e_{ni}^{\phi_n}\right]^{-1} \text{ and } \omega_i \;=\; \left[A_{m_i^* i}\; D_i\; e_{m_i^* i}\right]^{-1}$$

where m_i^* is the optimally chosen location for agent i given realized e_{mi} at the planner's optimal allocation of consumption c_m^* for all m locations. Including ω_i in utility in this way ensures that the average marginal utility of consumption in a location is constant across locations at the planner's chosen allocation, and thus the planner has no motive to redistribute to equate within-type marginal utilities of consumption.[9]

The impact of the adjustment on redistribution is shown by the dashed black lines in Figure 2. In the top panel, the version of the model with no externalities, the planner chooses no redistribution for all values of ϕ_1 and ϕ_n. In the bottom panel, the planner chooses to redistribute from households living in location $n > 1$ to households living in location 1 due to the production externality. The amount of redistribution does not depend on ϕ_1 or ϕ_n and is exactly equal to the impact on output of residents in location 1 from a marginal increase in the population in location 1 due to the externality, part "B" in equation (6).[10] This example shows our adjustment allows for the direct study of optimal place-based transfers in response to production externalities, without the size and direction of those transfers being influenced by differences in the marginal utility of consumption across locations that are not identifiable from location choice data.

3.4. Discussion of Uniqueness

In all simulations where we apply our adjustment, we use the 5-step procedure outlined in section 3.1 to find optimal allocations. The procedure finds an allocation with the following property: With weights ω_i set equal to the inverse of the marginal utility of consumption at the candidate allocation, the candidate allocation solves the adjusted planner's problem once adjusted to include ω_i. An allocation has this property if and only if it is a solution to equation (5). Therefore, if equation (5) has a unique solution our procedure also produces a unique solution. Uniqueness of equation (5) depends on researcher choices that determine elasticities, externalities, and Pareto weights.[11] If researchers can prove equation (5) has a unique solution, our experience suggests our procedure will find that solution as long as the dampening factor d is sufficiently small.

[9] The planner may still wish to redistribute across locations and types (in a multiple-type model) for reasons of across-type equity or efficiency.

[10] This wedge is slightly bigger than $0.2 = \delta * z_1 = 0.15 * (4/3)$ since $L_1 > L^*$ at the optimal allocation.

[11] For example, multiple candidate solutions may exist depending on the properties of agglomeration externalities in the model.

4. Additional Thoughts

At seminars, participants have asked if the problem we identify is relevant for researchers choosing a utility function of the form $\nu \log c_n + e_{ni}$, where c_n is consumption in location n and e_{ni} are drawn iid across households i and locations n from the Type 1 Extreme Value distribution. This specification has the virtue that the shocks determining location choice e_{ni} do not directly affect the marginal utility of consumption; additionally, the optimal policy arising from this specification is identical to the policy from the base case we considered earlier, $c_n - c_{n'} = \left(\frac{\nu}{1+\nu}\right)(z_n - z_{n'})$.

With this specification, the problem we document does not go away. Define a variable D_i that is constant for any household i but can vary across households. If utility is specified as $D_i [\nu \log c_n + e_{ni}]$ then each household's optimal location decision does not change, but optimal policy might depending on the correlation across households of D_i with the values of e_{ni}.[12] For all specifications of utility including this one, our proposed 5-step adjustment can be implemented exactly as written.

The possibility that the marginal utility of consumption may be low when measured income is low, and the implications of this for optimal policy, has been identified in other areas of economics. For example, Klevin et al. (2009) studies optimal taxation of married couples. In that paper, the secondary earner can choose not to work for one of two reasons: he/she either receives a bad draw of market earnings or a good draw of home productivity. Klevin et al. (2009) show that optimal policy depends on which of the two explanations caused the secondary earner to not work in the market.[13]

One path for future research may be to use data to estimate differences across locations in the average marginal utility of consumption of otherwise identical households. Researchers in other fields of economics have attempted to estimate state dependence in the marginal utility of consumption. For example, health economists have tried to identify how the state of a person's health affects their marginal utility of consumption. Finkelstein et al. (2009)

[12]There are other specifications in which households living in low-income locations may have a low marginal utility of consumption, on average. For example, suppose utility for household i in location n is $\nu \log(c_n + \epsilon_{ni}) + e_{ni}$, where ϵ_{ni} is drawn from some distribution. This specification allows that unobservable attachment factors that push households to live in certain locations may be substitutable for consumption.

[13]We thank Patrick Kline for suggesting this connection.

survey the various approaches and results in the literature and conclude, "Currently available estimates offer little in the way of a consensus on the sign or magnitude of health state dependence." The hurdle for estimation is high in location choice models, as researchers need to understand variation in the average marginal utility of consumption across locations and we believe this will be difficult to measure. Even if a policy experiment exogenously shifts location choices of some marginal households, optimal policy depends on the marginal utility of consumption of *all* households including – perhaps most importantly – those least likely to move. Until we have direct evidence on differences in the average marginal utility of consumption across locations, we advocate imposing our adjustment to the planning problem, which removes a planner's incentives for within-type transfers across locations absent motives of productive efficiency or externalities.

References

Arefeva, A., Davis, M.A., Ghent, A., Park, M., 2021. The effect of capital gains taxes on business creation and employment: The case of opportunity zones. Working Paper.

Busso, M., Gregory, J., Kline, P., 2013. Assessing the incidence and efficiency of a prominent place based policy. American Economic Review 103, 897–947.

Chetty, R., 2006. A general formula for the optimal level of social insurance. Journal of Public Economics 90, 1879–1901.

Davis, M.A., Gregory, J., Hartley, D.A., 2019. The long-run effects of low-income housing on neighborhood composition. Working Paper.

Fajgelbaum, P.D., Gaubert, C., 2019. Optimal spatial policies, geography and sorting. Forthcoming, Quarterly Journal of Economics.

Finkelstein, A., Luttmer, E.F.P., Notowidigdo, M.J., 2009. Approaches to estimating the health state dependence of the utility function. American Economic Review 99, 116–121.

Gaubert, C., Kline, P., Yagan, D., 2020. Placed-based redistribution. Working Paper.

Klevin, H., Kreiner, C., Saez, E., 2009. The optimal income taxation of couples. Econometrica 77, 537–560.

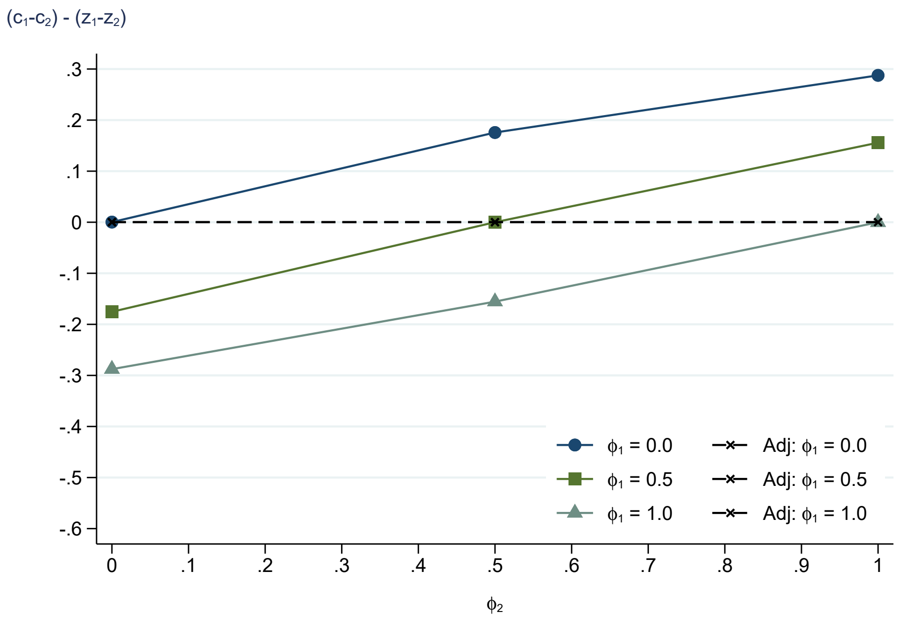
Kline, P., Moretti, E., 2014. Local economic development, agglomeration economies and the big push: 100 years of evidence from the tennessee valley authority. Quarterly Journal of Economics 129, 275–331.

Matzkin, R.L., 1993. Nonparametric identification and estimation of polychotomous choice models. Journal of Econometrics 58, 137–168.

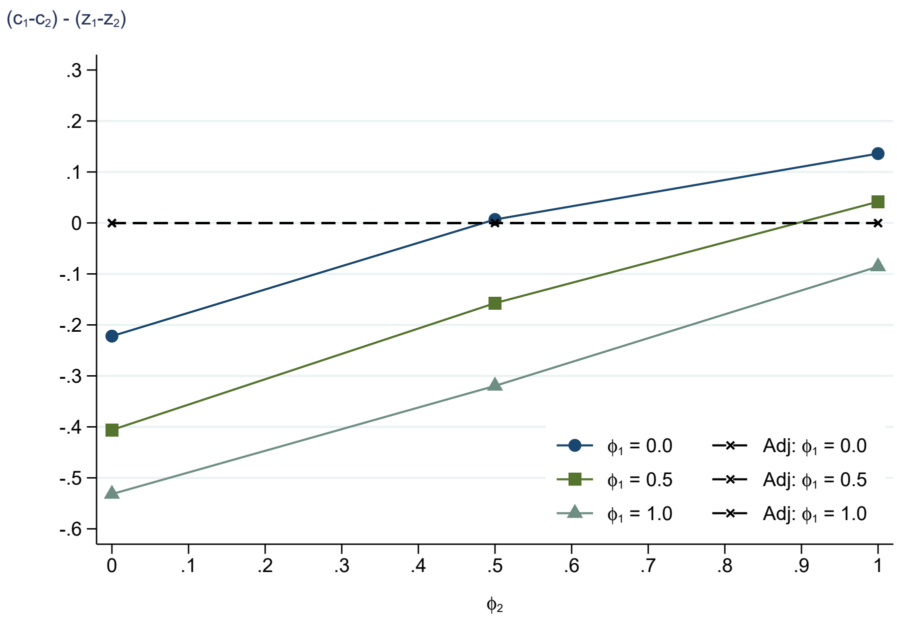
Roback, J., 1982. Wages, rents, and the quality of life. Journal of Political Economy 90, 1257–1278.

Rossi-Hansberg, E., Sarte, P.D., Schwartzman, F., 2020. Cognitive hubs and spatial redistribution. Working Paper.

Figure 1: Redistribution from Location 2 to 1, 2 Locations

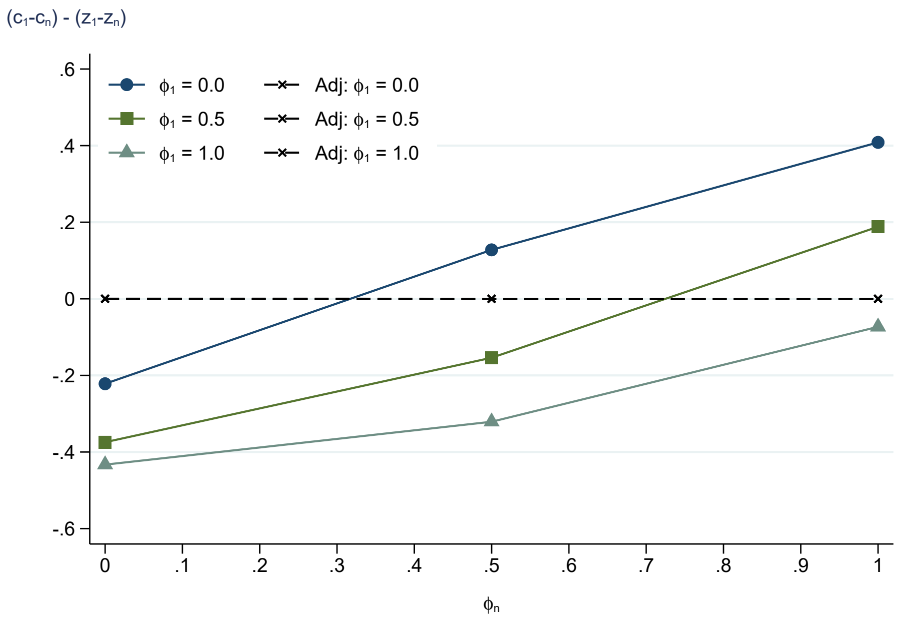


(a) $z_1 = z_2 = 1.0$

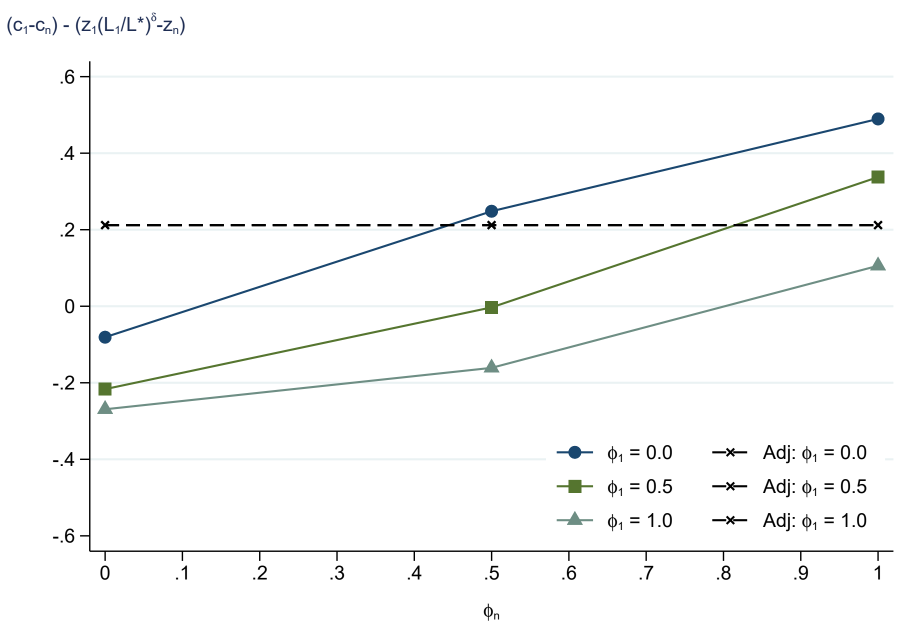


(b) $z_1 = \frac{4}{3}$, $z_2 = \frac{2}{3}$

Figure 2: Redistribution from Location $n > 1$ to Location 1, 26 Locations



(a) No Externalities: $z_1 = \dfrac{4}{3}$, $z_n = \dfrac{2}{3}$ for $n > 1$



(b) Externality in Location 1: $\delta = 0.15$, $z_1 = \dfrac{4}{3}$, $z_n = \dfrac{2}{3}$ for $n > 1$

Not-For-Publication Online Appendix

Appendix A. Planning Solution

Denote A_n as amenities in location n and c_n as consumption in location n such that the deterministic portion of utility in location n is $u_n = A_n c_n$ and utility for person i in location n is $u_{ni} = u_n e_{ni}$ where e_{ni} is drawn iid from the Fréchet distribution with parameter ν. Also denote L_n as the population in location n and let G denote the pre-determined amount of government expenditure that needs to be funded by taxation. The planner solves:

$$\max_{\{c_n, L_n\}_{n=1}^{N}} U$$

subject to the following constraints (Lagrange multipliers are to the left of the brackets)

$$\begin{array}{lrcl}
\text{Expected Utility} & \lambda\left[\left(\sum_n u_n^{\nu}\right)^{\frac{1}{\nu}} - U\right] & = & 0 \\
\text{Resource constraint} & P\left[\sum_n L_n z_n - \sum_n L_n c_n - G\right] & = & 0 \\
\text{Population:} & \mu\left[1 - \sum_n L_n\right] & = & 0 \\
\text{Utility n=1,...,N} & \theta_n\left[A_n c_n - u_n\right] & = & 0 \\
\text{Individual optimization n=1,...,N:} & W_n\left[\left(\frac{u_n}{U}\right)^{\nu} - L_n\right] & = & 0
\end{array}$$

First-order conditions are

$$\begin{array}{lrcl}
u_n: & 0 & = & \lambda L_n U - \theta_n u_n + \nu W_n L_n \\
c_n: & 0 & = & \theta_n u_n - P L_n c_n \\
L_n: & 0 & = & P\left(z_n - c_n\right) L_n - W_n L_n - \mu L_n \\
U: & 0 & = & 1 - \lambda - (\nu/U)\sum_n W_n L_n
\end{array}$$

From the FOC for U we have $(\nu/U)\left(\sum_n W_n L_n\right) = 1 - \lambda$. Add the Focs for u_n to get $1 = \sum_n \theta_n \left(u_n/U\right)$. Now add the FOCs for c_n to get $(U/P) = GDP - G$ where $GDP = \sum_n z_n L_n$.

Now start with the FOC for L_n

$$0 \;=\; P\left(z_n - c_n\right)L_n \;-\; W_n L_n \;-\; \mu L_n$$

Use FOC for u_n

$$W_n L_n \;=\; \frac{1}{\nu}\left(\theta_n u_n\right) \;-\; \frac{1}{\nu}\left(\lambda L_n U\right) \;=\; \frac{1}{\nu}\left(P L_n c_n\right) \;-\; \frac{1}{\nu}\left(\lambda L_n U\right)$$

Insert

$$\begin{aligned}
0 &= P L_n z_n \;-\; P L_n c_n \;-\; \frac{1}{\nu}\left(P L_n c_n\right) \;+\; \frac{1}{\nu}\left(\lambda L_n U\right) \;-\; \mu L_n \\
0 &= z_n \;-\; c_n \;-\; \frac{1}{\nu}\left(c_n\right) \;+\; \frac{1}{P\nu}\left(\lambda U\right) \;-\; \frac{\mu}{P} \\
&= z_n \;-\; \left[\frac{1+\nu}{\nu}\right] c_n \;+\; \left(\frac{U}{P}\right)\left(\frac{\lambda}{\nu} - \frac{\mu}{U}\right)
\end{aligned}$$

Rearrange terms and substitute for U/P to get

$$c_n \;=\; \left[\frac{\nu}{1+\nu}\right] z_n \;+\; \left(\frac{\lambda - \dfrac{\mu\nu}{U}}{1+\nu}\right)\left(GDP - G\right) \tag{A.1}$$

If we multiply the above equation by L_n and then sum over n, we get the expression

$$\begin{aligned}
\left(\lambda - \frac{\mu\nu}{U}\right)\left(GDP - G\right) &= \left(1+\nu\right)\left(GDP - G\right) \;-\; \nu GDP \\
&= GDP - \left(1+\nu\right)G \qquad\qquad (A.2)
\end{aligned}$$

After inserting equation (A.2) into (A.1), we get the following expression for optimal consumption in location n

$$\begin{aligned}
c_n &= \left(\frac{\nu}{1+\nu}\right) z_n \;+\; T \\
\text{where}\quad T &= \frac{GDP}{1+\nu} \;-\; G
\end{aligned}$$

Appendix B. Multiple Types of Households, Multiple Locations and Production Externalities

Appendix B.1. No Adjustment

We now consider an environment with $n = 1, \ldots, N$ discrete locations and $\tau = 1, \ldots, T$ types. We assume a planner can choose any level of consumption for any type in any location, as long as the allocation satisfies aggregate feasibility conditions and respects individual optimization, i.e. households optimally choose locations given their location attachment draws and given the allocation of consumption across locations.

The objective of the planner is as follows

$$\max_{\left\{\{t_n^\tau, L_n^\tau\}_{\tau=1}^T\right\}_{n=1}^N} \sum_\tau \Pi^\tau L^\tau U\left(V^\tau\right)$$

where U is a concave function, L_n^τ is the population of type τ in location n, L^τ is the total population of type τ and V^τ is the expected utility associated with type τ. Then planner maximizes this function subject to constraints listed below. Note that in the list of constraints the Lagrange multipliers are to the left of the brackets:

Expected Utility, by type: $\tau = 1, \ldots, T$	$\lambda^\tau \left[E_{e_{ni}}\left(\max_{n'} u_{n'i}^\tau\right) - V^\tau\right]$	$=$	0
Resource constraint:	$P\left[\sum_n \sum_\tau t_n^\tau L_n^\tau\right]$	$=$	0
Population, by type: $\tau = 1, \ldots, T$	$\gamma^\tau \left[L^\tau - \sum_n L_n^\tau\right]$	$=$	0
Optimization, by type and location: $\tau = 1, \ldots, T$ and $n = 1, \ldots, N$	$W_n^\tau \left[\rho_n^\tau L^\tau \; - \; L_n^\tau\right]$	$=$	0

u_{ni}^τ for agent i of type τ is the function $u_n\left(c_n, D_i, e_{ni}\right)$ with $c_n^\tau = z_n^\tau - t_n^\tau$ where z_n^τ is income generated by one type τ worker in location n which can be a function of $L_n^{\tau'}$ for $\tau' = 1, \ldots, T$, for example $z_n^\tau = z\left(z_n, L_n^1, L_n^2, \ldots, L_n^T\right)$ where z_n is TFP for location N. As specified, this framework allows for complementarities across types or externalities involving one or more types in production.[14] ρ_n^τ is the probability that $n = \text{argmax}_{n'} u_{n'i}^\tau$ for $n' = 1, \ldots, N$.

[14] As an example, in a decentralized economy firms may take as given in location n multifactor productivity of a_n where $a_n = z_n\left(L_n^{\tau *}\right)^\delta$, with $L_n^{\tau *}$ an externality in type $\tau *$ workforce. The planner explicitly takes into consideration the impact of allocations on the externality.

The first-order conditions are:

$$\begin{aligned}
V^\tau : \quad 0 &= \left(\frac{\partial U}{\partial V^\tau}\right)\Pi^\tau L^\tau - \lambda^\tau \\
L_n^\tau : \quad 0 &= \sum_{\tau'} \lambda^{\tau'} \left(\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial L_n^\tau}\right) + P t_n^\tau - \gamma^\tau - W_n^\tau \\
t_n^\tau : \quad 0 &= -\lambda^\tau \left(\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau}\right)}{\partial c_n^\tau}\right) + P L_n^\tau + \sum_m W_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau}\right)
\end{aligned}$$

where we have made use in the last equation that $\partial c_n^\tau / \partial t_n^\tau = -1$.

To reduce notation, for any given type τ' define the derivative of the expected value with respect to L_n^τ as

$$\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial L_n^\tau} = \frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial c_n^{\tau'}} \cdot \frac{\partial c_n^{\tau'}}{\partial L_n^\tau} = \frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau'}\right)}{\partial c_n^{\tau'}} \cdot \epsilon_n^{\tau \to \tau'}$$

Also define

$$\frac{\partial E_{e_{ni}}\left(\max_{n'} u_{n'i}^{\tau}\right)}{\partial c_n^{\tau}} = \left(\frac{L_n^\tau}{L^\tau}\right)\mu_n^\tau$$

where μ_n^τ is the average of the marginal utility of consumption of type τ agents that have chosen to live in location n:

$$\mu_n^\tau = E\left[\frac{\partial u_{ni}^\tau}{\partial c_n^\tau} \;\middle|\; n = \operatorname{argmax} u_{n'i}^\tau\right]$$

After substituting $\mathcal{U}^\tau = \partial U / \partial V^\tau$, this allows us to rewrite the FOCs as:

$$\begin{aligned}
V^\tau : \quad 0 &= \mathcal{U}^\tau \Pi^\tau L^\tau - \lambda^\tau \\
L_n^\tau : \quad 0 &= \sum_{\tau'} \lambda^{\tau'} \left(\frac{L_n^{\tau'}}{L^{\tau'}}\right) \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} + P t_n^\tau - \gamma^\tau - W_n^\tau \\
t_n^\tau : \quad 0 &= -\lambda^\tau \left(\frac{L_n^\tau}{L^\tau}\right)\mu_n^\tau + P L_n^\tau + \sum_m W_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau}\right)
\end{aligned}$$

Consider the FOC for L_n^τ after reducing for λ^τ:

$$0 = \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_n^{\tau'} \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} + P t_n^\tau - \gamma^\tau - W_n^\tau$$

Multiply everything by L_n^τ and sum over n

$$0 = \sum_n L_n^\tau \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_n^{\tau'} \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} \right] + P \sum_n t_n^\tau L_n^\tau - \gamma^\tau \sum_n L_n^\tau - \sum_n W_n^\tau L_n^\tau$$

Define total tax revenues collected for type τ residents as $\mathcal{T}^\tau$. After rearranging terms, this reduces to

$$\gamma^\tau = \sum_n \left(\frac{L_n^\tau}{L^\tau} \right) \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_n^{\tau'} \mu_n^{\tau'} \epsilon_n^{\tau \to \tau'} \right] + P \left(\frac{\mathcal{T}^\tau}{L^\tau} \right) - \sum_n W_n^\tau \left(\frac{L_n^\tau}{L^\tau} \right)$$

Insert this expression for γ^τ into the FOC for L_n^τ, rearrange terms, and replace n with m everywhere:

$$W_m^\tau =$$
$$P t_m^\tau - P \left(\frac{\mathcal{T}^\tau}{L^\tau} \right) + \sum_{m'} W_{m'}^\tau \left(\frac{L_{m'}^\tau}{L^\tau} \right) + \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} - \sum_{m'} \left(\frac{L_{m'}^\tau}{L^\tau} \right) \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_{m'}^{\tau'} \mu_{m'}^{\tau'} \epsilon_{m'}^{\tau \to \tau'} \right]$$

Now return to the t_n equation and substitute for λ^τ

$$\mathcal{U}^\tau \Pi^\tau L_n^\tau \mu_n^\tau = P L_n^\tau + \sum_m W_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)$$

Insert for W_m^τ

$$\mathcal{U}^\tau \Pi^\tau L_n^\tau \mu_n^\tau =$$
$$P L_n^\tau + \sum_m \left\{ P t_m^\tau - P \left(\frac{\mathcal{T}^\tau}{L^\tau} \right) + \sum_{m'} W_{m'}^\tau \left(\frac{L_{m'}^\tau}{L^\tau} \right) + \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} - \sum_{m'} \left(\frac{L_{m'}^\tau}{L^\tau} \right) \left[\sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_{m'}^{\tau'} \mu_{m'}^{\tau'} \epsilon_{m'}^{\tau \to \tau'} \right] \right\} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)$$

Note that since the overall population of type τ is fixed, this implies $\sum_m (\partial \rho_m^\tau / \partial t_n^\tau) = 0$.

Thus, the above can be reduced to:

$$\mathcal{U}^{\tau}\Pi^{\tau}L_n^{\tau}\mu_n^{\tau} = PL_n^{\tau} + P\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right) + \sum_m\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

Add across τ and rearrange:

$$\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_n^{\tau}\mu_n^{\tau} - \sum_{\tau}\sum_m\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right) = P\sum_{\tau}L_n^{\tau} + P\sum_{\tau}\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

Sum over n

$$\sum_n\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_n^{\tau}\mu_n^{\tau} - \sum_n\sum_{\tau}\sum_m\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right) = P\sum_n\sum_{\tau}L_n^{\tau} + P\sum_n\sum_{\tau}\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

Define $\overline{\mathcal{U}\Pi}$, $\bar{\mu}$, ℓ and Δ as follows

$$\begin{aligned}
\overline{\mathcal{U}\Pi} &= \sum_n\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_n^{\tau} \\
\bar{\mu} &= \left(\overline{\mathcal{U}\Pi}\right)^{-1}\sum_n\sum_{\tau}\mathcal{U}^{\tau}\Pi^{\tau}L_n^{\tau}\mu_n^{\tau} \\
\ell &= \sum_n\sum_{\tau}\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right) \\
\Delta &= \sum_n\sum_{\tau}\sum_m\sum_{\tau'}\mathcal{U}^{\tau'}\Pi^{\tau'}L_m^{\tau'}\mu_m^{\tau'}\epsilon_m^{\tau\to\tau'}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)
\end{aligned}$$

For economic interpretation, $\bar{\mu}$ is the Pareto-weighted average marginal utility of consumption in the economy; ℓ measures the impact on the tax base generated by the location responses to a marginal increase in taxes that is uniformly applied across locations and types; Δ measures the Pareto-weighted sum of the marginal change in economy-wide utility arising from spillovers generated by the location responses to a marginal increase in taxes that is uniformly applied across locations and types.[15]

[15] In the special case in which utility is linear in consumption and location-specific preferences are additive to utility then $\sum_n(\partial\rho_m^{\tau}/\partial t_n^{\tau}) = 0$ giving $\ell = \Delta = 0$.

With this notation, we write

$$P = \frac{\overline{\mathcal{U}\Pi\bar{\mu}} - \Delta}{1+\ell} = \overline{\mathcal{U}\Pi\bar{\mu}} \left(\frac{1 - \frac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}{1+\ell} \right)$$

Insert this definition of P and return to the FOC for t_n

$$\mathcal{U}^\tau \Pi^\tau L_n^\tau \mu_n^\tau - \sum_m \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right) = \overline{\mathcal{U}\Pi\bar{\mu}} \left(\frac{1 - \frac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}{1+\ell} \right) \left[L_n^\tau + \sum_m t_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right) \right]$$

Divide by L_n^τ

$$\mathcal{U}^\tau \Pi^\tau \mu_n^\tau - \left(\frac{1}{L_n^\tau} \right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right) = \overline{\mathcal{U}\Pi\bar{\mu}} \left(\frac{1 - \frac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}{1+\ell} \right) \left[1 + \left(\frac{1}{L_n^\tau} \right) \sum_m t_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right) \right]$$

Define $\kappa = \frac{1+\ell}{1 - \frac{\Delta}{\overline{\mathcal{U}\Pi\bar{\mu}}}}$. Then

$$\kappa \mathcal{U}^\tau \Pi^\tau \mu_n^\tau - \kappa \left(\frac{1}{L_n^\tau} \right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right) = \overline{\mathcal{U}\Pi\bar{\mu}} \left[1 + \left(\frac{1}{L_n^\tau} \right) \sum_m t_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right) \right]$$

Subtract $\overline{\mathcal{U}\Pi\bar{\mu}}$ and then divide.

$$\frac{\kappa \mathcal{U}^\tau \Pi^\tau \mu_n^\tau - \overline{\mathcal{U}\Pi\bar{\mu}}}{\overline{\mathcal{U}\Pi\bar{\mu}}} - \frac{\kappa \left(\frac{1}{L_n^\tau} \right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)}{\overline{\mathcal{U}\Pi\bar{\mu}}} = \left(\frac{1}{L_n^\tau} \right) \sum_m t_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)$$

Define ϵ_n^τ as the Pareto-weighted sum of the marginal change in economy-wide utility arising from spillovers generated by the location responses to a marginal increase in taxes that is applied in location n to type τ:

$$\epsilon_n^\tau = \left(\frac{1}{L_n^\tau} \right) \sum_m \sum_{\tau'} \mathcal{U}^{\tau'} \Pi^{\tau'} L_m^{\tau'} \mu_m^{\tau'} \epsilon_m^{\tau \to \tau'} L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau} \right)$$

Rewrite the above as

$$\underbrace{\frac{\kappa\mathcal{U}^{\tau}\Pi^{\tau}\mu_n^{\tau} - \overline{\mathcal{U}\Pi}\bar{\mu}}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(1)} - \underbrace{\frac{\kappa\epsilon_n^{\tau}}{\overline{\mathcal{U}\Pi}\bar{\mu}}}_{(2)} = \underbrace{\left(\frac{1}{L_n^{\tau}}\right)\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)}_{(3)} \tag{B.1}$$

For a given type τ in location n, the first term on the left-hand side captures the difference in the Pareto-weighted marginal utility of consumption of that type in that location from the economywide-average and the second term captures the economy-wide utility (net) benefit of production spillovers generated by that type in that location. The difference of these two terms is equated to the the marginal deadweight loss from increasing transfers for that type in that location, the third term.

We can rewrite this third term to gain some intuition. To start, note the following

$$\frac{\partial\rho_n^{\tau}}{\partial t_n^{\tau}} = -\sum_{m\neq n}\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}$$

Then the third term becomes

$$\left(\frac{1}{L_n^{\tau}}\right)\frac{\partial\rho_n^{\tau}}{\partial t_n^{\tau}}L^{\tau}\left(t_n^{\tau} - \frac{\sum\limits_{m\neq n} t_m^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)}{\sum\limits_{m\neq n}\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}}\right) \tag{B.2}$$

This "fiscal externality" is the amount by which the tax from type τ in location n exceeds the tax that the marginal leavers of type τ will be exposed to, on average, conditional on leaving location n.

Appendix B.2. With our Proposed Adjustment

It is convenient to rewrite equation (B.1) as follows

$$\frac{\kappa\mathcal{U}^{\tau}\Pi^{\tau}\left(\mu_n^{\tau} - \mu^{\tau}\right) + \kappa\mathcal{U}^{\tau}\Pi^{\tau}\mu^{\tau} - \overline{\mathcal{U}\Pi}\bar{\mu}}{\overline{\mathcal{U}\Pi}\bar{\mu}} - \frac{\kappa\epsilon_n^{\tau}}{\overline{\mathcal{U}\Pi}\bar{\mu}} = \left(\frac{1}{L_n^{\tau}}\right)\sum_m t_m^{\tau}L^{\tau}\left(\frac{\partial\rho_m^{\tau}}{\partial t_n^{\tau}}\right)$$

where $\mu^{\tau} = \sum\limits_n \left(L_n^{\tau}/L^{\tau}\right)\mu_n^{\tau}$. We have shown earlier that location data do not pin down within-type transfers that are only based on differences in within-type marginal utility of consumption across locations, the term involving $\mu_n^\tau - \mu^\tau$. For this reason, we advocate setting $\mu_n^\tau = \mu^\tau$, thereby eliminating the desire for a planner to redistribute for this motive.

If researchers wish to eliminate these transfers, they simply need to ensure that the average marginal utility of consumption for a given type does not vary across locations. There are many possible ways to generate this outcome. We propose simply setting $\mu_n^\tau = \mu^\tau = 1$ for all households, which also implies $\bar{\mu} = 1$. After this adjustment, the optimal tax on type τ at location n satisfies:

$$\left(\kappa \mathcal{U}^\tau \Pi^\tau - \overline{\mathcal{U}\Pi}\right) \;-\; \kappa \epsilon_n^\tau \;=\; \left(\frac{1}{L_n^\tau}\right) \sum_m t_m^\tau L^\tau \left(\frac{\partial \rho_m^\tau}{\partial t_n^\tau}\right) \tag{B.3}$$

With one type and no externalities in production, $\epsilon_n^\tau = 0$ and $\Delta = 0$. The condition for optimality can be written as

$$\ell \cdot \mathcal{U} \;=\; \left(\frac{1}{L_n}\right) \sum_m t_m \left(\frac{\partial \rho_m}{\partial t_n}\right)$$

Notice that the left-hand side does not vary across locations. The only solution that satisfies this equation for every location is $t_n = 0$ for all n.[16]

Returning to the multiple-type case of equation (B.3) , the framework has the capacity to deliver both transfers across and within types. The term $\kappa \mathcal{U}^\tau \Pi^\tau - \overline{\mathcal{U}\Pi}$ is constant across locations for any given type, but allows transfers of consumption across types based on differences in Pareto weights and the slope of the concave function U evaluated at the optimal policy. The term $\kappa \epsilon_n^\tau$ measures the impact of spillovers and externalities in production (which we believe the data can identify).[17] Within-type variation in this term determines across-location, within-type transfers.

[16] $\ell = 0$ when $t_n = 0$ at every n.

[17] Recall we have set $\mu_n^\tau = 1$, such that ϵ_n^τ contains Pareto weights, elasticities of location choices with respect to income, and production-function spillovers and externalities.

Appendix C. Multiple Locations and a Production Externality

In this section, we solve for optimal consumption in an $N+1$ location model where locations $n = 2, \ldots, N$ are all identical and location 1 is subject to an agglomeration externality. We assume utility for household i in location m is $c_m e_{mi}$ where c_n is consumption in m and and e_{mi} is an *iid* draw from the Fréchet distribution with parameter ν. Given these assumptions, the planner maximizes U which we define as

$$\left(\sum_m c_m^\nu\right)^{1/\nu}$$

subject to the following first-order conditions:

$$\begin{array}{lr}
\text{Resource:} & P\left[z_1\left(\frac{L_1}{L^*}\right)^\delta L_1 + \sum_{n\neq 1} z_n L_n - \sum_m c_m L_m\right] \\
\text{Incentive Compatibility}, \forall m & W_m\left[L_m - \frac{c_m^\nu}{\sum_{m'} c_{m'}^\nu}\right] \\
\text{Population:} & \mu\left[1 - \sum_m L_m\right]
\end{array}$$

The first order conditions are

$$\begin{array}{lcl}
c_m: & 0 = & \left(\sum_m c_m^\nu\right)^{\frac{1}{\nu}-1} c_m^{\nu-1} - PL_m - W_m L_m\left(1-L_m\right)\left(\frac{\nu}{c_m}\right) + \sum_{n\neq m} W_n L_n L_m\left(\frac{\nu}{c_m}\right) \\
L_1: & 0 = & P\left[\left(1+\delta\right) z_1\left(\frac{L_1}{L^*}\right)^\delta - c_1\right] + W_1 - \mu \\
L_n, n>1: & 0 = & P\left[z_n - c_n\right] + W_n - \mu
\end{array}$$

Define the externality wedge as

$$\Delta \equiv \delta z_1\left(\frac{L_1}{L^*}\right)^\delta L_1$$

Note that if we multiply the FOC for L_1 by L_1, multiply the FOC for L_n by L_n, and then add the FOCs for L_m for all m, we get the expression

$$\mu = P\Delta + \sum_m W_m L_m \tag{C.1}$$

Now multiply the FOC for c_m by c_m and use definition of U to get

$$\begin{aligned}
0 &= UL_m - PL_m c_m - \nu W_m L_m (1 - L_m) + \nu \sum_{n \neq m} W_n L_n L_m \\
&= UL_m - PL_m c_m - \nu W_m L_m + \nu L_m W_m L_m + \nu L_m \sum_{n \neq m} W_n L_n \\
&= UL_m - PL_m c_m - \nu W_m L_m + \nu L_m \sum_{n} W_n L_n \\
&= UL_m - PL_m c_m - \nu W_m L_m + \nu \mu L_m
\end{aligned}$$

Sum this FOC for all m and use equation (C.1) to get

$$U = P \cdot GDP - \nu P \Delta$$

From the FOC for L_n for $n > 1$ we have

$$W_n L_n = \mu L_n - PL_n [z_n - c_n]$$

Insert into the FOC for c_n for $n > 1$

$$\begin{aligned}
0 &= UL_n - PL_n c_n - \nu \mu L_n + \nu PL_n [z_n - c_n] + \nu \mu L_n \\
&= U - Pc_n + \nu P [z_n - c_n] \\
&= GDP - \nu \Delta - c_n + \nu [z_n - c_n] \\
&= GDP - (1 + \nu) c_n + \nu [z_n - \Delta]
\end{aligned}$$

Implying

$$c_n = \left(\frac{\nu}{1 + \nu} \right) [z_n - \Delta] + \frac{GDP}{1 + \nu} \tag{C.2}$$

From the FOC for L_1 for we have

$$W_1 L_1 = \mu L_1 - PL_1 \left[(1 + \delta) z_1 \left(\frac{L_1}{L^*} \right)^{\delta} - c_1 \right]$$

Insert into the FOC for c_n for $n > 1$

$$\begin{aligned} 0 &= UL_1 - PL_1 c_1 - \nu\mu L_1 + \nu P L_1 \left[(1+\delta) z_1 \left(\frac{L_1}{L^*}\right)^\delta - c_1 \right] + \nu\mu L_1 \\ &= U - Pc_1 + \nu P \left[(1+\delta) z_1 \left(\frac{L_1}{L^*}\right)^\delta - c_1 \right] \\ &= GDP - \nu\Delta - c_1 + \nu \left[(1+\delta) z_1 \left(\frac{L_1}{L^*}\right)^\delta - c_1 \right] \\ &= GDP - (1+\nu) c_n + \nu \left[(1+\delta) z_1 \left(\frac{L_1}{L^*}\right)^\delta - \Delta \right] \end{aligned}$$

Implying

$$c_1 = \left(\frac{\nu}{1+\nu}\right) \left[(1+\delta) z_1 \left(\frac{L_1}{L^*}\right)^\delta - \Delta \right] + \frac{GDP}{1+\nu} \tag{C.3}$$

Combining equations (C.2) and (C.3) gives

$$c_1 - c_n = \left(\frac{\nu}{1+\nu}\right) \left[(1+\delta) z_1 \left(\frac{L_1}{L^*}\right)^\delta - z_n \right]$$

which we can write as

$$c_1 - c_n = \left(\frac{\nu}{1+\nu}\right) \left[\underbrace{z_1 \left(\frac{L_1}{L^*}\right)^\delta - z_n}_{A} + \underbrace{\delta z_1 \left(\frac{L_1}{L^*}\right)^\delta}_{B} \right]$$

A is the TFP differential of locations 1 and n at the optimal allocation and B is the impact on output of existing residents at location 1 from a marginal increase in the population in location 1 due to the externality.