

Intermediate Micro (Econ 311)

Cost Function

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Profit maximization and cost minimization

- Firm's profits as a function of inputs,

$$\Pi(L, K) = pF(L, K) - wL - rK.$$

- Profit maximization would then be a question of picking L and K so as to maximize this expression.
- Alternatively, state the profit maximization problem as a question of picking optimal scale of production, Q , to maximize profits,

$$\Pi(Q) = pQ - C(Q),$$

where $C(Q)$ is the cheapest possible way of producing output Q . This is the cost function.

The cost function

- Define the cost function $C(Q)$ as the cheapest way of producing output level Q ,

$$C(Q) = \min_{L,K} [wL + rK]$$
$$s.t. : F(L, K) = Q.$$

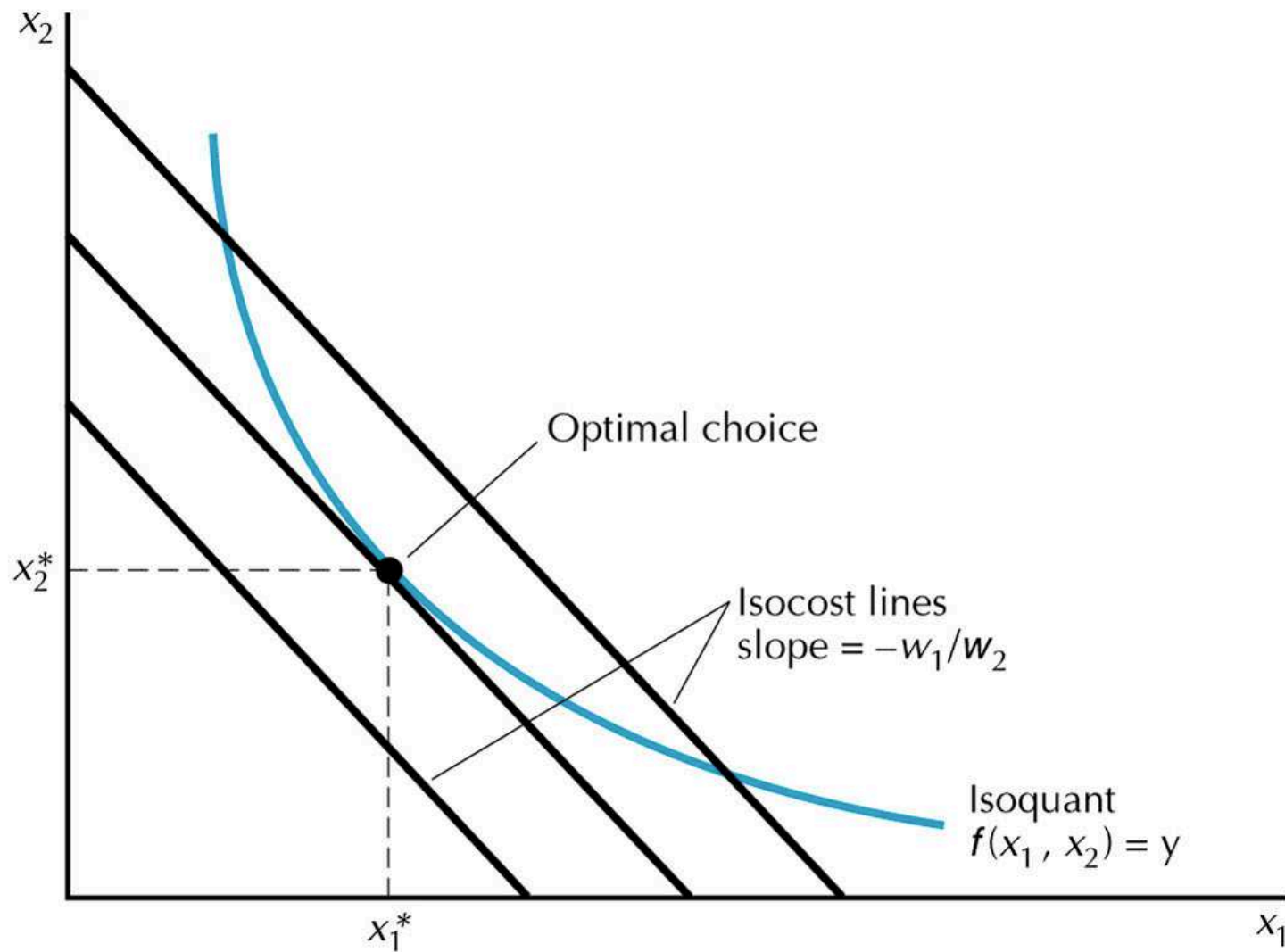
- Among the input combinations (L, K) that are on the Q -isoquant, find the cheapest one.
- By now, it should not be surprising that we can illustrate the first order condition for the optimal choice of (L, K) as a tangency point. In this case it is the tangency between our isoquant and **isocost** curves.
- Define an isocost curve as input bundles (L, K) that all cost the same,

$$\bar{c} = wL + rK$$
$$\Updownarrow$$
$$K = \frac{\bar{c}}{r} - \frac{w}{r}L.$$

- Has slope $-w/r$.

Cost minimization

Translation: $x_1 = L$, $x_2 = K$, $w_1 = w$, $w_2 = r$.



Cost minimization - first order condition

- Hence, the equations that characterize the optimal input choice, (L^*, K^*) subject to the constraint of producing output level Q are,

$$\begin{aligned}MRTS(L^*, K^*) &= \frac{MP_L(L^*, K^*)}{MP_K(L^*, K^*)} = \frac{w}{r} \\ F(L^*, K^*) &= Q.\end{aligned}$$

- The cost function is,

$$C(Q) = wL^* + rK^*.$$

- Slope of isoquant is $-MRTS = dK/dL$ conditional on keeping output fixed.

$$dQ = 0 = MP_L \times dL + MP_K \times dK \Rightarrow \frac{dK}{dL} = -\frac{MP_L}{MP_K}.$$

Cost minimization - first order condition

- Tangency condition can also be written as:

$$\frac{MP_L(L^*, K^*)}{w} = \frac{MP_K(L^*, K^*)}{r}.$$

- Another dollar spent on labor buys $1/w$ units of labor. Each additional unit of labor adds MP_L to production. Therefore, an additional dollar spent on labor adds MP_L/w to production.
- Similar argument says that another dollar spent on capital adds MP_K/r to production.
- Necessary condition for cost minimization: Cannot shift expenditure from either input to the other and thereby increase production. Achieved at tangency condition.

Marginal and Average cost

- Marginal cost: Additional input cost from increasing quantity produced.
 - Simply the derivative of the cost function, $MC(Q) = C'(Q)$.
- Average cost: Average cost per unit of output produced,

$$AC(Q) = \frac{C(Q)}{Q}.$$

Short and long run cost functions: Example

- Hannah and Sam run a house renovation business using labor and capital. Output is renovated sq feet.
- $Q = F(L, K) = 10\sqrt{L}\sqrt{K}$.
- Prices, $(W, R) = (1000, 250)$.
- In long run both inputs variable. In short run, only labor variable. Capital fixed.
- Constant returns in long run.

Hannah and Sam: Long Run Cost

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$$F(L^*, K^*) = Q \Leftrightarrow 10\sqrt{L^*}\sqrt{K^*} = Q \Leftrightarrow L^*K^* = \left(\frac{Q}{10}\right)^2.$$

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- Take $K^* = \frac{4}{L^*}$ and substitute into isoquant,

$$4L^*L^* = \left(\frac{Q}{10}\right)^2 \Leftrightarrow L^* = \frac{Q}{20},$$

which then gives

$$K^* = 4L^* \Leftrightarrow K^* = \frac{Q}{5}.$$

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- $C(Q) = 1000L^* + 250K^* = 50Q + 50Q = 100Q.$

Hannah and Sam: Short Run Cost

- Short run cost function $C_{SR}^{\hat{Q}}(Q)$: Cost of producing Q units of output with a fixed capital stock that is optimally chosen with the long run aim of $\hat{Q}$ units of output, that is, $K = \hat{Q}/5$.

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- Thus, for a given long run production aim of $\hat{Q}$, the short run production function is variable only in labor and has capital $K = \hat{Q}/5$,

$$F_{SR}^{\hat{Q}}(L) = 10\sqrt{L}\sqrt{\hat{Q}/5} = 2\sqrt{5}\sqrt{L}\sqrt{\hat{Q}}.$$

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- In the short run, a production of Q units is then obtained with the labor input, $Q = F_{SR}^{\hat{Q}}(L)$,

$$Q^2 = 4 \times 5L\hat{Q}$$

$$\Updownarrow$$

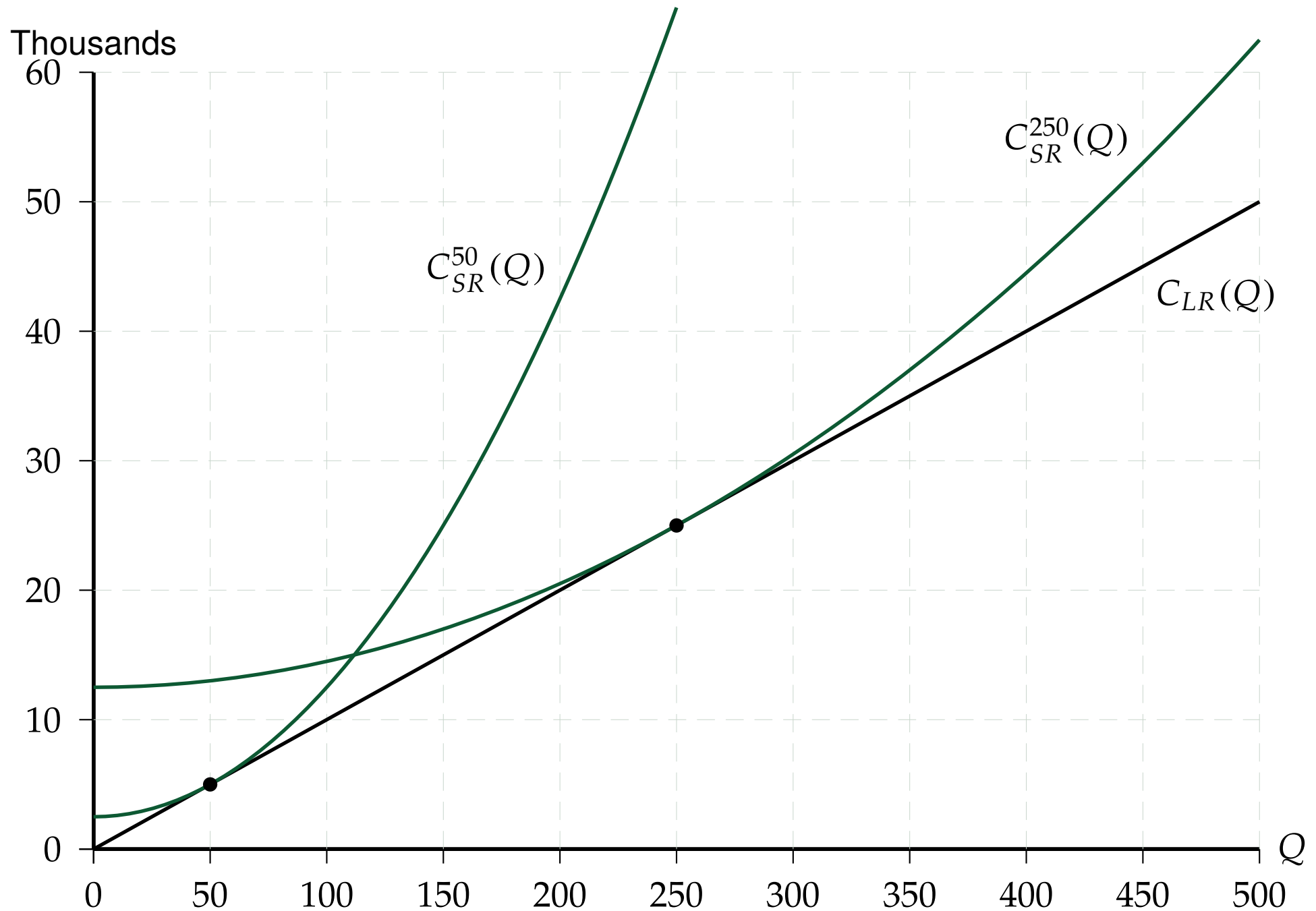
$$L_{SR}^* = \frac{Q^2}{20\hat{Q}}.$$

Hannah and Sam: Short Run Cost

- Thus, putting together the labor and capital costs for the short run production, $(L_{SR}^*, K^*) = (Q^2/(20\hat{Q}), \hat{Q}/5)$,

$$C_{SR}^{\hat{Q}}(Q) = 1000 \frac{Q^2}{20\hat{Q}} + 250 \frac{\hat{Q}}{5} = 50 \frac{Q^2}{\hat{Q}} + 50\hat{Q} = 50 \frac{Q^2 + \hat{Q}^2}{\hat{Q}}.$$

Short and long run cost functions: Hannah and Sam



Hannah and Sam: Short and long run MC and AC

- Short run marginal cost,

$$MC_{SR}^{\hat{Q}}(Q) = \frac{d}{dQ} \left(50 \frac{Q^2 + \hat{Q}^2}{\hat{Q}} \right) = \frac{100}{\hat{Q}} Q.$$

- Short run average cost,

$$AC_{SR}^{\hat{Q}}(Q) = 50 \frac{Q^2 + \hat{Q}^2}{Q\hat{Q}}.$$

- Long run marginal cost,

$$MC_{LR}(Q) = C'(Q) = 100.$$

- Long run average cost,

$$AC_{LR}(Q) = 100.$$

Hannah and Sam: Short and long run MC and AC

