LONG-TERM PARTNERSHIP FORMATION: MARRIAGE AND EMPLOYMENT*

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Of course, marriage and employment are different. Nevertheless, a worker looking for a job, a firm looking for worker, or a single person looking for a marriage partner face similar problems as all are seeking a long-term partner. Indeed, forming long-term partnerships is a common occurrence in life. There are many other examples – business people search for other business people to form a profitable relationship, bridge players seek to find a suitable partners, students search for a good university, we would all like to find a good friend, etc. The problem becomes significant if there are substantial differences in the return obtained from forming a partnership with different partners. For example, employers differ in the wages they offer, or in the work environment they provide. In such a situation a worker may reject some job offers. Similarly, as many have learned to their cost, some make better marriage partners than others.

The problem is two-sided. While a worker is evaluating a potential employer, the employer is also evaluating the worker. It is this two-sided aspect of the problem that generates a significant interest. A worker's willingness to accept employment at a firm depends not only on the characteristics of the firm but also the other possible options open to the worker. The better an individual's opportunities elsewhere, the more selective he or she will be in evaluating a potential partnership. An academic who believes Harvard may make an offer in the near future, will be more selective in evaluating offers from lesser universities. In this way expectations play a role. If a single man believes that few, if any, women will find him an acceptable marriage partner, then he may accept the first opportunity that presents itself.

Partnership formation, typically, does not comply with a classic market situation, where all participants know everything and all trades take place at zero cost. Finding a job, finding a husband or wife, or finding a business partner is a time consuming activity where opportunities typically arrive over time at uncertain intervals of time. Of course, we can act in ways that influence the arrival rate of potential partner. Workers go to employment agencies, or read help wanted advertisements in newspapers, singles of a certain age go to discos, or join tennis clubs.

The literature on search and matching (SM) (see Mortensen (1982), and Pissarides (1990) for early examples)[1] provides an excellent framework for

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[1] Diamond (1971) and Albrecht and Axell (1984) provide early examples of closely related literature on equilibrium search.

analysing long-term partnership formation. In this literature agents only face trading opportunities from time to time. Anybody contacted is viewed as a random daw from the set of potential partners. This seems a reasonable approximation of what happens in many partnership formation situations.

In the vast majority of SM studies, however, participants are assumed to be the same. For example, in most labour market studies homogenous worker and homogenous employers are typically assumed to participate the markets considered (see, for example, Pissarides (1990)). In this case a worker, for example, will always accept (or, in the trivial case, always reject) job offers; there is no real matching problem in the commonly understood meaning of the term.

In the last few years, a literature has developed which focuses on issues raised due to heterogeneity of agents. Following the pioneering work of Becker (1973), who considered frictionless markets, this literature has centred on highly stylised marriage markets (see, Smith (1993), Bloch and Ryder (1994), Burdett and Coles (1994) for early examples). These studies are somewhat technical, which has made them inaccessible to many. A major goal of this study is to show how this framework when there is heterogeneity can be used to add new and important insights into labour economics. To achieve this goal, simple examples of heterogeneous partnership formation are explored in some detail.

It should be noted that the 'decision theory' aspect of the problem will be kept simple. Deciding who to marry, or choosing a job is a highly complicated problem with much uncertainty. Here, most of this complexity (and practically all of the uncertainty) will be ignored. The objective is not to construct a realistic model of marriage or job search but to make some preliminary steps towards understanding the equilibrium process. Particular attention is paid to the patterns formed by equilibria in the sense of how the participants sort themselves out by matching. This problem has had a relatively long history in biology and sociology but was introduced to economists by Gronau (1970) and Becker (1973).[2]

The concept of assortative matching has been much used in this literature. Positive assortative matching is said to hold if the traits of those that match are positively correlated, whereas negative assortative matching holds if they are negatively correlated. It is now well known, for example, that intelligence, height, age, education, family background, etc. are positively correlated among married couples. With other traits such as dominance we might observe negative assortative matching – the highly dominant marry less dominant individuals on average. Becker (1973) stressed that if the idea that if a trait (such as intelligence) are complements in marriage, then positive assortative matching should hold, whereas if a trait (such as the inclination to succor) are substitutes in marriage, then negative assortative matching should hold. A

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[2] Earlier examples of sorting in a labour market context are provided by Black (1926) and Hicks (1948).

major element of this survey is to demonstrate within a context of matching models when negative or positive assortative matching is predicted to hold.

We start by discussing the basic framework used throughout.

1. The Basic Framework

Market frictions are perhaps the defining characteristic of the SM literature -. traders face a cost in contacting other trading partners. Recognising this friction yields significantly different equilibrium consequences than in the standard competitive model. For example, within the standard framework it is assumed traders can freely contact others. Hence, one seller charging a higher price than another cannot exist in equilibrium. But if there is a cost to moving from one trader to another, the law of one price may not hold. In what follows it will be assumed that finding trading partner always involves a time cost.

Of course, there are lots of ways market frictions can be modelled. Nevertheless, most have followed those used in the pioneering works of Diamond (1981), Mortensen (1980) and Pissarides (1984). Four restrictions have become standard.

(A) *Bilateral Meetings.* Most, if not all, studies have assumed that any particular meeting involves only two agents. Although it is possible to envisage environments where more than two meet, this restriction will be used throughout this study.

(B) *Poisson Arrival Rates.* Throughout, time is assumed to be a continuous variable. Although some studies in the SM literature have utilised discrete time models, in recent years practically all studies have considered continuous time models as this leads to a sharper analysis and simpler empirical specification.

Contacts occur according to a Poisson process with parameter α. Let $A(n, \Delta)$ be the probability an individual contacts n traders over time period Δ. Then the expected number of contacts is $\alpha\Delta$, and

$$A(n, \Delta) = \frac{(\alpha\Delta)^n e^{-\alpha\Delta}}{n!},$$

which is the Poisson density function. Let $o(\Delta)$ indicate the probability more than one trader is contacted in interval Δ. An important property of this formulation is that $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$.

(C) *Random Matching.* If a single person contacts another, it is assumed the trader encountered is the realisation of a random draw from the set of all possible contacts. For example, if $G(.)$ denotes the distribution of wage offers in a labour market, then $G(z)$ indicates the probability an unemployed worker obtains a wage offer no great than z, given an offer is received.

(D) *The Encounter Function.* Deep in the heart of all SM models is what is termed here an encounter function which relates the number of encounters per unit of time as a function of the number of participating agents. In a labour market context, $M(u, v)$ indicates the number of encounters between unemployed workers and employers posting a vacancy as a function of the

stock of unemployed (u) and vacancies (v). This function is assumed to be continuous and increasing in both its arguments.

This setup is slightly different from most studies in this area. Most define a matching function which specifies, in the labour market context, the number of hires given the stocks v and u. In a world where all are homogeneous and therefore all encounters lead to a match (in the non-trivial case) there is no difference between the two functions. However, this is not the case when agents are heterogenous as some encounters may not lead to a match.

The above four restrictions which play a central role in the SM literature will be used throughout this survey. Of course, the SM literature is huge and the objective here is not to discuss it all. Indeed, we limit the review further by imposing 3 additional restrictions.

(E) *Long-term Partnerships.* Those participating in the market wish to form a long-term relationship with another agent. Hence, in the two primary examples used here workers want to find a good employer and employers seek workers to employ in labour markets, whereas single men and women seek marriage partners in marriage markets.

This restriction rules out a large number studies using the SM framework. For example, in the multitude of studies on money using the framework developed by Kiyotaki and Wright (1989), and the large number of trading models using the framework developed by Diamond (1981), the objective of individuals is to find another to trade with. After trade they separate and return to the market. Such trading behaviour is not considered here.

(F) *No Learning.* When an individual contacts another they both observe the payoff to the match. Of course, this restriction is infrequently satisfied. However, given all is known about a partner when a match is formed, there are few reasons for wanting to separate later on.

(G) *Steady-State.* There is a small but significant literature on non-steady-state dynamics in the search and matching literature (see, for example, Diamond and Fudenberg (1989), Mortensen and Pissarides (1998), Smith (1997a), and Burdett and Coles (1996, 1998)). Nevertheless, following most studies in this area, we shall only identify steady-state solutions.

Given the matching framework defined by restrictions (A)–(G), we shall focus on two particular issues.

(1) *Heterogeneous Agents.* In most studies on SM it has been assumed that agents (of the same group) are homogenous. Indeed, when models with heterogeneity have been considered, practically all have analysed markets where there is, what is termed here, *match heterogeneity*. In a labour market context, match heterogeneity implies that all workers are essentially the same, as are all employers, but some worker-employer pairs are more productive that others. In that case, all unemployed workers face the same chances that they will find a good match with an employer as do all employers with a vacancy.

But here we shall also consider a second type of heterogeneity which we refer to as *ex-ante heterogeneity*. In this world, some workers are more productive than others while some firms have more productive capital. Ex-ante heterogeneity here implies that all agree on who makes a better partner than

another, i.e. all have the same ranking of potential partners. A major element of this review is to show how the results developed in this rather technical literature can be readily applied to the traditional labour market models of SM.

(2) *Transferable Utility and Non-Transferable Utility*

Suppose a worker more productive than most and an employer contact each other. The no learning restriction implies both realise what they would obtain if they form a partnership. Further, both are cognisant that if they do not form a partnership they will experience costly search. Also assume a union wage determines the wage paid if he or she is hired, which also determines the revenue accruing to the employer. Suppose they accept this split when deciding if they should form a partnership, or not. This is an example of *non-transferable utility*. With non-transferable utility there is a 'natural' split of the total payoff generated by forming a match and both parties accept this division. Of course, the skilled worker in the example above may attempt to obtain more than the union wage by bargaining with the employer (perhaps negotiating longer holidays, for example). Further, in many other situations there is not a 'natural' split of the payoff generated if they form a match and thus it must be negotiated in some way. Indeed, even if there is a 'natural' split of the total payoff, the potential partners may still negotiate another division. If they do, it will be said there is *transferable utility*, and following the standard matching approach, we shall assume Nash bargaining determines the terms of trade.[3]

In what follows, we shall consider equilibrium matching for 4 separate cases. In Section 3 we consider matching when there is ex-ante heterogeneity and depending on whether (i) utility is non-transferable, or (ii) utility is transferable and the terms of trade are determined by Nash bargaining. In Section 4 we characterise matching equilibria when there is match specific heterogeneity and depending on whether utility is transferable or not. However, first we review the optimal search problem and Section 2 shows how to identify market equilibrium.

1.1. *Single-Sided Search*

At the heart of SM models is a single-sided search problem faced by those on both sides of the market. The objective here is to review this problem briefly. The essential result obtained is that faced with a sequential search problem, a searcher utilises a cut-off rule; accepting offers if and only if the utility obtained is at least as great as this cut-off.

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[3] There is a literature on wage posting that is ignored here. In this literature (see, for example, Albrecht and Axell (1984) and Burdett and Mortensen (1998)), firms post wages to transfer utility from matches to their workers.

To illustrate essentials we analyse a highly stylised marriage market and consider the problems faced by Mr J finding a wife. Let α_J denote the rate Mr J receives offers of marriage from women, where α_J is the parameter of a Poisson process. If such a woman is encountered, he can tell instantly the utility, x, per unit of time he obtains if they marry. As all are assumed to live forever and divorce is banned, Mr J's discounted lifetime utility in this case is x/r, where r is the discount rate. Of course, some women would yield different utility than others. Mr J obtains utility b per unit of time while single.

Let V_J denote Mr. J's maximum expected discounted lifetime when single. Over the next short interval of time Δ, he obtains utility $b\Delta$. Further, over this time interval, $\alpha_J \Delta$ denotes the probability he contacts a woman willing to marry him. Given he meets a woman willing to marry him, let $F_J(x)$ denote the probability he meets a woman who yields no more than x per unit of time if they marry. Clearly, he will marry the woman contacted if and only if the utility obtained is at least as great as V_J. It follows that

$$V_J = \frac{1}{1 + r\Delta} \left[b_J \Delta + (1 - \alpha_J \Delta) V_J + \alpha_J \Delta E_J \max \left(V_J, \frac{x}{r} \right) \right] + o(\Delta) \qquad (1)$$

where E_J is the expectations operator given x has distribution F_J. The $o(\Delta)$ term captures what happens if more than one single is contacted.

It follows from (1) that Mr. J's optimal strategy has the reservation payoff property – accept all offers $x \geqslant R_J$ where $R_J = rV_J$. Using this fact, substituting out $V_J = R_J/r$ in (1), rearranging and dividing by Δ, and then letting $\Delta \to 0$ we obtain the standard reservation equation

$$R_J = b_J + \frac{\alpha_J}{r} \int_{R_J}^{\bar{x}} (x - R_J) \, dF_J(x). \qquad (2)$$

This implies that the flow value of search $(rV_J = R_J)$ equals b_J plus the expected surplus generated by the optimal search strategy R_J. Integrating by parts implies (2) can also be written as

$$R_J = b_J + \frac{\alpha_J \varphi_J(R_J)}{r}, \qquad (3)$$

where

$$\varphi_J(R_J) = \int_{R_J}^{\bar{x}} [1 - F_J(x)] \, dx = \int_{R_J}^{\bar{x}} (x - R_J) \, dF_J(x).$$

As φ_J is decreasing in R_J, (3) implies a unique solution for R_J given α_J and F_J. Equation (3) describes the best strategy of Mr. J given his beliefs about (a) the arrival rate of offers, α_J, and (b) the distribution of offers from women who wish to marry him, F_J.

Note, at this stylised level, we can interpret the model as one of Mr. J looking for a job. Assuming α_J now indicates the rate at which Mr. J receives job offers while unemployed, F_J as the distribution of wages offered by employers willing to hire him, and b_J as Mr. J's flow utility while unemployed,

we have the classic job search model. In this case we can interpret R_J in (3) as the reservation wage of Mr J. He will accept the first offer received with a wage at least as great as R_J.

The above model has turned out to be remarkably robust. As the literature on search demonstrates, by adding all sorts of realistic complexities to the above model the basic analysis remains essentially the same – searchers use a reservation number, R, and will only accept offers if it is at least as great as R.

2. Two Sided Search

In the Section above we determined the best strategy of Mr. J, given the arrival rate of offers, α_J, and the distribution of utility payoffs associated with those offers, F_J. Clearly, this can be done for all single men. Of course, the reservation utility levels used by different men may differ as they may face different arrival rates of offers and different distributions. Nevertheless, knowing their reservation utility levels, we can calculate the set of men willing to marry any woman in this marriage market. Further, a simple change of notation implies we can determine the reservation utility level, R_K, of any particular woman, Ms. K, as a function of the arrival rate of offers by men, α_K, and the distribution of utilities associated with those offers, F_K. Equilibrium in this marriage market, given (and this is a big given) things remain in steady state, requires that the reservation utility strategies of men (women) defines the arrival rate of offers and the associated distribution of offers faced by women (men).

To characterise an equilibrium we break the general problem down into two smaller problems. First, assume the market is in steady state, where the number and distribution of types on both sides of the market remain constant through time, which is taken as given by all in the market. Given the assumed steady state number and distribution of types, we consider a Nash equilibrium in proposal strategies where an individual's proposal strategy specifies who she or he is willing to marry given they make contact. Obviously a Nash equilibrium requires that all use utility maximising proposal strategies, given the strategies of everyone else.

Given the assumed steady state number and distribution of types and the resulting Nash equilibrium, we can then calculate the number and types of both women and men who marry per unit of time and hence leave the market. Of course this can only describe a steady state if this outflow is balanced by an equal inflow. Closing the model therefore requires describing the flow of new singles into the market. Below we describe 4 typical cases. Whichever is the assumed inflow, a steady state can only exist if the outflow generated by the above Nash equilibrium maintains the steady state; i.e. equals the number and type flowing in. If such a happy coincidence exists, we define it as a Market Equilibrium.

Identifying a Market Equilibrium therefore involves solving two problems:

(NE) Given a steady state number and distribution of types in the market, a Nash Equilibrium describes who is willing to match with whom (and also the

equilibrium terms of trade). Nash Equilibrium requires that all are using optimal search (and bargaining) strategies.

(ME) A Market Equilibrium requires finding a steady state number and distribution of types in the market so that the corresponding Nash Equilibrium defined above generates an exit flow for each type equal to the entry rate of that type.

Notice that we do not consider non steady state dynamics, and so do not consider whether any particular steady state is stable or otherwise.

2.1. *Possible Inflow Specifications*

Clearly we cannot solve for a Market Equilibrium until we have described how new singles enter the market over time. Typically, 4 cases have been considered in the literature. The simplest is the Clone Restriction.

Clones. If two partners match and leave the market, two identical singles enter the market.

This restriction is possibly the least reasonable case but is also the most tractable. By closing down the steady state problem – the distribution of types is unaffected by the proposal strategies of agents as those who pair off are immediately replaced – characterising a market equilibrium is reduced to characterising (NE). A more reasonable approach is

Exogenous Inflows. $g\Delta$ new singles enter the market per period Δ, where the productivity/fitness/charm x of an entrant has (exogenous) distribution $H(x)$.

Assuming exogenous inflows is perhaps more reasonable but complicates the framework by introducing a double infinity of agents – there is a continuum of agents in the market at any point in time, and over time, a second infinity of agents passes through the market. A (mathematically) simpler approach is to assume turnover is generated by an exogenous separation rate.

Exogenous Separations. There is no entry of new singles. However partnerships are destroyed at some exogenous rate $\delta > 0$ whereupon both return to the singles market.

By removing the second infinity problem, this approach implies a general existence proof of market equilibrium can be given (see Shimer and Smith (1997), Smith (1997*b*) for example).

Endogenous Entry. Singles on one side of the market enter until the expected return to entry is zero.

Endogenous entry is typically used in the standard matching framework to determine the number of vacancies in the market (e.g. Pissarides (1990)).

3. **Ex-Ante Heterogeneity**

3.1. *Non-Transferable Utility*

We start by considering the simplest case which illustrates the relevant issues. Suppose there are equal numbers of single men and single women, N, who participate in a marriage market. Constant returns to scale of the encounter

function implies $\alpha = M(N, N)/N = M(1, 1)$ describes the rate at which each contacts singles of the opposite sex and α does not depend on N.

Keeping things as simple as possible, suppose there two types – goods and bads. The payoff to marrying a good is x_G/r, whereas the payoff to marrying a bad is x_B/r where $x_G > x_B > 0$. No bargaining is allowed and there is no divorce. Further, assume while single both sexes obtain zero utility flow (i.e., $b_w = b_m = 0$).

If a single man and woman meet, both observe the other's type. If both agree, they marry and leave the market for good. If however at least one does not agree, they separate and continue to look for a suitable partner. Assume the proportion of both men and of women who are good is λ.

In order to characterise a market equilibrium, we first solve for the Nash Equilibrium in proposal strategies (given a steady state).

(NE) *The Nash Equilibrium*

For now, assume a steady state exists where N and λ are given. Clearly, in any equilibrium, all are willing to marry a good – they can do no better. The central issue is whether a good is willing to marry a bad.

Let V_G denote the value of being an unmatched good of either sex. As all will propose to a good, then putting $J = G$ in (1) and noting $\alpha_G = \alpha$ implies

$$rV_G = \alpha\lambda\left(\frac{x_G}{r} - V_G\right) + \alpha(1 - \lambda)\left[\max\left(V_G, \frac{x_B}{r}\right) - V_G\right]. \qquad (4)$$

With probability λ any contact is with another good and they marry. However, with probability $(1 - \lambda)$ a bad is contacted and the optimal matching decision depends on whether x_B/r exceeds V_G, or not. After some manipulation it follows that (4) can be expressed as:

$$V_G = \begin{cases} \dfrac{\alpha}{\alpha + r}\left[\dfrac{\lambda x_G}{r} + \dfrac{(1 - \lambda)x_B}{r}\right], & \text{if } \alpha\lambda \leqslant \dfrac{rx_B}{x_G - x_B} \\[3mm] \dfrac{\alpha\lambda}{\alpha\lambda + r}\left(\dfrac{x_G}{r}\right) & \text{if } \alpha\lambda > \dfrac{rx_B}{x_G - x_B} \end{cases} \qquad (5)$$

where $V_G = x_B/r$ when $\alpha\lambda = rx_B/(x_G - x_B)$. Note, $\alpha\lambda$ describes the rate at which each good meets (and marries) other good. Holding α fixed, if λ is small (i.e., there are few goods) goods will marry bads if they make contact. However, if λ is large enough, goods reject bads.

If goods reject bads, the only option for bad is to marry another bad. In that case, the arrival rate of proposals faced by a bad partner is $\alpha_B = \alpha(1 - \lambda)$. On the other hand, suppose goods are willing to marry bads, i.e., $V_G \leqslant x_B/r$. As all use a reservation rule, it follows that goods receive at least as many offers as bads and so $V_G \geqslant V_B$. Hence $V_B \leqslant x_B/r$ and therefore bads will also marry each other. In this case, all marry the first person of the opposite sex they encounter and $\alpha_B = \alpha$. Using these facts the expected payoff to a bad of either sex can be written as

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V_B = \begin{cases} \dfrac{\alpha}{\alpha + r} \left[\dfrac{\lambda x_G}{r} + \dfrac{(1-\lambda) x_B}{r} \right] & \text{if } \alpha\lambda \leqslant \dfrac{r x_B}{x_G - x_B} \\[3ex] \dfrac{\alpha(1-\lambda)}{\alpha(1-\lambda) + r} \left(\dfrac{x_B}{r} \right) & \text{if } \alpha\lambda > \dfrac{r x_B}{x_G - x_B} \end{cases}
$$

Given the above analysis, it is straightforward to specify Nash equilibria. If λ is small enough $(\lambda \leqslant r x_B / [\alpha(x_G - x_B)])$, there is a *mixing Nash equilibrium* where all marry the first person of the opposite sex they meet and all obtain the same expected payoff $V_B = V_G < x_B/r$. However, if λ is large enough $(\lambda > r x_B / [\alpha(x_G - x_B)])$, there is an *elitist Nash equilibrium* where goods reject bads. The equilibrium payoffs in this case satisfy $V_G > x_B/r > V_B$. At the boundary between the two equilibria, bads strictly prefer the mixing equilibrium – they are worse off when goods reject them.

These Nash equilibria imply that given any steady state values (λ, N), we know who will marry who and can compute the exit rates of good and bads. We now turn to problem (ME) to determine those values of λ, N where the corresponding Nash equilibrium matching strategies imply a steady state.

(ME) *The Market Equilibria*

We assume there are exogenous inflows – the flow in of single men (and single women) is $g\Delta$ per interval Δ, where proportion π of those singles are goods. In a steady state, the exit rate of each type must equal the entry rate of new singles of that type.

(a) *A Mixing Market Equilibrium*

Consider the mixing Nash equilibrium as described above. As there are λN goods in a steady state, where each marries the first person of the opposite sex contacted in a mixing equilibrium, the implied exit flow of goods is $\alpha\lambda N$, whereas the exit flow of bads is $\alpha(1-\lambda)N$. A mixing Market Equilibrium requires that this exit flow equals the exogenous inflow. Hence we must have $\pi g = \lambda N \alpha$, and $(1-\pi)g = (1-\lambda)N\alpha$. This implies $\lambda = \pi$ and $N = g/\alpha$. Of course, this is consistent with a mixing Nash equilibrium if and only if $\pi \leqslant r x_B / [\alpha(x_G - x_B)]$.

(b) *An Elitist Market Equilibrium*

As before there are λN goods in a steady state, but an elitist Nash equilibrium implies these now match at rate $\alpha\lambda$ as goods only marry each other. The exit flow of goods is therefore $\alpha\lambda^2 N$. The steady state number of bads is $(1-\lambda)N$. As bads marry at rate $\alpha(1-\lambda)$, the exit flow of bads is $\alpha(1-\lambda)^2 N$. An elitist Market Equilibrium requires that (λ, N) satisfy $\pi g = \lambda N \alpha\lambda$, and $(1-\pi)g = (1-\lambda)N\alpha(1-\lambda)$. In this case the steady state proportion of goods, denoted $\bar{\lambda}(\pi)$, is defined by

$$\frac{\overline{\lambda}(\pi)^2}{\overline{\lambda}(\pi)^2 + [1 - \overline{\lambda}(\pi)]^2} = \pi.$$

Note, $\overline{\lambda}(0) = 0$, $\overline{\lambda}$ is strictly increasing in π where $\overline{\lambda}(\pi) > \pi$ if $\pi \in (0, 0.5)$ and $\overline{\lambda}(\pi) < \pi$ if $\pi \in (0.5, 1)$. The intuition is that if π is small and equilibrium is elitist, then the exit rate of each good partner is less than the exit rate of each bad. This implies that the number of goods builds up relative to the number of bads and in a steady state $\overline{\lambda}(\pi) > \pi$. Of course, this is consistent with an elitist Nash equilibrium if and only if $\overline{\lambda}(\pi) > r x_B / [\alpha(x_G - x_B)]$.

Note, if $\pi \in (0, 0.5)$ and $\pi \leqslant r x_B / [\alpha(x_G - x_B)] < \overline{\lambda}(\pi)$, there are multiple equilibria in that both an elitist and a mixing market equilibrium exist. This multiple equilibria is generated by a sorting externality that is discussed in Section 3.4. As this externality is quite general, we shall first review the literature on ex-ante heterogeneity before discussing this externality at the end.

3.2. *The General Case – Non-Transferable Utility*

Suppose that associated with each man is a number, x_m – his charm. A woman's utility from a marriage is a strictly increasing function of the man's charm. Let G_m denote the distribution of charm among all men in the market, i.e., $G_m(z)$ is the probability that a randomly selected man's charm is no greater than z. Similarly let x_w denote the charm associated with a given woman, and G_w denote the market distribution of charm among single women. The only restriction required on these two distribution functions is that they have finite support.

Not all men may be willing to marry a woman of charm x_w. Let $\mu_m(x_w)$ denote the proportion of men who will marry a woman with charm x_w, if they make contact, and let $F_m(.|x_w)$ indicate the distribution of charm among these men. Hence, if $\hat{\alpha}_w = M(N_m, N_w)/N_w$ denotes the rate at which each woman encounters single men, then $\alpha_w = \hat{\alpha}_w \mu_m(x_w)$ is the rate at which this woman receives offers.

Suppose $u_w(x_w, x_m)/r$ describes this woman's expected lifetime utility if she marries a man with charm x_m, and assume u_w is increasing in x_m. Letting $V_w(x_w)$ denote the expected discounted lifetime utility of this woman, it follows

$$V_w(x_w) =$$

$$\frac{1}{1 + r\Delta} \left[b_w \Delta + (1 - \alpha_w \Delta) V_w(x_w) + \alpha_w \Delta E_w(x_w) \frac{u_w(x_w, \tilde{x}_m)}{r} \right] + o(\Delta) \quad (6)$$

where $\tilde{x}_m$ is distributed according to $F_m(.|x_w)$.

As before, the optimal strategy of a woman is to use a reservation match strategy, $R_w(x_w)$. The reservation match is defined by $u_w(x_w, R_w(x_w)) = r V_w(x_w)$. Rearranging and substituting out $V_w(x_w)$ implies

$$u_w(x_w, R_w(x_w)) = b_w + \frac{\hat{\alpha}_w \mu_m(x_w)}{r} \int_R \overline{w}(x_w) x [u_w(x_w, \tilde{x})$$

$$- u_w(x_w, R_w(x_w))] \, dF_m(\tilde{x}|x_w). \tag{7}$$

A woman's reservation match depends on her own charm as this affects the arrival rate of proposals and her own utility through being married. Of course, the argument is symmetric and the reservation match of a man of charm x_m, $R_m(x_m)$ is given by

$$u_m(x_m, R_m(x_m)) = b_m + \frac{\hat{\alpha}_m \mu_w(x_m)}{r} \int_R \overline{m}(x_m) x [u_m(x_m, \tilde{x})$$

$$- u_m(x_m, R_m(x_m))] \, dF_w(\tilde{x}|x_m). \tag{8}$$

Describing an equilibrium requires finding a pair of functions $\{R_m(.), R_w(.)\}$ which satisfy (7), (8) where these reservation match strategies define the flow of proposals received by singles of the opposite sex.

This general case is obviously somewhat complicated. Nevertheless, there are only two central forces of interest. As all men use reservation match strategies, a woman of greater charm than another will expect to receive at least the same number of proposals than the other. This implies women with greater charm are better off and hence tend to have a reservation match at least as great as those with less charm. This effect promotes positive assortative matching – people with greater charm tend to marry each other. However, if $u_w(., .)$ is increasing in x_w, there is a countervailing effect. In this case a woman of greater charm than another may be in a bigger hurry to marry as she obtains greater utility from being married to any given man. This makes her less selective and promotes negative assortative matching. In general, the reservation match $R_w(.)$ may be increasing or decreasing in x_w.

A simple case arises if we restrict attention to preferences of the form $u_w(x_w, x_m) = x_m$ and $u_m(x_w, x_m) = x_w$. In this case, an individual's payoff to marriage depends only on the charm of his or her spouse and her reservation match reduces to $R_w = rV_w(x_w)$.[4] The argument that those with greater charm receive more proposals immediately implies that reservation match strategies are increasing in own charm – positive assortative matching is guaranteed. However, this case is particularly interesting as it generates a class outcome.

Consider the most charming woman, the one with $x_w = \overline{x}_w$. As all men will propose to her (she is ideal), $\mu_w(\overline{x}_w) = 1$ and $F_m(.|\overline{x}_w) = G_m(.)$. Hence, (7) implies her reservation match $R_w = R_w(\overline{x}_w)$ is

$$R_w = b_w + \frac{\hat{\alpha}_w}{r} \int_{R_w}^{\overline{x}_m} (x - R_w) \, dG_m(x). \tag{9}$$

The analogous condition for the most charming man is

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[4] The same results hold if we assume men with charm x_m obtain flow value $b_m(x_m)$ while single, and obtain flow payoff $u = b_m(x_m) + \gamma_m(x_w)$ if married to a woman of charm x_w; i.e. narcissism is not necessarily ruled out.

$$R_m = b_m + \frac{\hat{\alpha}_m}{r} \int_{R_m}^{\bar{x}_w} (x - R_m) \, dG_w(x). \tag{10}$$

However, reservation match strategies are non-decreasing in own charm. As the most charming man is willing to propose to all women with charm $x_w \geqslant R_m$ as defined in (10), then all men are willing to propose to these women. As all women with charm $x_w \in [R_m, \bar{x}_w]$ receive the same offers they are equally selective – they use the same reservation match R_w as the most charming woman. Similarly men with charm $x_m \in [R_w, \bar{x}_m]$ have the same reservation match payoff R_m as the most charming man. These types form a class, called class 1. Class 1 women with charm $x_w \in [R_m, \bar{x}_w]$ will only marry men with charm $x_m \in [R_w, \bar{x}_m]$, and these men will only marry women with charm $x_w \in [R_m, \bar{x}_w]$. This is illustrated in Fig. 1, where $\underline{x}_j$ and $\bar{x}_j$ indicate the infimum and supremum of the support of F_j, $j = m, w$.

The next question is what is the optimal matching strategy of those not in class 1. Obviously $\hat{\alpha}_w G_m(R_w)$ is the rate at which each non-class 1 woman contacts men who are not in class 1. Similarly $\hat{\alpha}_m G_w(R_m)$ is the rate at which each non-class 1 man meets women who are not in class 1. The same matching structure holds and a second class, class 2, is generated which is defined by two intervals, $[A_w, R_w)$ and $[A_m, R_m)$, where a man with charm $x \in [A_w, R_w)$ is a member of class 2 and has reservation utility A_m (defined in an analogous way to (10)). Class 2 types reject all those who are not in class 2 (except for class 1
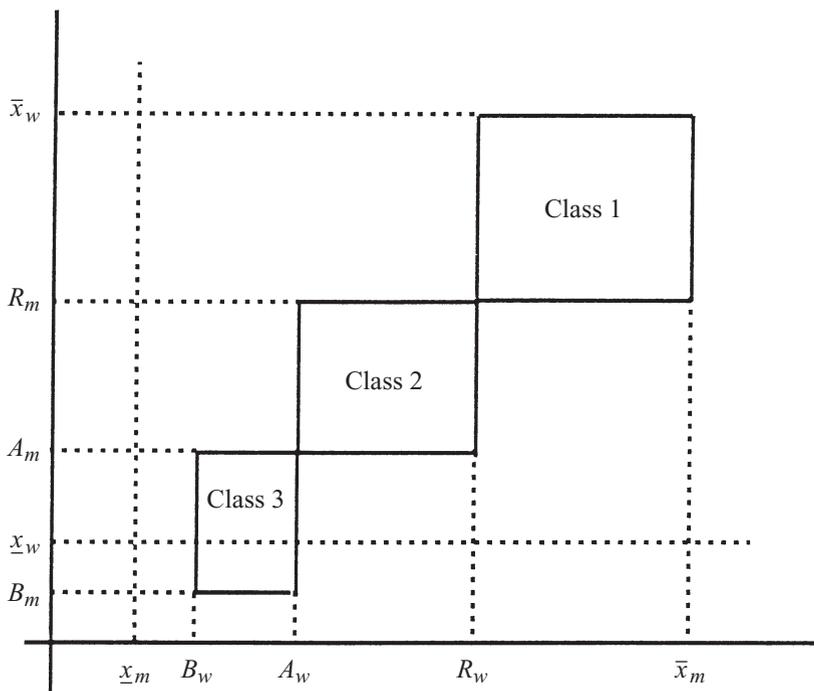

Fig. 1.

types who instead reject them). More classes can be derived in this way – and an induction argument implies a class partition (see Burdett and Coles (1997a)). In Fig. 1 we illustrate a situation where there are 3 classes. Note, in this Figure there are some men with charm $x \in [\underline{x}_m, B_w]$ who are not acceptable to any women in the market and therefore never marry. It can be shown that if the given arrival rates faced by singles is increased, this will generate more classes of smaller sizes. Further, if the given arrival rates faced by singles becomes unbounded large, the classes converge to a 'straight line', where the *nth* ranked woman marries with the *nth* ranked man, $n = 1, 2, 3 \ldots$.

This is a startling result (see, for example, Collins and McNamara (1990), Smith (1993), Bloch and Ryder (1994), Burdett and Coles (1997a), and Eeckhout (1996)). Of course, it only takes a little imagination to transform this result to a labour market context. For example, a unionised labour market may specify an industry-wide wage w. Further, suppose some employers are more desirable to work for than others – perhaps some are located at more convenient positions than others, e.g. close to a subway station, or are in more pleasant surroundings. Let x_f denote the value to a worker of the firm's location. If the productivity of an employee is denoted x_e then the payoff to a firm and employee by forming a match is

$$\pi_f(x_f, x_e) = (x_e - w)/r, \pi_e(x_e, x_f) = (x_f + w)/r.$$

These preferences correspond to the above scenario and a class partition describes the equilibrium outcome. The most attractive firms only employ workers of a certain minimum standard, the most productive workers only accept employment at firms with a certain minimum quality. Of course, changes in the union wage will change those reservation match boundaries.

Smith (1997a) shows that when $b_w = b_m = 0$, the class result holds wherever payoffs are multiplicatively separable; i.e. $u_i(x_i, x_{-i}) = \gamma_i(x_i)\eta_i(x_{-i})$. Characterising equilibrium for the general utility function case $u_i = u_i(x_i, x_{-i})$, however, is much more complicated. A basic result of interest is under what restrictions on u_i can we guarantee that those with greater charm will be more selective; i.e. that the reservation match strategies are increasing in own charm. In the sense of Becker (1973) we might consider this as positive assortative matching – that charm is positively correlated across matched pairs. Smith (1997b) assumes a partnership framework where $u_m = u_w = u(x, y)$ where x is own charm and y is partner charm. Sufficient conditions which guarantee (our) definition of positive assortative matching is that $log[u(x, y)]$ is supermodular; i.e., for all $x_1 < x_2$ and $y_1 < y_2$

$$u(x_1, y_1) u(x_2, y_2) \geqslant u(x_1, y_2) u(x_2, y_1).$$

Note the case considered above, $u(x, y) = y$, satisfies this condition exactly.

3.3. *Transferable Utility and Nash Bargaining*
Clearly, allowing transferable utility changes matching behaviour. With non-transferable utility, an individual who would obtain a large payoff to a match

with a given partner, cannot compensate that potential partner to ensure the match is consummated. Allowing transferable utility implies a price mechanism – sidepayments (such as offering a higher wage) ensure that potential matches, which are jointly efficient, are made. Remarkably, although this appears to be a natural setting, few papers have investigated this issue, perhaps because of the technical difficulty associated with analysing this problem (see Lockwood (1986), Sattinger (1995), Shimer and Smith (1997), Burdett and Coles (1998), and Delacroix (1997) for the only examples we could find).

Following Shimer and Smith (1997) we assume a partnership framework. In this case a group of individuals participate in a market. Each is unproductive on his or her own. To generate revenue, an individual must form a partnership with another (partnerships involving more than two are unproductive). The revenue generated by a partnership depends on the individuals involved. In particular, assume an individual can be described by a real number, x, let us say his or her productivity. The revenue generated if an individual with productivity x matches with another with productivity y is $Q(x, y)$. When two meet and form a partnership they divide the output according to a Nash bargain.

Shimer and Smith (1997) consider sufficient restrictions on Q which ensure positive assortative matching. Unfortunately their analysis is somewhat complex and there is no simple answer. However, we can provide some insight into the nature of matching in such markets by considering a much simpler two types example.

Let N denote the number participating in this market, and let α denote the rate at which they meet each other. There are two types of partners – good and bad. If two bads form a partnership revenue $2Q^L$ is generated, if two goods form a partnership they generate revenue $2Q^H$, whereas if a good and a bad form a match revenue $2Q^M$ results, where $Q^H > Q^M > Q^L > 0$. Assume $b = 0$ and the fraction λ of the unmatched are good.

Let V_G and V_B denote the value of being an unmatched good and bad. When two meet they recognise instantly the revenue resulting if they form a partnership. If the match is rejected they continue search, obtaining payoffs V_i, $i = B, G$. If they form a match they split the pie according to a Nash bargaining solution. Let π^{ij} denote the payoff to a partner of type i who matches with a type j. Further, we use β^{ij} to indicate whether a match forms or not; i.e. $\beta^{ij} = 1$ if an i who meets a j form a match, and $\beta^{ij} = 0$ if they do not form a match. For simplicity, mixed strategy equilibria are not considered here (though these often exist).

Assuming Nash bargaining implies the following.

(a) Two goods form a partnership (i.e., $\beta^{GG} = 1$) if and only if $Q^H \geqslant V^G$ and each obtains $\pi^{GG} = Q^H$.

(b) Two bads form partnerships (i.e., $\beta^{BB} = 1$) if and only if $Q^L \geqslant V^B$ and each obtains $\pi^{BB} = Q^L$.

(c) If a good contacts a bad, they form a partnership (i.e., $\beta^{BG} = 1$) if and only if $2Q^M \geqslant V^B + V^G$ and the negotiated payoffs are $\pi^{GB} = V^G + (2Q^M - V^G - V^B)/2$ and $\pi^{BG} = V^B + (Q^M - V^G - V^B)/2$.

The issue is to relate equilibrium matching behaviour to the properties of the production function. In particular, it seems important whether $Q^L + Q^H > 2Q^M$ or not. If this inequality is satisfied it is more efficient that goods do not match with bads. Conversely, if the inequality is reversed, it is more efficient that goods match with bads. The problem is that decentralised trade and Nash bargaining may not generate the efficient outcome.

The expected discounted lifetime utility of a good, V_G, can be written as

$$rV_G = \alpha\lambda\beta^{GG}(\pi^{GG} - V_G) + \alpha(1 - \lambda)\beta^{GB}(\pi^{GB} - V_G). \qquad (11)$$

The flow value of being an unmatched good depends on the rate at which she meets other goods and the surplus realised by such contacts (if any) and on the rate at which she meets bads. The flow value of being an unmatched bad is given by

$$rV_B = \alpha\lambda\beta^{BG}(\pi^{BG} - V_B) + \alpha(1 - \lambda)\beta^{BB}(\pi^{BB} - V_B). \qquad (12)$$

Nash bargaining implies (11) and (12) become

$$rV_G = \alpha\lambda\beta^{GG}(Q^H - V_G) + \alpha(1 - \lambda)\beta^{GB}[Q^M - (V_G + V_B)/2] \qquad (13)$$

$$rV_B = \alpha\lambda\beta^{GB}[Q^M - (V_G + V_B)/2] + \alpha(1 - \lambda)\beta^{BB}(Q^L - V_B). \qquad (14)$$

Given α, λ and Q^k, $k = L, M, H$, a Nash equilibrium (NE) requires solving for V_G, V_B defined in (13), (14), and β^{ij} given by the bargaining conditions (a)–(c) defined above.

An equilibrium can be described by the matches that form. For example, {GG, GB} describes an equilibrium where only good/good and bad/good partnerships form. However, $b = 0$ implies that in any equilibria all match at a strictly positive rate. Hence, {BB}, {GG} and $\varnothing$ cannot describe Nash equilibria. This implies there are 5 possible types of (pure strategy) equilibria.

(1) A mixing equilibrium; $\beta^{GG} = \beta^{BB} = \beta^{GB} = 1$, where matches {GG, GB, BB} form.

(2) An elitist equilibrium; $\beta^{GG} = \beta^{BB} = 1$, $\beta^{BG} = 0$, where matches {GG, BB} form.

(3) Negative Assortative Matching I; $\beta^{GG} = 0$, $\beta^{BG} = 1$, $\beta^{BB} = 1$, where matches {BG, BB} form.

(4) Negative Assortative Matching II; $\beta^{GG} = 1$, $\beta^{BG} = 1$, $\beta^{BB} = 0$, where matches {GG, BG} form.

(5) Negative Assortative Matching III; $\beta^{GG} = \beta^{BB} = 0$, $\beta^{BG} = 1$, i.e, where match {BG} forms.

The last three cases borrow the Shimer/Smith taxonomy that negative assortative matching is said to occur if a type exists which will reject others of the same type.

Characterising equilibria is a labourious process. First pick an equilibrium configuration $\{\beta^{ij}\}$ and solve (13) and (14) for V_B, V_G. Given this solution, characterise the set of parameters α, λ and Q^k so that the implied values of V_B, V_G and the bargaining equations (a)–(c) imply the assumed equilibrium

matching configuration $\{\beta^{ij}\}$. That equilibrium then exists for those parameter values.

For example, consider an elitist equilibrium where $\beta^{GG} = \beta^{BB} = 1$, $\beta^{GB} = 0$. Equations (13) and (14) imply

$$V_G = \frac{\alpha\lambda Q^H}{(r + \alpha\lambda)}, \ V_B = \frac{\alpha(1 - \lambda)Q^L}{r + \alpha(1 - \lambda)}.$$

The bargaining equations (a)–(c) imply that these value functions are consistent with an elitist equilibrium if and only if $V_G + V_B > 2Q^M$ so that there is no gain to trade between these types. This implies parameter restrictions

$$Q^L \geqslant \frac{2[\alpha(1 - \lambda) + r]}{\alpha(1 - \lambda)} Q^M - \frac{\lambda[\alpha(1 - \lambda) + r]}{(1 - \lambda)(\alpha\lambda + r)} Q^H.$$

Hence if Q^L is large enough (but $Q^L \leqslant Q^M$), an elitist matching equilibrium exists.

Repeating for each possible equilibrium, the Nash equilibria can be partitioned as shown in Figs. 2 and 3. Fig. 2 graphs the partition of Nash equilibria when $r = 0$ and $\lambda < 0.5$. This case corresponds to the frictionless case when
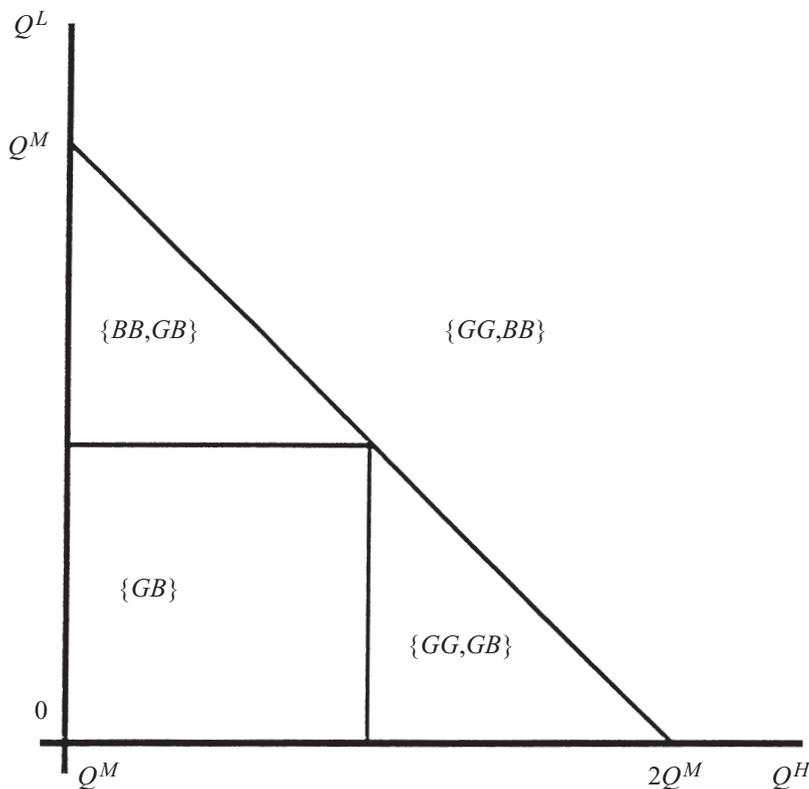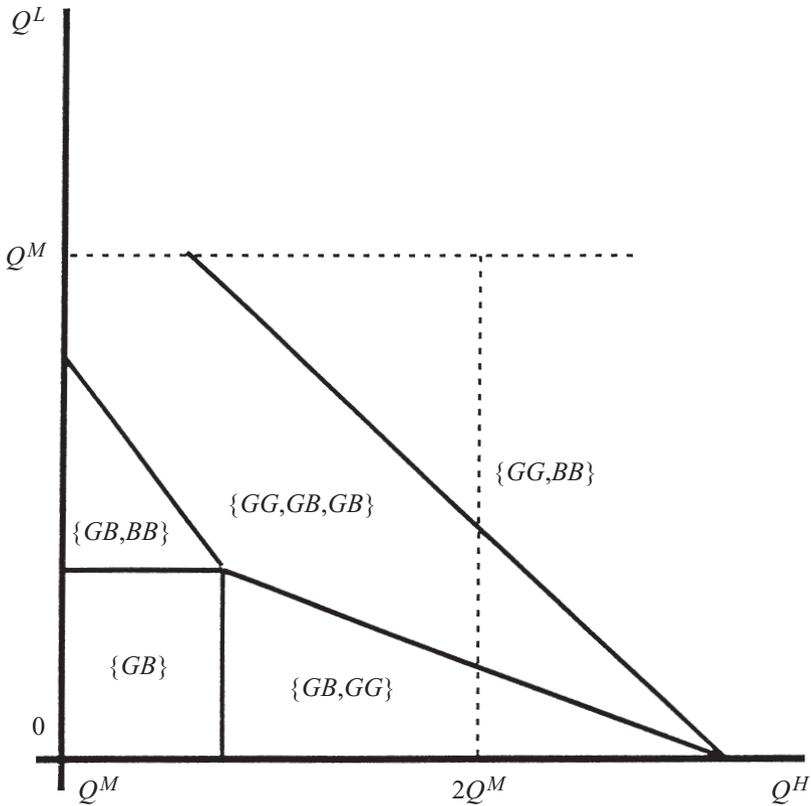

Fig. 2.

Fig. 3.

there are relatively few goods.[5] Note, the mixing Nash equilibrium only exists along the line where $Q^L + Q^H = 2Q^M$. For $Q^L + Q^H > 2Q^M$, the only equilibrium is the elitist equilibrium which generates positive assortative matching. This outcome coincides with Becker's notion that positive assortative matching occurs when inputs are complements. For $Q^L + Q^H < 2Q^M$, there are 3 possible equilibria but each yield negative assortative matching. As $\lambda \to 0$, the only equilibrium is {GB, BB} where $V_B \to Q^L$ and $V_G \to 2Q^M - Q^L$. This ensures that $V_G > Q^H$ and therefore goods choose not to match with each other – we obtain negative assortative matching. As $\lambda \to 0.5$, the only equilibrium is {GB, GG} where $V_G \to Q^H$ and $V_B \to 2Q^M - Q^H$, which ensures that bads choose not to match with each other.

Fig. 3 illustrates how the partition changes when there are matching frictions, i.e., when $r > 0$. In that case (when $\lambda < 0.5 - r/\alpha$),[6] all 5 types of Nash equilibria can hold for particular values of Q^H and Q^L (given Q^M).

As r increases, the set of parameters for which the mixing Nash equilibrium

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[5] In this framework, letting $r \to 0$ is equivalent to letting $\alpha \to \infty$.
[6] A different partition applies for λ high, but the insights offered below continue to hold.

exists becomes large. Indeed, for r arbitrarily large only the mixing Nash equilibrium exists – each single matches with the first partner who is contacted. In that case there is no correlation of types across matched pairs. A significant result here is that Nash equilibria which generate negative assortative matching may occur even when $Q^L + Q^H > 2Q^M$. It is no longer the case that complementary inputs necessarily generate positive assortative matching. For example if $Q^H = 2Q^M$ and $Q^L = \varepsilon$ where $\varepsilon > 0$ but small, the unique Nash equilibrium is $\{GG, GB\}$ where

$$V_G = \frac{\alpha^2\lambda^2 + \alpha r(1+\lambda)}{\alpha^2\lambda^2 + \alpha r(1+2\lambda) + 2r^2} 2Q^M$$

$$V_B = \frac{2\alpha\lambda r}{\alpha^2\lambda^2 + \alpha r(1+2\lambda) + 2r^2} Q^M.$$

Inspection shows that $V_G < Q^H$ (so that GG matches form) and $V_B + V_G < 2Q^M$ (so that GB matches form). However, for $r > 0$ and ε small enough $V_B > \varepsilon/2$ and so BB matches never form, even though $Q^L + Q^H > 2Q^M$. The intuition is that the bargaining friction allows the bads to extract sufficient surplus from goods that bads prefer not to match with each other as joint production is very small.

Of course this example only establishes a Nash equilibrium. To show that such behaviour also survives as a market equilibrium, we can assume exogenous inflows. Given (λ, N), this Nash equilibrium implies an exit flow of goods equal to $(\lambda N)\alpha$ where goods match with the first person they meet. The exit flow of bads is $[(1 - \lambda)N]\alpha\lambda$ as these only match with goods. Exogenous inflows, where proportion π are goods imply $\lambda = 2 - 1/\pi$ in a steady state. Hence $\lambda \in (0, 0.5 - r/\alpha)$ is consistent with a market equilibrium as long as $r/\alpha < 0.5$ and $\pi \in (0.5, 2/(3 + 2r/\alpha))$. In that case we obtain a market equilibrium with negative assortative matching, even when inputs are complements.

An Example

An interesting interpretation for Fig. 3 can be presented by the following interpretation. Suppose a partnership has to complete two tasks. For concreteness, suppose these partners write economic articles which involves two tasks; (a) constructing a new model, and (b) writing up the results. Each partner is allocated one task. Suppose goods are good at both tasks. However, there are two interesting possibilities for bads.

(i) Bads are bad at both tasks, i.e., $Q^H \gg Q^M = Q^L = \varepsilon$, where $\varepsilon > 0$ but small.

(ii) Bads are good at one task. Say they are good at writing up but not at constructing new models. In that case, by allocating tasks appropriately, we can assume $Q^H = Q^M \gg Q^L = \varepsilon$.

Case (i) implies we are towards the top right of Fig. 3. With $\lambda < 0.5 - r/\alpha$, we

obtain an elitist equilibrium for Q^H large enough[7] where goods only match with each other. Of course, if search frictions become large enough, we obtain the mixing equilibrium. In either case, there is positive assortative matching.

The opposite occurs for case (ii). Now it is more efficient that good partners do not match with each other (with $\lambda < 0.5$). Of course one might have anticipated that in the decentralised equilibrium, goods would still choose to match with each other if they contacted each other. But this does not occur for λ small enough. Bads are in a weak bargaining position - they must match with a good in order to be productive. Goods therefore extract most of the rents when bargaining with a bad and it is this effect which ensures goods reject other goods when they meet. Hence we obtain negative assortative matching.

3.4. *Matching Efficiency and Sorting Externalities*

An interesting (and desirable) feature of the above model is it implies the market outcome is efficient as we let search frictions go to zero. $Q^H + Q^L > 2Q^M$ guarantees positive assortative matching where it is more efficient that types match with their own type. Conversely $Q^H + Q^L < 2Q^M$ generates negative assortative matching where one type only matches with the other type in the market.

However, this does not mean that matching is socially efficient when search frictions are present. As discussed in detail by Burdett and Coles (1997*a*), there are sorting externalities in a market equilibrium.[8] Agents do not take into account that when forming a match and leaving the market, they change the composition of types in the market, which then changes the expected return to search for unmatched singles in the market.

For example, consider the multiple market equilibria result in Section 3.1. If goods are not elitist, they match relatively quickly. Because they match relatively quickly, steady state implies they are relatively few in number. Because they are relatively few in number, goods prefer to match with bads than continue search for a good and a mixing market equilibrium exists. The converse is the case in the elitist market equilibrium – if goods are elitist they match more slowly, their steady state number increases and goods then prefer to continue search for a good than marry a bad. These equilibria have clear welfare implications - goods prefer the elitist equilibrium, bads prefer the mixing equilibrium. The sorting externality is that when marrying a bad, a good does not take into account this reduces the payoff to other unmatched goods.

Not surprisingly, this same insight also holds for the transferable utility case. When two meet, they only match if it is jointly efficient to do so (efficient bargaining). However, by exiting the market, they change the composition of types in the market which affects the welfare of those unmatched. Using the

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[7] $Q^H > 2[1 + r/(\alpha\lambda)]Q^M$ is sufficient.
[8] unless clones are assumed.

same methodology demonstrated in Section 3.1, it is straightforward to show that multiple steady state equilibria are possible whose qualitative properties are identical to those obtained with non-transferable utility.

4. Match Specific Heterogeneity

4.1. *Non-transferable Utility*

In one sense this is the easier case as all singles are ex-ante identical and hence will use the same search strategy. There are no complicating composition effects.

To illustrate essentials, consider a marriage market which is again characterised by an equal number of men and women and there are constant returns to the encounter function, i.e., $\alpha = M(N, N)/N$, is the arrival rate of encounters faced by any single which does not depend on the number of singles in the market.

Given a man and woman meet, their payoffs are considered as independent random draws from two distributions, F_w and F_m. In particular, $F_w(x)$ denote the probability a woman will obtain utility no greater than x per unit of time if she marries the next man she meets. In a similar way we may define F_m and let $\underline{x}_j$ and $\overline{x}_j$ denote the infimum and supremum of the support of F_j, and assume $\underline{x}_j \geqslant 0$, $j = w$, m.

To complete the model we assume exogenous separations. Hence, $\delta\Delta$ denotes the probability any partnership ends in small time period Δ. As all men are ex-ante the same, the expected discounted lifetime utility of a single man, V_m, can be written as

$$V_m = \frac{1}{1 + r\Delta} \{ b_m\Delta + \alpha_m\Delta \mathrm{E} \max[J_m(x), \ V_m] + (1 - \alpha_m\Delta) V_m + o(\Delta) \}$$

where $J_m(x)$ indicates the payoff to marriage with a woman yielding utility x per unit of time, and α_m ($\alpha_m \leqslant \alpha$) denotes the arrival rate of offers faced by any man. Manipulation implies

$$rV_m = b_m + \alpha_m \{ \mathrm{E} \max[J_m(x), \ V_m] - V_m \}.$$

Further, exogenous separations imply $J_m(x)$ can be written as

$$J_m(x) = \frac{1}{1 + r\Delta} [x\Delta + \delta\Delta V_m + (1 - \delta\Delta)J_m(x)] + o(\Delta).$$

Manipulation implies that

$$J_m(x) = \frac{x + \delta V_m}{r + \delta}.$$

Clearly, all single men will use a reservation utility level, R_m, where $V_m = J_m(R_m)$. Substituting out $J_m(x)$ and V_m implies

$$R_m - b_m = \frac{\alpha_m}{\delta + r} \varphi_m(R_m). \tag{15}$$

Similarly, single, women will use reservation R_w, which satisfies

$$R_w - b_w = \frac{\alpha_w}{\delta + r} \varphi_w(R_w). \tag{16}$$

Note, the above implies that $1 - F_w(R_m)$ denotes the probability a woman will find a randomly chosen man a suitable marriage partner before they meet. As α denotes the arrival rate of a single of the opposite sex, it follows that the arrival rate of offers can be written as $\alpha_w = \alpha[1 - F_m(R_m)]$ and $\alpha_m = \alpha[1 - F_w(R_w)]$. Hence, although singles take the arrival rate of proposals as exogenous, they are the choice variable of singles of the opposite sex,

$$\frac{R_m - b_m}{\varphi_m(R_m)} = \frac{\alpha[1 - F_w(R_w)]}{\delta + r} \tag{17}$$

$$\frac{R_w - b_w}{\varphi_w(R_w)} = \frac{\alpha[1 - F_m(R_m)]}{\delta + r}. \tag{18}$$

Note, (17) and (18) can be interpreted as reaction functions, $R_m = \sigma_m(R_w)$ and $R_w = \sigma_w(R_m)$. A matching equilibrium exists if there exist R_m and R_w that solve (17) and (18) simultaneously. Clearly, multiple equilibria are possible. In Fig. 4 we illustrate the case where there are three equilibria. In one of these equilibria, indicated by (R_{m1}, R_{w1}), women are relatively picky in choosing a spouse, whereas with the equilibrium (R_{m2}, R_{w2}), men are relatively picky.

It should be noted that different equilibria have a consequence on the steady-state numbers of singles. This follows as the steady state number of
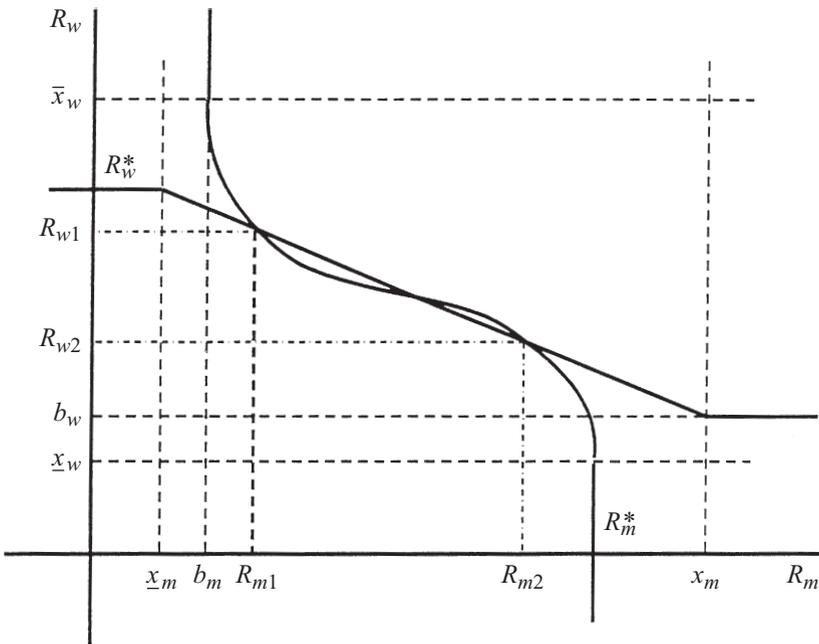

Fig. 4.

singles of either sex, U, implies the flow into singleness, $\delta[N - U]$ equals the flow out, ϕU, where $\phi = \alpha[1 - F_w(R_w)][1 - F_m(R_m)]$, i.e.,

$$U = \frac{\delta N}{\phi + \delta}.$$

Of course, it is useful to know when a unique equilibrium can be guaranteed. To develop sufficient conditions for a unique equilibrium first note $log \, \varphi(.)$ is concave if and only if

$$\varphi'(x)^2 - \varphi(x)\varphi''(x)$$

for all x. As $\varphi'(x) = -[1 - F(x)]$ and $\varphi''(x) = F'(x)$. Hence, assuming $log \, \varphi(.)$ is concave is a restriction on F. Indeed, it can be shown $log \, F$ concave $log \, \varphi(.)$ is concave (see Burdett (1996)).[9] Assuming $log[\varphi_i(.)]$ is concave $i = w, m$, it can be shown that the slopes of the reaction functions are such that a unique equilibrium is guaranteed (see Burdett and Wright (1998) for details).

4.2. *Transferable Utility*

Of course, the above assumed non-transferable utility. With transferable utility the agents bargain over the terms of trade. This framework is therefore more closely related to the standard matching framework. Here we consider a labour market model of this type. In this model, a participating firm (f) is either matched with a worker, or posts a single vacancy, whereas a worker (w) is either employed, or single looking for a job. All live forever.

As there may be a different number of workers than firms, let α_w denote the rate at which a worker contacts a firm, and α_f the rate at which a firm contacts an unemployed worker. For the present, all we assume about the encounter function, $M(U, V)$ is that it is increasing in both its arguments.

Suppose a worker and a firm make contact. They recognise instantly the total payoff generated if they form a match. Different worker/firm pairs, however, generate different payoffs. In particular, let $F(x')$ denote the probability the next worker/firm contact generates total payoff no greater than x'/r if the worker becomes employed at the firm.

Given a worker and firm have made contact and both recognise total payoff x/r is generated if they form a match, they decide either to form a match or not, and if they do, how much of the total payoff will go to each. As is typical within this framework, assume the terms of trade are described by Nash bargaining where each threat points are their expected payoff through continued search.

Let V_i denote the value of being unmatched, $i = f, w$ and η denote the bargaining power of the worker. In such a situation the outcomes associated with Nash Bargaining can be described as follows

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[9] For example, a Normal distribution implies $log \, F$ is concave, whereas if F is a Pareto distribution function, $log \, F$ is convex.

(i) If $x/r < V_w + V_f$ there is no gain to trade and both remain single with payoffs V_w, V_f.

(ii) If $x/r \geqslant V_w + V_f$, a match is formed, where the payoff to the worker is $w(x)/r = V_w + \eta(x/r - V_w - V_f)$ and the firm obtains $[x - w(x)]/r = V_f + (1 - \eta)(x/r - V_w - V_f)$.

Given it is known a partnership will form if and only if $x \geqslant R$, where $R = r(V_f + V_w)$, then $\alpha_w \Delta[1 - F(R)]$ is the arrival rate match faced by an unemployed worker. Hence, the expected discounted lifetime payoff to an unemployed worker can be written as

$$V_w = \frac{1}{1 + r\Delta}\left(b_w\Delta + \alpha\Delta\left\{ [1 - F(R)]\mathrm{E}\max\left[V_w, \frac{w(x)}{r}\right] + F(R)\,V_w\right\}\right.$$

$$\left. + (1 - \alpha\Delta)\,V_w\right) + o(\Delta).$$

Rearranging and letting $\Delta \to 0$ implies

$$rV_w = b_w + \frac{\alpha\eta}{r}\varphi(R).$$

As before, the flow value of search depends on the flow utility while unmatched, plus the expected surplus generated by optimal search. Of course with transferable utility, this surplus depends on the outcome of the underlying bargaining game. The analogous condition for firms is that

$$rV_f = b_f + \frac{\alpha_f(1 - \eta)}{r}\varphi(R).$$

A neat simplifying trick is to add up the above equations to get

$$R = (b_w + b_f) + \frac{\eta\alpha_w + (1 - \eta)\alpha_f}{r}\varphi(R).$$

This condition is clearly analogous to (2), except the appropriate 'matching rate' is a weighted average of the individual matching rates, where those weights are the agents' bargaining powers.

Closing the model requires specifying restrictions about turnover. If we assumed clones, then α_i are exogenously fixed and it follows there is a unique solution for R and hence a unique matching equilibrium. The approach most often used with these type of models, however, is to assume fixed entry rates for job seekers and perfectly elastic entry of vacancies (see, for example, Pissarides (1990)). In particular, assume

(i) $V_f = 0$ where $b_f = -a < 0$ is the flow cost of advertising a vacancy, while

(ii) job seekers enter at rate $g > 0$.

The entry assumption (i) implies $R = rV_w$ and therefore

$$R - b_w = + \frac{\alpha_w \eta}{r} \varphi(R) \tag{19}$$

and

$$a = \frac{\alpha_f(1 - \eta)}{r} \varphi(R). \tag{20}$$

Finally, entry restriction (ii) implies in steady state we have

$$g = M(U, V)[1 - F(R)] \tag{21}$$

$\alpha_w = M(U, V)/U$, and $\alpha_f = M(U, V)/V$. Equilibrium reduces to solving for (U, V, R). Substituting into (19), (20) and (21) implies

$$U(R) = -\frac{g\eta}{(R - b_w)r} \frac{\varphi(R)}{\varphi'(R)}$$

$$V(R) = -\frac{g(1 - \eta)}{ar} \frac{\varphi(R)}{\varphi'(R)}.$$

$$g = \Psi(R) = M(U(R), V(R))[1 - F(R)]$$

where Hence, if $\Psi'(.) < 0$, uniqueness of the equilibrium is guaranteed. It is well known that by assuming $M(., .)$ has constant returns to scale we have a unique equilibrium. A slightly different question was posed by Burdett and Wright (1998): what conditions on the distribution function, F, are required for a unique equilibrium, given any matching function that is increasing in both its arguments? To answer this question, note if $\log \varphi(.)$ concave then $d[\varphi(R)/\varphi'(R)]/dR < 0$ and therefore, after taking the relevant derivatives, $U'(R) < 0$ and $V'(R) < 0$. This in turn guarantees $\Psi'(R) < 0$ which ensures a unique equilibrium. Hence, if the distribution function, F, is such that $\log \varphi(.)$ concave, there is a unique equilibrium no matter if the returns to scale of the encounter function are increasing or decreasing.

5. Conclusion

Above we have outlined some of the basic ingredients required for a theory of long-term partnership formation and how these combine within an equilibrium setting. Of course, to develop a satisfactory theory of marriage formation or job search there are many gaps remaining that have to be filled. Adding institutional detail to the frameworks developed above should not be too difficult. There are, however, other aspects of long-term partnership formation that need to be embedded into the general framework. Below we briefly discuss two of these: uncertainty about quality and investment in desirability. There are, of course, several more possible developments which could be discussed.

   The decision to form a long-term partnership is typically clouded in uncertainty. The uncertainty about the utility to be obtained in a partnership can be reduced (but not eliminated) by accumulating information about the potential partner. Hence, couples date before deciding to marry, firms inter-

view and test in various ways possible job candidates, whereas workers try to find out as much as they can about a particular job before accepting employment (see Chade (1997) and Burdett and Wright (1998) for analyses of marriage formation when information is uncertain). Even after this accumulation of information, the decision to form a partnership is still a risky one as there is much information that can only be obtained by living the partnership. Divorce, or separation is a possibility. Clearly, some learn aspects about their partners which implies they either quit the partnership, or start to search for another partner while remaining in the partnership (see Sahib and Xinhua (1998) and Burdett and Wright (1998) for studies of separations when individuals learn characteristics after marriage). A complete treatment of learning about your partner in a partnership is yet to be attempted.

Some seemingly long-term partnerships may be only temporary partnerships. Many accept jobs knowing they are going to leave when something better comes along. Further, employers may know that workers will leave in the not too distant future. This can even happen in marriage. If the cost of search in a partnership is not too great individuals may choose to form temporary partnerships until something more desirable comes along (Webb (1998) for an analysis of on-the-job search in partnerships). Burdett and Coles (1998a) consider another reason for separation – changes through time in the alternatives available to individuals.

Long before a individual decides to obtain a job, investments are made at school and college which makes that individual more or less desirable to firms. On the other hand, a firm may invest in pleasant surroundings, or pre-commit to paying a high wage, as a way of making itself more desirable to workers. Similarly, some individuals invest in various things (good clothes, good haircut, lippo-suction, etc) to make themselves more desirable to potential marriage partners. An example of an individual's problem is in this case, is that, given a particular endowment of say charm, how much should he or she invest in accumulating more charm (at a cost) before entering a marriage market. Of course, the answer to this question depends on how the marriage market is expected to work and the preferences of the participants. Burdett and Coles (1998b) have considered such a set up and derive the equilibrium in a two-stage game. In the first stage individuals make investment decisions which determine how desirable they will be to those of the opposite sex. The second stage describes equilibrium behaviour in the marriage market (which is in this case a model with non-transferable utility). At the equilibrium, individuals make utility maximising investments given they have correct beliefs about the marriage market. Wage posting by firms can also be viewed as an investment, or, or correctly a pre-committment. In this case before entering the market firms pre-commit to a wage it will pay any employee (given it is willing to hire that individual). A higher wage will imply the firm is more desirable to the employee. Such a framework can be shown to lead to some fascinating insights. However, the relatively large literature on wage posting is not discussed here. (but see, for example, Albrecht and Axell (1984) Burdett and Mortensen (1998), and Mortensen and Pissarides (1998)).

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