Credit Lines and Bank Risk*

Jason Choi†

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Abstract

This paper studies how much providing credit lines to firms contributes to bank risk and its welfare implications. I develop a quantitative model in which banks lend to heterogeneous firms both through term loans and credit lines. Credit lines give firms liquidity insurance against crisis times and help alleviate financing frictions. At the same time, credit lines also introduce a new channel for banks to be exposed to excessive risk. I estimate the model to match both aggregate and distributional moments. I find that 20% of bank losses in crisis times can be attributed to credit lines, and that credit lines help stabilize banks during crises, but only to moderate shocks. My model suggests banks are overlending in both contracts compared to a planner, but the relative shares of contracts are close optimal. I find bank capital ratios should be 3% higher and show how to implement optimal policy. Additionally, I show how a model with only term loans would underpredict optimal capital ratios.

Keywords: Credit lines, bank capital, loan supply, macroprudential policy

JEL Classification Numbers: E44, G21, G28.

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†University of Wisconsin–Madison. Email: jason.choi@wisc.edu.

1 Introduction

Excessive risk in the banking sector is of critical interest to regulators and has generated a rich literature in banking. The conventional approach to studying banks is to model bank lending to firms as *term loans*, which require firms to sign new contracts each time they want to raise funds. Since the decision to issue new loans by the bank is discretionary, firms may be unable to raise funds in some states of the world. This view of bank lending overlooks the important fact that more than half of bank loans to firms in the US originate through *credit lines* (Greenwald et al. (2021)). Unlike term loans, credit lines constitute a commitment by banks to lend to firms at pre-determined rates and up to some amount. As part of credit line contracts, firms lock in relatively low interest rates from which they can access funds during the duration of the contracts[1]. In return, firms pay banks an annual fee, even when the credit line is not used, called a commitment fee — analogous to insurance premiums.[2]

Credit lines are valuable to firms. For example, firms benefited from having access to pre-arranged credit lines during the COVID-19 pandemic when capital market funding froze. Chodorow-Reich et al. (2021) show firms that did not have credit lines to draw on were more likely to sharply reduce investment, R&D expenditures, dividend payouts and total debt following these crises. While credit lines provide valuable insurance to firms, they also make banks more susceptible to aggregate risk. In times of crises[3] large numbers of firms draw upon these credit lines which puts severe pressure on bank balance sheets. In a recent op-ed, Acharya and Steffen (2021) suggest "[...] bank credit lines [are] the new source of financial fragility" and call for higher bank capital buffers.

In this paper, I consider an environment that builds on Holmström and Tirole (1998) in which banks offer lending contracts to firms who have access to productive investment projects. Firms experience liquidity shocks and are subject to moral hazard frictions. Depending on firm characteristics, optimal contracts take the form of either term loans, which offer no insurance against these liquidity shocks, or credit lines, which offer full insurance. Banks are subject to limited liability and I assume that bank defaults generate social costs which are not internalized by banks. This moral hazard friction incentivizes banks to take on more risk than is socially optimal and also allocate this risk between term

[1] Sufi (2009) finds that the median maturity on credit line contracts is approximately 3 years.

[2] While spreads can in principle be renegotiated, Greenwald et al. (2021) show that spreads on more than 90% of credit lines remain completely unchange from 2012-2019.

[3] For Great Financial Crisis, see Cornett et al. (2011); Ivashina and Scharfstein (2009). For COVID-19, see Li et al. (2020).

loans and credit lines in a sub-optimal fashion. I first use a two-period model to show theoretically that the bank moral hazard friction unambiguously increases the supply of both term loans and credit lines. How the relative share of contract types is distoreted is, however, theoretically ambiguous because limited liability and social cost of default each work in opposing directions. Therefore, I turn to a quantitative model to determine whether contracts in equilibrium are biased towards credit lines or term loans. I estimate this model using data from FRY-14 and Call Reports, which are bank supevisory data from the Federal Reserve, and find that 20% of bank risk can be attributed to credit lines. I then study the problem of how best to regulate banks. Imposing excessive regulation on credit lines can increase bank default since uninsured firms suffer significant losses in the face of liquidity shocks which in turn hurts bank balance sheets. I find that the optimal allocation can be implemented with a capital requirement of 11% and a constraint on undrawn credit lines.

My quantitative model is a dynamic general equilibrium in which banks intermediate lending between households and firms who need to raise funds in order to invest in a project. In addition, firms are subject to liqudity shocks which require additional funds to be raised. If these funds are not raised, firms suffer output losses. Firms are also subject to a moral hazard friction which limits the surplus that banks can extract from them. The combination of the moral hazard friction along with the need for insurance implies that for some firms optimal contracts take the form of credit lines which fully insure firms against liquidity shocks. A key feature of my model is that banks choose which firms to offer credit lines to. Consistent with empirical evidence, banks do not find it optimal to insure smaller firms with severe liquidity needs and instead offer them terms loans. Banks lend to a heterogeneous distribution of firms and are subject to aggregate shocks which affect the fraction of firms that are hit with liquidity shocks. I assume that banks have limited liability and can default on households. As in Karaken and Wallace (1978), I assume that the government insures household deposits in the event of bank default. This generates a moral hazard problem on the part of banks and leads to excessive risk taking by distorting both the level and share of credit lines and term loans.

To quantify excessive risk and characterize optimal regulatory policy, I estimate the model via simulated method of moments (SMM). At the micro level, I estimate the distribution of firm characteristics by using moments from the Federal Reserve's Y-14Q supervisory loan-level micro data. The optimal contract framework of my model makes predictions about the size, interest rate, and type of loan contracts that firms receive. I use the observed heterogeneity in loan characteristics to back out the distribution of firm characteristics that best explains the loan data. At the macro level, I discipline aggregate moments, such as overall bank leverage, default frequency, etc., by using the Federal Reserve's Consolidated Report of Condition and Income, also known as Call Reports. By targeting these banking industry moments, I use bank balance sheet data from Call Reports to inform bank parameters.

I find that 20% of bank losses during times of crisis are from credit line lending and the remaining 80% of losses are from term loan lending. While the losses arising from credit lines are quantitatively significant, they provide valuable insurance to firms which in turn help bank balance sheets. To understand this trade-off, I consider an environment in which banks are restricted to only offering term loans and find that bank default rates increase — from 1% to 2%. This is because while there is a direct effect of eliminating the risk from credit lines, there is also a substitution effect in which the banks are forced to offer term loans to firms who previously received credit lines. These firms were cheap to insure against liquidity shocks, but are now forced to suffer losses if they are hit with liquidty shocks which have adverse conseqeuences for bank balance sheets. I find that the term loan part of bank portfolios generates negative gross returns, thereby increasing bank default risk, at moderate shock sizes, while credit lines only generate negative returns for large shocks. Therefore, credit lines help stabilize banks at moderate shock sizes. However, I also find that as the bank grows in size following a sequence of low aggregate shocks, its probability of defaulting increases from near 0% to the steady state value of 1% with most of this increase arising from the new issuance of credit lines.

To characterize the optimal regulatory policy, I study a planner who is subject to the same frictions as private agents, but internalizes the social cost of bank default. While banks take on excessive risk in the aggregate by overlending, I find that they choose the relative shares of term loans and credit lines in an approximately efficient manner. I show that that optimal allocation can be implemented using a conventional capital requirement, a leverage requirement, and a loan commitment constraint. However, the majority of the gains (95%) from optimal policy can be achieved with a single risk-weighted capital requirements of 12%. This is a significant increase from the existing Basel III requirements of 8%. Finally, I show that if we use a model with only term loans, as in conventional models, we would underpredict optimal capital ratios and conclude that that the current Basel III 8% is sufficient, leaving us with lower welfare and a higher bank default probability than is desirable. This happens because the cost of increasing the borrowing constraint, in the form of capital requirements, is higher in a model with only term loans. Therefore, model misspecification leads the planner to overestimate the cost of increased capital ratios.

Related literature

This paper relates to a growing literature of analyzing banking regulation in quantitative general equilibrium models. Seminal papers include Van den Heuvel (2008), Corbae and D'Erasmo (2021), Begenau (2019); Begenau and Landvoigt (2018)[4]. Recent contributions include Pancost and Robatto (2022) which studies how capital requirements can increase risk in non-financial firms, and Dempsey (2020) which shows how aggregate risk in the economy can shift to non-banks. In these papers, banks lend through term loan contracts — banks do not have committed long-term loan obligations to borrowers. I argue that these papers miss an important feature of bank lending — that half of bank lending to firms occur through credit line contracts and that these loan commitment exposures are large and important to bank risk. In this paper, I explicitly model a bank's choice to lend through either term loan and/or credit line contracts, thereby allowing the bank to trade-off higher returns and higher risk and allowing risk shifting across contract types in response to regulation. In contrast to much of the quantitative banking literature, I apply an optimal contracting framework to allow contracts to change in response to regulation, and explicitly model the benefits of credit lines as well as the costs. The micro-foundation of the contracts is an extension of seminal work by Holmström and Tirole (1998), which shows that firm moral hazard necessitates loan contracts with commitments to avoid inefficient liquidations.

There is a large corporate finance literature studying the prevalence and use of credit lines by firms[5]. Complementing this literature, this paper focuses on the supply side of credit lines, endogenous choice in contracts, and the implications of drawdown risk for banks and its welfare consequences. Several empirical papers examine bank exposure to credit line risk during market turmoils. Ivashina and Scharfstein (2009) and Cornett et al. (2011) show that following the failure of Lehman Brothers, banks with more credit line exposure decreased their new lending more. Firms drawing on existing credit lines acts similarly to a liquidity shock and constrains both new term loan lending and new credit line extensions. More recently, Li et al. (2020) document the same phenomenon during the

[4]See also for quantitative banking models, Bianchi and Bigio (2022), Pandolfo (2021), Nguyen (2018), Davydiuk (2019), and others.

[5]See Bolton et al. (2011), Nikolov et al. (2019), Sufi (2009), and others.

COVID market panic of March 2020. In both these episodes, bank deposits inflow increased as banks were simultaneously perceived to be safe institutions, a phenomenon Gatev and Strahan (2006) first recorded[6]. Despite these inflows, papers conclude that government intervention played a significant role in managing bank stress. Acharya et al. (2021) and Kapan and Minoiu (2021) empirically study the mechanism through which banks were affected by credit line draws, and suggest capital constraints and lender risk preference were the driving factors, respectively.

Recent papers such as Greenwald et al. (2021) and Chodorow-Reich et al. (2021) use the Federal Reserve's Y-14 data to outline firms' credit line usage during crises and show that firm heterogeneity in credit lines has important implications for how we think about policy. They show that only large firms had access to credit lines during COVID, and that virtually all the increase in corporate cash occurred specifically through these existing credit line contracts. I extensively rely on the data moments reported in these two papers in the estimation of my model. In a closely related work, Payne (2020) shows bank-firm search frictions amplifies shocks to banks to the real economy. While this paper also uses an optimal contract framework to highlight the role of long term loan contracts, like credit lines, in contrast, my paper features a bank with a rich balance sheet problem and defaults on-the-equilibrium path, making it more suitable to study banking regulation. In recent work, Benetton et al. (2022) also model multi-product banks which extend both term loans and credit lines. They use an IO approach to estimate the benefits of economies of scope across products versus the cost of market power exploitation by these banks. In this paper, I focus on the risk implications of credit lines to banks and incorporate endogenous contracts.

Roadmap

The remainder of the paper is organized as follows: Section 2 outlines a simplified 2-period environment to examine theoretical results, Section 3 characterizes the equilibrium, Section 4 describes the dynamic quantitative model, Section 5 characterizes the dynamic equilibrium, Section 6 describes how I map the model to the data, Section 7 conducts a quantitative analysis of the model, and Section 8 concludes.

[6]Kashyap et al. (2009) show that synergy with deposit taking make credit lines a uniquely bank product.

2 Two-period model

I first consider a simplified two-period general equilibrium environment with heterogeneous firms, a bank, households, and a government. There are two periods and all agents are risk neutral. Firms have productive projects to invest in, experience liquidity shocks and are subject to moral hazard frictions. The optimal lending contract builds on Holmström and Tirole (1998), where depending on fundamentals, the optimal contract can be implemented through credit lines or with term loans. Banks also experience moral hazard due to limited liability and deposit insurance. Households own both firms and banks, but suffer the social cost of bank default. The government simply operates a deposit insurance fund. In this section, I present model mechanics and theoretical results. In Section 4, I extend this model to an infinite horizon quantitative model and evaluate the theoretical results quantitatively.

2.1 Environment

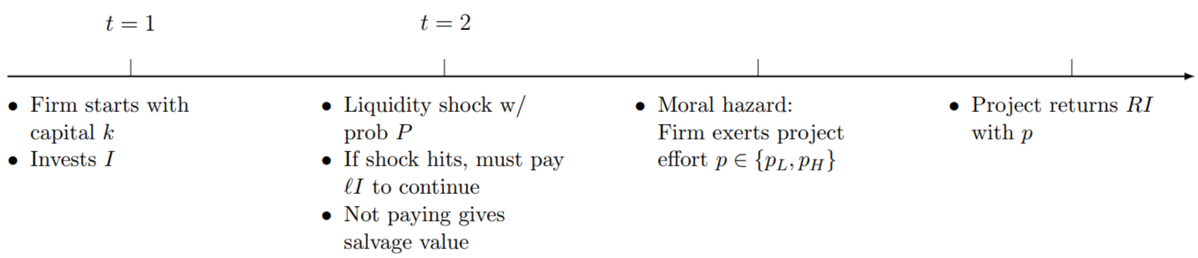
2.1.1 Firms

There is a continuum of firms indexed by (i, j) that live for two periods $t = 1, 2$. The firms are heterogeneous along two orthogonal dimensions: liquidity cost ℓ_i, drawn from a distribution $F(\ell)$, and project technology R_j drawn from $G(R)$. At $t = 1$, firms starts with capital k and choose investment sizes $I \in \mathbb{R}^+$ to a risky project with linear technology: the project returns $R_j \cdot I$ at the end of $t = 2$ with probability $p \in \{p_L, p_H\}$ and 0 with $1 - p$, where $\Delta \equiv p_H - p_L > 0$. This probability p is a hidden effort choice of the firms. At the beginning of $t = 2$, an aggregate state $P \in [0, 1]$ is drawn from $H(P)$; this P is common to all firms. Then, firms receive an iid liquidity shock of $\ell_i \cdot I$ with probability P, or 0 with probability $1 - P$. There is ex-ante heterogeneity in liquidity type $\ell_i \sim F(\ell)$. The project continues if the liquidity shock is 0. If the liquidity shock is $\ell_i \cdot I$, then the liquidity cost must be paid to continue. If it is not paid for, the project is destroyed, there is a salvage value of χI, and the model ends. If the project continues, then the firm chooses its project effort $p \in \{p_L, p_H\}$[7]. This effort is subject to moral hazard; choosing low effort p_L gives the firm a private benefit $B \cdot I$. At the end of period 2, project returns are realized. The firm has limited liability and only has to pay the bank if

[7]It's important the moral hazard happens at the end, after the liquidity shock. If it happens before the shock, the moral hazard choice is already done by the time the shock arrives, so agency friction plays no role in the continuation decision of the project.

the project suceeds.

Figure 1: Timing: Firms



2.1.2 Households

Two-period lived risk neutral household is endowed with ω units of goods at $t = 1$. The objective of the household is to maximize end of period 2 consumption only C. At the beginning of period 1, households can save their endowment ω through either deposits D or riskless storage technology a. At the end of the period 2, households earn returns from deposit $R_D D$, storage technology $R_f a$, earn profits from the bank, own both the bank and firms, and consume C. While households own the bank, they pay for the resource costs ξ of bank assets when bank default.

2.1.3 Bank

A risk neutral bank with commitment technology maximizes end of period 2 profits. At the beginning of period 1, the bank chooses to raise deposits D from households at interest rate R_D. Using deposit funds D and bank capital K, the bank offers optimal loan contracts to each individual firm (i, j). L_{TL} denotes the aggregate amount of term loan contracts given to firms, and L_{CL} denotes the aggregate amount of credit line contracts. At the beginning of period 2, the bank also realizes the aggregate state P. Because the liquidity shock draws are iid to firms, the bank knows that exactly P share of firms received a liquidity shock. As per the credit line contract, the bank pays $P\ell_i I_{CL}$ to every firm that received credit line contracts; the aggregated amount is $P\mathcal{L}$, where $\mathcal{L}$ is the aggregate amount of liquidity the bank has committed. At the end of period 2, the bank earns the returns from its investments L_{TL} and L_{CL}, which are $(1 - P)\, R_{TL} + P\chi R_S$ and R_{CL}, respectively. $R_{TL}(R_{CL})$ denotes the aggregate total returns from all term loan (credit line) contracts $L_{TL}(L_{CL})$, and R_S denotes the aggregate project value from liquidated firms. It pays depositors $R_D D$ and realizes its profits.

Importantly, there is deposit insurance for bank deposits which generates moral hazard for the bank against households — households are guaranteed their deposits by the government even if the bank fails. This is an institutional feature of the real world that tries to address bank runs, as in Diamond and Dybvig (1983), which is unmodeled in this paper. Because of limited liability, the bank therefore has a convexified payoff. Furthermore, when the bank defaults, only a fraction ξ of their assets are recovered. The bank will choose an expected default probability that does not internalize the social cost of its default. If the realization of profits is negative $\pi < 0$ and the bank cannot fully repay its depositors, it receives a value of 0. Once defaulted, the government steps in and takes over the bank and liquidates its assets to pay depositors. Through monitoring technology, there is no private information — all parameters in the environment are known to the bank, except for the single hidden effort choice p of the firm.

2.1.4 Timing

1. At the beginning of period 1, the bank raises deposits D from households.

2. The bank offers optimal contracts $\left(I, R^F, \iota\right)_{ij}$ to each individual firm (i, j).

3. Each firm starts with assets k, borrows $I_{ij} - k_i$ from bank, and invests in project size I_{ij}. The bank has given an aggregate term loan amount of L_{TL} , aggregate credit line amount of L_{CL} with $\mathcal{L}$ amount of total liquidity commitments.

4. At the beginning of period 2, aggregate state $P \in [0, 1]$ is realized by all agents.

5. Firms receive liquidity shock $\ell_i I_{ij}$ with probability P and 0 with $(1 - P)$ which is iid across firms. In the aggregate, exactly P share of firms received a liquidity shock $\ell_i I_{ij}$.

6. Of those who received a liquidity shock, firms who received a credit line contract get $\ell_i I_{ij}$ from the bank and continue projects; firms who received term loans liquidate their projects and receive salvage value χI_{ij}. In the aggregate, the bank pays $P\mathcal{L}$ to firms in total.

7. Firms that continue exert project effort $p \in \{p_L, p_H\}$.

8. Project returns $R_j I_{ij}$ are realized with p. The bank receives a total of $(1 - P) R_{TL} + \chi P R_S$ and R_{CL}.

9. If bank profit is negative, the bank defaults and is liquidated by the government.

10. Households are paid deposits back at $R_D D$ and earn $R_f a$ from their riskless storage. Taxes T are levied if necessary.

11. Households consume C.

2.2 The optimal contract

The firm and bank are free to choose from a general contract space.

Assumption 1. *The project is NPV positive to invest in, but the entrepreneur is constrained in her borrowing by moral hazard:* $p_H\left(R - \frac{B}{\Delta p}\right) < min\left\{1 + P\ell, \frac{1-P\chi}{1-P}\right\} < p_H R.$

We first start by assuming the particular project is a worthwhile investment and investigate how individual contracts to firms are chosen.

Definition 1. A contract $\mathcal{C}$ is a 3-tuple $\left(I, R^F, \iota\right)$ where

1. I: Project investment size

2. R^F: Per unit returns to the firm

3. ι: Funds to be paid by the bank in the event of a liquidity shock

We can show that contracts can be summarized by a 3-tuple of project investment size I, per unit return to the firm R^F, and how much liquidity insurance ι the contract gives where these three objects map to loan quantity, loan price and contract type, respectively, when taken to the data. The optimal contract is solved for over this 3-tuple space.

Lemma 1. *In the optimal contract,*

1. *The split of the project returns RI between firm and bank is not state-contingent*

2. *Any salvage value χ goes to the bank*

Appendix A contains the proofs. Intuitively, for the non-state contingency of the split of project returns, the bank will find it better to always minimize the firm's share of returns for all states. It is better to induce the firm to particpate through increasing the project size I, rather than through increasing the return share R^F. Any salvage value that is earned *before* the project effort choice by the firm cannot be used to discipline the firm's moral hazard. This allows us to state contracts in just a 3-tuple form $\left(I, R^F, \iota\right)$.

Lemma 2. *The optimal contract must take one of two forms, either* $\mathcal{C}_{TL} = \left(I_{TL}, R^F_{TL}, 0\right)$ *or* $\mathcal{C}_{CL} = \left(I_{CL}, R^F_{CL}, \ell\right)$*, referred to as a term loan contract or a credit line contract, respectively.*

Lemma 2 shows that we only need to solve for two types of contracts: one where there is no insurance against the liquidity shock, which I refer to as a *term loan* contract, and one with full insurance against the liquidity shock, which I refer to as a *credit line* contract. The intuition here is that the support of the liquidity shock is only two values $\{0, \ell\}$, therefore any partial insurance will be useless — the liquidity cost must be paid in full to continue. In credit line contracts with full insurance, for simplicity we model the bank as giving a transfer to firms and getting negative cash flow. However, in the real world firms are eventually paying back these additional loans drawn from credit lines. I show in Appendix B that this environment is isomorphic to one in which the bank has an outside investment opportunity, but is unable to invest in it since it provides the funds to the firm as promised. The bank would suffer an opportunity cost loss in the state of the world in which the firm receives a liquidity shock, as in the original model. The real world situation this models would be one in which the bank provides loans through the credit line contract at the contracted fixed interest rate, and is capacity constrained to provide funds at the currently higher market rate somewhere else.[8]. Furthermore, while I label $\mathcal{C}_{CL}$ as the *credit line* contract, the implementation of it can be through a term loan to fund the project plus a credit line, or only with a credit line where there is an initial draw to fund the project. The crucial point is that it *must* include a credit line, as explained further in Lemma 1.

Taking Lemma 1, we compare the optimal term loan contract solution and the optimal credit line contract solution to determine the optimal contract. The optimal term loan contract $\mathcal{C}_{TL} = \left(I_{TL}, R^F_{TL}, 0\right)$ solves,

$$\pi_{TL} = \max_{I_{TL}, R^F_{TL}} (1-P)\, p_H \left(R - R^F_{TL}\right) I_{TL} + P\chi I_{TL} - (I_{TL} - k) \tag{1}$$

[8] Acharya et al. (2021) show that there may be several reasons why banks would be constrained in lending during credit line drawdowns. Funding liquidity to source new loans can become a binding constraint if the bank does not have enough deposits or liquid assets. Also, the credit line drawdowns can lock up scarce bank capital through regulatory constraints such as capital requirements.

where the firm incentive compatibility constraint (IC) is

$$p_H R_{TL}^F I \geq p_L R_{TL}^F I + BI \tag{2}$$

and the firm participation constraint (PC) is

$$(1 - P)\, p_H R_{TL}^F I_{TL} \geq k \tag{3}$$

The objective function shows, the return to the bank in the event of no liquidity shock, the salvage value it receives if there is a shock, and the initial loan quantity. The incentive compatibility constraint shows that the firm's earnings upon its exerting high effort must be greater than if it had exerted low effort. Finally, the participation constraint shows that the firm must be willing to put in its initial capital k instead of consuming it. Similarly, the optimal credit line contract $\mathcal{C}_{CL} = \left(I_{CL}, R_{CL}^F, \ell\right)$ solves,

$$\pi_{CL} = \max_{I_{CL}, R_{CL}^F} p_H \left(R - R_{CL}^F\right) I_{CL} - \left(I_{CL} - k\right) - P\ell I_{CL} \tag{4}$$

where the firm incentive compatibility constraint (IC) is

$$R_{CL}^F \geq \frac{B}{\Delta p} \tag{5}$$

and the firm participation constraint (PC) is

$$p_H R_{CL}^F I_{CL} \geq k \tag{6}$$

The objective function shows that the project return to the bank is always the same since the project always continues. However, there is a potential cost for the bank of insuring the firm against the liquidity cost, ℓI_{CL}. These two problems clearly show the trade-off the bank faces when deciding which contract to give. In the term loan contract, there is a potential liquidation of the project and therefore lower return; in the credit line contract, there is a guarantee of the project surviving, but at a potential additional cost to the bank.

Lemma 3. *The optimal term loan contract is* $\mathcal{C}_{TL} = \left(I_{TL}, R_{TL}^F, 0\right) = \left(\frac{k}{(1-P)p_H \frac{B}{\Delta p}}, \frac{B}{\Delta p}, 0\right)$, *while the* *optimal credit line contract is* $\mathcal{C}_{CL} = \left(I_{CL}, R^F_{CL}, \ell\right) = \left(\frac{k}{p_H \frac{B}{\Delta p}}, \frac{B}{\Delta p}, \ell\right)$.

The bank chooses the contract that maximizes its profits, $\mathcal{C}^* = argmax\left\{\pi_{TL}, \pi_{CL}\right\}$.

Lemma 4. *The optimal contract offered by the bank is the credit line contract* $\mathcal{C}_{CL}$ *iff*

$$\ell < \frac{1-\chi}{1-P} \tag{7}$$

In this simple version, there is a clear cutoff value for the firms per-unit liquidity cost ℓ. If the cost is sufficiently low, it is better ex-ante to insure the firm against this risk and prevent liquidation. If the cost is sufficiently high, it is better to not pay the cost and allow the project to liquidate. The cutoff value depends also on how generous the salvage value is and how likely the liquidity shock will hit. When the bank chooses the credit line contract $\mathcal{C}_{CL}$, there are some cases where the bank would receive negative profits from paying the liquidity shock and continuing the project, and therefore find it time-inconsistent. The bank *commits* to paying and continuing the project in these states of the world. Therefore, this *commitment* technology[9] is key to being able to convince the firm that its participation constraint is satisfied, and to be able to extend the efficient optimal contract.

Proposition 1. *If* $p_H R > \ell > p_H\left(R - \frac{B}{\Delta p}\right) - \chi$, *optimal contracts are renegotiation-proof:*

1. *The credit line contract cannot be implemented through two term loans in a wait-and-see policy, and can only be implemented through a committed long term contract.*

2. *The term loan contract cannot be renegotiated in the event a liquidity shock is realized. That is, the firm cannot get a second loan to continue the project.*

The optimal contract implementations are distinct from each other because the moral hazard friction necessitates long-term committed contracts. The intuitive argument here is that if moral hazard did not exist, the firm and the bank would always value the project equally. Whenever the project is net present value positive, both the firm and the bank will want to continue by paying the liquidity cost. Therefore, we don't need prior commitments. We can achieve the first best without commitments.

[9]Discussed earlier, empirically we see that banks follow through on these commitments in times of crisis. Theoretically, I am abstracting from modelling dynamic reputations. If reneging on credit lines causes firms to not accept particpation in future periods, this off-equilibrium-path would discipline bank behavior on-the-equilibrium path. See for example, Boot et al. (1993).

However, when moral hazard is sufficiently high, there are situations where it is ex-ante efficient to proceed with the project, but if the liquidity shock arrives, the bank's end share of the project returns $p_H\left(R-\frac{B}{\Delta p}\right)$ are lower than the liquidity cost ℓ, so the bank will not want to continue. That is, the liquidity cost is even higher than what the bank would receive from the successful project. Then, the firm will want to give the bank some of its share of the project $R^F=\frac{B}{\Delta p}$ to help convince the bank to continue the project. However, since the optimal contract already has the incentive compatibility binding, the firm will not be able to credibly give part of its share of returns to the bank — the bank knows that the firm will be tempted to put in low effort and to earn private benefit. Therefore, renegotiations will fail, and the bank will not want to continue. This is ex-post inefficient because the total surplus of the project is still positive since the firm's share is not only positive, but larger than the bank's negative return $p_H\left(R-\frac{B}{\Delta p}\right)-\ell$. Therefore, in the presence of moral hazard, credit line contracts must be implemented through *committed* long term contracts, instead of through a wait-and-see policy of two separate loans. This commitment technlogy is what enables the bank to credibly satisfy the firm's participation constraint under the credit line contract. The intuition is similar that a term loan contract will not lead to renegotiation with the firm receiving a second loan it was not promised initially.

Finally, this simple contract has sharp predictions on how the optimal contract terms will vary according to firm characteristics — the underlying parameters. In particular, Lemma 5 introduces three comparative static which I specifically introduce because they will play a critical role in the estimation of the model.

Lemma 5. *The optimal contracts have the following features:*

1. *Firms with lower ℓ get credit line contracts, as shown in Lemma 4*
2. *Firms with higher k get larger loans, i.e., loan size $(I-k)$ is increasing in firm size k*
3. *Firms with lower B pay lower interest rates, i.e., the implied interest rate r is increasing in moral hazard B, i.e. $\frac{\partial r}{\partial B}>0$.*

In Section 6 where we take the model to the data, we will see that the optimal contract correctly predicts the contract data across the firm distribution. In fact, the optimal contract can be seen as a mapping from the set of firm characteristics to the set of contract characteristics. This mapping will be exploited later in the estimation procedure and illuminates why we see heterogeneous contracts in equilibrium.

3 Equilibrium characterization

3.1 Decentralized equilibrium

3.1.1 Bank problem

The bank maximizes expected profits by choosing optimal contracts $\{\mathcal{C}_{ij}\}_{\forall ij}$, and deposits D to fund these. Bank's expected profits are,

$$V^B = \max_{\left\{\{\mathcal{C}_{ij}\}_{\forall ij}, D\right\}} \int_0^1 max\left\{\pi, 0\right\} dH\left(P\right) \tag{8}$$

s.t.

$$L_{TL} + L_{CL} \leq D + K \tag{9}$$

$$\pi = \left(1 - P\right) R_{TL} + R_{CL} - P\mathcal{L} - R_D D \tag{10}$$

$$R_i^F \geq \frac{BI_{ij}}{\Delta p} \; \forall i, j \tag{11}$$

$$\begin{cases} \left(1 - \bar{P}\right) p_H R_{ij}^F I_{ij} \geq k_i & \text{if } \mathcal{C}_{ij} \in \mathcal{C}_{TL} \\ p_H R_{ij}^F I_{ij} \geq k_i & \text{if } \mathcal{C}_{ij} \in \mathcal{C}_{CL} \end{cases} \tag{12}$$

where

$$L_{TL} \equiv \int_{\mathcal{C}_{TL}} \left(I_{ij} - k_i\right) \qquad L_{CL} \equiv \int_{\mathcal{C}_{CL}} \left(I_{ij} - k_i\right) \tag{13}$$

$$R_{TL} \equiv \int_{\mathcal{C}_{TL}} \left(R_j - R_{ij}^F\right) I_{ij} \qquad R_{CL} \equiv \int_{\mathcal{C}_{CL}} \left(R_j - R_{ij}^F\right) I_{ij} \tag{14}$$

$$\mathcal{L} \equiv \int_{\mathcal{C}_{CL}} \ell_i I_{ij} \tag{15}$$

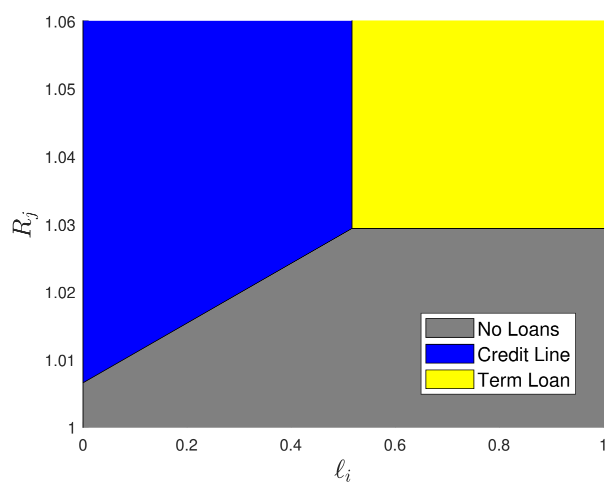
The bank's problem over a general contract space may seem abstract, but a visual representation of optimal contracts illustrates how monotonicity gives a clear characterization of regions in the firm characteristics space. In Figure 2 we see how firm characteristics determine which type of contract firms receive. The relevant characteristic for determining contract type is the liquidity cost ℓ, which is plotted on the x-axis. On the y-axis is the project quality R_j dimension. Firms in the top right quadrant have high project quality and are therefore worth funding, but their liquidity cost ℓ_i is too high such that insuring them against liquidity shocks is not profitable. Firms on the top left are similarly of high project quality, but they also have low liquidity cost. Therefore, the optimal contract is to insure them with credit lines. The boundary of the term loan region and the unfunded region is flat because conditional on receiving a term loan, the liquidity cost ℓ_i does not matter and only the project quality R_j matters. However, for credit line optimal firms, the liquidity cost is also relevent for accessing project profitability. Therefore, as the project quality decreases, if the firm has sufficiently low liquidity cost, the bank will to fund it, giving a downward slope. The bottom right region in grey shows the firms who have too high of a liquidity cost and/or too low of a project quality and are not funded at all. We formalize this below in Lemma 6.

Lemma 6. *Bank chooses cut off values (ℓ^*, R^*) such that*

1. *For firms $(\ell_i \geq \ell^*)$ and $(R_j \geq R^*)$, the optimal contract is $\left(I_{TL}, R^F_{TL}, 0\right)$*
2. *For firms $(\ell_i < \ell^*)$ and $\left(R_j \geq R^* + \frac{\bar{P}}{p_H}(\ell_i - \ell^*)\right)$, the optimal contract is $\left(I_{CL}, R^F_{CL}, \ell\right)$*
3. *For all other firms, the optimal contract is to not be funded*

where $R^F_{TL} = \frac{B}{\Delta p}$, $I_{TL} = \frac{k}{(1-\bar{P})p_H \frac{B}{\Delta p}}$ *and* $I_{CL} = \frac{k}{p_H \frac{B}{\Delta p}}$, $R^F_{CL} = \frac{B}{\Delta p}$

Figure 2: Firm type and optimal contracts



3.1.2 Household problem

The problem of a representative houshold is

$$V^H = \max_{D,a,C} u\left(C\right) \tag{16}$$

such that

$$D + a \leq \omega \tag{17}$$

$$C \leq R_f a + R_D D - T + V^B + V^F \tag{18}$$

Therefore, for households to hold bank deposits it must be that

$$R_D \geq R_f$$

and households' deposit supply function is

$$D_S = \begin{cases} 0 & \text{if } R_D < R_f \\ [0, \omega) & \text{if } R_D = R_f \\ \omega & \text{if } R_D > R_f \end{cases} \tag{19}$$

3.1.3 Government problem

The government operates a balanced budget. When the bank enters default, the government liquidates assets at a resource cost of ξ and pays depositors - remaining amount is drawn from deposit insurance fund. Deposit insurance is funded by taking lump sum tax T from household.

$$T = max\left\{0, R_D D - \left[\xi\left(\left(1-P\right)R_{TL} + P\chi R_S + R_{CL}\right) - P\mathcal{L}\right]\right\} \tag{20}$$

3.2 Definition of equilibrium

A *general equilibrium* is a set bank functions $\left\{\{\mathcal{C}_{ij}\}_{\forall ij}, D^D\right\}$, household functions $\left\{C, a, D^S\right\}$, and deposit rate R_D such that

1. Given the deposit rate, $\left\{\{\mathcal{C}_{ij}\}_{\forall ij}, D^D\right\}$ solves the maximization problem of the bank
2. Firm incentive compatibility (IC) and participation constraints (PC) are satisfied
3. Given the deposit rate, $\left\{C, a, D^S\right\}$ solves the maximization problem of households
4. Given the deposit rate, the government budget constraint holds
5. The deposit market clears: $D^D = D^S$

3.3 Planner's problem

We consider a planner whose objective is to maximize household consumption. The planner is subject to the same frictions as private agents — we want to consider an equilibrium that is achievable through financial regulation. The key difference is that the planner internalizes the cost of bank default. The planner also chooses to extend optimal contracts to firms, but bears the negative profits and any resource costs. Equation 22 represents the resource constraint from the household, Equation 25 the resource constraint from financial intermediation, and Equations 23 and 24 show the cost of default and how capital is updated, respectively. The planner's problem is

$$V^{SP} = \max_{\left\{C,D,\{\mathcal{C}_{ij}\}_{\forall ij}\right\}} u(C) \tag{21}$$

s.t.

$$C \leq R_f(\omega - D) + R_D D - T + V^B + V^F \tag{22}$$

$$T = max\left\{0, R_D D - \left[\xi\left((1-P) R_{TL} + P\chi R_S + R_{CL}\right) - P\mathcal{L}\right]\right\} \tag{23}$$

$$V^B = \int_0^1 max\{\pi, 0\}\, dH(P) \tag{24}$$

$$L_{TL} + L_{CL} \leq D + K \tag{25}$$

$$\pi = (1-P) R_{TL} + R_{CL} - P\mathcal{L} - R_D D \tag{26}$$

$$R_i^F \geq \frac{BI_{ij}}{\Delta p}\ \forall i, j \tag{27}$$

$$\begin{cases} \left(1-\bar{P}\right) p_H R_{ij}^F I_{ij} \geq k_i & \text{if } \mathcal{C}_{ij} \in \mathcal{C}_{TL} \\ p_H R_{ij}^F I_{ij} \geq k_i & \text{if } \mathcal{C}_{ij} \in \mathcal{C}_{CL} \end{cases} \tag{28}$$

3.4 Distortions from bank moral hazard

In this section I study how bank moral hazard friction distorts the bank's choices. It is well known in the banking literature that bank moral hazard against depositors causes banks to over leverage by overborrowing and overlending — I confirm this same level effect in my model. In addition, I show that the contract choice itself is also distorted albeit in an ambiguous direction. Following the literature, the bank moral hazard friction in my model enters technically in two forms, through limited liability and resource cost of default. Let us consider a bank's problem in which the bank has *partial limited liability* as defined as follows.

$$V^B = \max_{\ell^* \in \left[0,\bar{\ell}\right], R^* \in \left[0,\bar{R}\right]} \int_0^{P^*} \pi_G dH\left(P\right) + \theta \int_{P^*}^1 \pi_B dH\left(P\right)$$

subject to the budget constraint

$$L_{TL} + L_{CL} \leq D + K$$

where

$$\pi_G = \left(1-P\right) R_{TL} + P\chi L_{TL} + R_{CL} - P\mathcal{L} - R_D D$$

$$\pi_B = \xi\left[\left(1-P\right) R_{TL} + P\chi L_{TL} + R_{CL}\right] - P\mathcal{L} - R_D D$$

Note that this problem nests the market bank's problem with $\theta = 0$, in which the bank does not internalize any of the negative profit states, and the planner's problem with $\theta = 1$, in which the bank internalizes all the negative profit states. Using this setup we separately study the distortions that come from the limited liability θ and resource cost of default ξ. In the following analytical exercises we assume $F\left(\ell\right)$ and $G\left(R\right)$ are uniformly distributed.

Limited liability

Proposition 2. *Where θ governs the degree of the limited liability friction,*

1. *Bank increases loan supply compared to the planner: $R^*_{\theta<1} < R^*_{\theta=1}$*

2. *Bank skews contract supply towards term loans compared to planner: $\ell^*_{\theta<1} < \ell^*_{\theta=1}$*

When $\theta < 1$ and the bank no longer fully internalizes negative profits, the bank sets a lower R^* cutoff. This means, all else equal, the bank is increasing its loan supply to fund more firms. Crucially these marginal firms have low returns, but must be funded with the constant cost of deposits. A bank that fully internalizes losses would have found the expected value of these loans to be too low, but one that ignores the negative states would find the expected value to be artificially higher.

$$\frac{\partial R^*}{\partial \theta} > 0 \tag{29}$$

The limited liability friction also skews the share of contracts. The bank finds term loans benefit more from limited liability than credit lines do. This is because, all else equal, term loan project sizes are larger than credit line projects, $I_{TL} > I_{CL}$. Firms have to enticed with larger project sizes to satisfy the participation constraint (PC) under term loans. Therefore, both the returns and potential costs of term loans are larger for each firm. In essence, term loans have higher variance compared to credit lines, and therefore benefit more from the convexification of payoff coming from limited liability.

$$\frac{\partial \ell^*}{\partial \theta} > 0 \tag{30}$$

Overall, the limited liability distorts total loan supply to be larger on both contracts and skews contracts *towards* term loans.

Social cost of default

To study the distortions from the social cost of default ξ, we consider again the same problem with $\theta = 1$. The planner internalizes losses, but faces no social cost $\xi = 1$, and see how ξ changes her allocations.

Proposition 3. *Where ξ governs the social cost of bank default ,*

1. *Planner lowers loan supply compared to no default cost:* $R^{SP}_{\xi<1} < R^{SP}_{\xi=1}$

2. *Planner skews contract supply towards term loans by setting* $\ell^{SP}_{\xi<1} < \ell^{SP}_{\xi=1}$

Note the market equilibrium does not change as resource cost ξ changes — only the planner's allocation changes. Only the planner considers ξ in her problem; it is wholly absent in the market bank's problem. Therefore, to understand the inefficiency of the market equilibrium, we can look at how the planner's allocation changes from $\xi = 0$ to $\xi > 0$, and compare it to the constant market equilibrium.

$$\frac{\partial R^{SP}}{\partial \xi} > 0 \tag{31}$$

Increasing the resource cost of default decreases the expected value of projects because they are worth less in the default states. Since the planner maximizes expected profits, even though positive profits are not changing, the states of the world in which the planner makes negative profits become worse. Therefore, the planner increases her cutoff R_{SP} and becomes more conservative in lending to firms. As a result, aggregate loans decrease. Secondly, we can see that

$$\frac{\partial \ell^{SP}}{\partial \xi} > 0 \tag{32}$$

The resource cost not only affects return of all assets, it distorts the relative cost of the two contracts. In the profit equation, note that ξ only multiplies the asset returns, $(1-P)\,R_{TL} + P\chi L_{TL} + R_{CL}$, but not the liabilities/cost, $(-P\mathcal{L} - R_D D)$. The cost of a term loan is the potential depreciation in the value of the asset through firm default. However, with a credit line, the cost is the opportunity cost of using funds to invest in other assets. Resource cost upon default affects the value of bank assets, term loans and credit lines, but it doesn't affect the cost of credit line drawdowns. If anything, it would amplify the cost because the drawn loans would lose value as well. Therefore, all else equal, resource costs makes credit lines less attractive to the planner. The market equilibrium will feature overprovision of credit lines relative to term loans.

Therefore, in the 2-period model we can theoretically characterize how the bank moral hazard friction affects loan decisions. There will be an oversupply of both loan types since both limited liability and default costs work in the same direction. However, when it comes to the contract choice, the two parts that create bank moral hazard work in opposite direction. This ambiguity on whether there is too much term loans relative to credit lines and vice versa, is investigated further with a quantitative model.

3.5 Implementing optimal policy

How do we implement the planner's allocation? A simple implementation in this two-period model is achieved through two regulatory constraints — a capital requirement and a loan commitment constraint. The capital requirement is a standard instrument studied in the literature and the main regulatory instrument in the real world. The loan commitment constraint is a new instrument I propose to limit the off-balance sheet exposure stemming from credit lines. The constraint aims to ensure that loan commitments are a certain fraction of the bank's own capital to minimize excessive risk.

$$\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi^{CR} \tag{33}$$

$$\frac{\mathcal{L}}{L_{TL} + L_{CL} - D} \leq \phi^{CC} \tag{34}$$

Only using a capital requirement, as in conventional model, would lead to risk shifting. Since the capital requirement only considers already issued loans and not commitments, raising it corrects for the level distortion of bank moral hazard, but would skew contracts towards credit lines. The loan commitment constraint helps correct the distortion to contract share. Intuitively, since the bank has two degrees of freedom in R^* and ℓ^*, the regulator also needs two degrees of freedom to implement the planner's allocation. In the following quantiative section, we investigate how tight these constraints should be in our world.

4 Quantitative model

We now extend the two period model into an infinite horizon general equlibrium environment. Time is discrete and infinite. Each period is divided into two subperiods: an investment stage and a drawdown stage, which correspond to the periods in the two-period model of Section 3. There are four types of agents in the economy: firms, a bank, households, and a government. Each period, the bank intermediates between firms who have profitable projects, and households who want to save. The government operates a deposit insurance fund with a balanced budget. All consumption occurs at the end of the period.

4.1 Environment

4.1.1 Firms

Each period t, a distribution of firms indexed by (i, j) are born. The firms are heterogeneous along two orthogonal dimensions: a 3-tuple $(k, B, \ell)_i$ of initial asset k_i, private benefit B_i, liquidity cost ℓ_i, drawn from a distribution $F(k, B, \ell)$, and project technology R_j drawn from $G(R)$. At the beginning of the investment stage, each firm starts with initial assets k_i and choose an investment size I_{ij} to a risky project that returns $R_j I_{ij}$ at the end of the model period with probability $p \in \{p_L, p_H\}$ and 0 with $1 - p$. In the drawdown stage, an aggregate state $P \in [0, 1]$ is drawn from $H(P)$; this P is common to all firms. Then, firms receive an iid liquidity shock of $\ell_i I_{ij}$ with probability P, or 0 with probability $1 - P$. The liquidity shock must be paid for the project to continue. If it is not paid for, there is a salvage value of χI_{ij}. If the liquidity shock is paid for, firms choose their project effort $p \in \{p_L, p_H\}$. There is no private information except for the hidden action choice p which is subject to moral hazard — the bank knows the full distributions of firm characteristics. Choosing low effort p_L gives firms private benefit $B_i I_{ij}$. At the end of the period, project returns are realized. Then, the period t cohort of firms dies, and a new identical cohort is born at the beginning of period $t + 1$, giving a stationary distribution of firms.

4.1.2 Households

An infinitely lived, risk neutral household with discount factor β is endowed with ω units of goods each period. The objective of the household is to maximize the presented discount value of consumption C. At the beginning of the period, households start with endowment ω and can save through either deposits D or riskless storage technology a. At the end of the period, households earn returns from deposit $R_D D$, storage technology $R_f a$, pay state-contingent lump sum taxes T, and earn dividends from the bank, own both the bank and firms, and consume C. The state-contingent tax T is used to cover deposit insurance for failing banks. While households own the bank, they pay for the resource costs of default ξ through deposit insurance - the bank does not internalize these resource costs.

4.1.3 Bank

An infinitely lived, risk neutral bank, with commitment technology and the same discount factor as households β, maximizes the presented discount value of dividends Div. At the beginning of the investment stage, the bank starts with capital K and may further choose to raise deposits D from households at interest rate R_D. Using capital K and deposits D, the bank issues dividends $Div > 0$, or raises equity from households $Div < 0$, and offers optimal loan contracts to each individual firm (i, j). L_{TL} denotes the aggregate amount of term loan contracts given to firms, and L_{CL} denotes the aggregate amount of credit line contracts. At the beginning of the drawdown stage, the bank also realizes the aggregate state P. Because the liquidity shock draws are iid to firms, the bank knows that exactly P share of firms received a liquidity shock. As per the credit line contract, the bank pays $P\ell I_{CL}$ to every firm that received credit line contracts; the aggregated amount is $P\mathcal{L}$, where $\mathcal{L}$ is the aggregate amount of liquidity the bank has committed. At the end of the period, the bank earns the returns from its investments L_{TL} and L_{CL}, which are $(1 - P) R_{TL} + P\chi R_S$ and R_{CL}, respectively. $R_{TL}(R_{CL})$ denotes the aggregate total returns from all term loan (credit line) contracts $L_{TL}(L_{CL})$, and R_S denotes the aggregate salvage value from liquidated firms. It pays depositors $R_D D$ and updates its next period capital K' .

Importantly, there is deposit insurance for bank deposits which generates moral hazard for the bank against households — households are guaranteed their deposits by the government even if the bank fails. This is an institutional feature of the real world that tries to address bank runs, as in Diamond and Dybvig (1983), which is unmodelled in this paper. Because of limited liability, the bank therefore has a convexified payoff. Furthermore, when the bank defaults, a fraction ξ of their assets are deadweight loss. The bank will choose an expected default probability that does not internalize the social cost of its default. If the realization of next period capital is negative $K' < 0$ and the bank cannot fully repay its depositors, it receives a value of 0. Once defaulted, the government steps in and takes over the bank and liquidates its assets to pay depositors.

Following Gomes (2001) and Jermann and Quadrini (2012), external financing by raising equity through $Div < 0$ has a convex cost. Specifically, I follow Dempsey (2020) by using a valuation function $\zeta(\cdot)$ that captures any direct and agency costs from issuing equity. This function is strictly increasing for all $Div \in \mathbb{R}$, strictly concave for negative $Div < 0$ with $\zeta'(Div) > 1$, but linear ($\zeta'(Div) = 1$) for positive $Div > 0$.

Finally, the bank is subject to a standard capital requirement which states that its equity, total assets minus total liabilities, must be some fraction $\phi^{CR} \in [0,1]$ of total loans.

$$\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi^{CR} \tag{35}$$

4.1.4 Government

The government operates a per-period balanced budget. When the bank enters default, government liquidates assets at a resource cost of ξ and pays depositors; remaining amount is drawn from deposit insurance fund. Deposit insurance is funded by levying a state-contingent lump sum tax T on households.

4.2 Timing

1. At the beginning of the investment stage, the bank starts with state variable capital K, and raises deposits D from households.

2. The bank issues dividends Div to households, and offers optimal contracts $\left(I, R^F, \iota\right)_{ij}$ to each individual firm (i, j).

3. Each firm starts with assets k_i, borrows $I_{ij} - k_i$ from bank, and invests in project size I_{ij}. The bank has given an aggregate term loan amount of L_{TL} , aggregate credit line amount of L_{CL} with $\mathcal{L}$ amount of total liquidity commitments.

4. At the beginning of the drawdown stage, aggregate state $P \in [0,1]$ is realized by all agents.

5. Firms receive liquidity shock $\ell_i I_{ij}$ with probability P and 0 with $(1-P)$ which is iid across firms.

6. In aggregate, exactly P share of firms received a liquidity shock $\ell_i I_{ij}$.

7. Of those who received a liquidity shock, firms who received a credit line contract get $\ell_i I_{ij}$ from the bank and continue projects; firms who received term loans liquidate their projects and receive salvage value χI_{ij}. The bank pay $P\mathcal{L}$ to firms in total.

8. Firms who continue exert project effort $p \in \{p_L, p_H\}$.

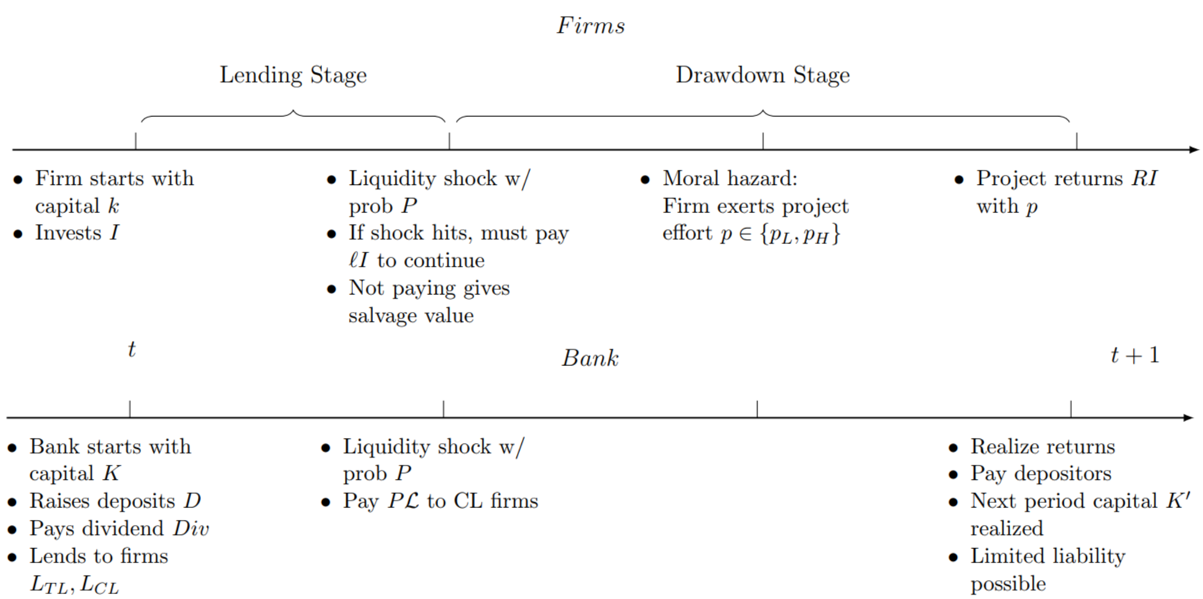
9. Project returns $R_j I_{ij}$ are realized with p. The bank receives a total of $(1-P)\, R_{TL} + \chi P R_S$ and R_{CL}.

10. If next period capital K' for the bank is negative, the bank defaults and is liquidated by the government. Otherwise, the bank continues to next period.

11. Households are paid deposits back at $R_D D$ and earn $R_f a$ from their riskless storage. Taxes T are levied if necessary.

12. Old firms die and new firms are born.

Figure 3: Timing: Firms and Bank



5 Equilibrium characterization

5.1 Decentralized equilibrium

5.1.1 Bank problem

The bank starts with capital K, and chooses dividends Div, contracts $\{\mathcal{C}_{ij}\}_{\forall ij}$, and deposits D to maximize profits. The value of the bank is

$$V^B(K) = \max_{\{Div, \{\mathcal{C}_{ij}\}_{\forall ij}, D\}} \zeta(Div) + \beta \mathbb{E} V^B(K') \tag{36}$$

s.t.

$$Div + L_{TL} + L_{CL} \leq D + K \tag{37}$$

$$K' = max\{0, (1-P) R_{TL} + \chi P R_S + R_{CL} - R_D D - P\mathcal{L}\} \tag{38}$$

$$\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi^{CR} \tag{39}$$

$$R_i^F \geq \frac{BI_{ij}}{\Delta p} \; \forall i, j \tag{40}$$

$$\begin{cases} \left(1-\bar{P}\right) p_H R_{ij}^F I_{ij} \geq k_i & \text{if } \mathcal{C}_{ij} \in \mathcal{C}_{TL} \\ p_H R_{ij}^F I_{ij} \geq k_i & \text{if } \mathcal{C}_{ij} \in \mathcal{C}_{CL} \end{cases} \tag{41}$$

where

$$L_{TL} \equiv \int_{\mathcal{C}_{TL}} (I_{ij} - k_i) \qquad L_{CL} \equiv \int_{\mathcal{C}_{CL}} (I_{ij} - k_i) \tag{42}$$

$$R_{TL} \equiv \int_{\mathcal{C}_{TL}} \left(R_j - R_{ij}^F\right) I_{ij} \qquad R_{CL} \equiv \int_{\mathcal{C}_{CL}} \left(R_j - R_{ij}^F\right) I_{ij} \tag{43}$$

$$\mathcal{L} \equiv \int_{\mathcal{C}_{CL}} \ell_i I_{ij} \tag{44}$$

5.1.2 Household problem

The problem of a representative houshold is

$$V^H = \max_{D,a,C} C + \beta \mathbb{E} V^H \tag{45}$$

s.t.

$$D + a \leq \omega \tag{46}$$

$$C \leq R_f a + R_D D - T + Div + V^F \tag{47}$$

Therefore, for households to hold bank deposits it must be that

$$R_D \geq R_f$$

and households' deposit supply function is

$$D_S = \begin{cases} 0 & \text{if } R_D < R_f \\ [0, \omega) & \text{if } R_D = R_f \\ \omega & \text{if } R_D > R_f \end{cases} \tag{48}$$

5.1.3 Government problem

The government operates a balanced budget. When the bank enters default, the government liquidates assets at a resource cost of ξ and pays depositors - remaining amount is drawn from deposit insurance fund. Deposit insurance is funded by taking lump sum tax T from household.

$$T = max\left\{0, R_D D - \xi\left[(1-P)\, R_{TL} + P\chi R_S + R_{CL} - P\mathcal{L}\right]\right\} \tag{49}$$

5.2 Definition of equilibrium

A *recursive general equilibrium* is a set bank functions $\left\{Div, \{\mathcal{C}_{ij}\}_{\forall ij}, D^D\right\}$, household functions $\left\{C, a, D^S\right\}$, and deposit rate R_D in aggregate state K such that

1. Given the deposit rate, $\left\{Div, \{\mathcal{C}_{ij}\}_{\forall ij}, D^D\right\}$ solves the maximization problem of the bank
2. Firm incentive compatibility (IC) and participation constraints (PC) are satisfied
3. Given the deposit rate, $\left\{C, a, D^S\right\}$ solves the maximization problem of households
4. Given the deposit rate, the government budget constraint holds

5. The deposit market clears: $D^D = D^S$

5.3 Constrained planner's problem

We consider a planner whose objective is to maximize household consumption. The planner is subject to the same frictions as private agents — we want to consider an equilibrium that is achievable through financial regulation. The key difference is that the planner internalizes the cost of bank default. The planner also chooses to extend optimal contracts to firms, but bears the negative profits and any resource costs. Equation 51 represents the resource constraint from the household, Equation 52 the resource constraint from financial intermediation, and Equations 53 and 54 show the cost of default and how capital is updated, respectively. The planner's recursive problem is

$$V^{SP}(K) = \max_{\left\{C, Div, D, \{\mathcal{C}_{ij}\}_{\forall ij}\right\}} C + \beta \mathbb{E} V^{SP}\left(K'\right) \tag{50}$$

s.t.

$$C \leq R_f(\omega - D) + R_D D - T + \zeta(Div) + V^F \tag{51}$$

$$Div + L_{TL} + L_{CL} \leq D + K \tag{52}$$

$$T = max\left\{0, R_D D - \left[\xi\left((1-P) R_{TL} + P\chi R_S + R_{CL}\right) - P\mathcal{L}\right]\right\} \tag{53}$$

$$K' = max\left\{0, (1-P) R_{TL} + P\chi R_S + R_{CL} - R_D D - P\mathcal{L}\right\} \tag{54}$$

6 Mapping the model to the data

In order to discuss counterfactual changes to macroprudential policies, I match the quantitative model with key "micro"-moments on firm heterogeneity and "macro"-moments on the aggregate banking industry. Matching the micro moments ensures that the bank is facing a distribution of firms similar to that in the data. The distribution of firm fundamentals disciplines what contracts the bank can give off-the-equilibrium path, crucial for counterfactuals. I do this by ensuring the heterogeneity in the quantity, price, and type of model contracts are consistent with the data. Matching macro moments ensures that the model bank has an accurate balance sheet and risk choices as observed in the data.

6.1 Data sources

In order to match the heterogeneity in the loan contracts given to firm, I use moments from FR Y-14Q H.1, which is a supervisory data set maintained by the Federal Reserve to assess capital adequacy and to support stress testing. The Y-14 data consists of information on loan facilities with $1 million in committed amount or more, held by bank holding companies (BHCs) subject to the Dodd-Frank Act Stress Tests. The advantages of the Y14 dataset are that coverage is wide and includes detailed loan level information to small firms; the supervisory data covers 60% of all corporate loans, including 50,000 SMEs. Using traditional datasets on bank loans that cover mostly large firms, such as Compustat/Capital IQ or DealScan, will incorrectly imply that most firms have credit lines and obscure the fact that loan contracts to smaller firms account for 1/3 of C&I lending by banks. Since I do not have direct access to this confidential supervisory data set, I construct and use moments from Chodorow-Reich et al. (2021) and Greenwald et al. (2021), two papers that utilize the full dataset. For bank moments, I use the FDIC's Consolidated Reports of Condition and Income ("Call reports") for commercial banks in the US. Since this model is specifically about investing in firms, I aggregate the banks to the bank holding company (BHC) level and only use commercial and industrial (C&I) loan numbers. The data sample period is 2012Q3 to 2020:Q3, unless otherwise noted. Detailed definitions of data and model moments are in Appendix B.

6.2 Estimation strategy

Model estimation occurs in two stages: an external calibration, where a subset of parameters are chosen outside the model, and an internal calibration, where parameters are chosen to match a set of moments in the data via simulated method of moments (SMM). The internal calibration is divided into two categories: micro firm parameters and macro bank parameters. Table 1 summarizes the baseline parameters of the model.

External calibration

The model period is one year. The discount factor that all agents in the economy share in common is β which I set to 0.99 to get a risk-free interest rate R_f of 1%. The household storage technology pins down the deposit rate that households are willing to accept; in equilibrium $R_D = R_f$, therefore, I set this to be the observed average bank deposit rate. I normalize the project success rates p_H and p_L to be

1 and 0, respectively[10]. Resource loss upon bank default is 20% and comes from FDIC data estimates related to the liquidation expense and cost of maintaining the FDIC deposit insurance fund. This gives resource loss value ξ of 0.8. Lastly, since the Basel II framework is already in place in the data period, the data reflects banks who are already constrained by an 8% capital requirement. Therefore, for the estimation we set $\phi^{CR} = 0.08$.

Micro firm parameters

These set of parameters rely on the Y14 data with heterogeneous contract information.We exploit the fact that the optimal contract is a mapping from the set of firm characteristics (k, B, ℓ) to the set of contract characteristics $\left(I, R^F, \iota\right)$. We use heterogeneous contract data to essentially back out the implied firm characteristics. In the Y14 data, the firm asset size distribution is heavily skewed, as it is in the universe of firms. Chodorow-Reich et al. (2021) and Greenwald et al. (2021) report that firms with asset size less than $250 million either do not have credit lines or have credit lines that cannot be reliably drawn following aggregate market shocks. I define *term loan firms* to be firms with asset size less than $250 million and consider them to not have credit lines. These firms account for 86% of the mass of firms, but account for approximately 2/3 of loans in the banking sector. On the other hand, firms with asset size greater than $250 million are defined as *credit line firms*. These firms are 14% of the mass of firms, account for 1/3 of bank loans, and have credit lines with banks that do insure against aggregate market shocks. For the estimation, I take the capital k distribution of firms to be a pareto distribution which is governed by two parameters: a scale parameter x_m and a shape parameter σ. The optimal contract shows that loan quantity is increasing in firm capital k. Therefore, we use the two parameters that govern the firm capital distribution to target loan sizes: total loans to term loan firms are $539 billion and total loans to credit line firms are $277 billion. For tractability, we assume that liquidity cost ℓ and moral hazard B are direct mappings of firm size k: $\ell = \ell_0 k^{\ell_1}$ and $B = B_0 k^{B_1}$. This gives a flexible functional form and asks the model to tell us the relationship between characteristics: positive/negative, extent of curvature, etc. I target the two parameters of the moral hazard function B_0, B_1 such that the average interest rate to term loan firms is 415 bp while the credit line firms get 378 bp (inclusive of any fees). For ℓ_0, ℓ_1, we use two moments. First, an estimate of the cost of the total unused commitments by banks. The Y14 data shows that banks have outstanding

[10] There is a technical restriction in these parameters. If $p_H = 1$ and $p_L = 0$ then the project effort is trivially verifiable from whether the project succeeded or not. As long as p_H, p_L are limiting to 1 and 0, this issue does not arise.

unused commitments of \$2.77 billion dollars in total. I take the opportunity cost of these loans to be the increase in spreads during these market downturns, which I set at 250bp. Second, the liquidity cost of the firms that receive term loans on the equilibrium path cannot be observed in the data because it is strictly off the equilibrium path. So instead we use the moment that only the top 14% mass of firms receive credit lines - we estimate the off equlibrium path costs to be such that the bank endogeneously chooses to give the correct mass of firms credit lines.

Macro bank parameters

This set of parameters relies on Call Reports. Since the aggregate liquidity shock P is a probability, which in the aggregate becomes the credit line drawdown rate, it must be a distribution that is bounded between $[0, 1]$. I use a bounded pareto distribution, which is heavily skewed left as are aggregate credit line drawdowns in the data. The raw average drawdown rate in the data during non-crisis times is 11%. I demean the series with this such that P represents only insurance against aggregate shocks, not normal times usage. For the three pareto distribution parameters α, H, L, I target the average of the demeaned drawdown series to be 3%, an expected bank failure rate of 1% as in the data, and a normalization of $H = 1$. The project technology distribution is taken to be uniform $R \sim U\left[0, \bar{R}\right]$ where $\bar{R}$ is chosen to match the banks overall leverage 0.92. I choose the functional form for the dividend valuation function as in Dempsey (2020),

$$\zeta\left(Div\right) = \begin{cases} 1 - exp\left(-Div\right) & \text{if } Div < 0 \\ Div & \text{if } Div \geq 0 \end{cases} \tag{55}$$

which is concave for negative $Div < 0$. The clean feature of this particular functional form from Dempsey (2020) is that it captures convex costs of equity issuance while still imposing smoothness at the potential kink at $Div = 0$, since $lim_{Div \to 0} \zeta\left(Div\right) = 1 = \zeta'\left(0\right)$. Another key parameter is the salvage rate χ, which determines how costly term loan defaults are relative to credit line payments. It is not realistic to assume that, for example, a 30% draw on credit lines will result in a 30% loss on term loans. Therefore we introduce some salvage value that makes term loan loss less costly. Using the Y-14 data, Greenwald et al. (2021) empirically show that banks who were harder hit with credit line draws issued fewer new loans to firms that were borrowing from them only through term loans without credit lines.

These firms who did not have credit lines, in turn, sharply decreased their capital expenditures. Using their regression estimates, I calculate that the 40% aggregate credit line drawdown during COVID 2020 was associated with a 13% drop in investment by term loan firms. I match salvage rate χ such that term loan losses equals this when the model is hit with the exact same magnitude shock. Finally, I set the household endowment ω such that total interest and dividend income to household total income matches the data moment from BEA estimates for Personal Income of interest and dividend income being 13.2% of pre-tax disposable income. This ensures that we capture household income that is not related to return on assets and ownership of the bank and firms, such as labor income, etc. Without this we would overestimate the welfare implications of financial regulation.

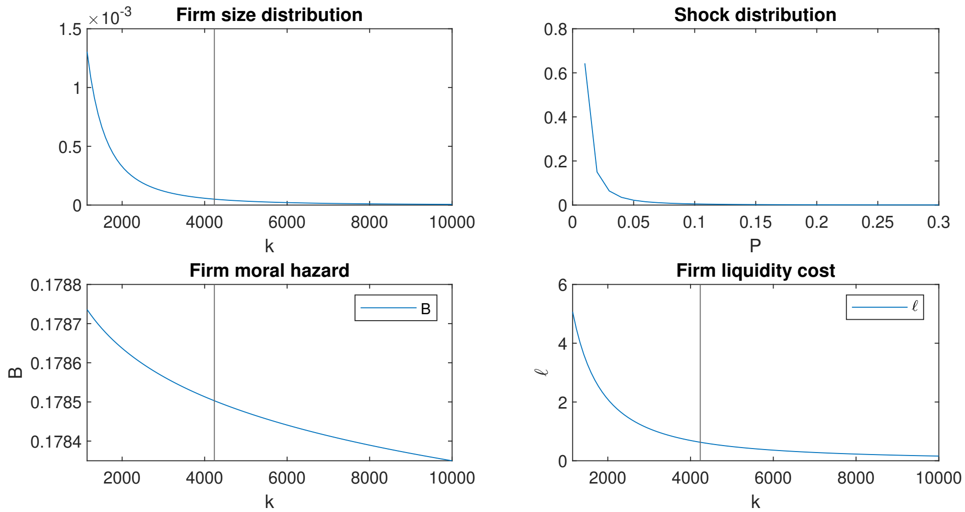
Table 1: Model parameters

Parameter	Value		Target	Data	Model
			Panel A: Micro firm parameters		
Firm size distribution k	x_m	1150	Total loans to TL firm	277	320
Firm size distribution k	σ	1.5	Total loans to CL firm	539	556
Moral hazard B	B_0	0.16	Average interest rate to TL firms	$415bp$	$500bp$
Moral hazard B	B_1	-0.001	Average interest rate to CL firms	$350bp$	$481bp$
Liquidity cost ℓ	ℓ_0	4×10^5	Share of firms: TL	84%	83%
Liquidity cost ℓ	ℓ_1	-1.651	Cost of unused commitments	72	68
			Panel B: Macro bank parameters		
Project quality distribution	$\bar{R}$	1.07	Bank leverage	0.92	0.90
Salvage rate	χ	0.5	TL loss in COVID	13%	14%
Shock distribution	α	1.04	Bank default rate	1%	0.95%
Shock distribution	L	0.01	Average excess drawdowns	3%	4%
Household endowment	ω	2932	Financial income/total assets	13.2%	13.2%
			Panel C: External calibration		
Discount factor	β	0.99	Risk-free rate	1%	1%
Household storage technology	R_f	1.01	Average deposit rate	–	–
Project success rates	(p_H, p_L)	$(1, 0)$	Normalized	–	–
Resource loss	ξ	0.2	FDIC liquidation cost	–	–
Capital requirement	ϕ^{CR}	0.08	Basel II baseline	–	–

Notes: Moments are in $ billions, unless otherwise noted.

In Figure 4 I show the estimated firm distribution and the relationship between firm characteristics. The model correctly picks up the inverse relationship between firm size k and firm liquidity cost ℓ as observed in the data. The vertical lines represent the endogenous cut-off value of contract types — left of the line get term loans and right of the line get credit lines.

Figure 4: Estimated firm distribution



7 Quantitative results

7.1 How much do credit lines affect bank risk?

In crisis times, the bank draws a high aggregate drawdown rate P. This aggregate shock causes losses on both the bank's term loan and credit line portfolios. Specifically, losses on term loans come because only $(1-P)$ share of firms survive and return R_{TL}, while P share of firms liquidate and only return a salvage value of χR_S. On credit lines, the bank always get R_{CL}, but has to pay out $P\mathcal{L}$. How much of its total losses comes from each contract type? We can rearrange the total returns from term loans to be:

$$\begin{aligned}&(1-P)\,R_{TL}+P\chi R_S\\ =&R_{TL}-P\left(R_{TL}-\chi R_S\right)\end{aligned} \tag{56}$$

where the first term represents a certain return from term loans, and the second term represents the variable losses from term loans. We can now easily compare the potential losses from term loans, $R_{TL}-\chi R_S$, with the potential losses from credit lines, $\mathcal{L}$. We find that term loans account for 80% of potential losses and credit lines account for 20% of potential losses. Since these losses are both linear in aggregate P, the decomposition stays the same for any realization P. When the bank defaults, total losses to the bank in dollars are \$83.5 million dollars, of which \$59.9 million dollars for term loans and \$23.6 million dollars from credit lines.

The above decomposition is using steady state values, that is, it answers what happens when the economy is in good times and gets hit with a large shock. Now we ask the opposite question, what does bank fragility look like as the bank experiences a series of good shocks? Interestingly, the model predicts that ex-ante bank default rate actually increases in good times. That is, financial fragility increases as risk builds up in good times. Specifically, the exercise we perform is to assume that the economy is off the steady state equilibrium because of large P shocks. Then, the bank receives a sequence of very low P shocks simulating good times. In Figure 5, we follow the transition of a bank with low capital $K_0 = 1$ and feed it a sequence of the average value of P. As the bank transitions back to the steady state, we see that it slowly builds up capital — it takes about 7 years to return to the steady state. The bank mostly builds up capital through retained earnings and only issues small amounts of equity due to costly issuance. During this time, the bank is borrowing constrained by the capital requirement and therefore must increase deposits slowly as well. As capital and deposits increase, the bank is able to lend more to firms and does so by increasing the issuance of both term loans and credit lines. In the left most panels in Figure 5, we see that as bank capital increases, and deposits and loans increase, the threshold P^* at which the bank would default decreases — the bank becomes increasingly susceptible to smaller P shocks. As a result, ex-ante bank default probability increases. This risk build up is quantitatively significant. We see that in the first several years the size of P shock needed to default the bank is much greater than 1, meaning bank default rate is 0%. As risk builds up, the threshold P^* eventually reaches the steady state value of 48%. What is driving this increasing financial fragility in good times? We examine this by constructing threshold P^* values for the term loan portfolio and credit line portfolio separately. That is, total bank threshold is

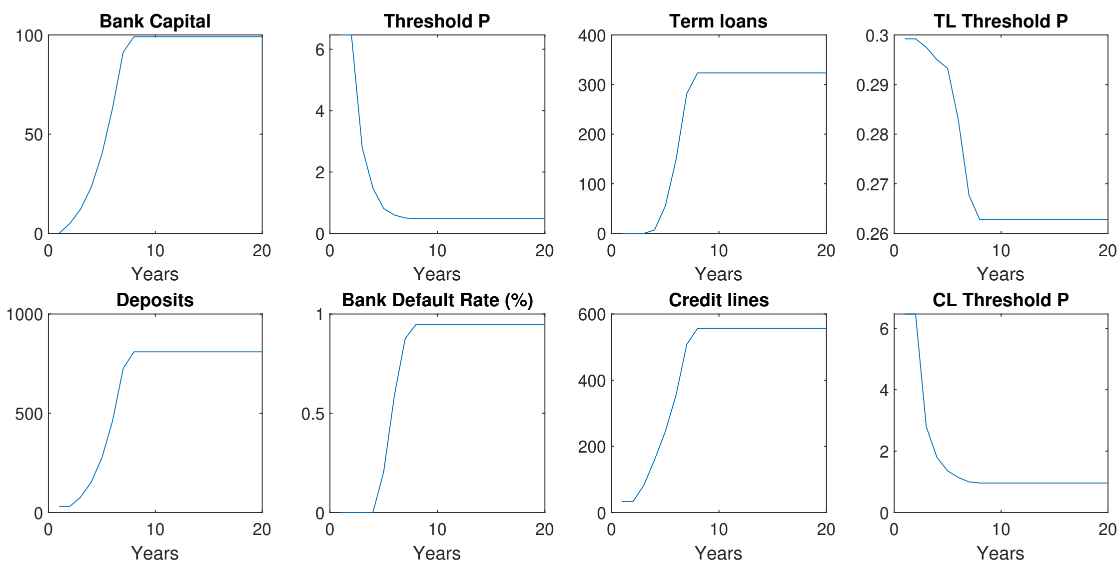
$$P^* = \frac{R_{TL} + R_{CL} - R_D D}{R_{TL} - \chi R_S + \mathcal{L}}$$

and the separated thresholds are

$$P^*_{TL} = \frac{R_{TL} - R_D D \frac{L_{TL}}{L_{TL}+L_{CL}}}{R_{TL} - \chi R_S} \qquad P^*_{CL} = \frac{R_{CL} - R_D D \frac{L_{CL}}{L_{TL}+L_{CL}}}{\mathcal{L}}$$

The exercise here is to pretend the term loans and credit lines are separate banks, and we split the liabilities according to relative portfolio size. In the right most panels of Figure 5, we make several observations. Term loans are much riskier — they default at P shocks of around 26% to 30%. However, the term loan riskiness does not change much. As the bank grows, the threshold only decreases from 26% to 30%. However, credit lines are the opposite. They start out very safe at a threshold of over 600%, but decreases rapidly to the steady state value of 95%. This means that in level terms term loans are consistently the riskiest part of the portfolio — they default easily. On the other hand, credit lines are very good at weathering small shocks, but are susceptible to big shocks. This is why credit lines only become a big liability to banks in big crises, not over the regular business cycle. Much of this feature is because banks select large firms with low cost to insure. However, in the transition from receiving series of good shocks, it is the credit line risk increasing dramatically that drives the overall bank default rate from going up from 0% to near 1%.

Figure 5: Increasing bank fragility in good times



Why is this happening? Recall the revenue of credit lines is R_{CL}, while its potential costs are $\mathcal{L}$, the total loan commitments. We see that R_{CL} is increasing at a decreasing returns to scale as does R_{TL}. However, the total loan commitment $\mathcal{L}$ is increasing at a constant rate. As the bank lends more to marginally lower project quality R_j, because of the orthogonality of the project quality dimension and the firm characteristics dimension, the marginal increasing in distribution of liquidity costs ℓ stays constant. At this point, it may seem the orthogonality assumption affects this result. However, we see that if we added correlation between the project quality R_j and liquidity cost ℓ, that correlation should be negative, that is, lower quality firms should have higher liquidity cost ℓ. That would make loan commitments $\mathcal{L}$ actually increasing returns to scale, further exacerbating the drop in revenue to cost for credit lines. Therefore, we see that there is build up of financial fragility in the model that is driven by credit lines, and that this is actually a conservative estimate given the orthogonality assumption.

7.2 Are we safer without credit lines?

By insuring firms through credit lines, the bank is assuming aggregate risk for firms - firm default risk is transferred to the bank. Does this then mean that eliminating credit lines will decrease bank default risk? If one eliminates credit lines, the bank will adjust and make new choices. Therefore, we need to perform an equilibrium analysis of such a counterfactual. In Table 2 we see the model moments from the baseline model compared to a counterfactual economy in which credit lines are eliminated and the bank can only lend through term loan contracts. The first thing we see is that the bank increases term loans by substituting the credit line lending with term loans. However, it does not substitute completely as total loans decreases by \$100 million. The bank borrows less to finance this smaller amount of lending by decreasing deposits by \$110 million. Because the term loan quantity increases, the interest rate on term loans decreases following the demand curves derived from the optimal contracts. What is striking is that bank default rate actually increases - it doubles from 0.95% to 2.1%. This is because, while the bank does not have credit line exposure, only lending through term loans increases the firm default rate. This increase in the firm default rate drives a higher bank default rate. Output decreases from the decreased lending, and welfare decreases even further because of lower output and higher taxes from higher bank default rates.

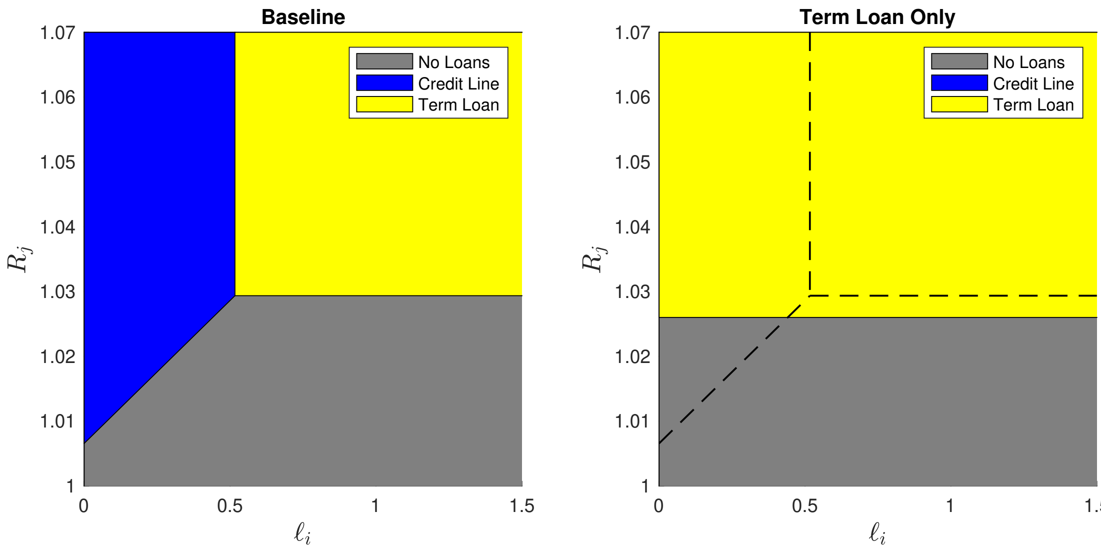
Table 2: Counterfactual equilibrium: Term loans only

	Baseline	Term loan only
Term Loans L_{TL}	320	651
Credit Lines L_{CL}	556	–
Total loans	876	651
Deposits D	813	505
Credit line exposure $\mathcal{L}$	68	–
TL interest rate r_{TL}	500bp	505bp
CL interest rate r_{CL}	481bp	–
Capital ratio	8%	8%
Bank default rate	0.95%	2.1%
Firm default rate	3.5%	4.1%
Share of firms: Term loans	83%	100%
Output Y	6439	4714
Welfare (c.e.)	–	−13%

Notes: Values are in $ billions. Total loans are the sum of term loans and credit lines. Welfare is calculated in consumption equivalence.

What is causing the substitution effect to dominate and increase bank default rate? Table 2 shows the aggregate moments, but we can look at the firm distribution through Figure 6. First thing we notice is that the formerly blue region of firms with credit lines are now partially replaced with term loans. These firms were of low liquidity cost ℓ type and were relatively cheap to insure for the bank. Now, the bank isn't allowed to cheaply insure these loans for the firms and must let these projects fail from liquidity shocks. Instead of scaling back lending from this increased risk, the bank instead doubles down by increasing loans. We see that the project quality cut off R^* is lower in the counterfactual, meaning the bank is reaching deeper into the pool of firms and lending to less efficient firms now. Furthermore, the lower triangular region from the formerly credit line firms are now unfunded. Those firms were not very efficient, but their cost of insurance was also very low. Now that the bank can't insure them, their inefficiency becomes relevant again.

Figure 6: Firm types and optimal contracts: Term loan only



7.3 The planner's allocation

Given that a term loan only economy has lower welfare and a higher default rate, what then is the efficient allocation? In Table 3 we again see the baseline economy, now compared to the planner's allocation from the earlier planner's problem. Again, the planner is subject to the same frictions as private agents, but chooses the bank's allocations by maximizing household welfare. This means the key distinction is that the planner internalizes the fiscal cost of bank default through taxes. The planner decreases both the amount of lending through term loans, by 6%, and credit lines, by 7%. The planner thinks the decentralized bank is overlending in credit lines a bit more than through term loans, but there doesn't seem to be large differences, suggesting that the bank is largely able to understand the risk coming from each type of contract. The larger channel is the quantity channel through which the bank is lending too much overall, by 12%. However, the 7% decrease in credit lines masks the fact that the planner wants to decrease credit line exposures by 11%. Because the bank optimally selects the lower liquidity cost firms first, the marginal decrease in credit lines will come first for the firms with the largest liquidity costs. Therefore decreasing the credit line quantity does not result in a linear decrease in potential losses. The interest rate on both term loans and credit lines increase following the aggregate demand curves since lending is decreased. The planner wants less lending, but also wants the funding composition of the loans to be more capital intensive. Deposits decrease by 13% and the capital ratio increases from 8% to 11%. This increase in the capital ratio helps the bank half its default rate from 1% to 0.5%. The decrease in the amount of credit lines being larger than the decrease in term loans is reflected in the fact that the share of firms who receive term loans increases from 85% to 86%. This causes the firm default rate to increase slightly from 3.3% to 3.34%. Output and welfare increase by 0.2% largely because the tax costs to household from bank default is going down.

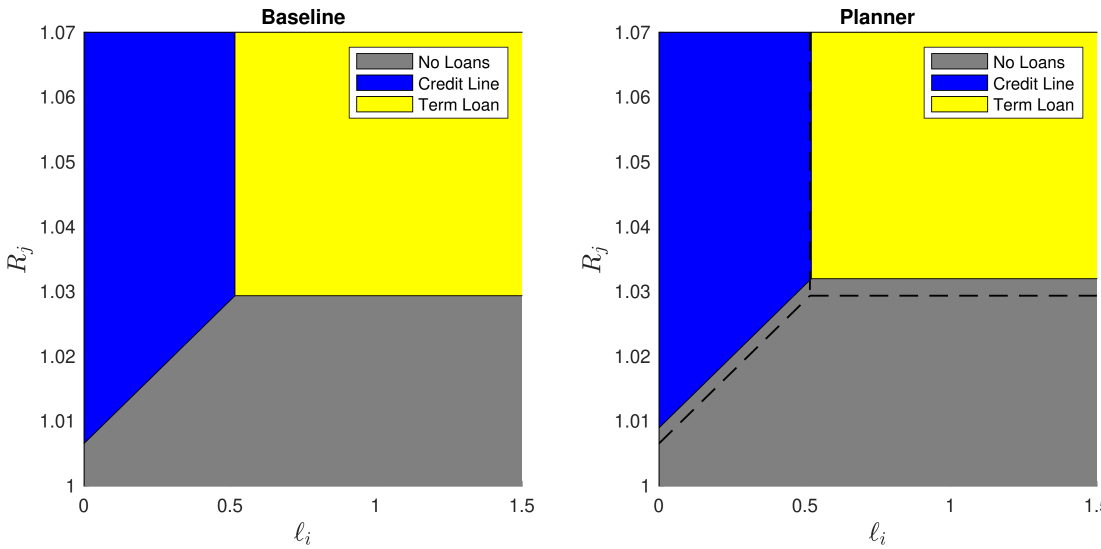
Table 3: Planner's allocation

	Baseline	Planner
Term Loans L_{TL}	320	301
Credit Lines L_{CL}	556	534
Total loans	876	835
Deposits D	813	745
Credit line exposure $\mathcal{L}$	68	66
TL interest rate r_{TL}	500bp	518bp
CL interest rate r_{CL}	481bp	496bp
Capital ratio	8%	11%
Bank default rate	0.95%	0.56%
Firm default rate	3.5%	3.3%
Share of firms: Term loans	83%	83%
Output Y	6439	6440
Welfare (c.e.)	–	+0.02%

Notes: Values are in $ billions. Total loans are the sum of term loans and credit lines. Welfare is calculated in consumption equivalence.

We can again look at the firm distribution to see how contracts are changing in Figure 7. We see that the vertical line that denotes the cut off value for ℓ^* shifts to the left for the planner. This means that the firms at the margin of the contract types that are receiving credit lines are viewed by the planner as not worth the risk given their high liquidity cost; they have, however, efficient projects and should still receive funding through term loans. We also see that the planner moves up the triangular bottom portion of the credit line region. Similar to firms at the margin of contract types, the firms at the boundary of receiving credit lines and no loan shifts and fewer firms are funded. These firms have low enough liquidity costs, but the planner views their project quality to be too low. Lastly, the project quality cut off R^* also moves up as the planner thinks the low project quality firms do not return enough in positive states. While the term loan region increases along the firm characteristic dimension, it decreases along the project quality dimension and ends up decreasing in the aggregate. The credit line region decreases along both dimensions and unambiguously decreases in total.

Figure 7: Firm types and optimal contracts: Planner's allocation



7.4 Optimal macropudential policy

How do we get to the planner's allocation in a market economy? We can decentralize the allocation by modifying the existing framework of macroprudential policy. We first show how to fully achieve the planner's allocation, and second, show how to maximize welfare when the set of policy tools at our disposal is limited.

The planner's allocation can be achieved through three instruments: a capital requirement, a leverage ratio, and a loan commitment constraint. The first two instruments are policies already in place under the Basel III framework. The capital requirement dictates how much capital the bank must have as a fraction of its total balance sheet. The leverage ratio constrains how much debt a bank holds relative to its capital stock. Intuitively, together these two constraints help pin down the total lending $L_{TL} + L_{CL}$ and total borrowing D of the bank to match that of the planner's. For the third instrument we propose a loan commitment constraint: a constraint on how much off-balance sheet exposure the bank has relative to its capital stock. This last constraint helps the planner split the total lending $L_{TL} + L_{CL}$ into the appropriate shares of L_{TL}, L_{CL}.

$$\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi^{CR} \tag{57}$$

$$\frac{D}{L_{TL} + L_{CL} - D} \leq \phi^{LR} \tag{58}$$

$$\frac{\mathcal{L}}{L_{TL} + L_{CL} - D} \leq \phi^{CC} \tag{59}$$

However, in the real world, the political process of passing legislation and introducing new regulatory policy, such as a loan commitment constraint, can be quite difficult and lengthy. We show that even if we limit ourselves to just a single instrument, a risk-weighted capital requirement, we are able to achieve 95% of the planner's welfare gains if implemented correctly.

$$\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL} + \theta^{LC}\mathcal{L}} \geq \phi^{CR} \tag{60}$$

Table 4: Optimal policy parameters

	Full implementation	Single instrument
Capital requirement ϕ^{CR}	11%	12%
Leverage requirement ϕ^{LC}	8%	–
Commitment constraint ϕ^{CC}	66	–
Risk-weight θ^{LC}	–	0.1

Notes: Optimal parameters are found through grid searching to maximize steady state welfare.

When constrained to only the risk-weighted capital requirement, the capital requirement is set higher than when using all three instruments (11% vs. 12%). This is because the capital requirement is used, inefficiently, to curb risk-shifting that occurs through leverage and credit lines. To be more concrete, when only the capital requirement is set at 11%, as in the full implementation, without setting leverage requirements and commitment constraints, then the bank tries to undo the decreasing default probability through other margins. Specifically, it increases leverage by increasing deposits and increasing the relative share of credit lines. Therefore, to compensate for this, the optimal policy increases capital requirements by a full percent. The regulator then uses the risk weight on loan commitments θ^{LC} to correct the share of credit lines vs. term loans. However, we see that this risk-weight is not very large — a weight of $\theta^{LC} = 0.1$ on $\mathcal{L}$, which is the opportunity cost on undrawn credit lines, translated into a weight of 0.0025 in the Basel III framework. This shows that the distortionary effects of risk-weights actually outweighs any inefficient substitution into credit lines. This is in line with the difference in the baseline equilibrium and the planner equilibrium in which the relative share of contracts is not very different — as we already saw, the decentralized bank is already setting the efficient *share* of contracts in equilibrium, but *over* lending in both.

7.5 Optimal policy from term loan only models

How can policy be inaccurate if we do not consider models with credit lines? To create the most apples-to-apples comparison, we conduct the following exercise. First, we take our model and shut down credit lines: the bank is only allowed to lend through term loan contracts. This model has the same mechanisms as conventional banking models: the bank takes excessive risk through overleveraging term loans, and the main risk comes from non-performing loans (term loan defaults). We take this model and re-estimate to match the same data moments as our benchmark model (estimation results in table B.1 of Appendix B). Then, using the newly estimated parameters, we solve the same planner problem, but one in which the planner also thinks the economy features only term loans. When we compare the optimal capital ratio that comes from this term loan only model, we find that it significantly underpredicts what the optimal capital ratio should be from 11.2% to 8.2%, and is remarkable close to the current captial requirement of 8%. This might lead policymakers to think that the current capital ratio is sufficient.

Table 5: Optimal policy across models

	Optimal capital ratio
Basel III	8%
Model with both contracts	11.2%
Model with only term loans	8.2%

Notes: Optimal parameters are found through grid searching to maximize steady state welfare.

Why is the model underpredicting the optimal capital ratio? In both models, bank default is socially costly and the planner would like to avoid it. Both models are calibrated to the same data moments, so both models feature 1% default probability. The planner can curb this costly 1% default by increasing capital ratios. What is the cost of capital ratios though? With higher capital ratios, this increases the cost to financing loans — a well known phenomenon in the literature. Therefore the planner must trade off lower profits against lower defaults. It is here where the models diverge. This borrowing constraint, in the form of capital requirements, is worse for term loans because, all else equal, term loans are larger than credit lines. This is because the project size of term loans are larger to satisfy firms particpation constraint under term loans. Therefore, term loans “fill” the banks balance sheet quicker than credit lines do. With a term loan only model, the planner would think raising capital requirements will capacity constrain lending and bank profits more than if banks issued credit lines. Thus while the planner would like higher capital ratios to lower default risk, the negative impact it would have on bank lending and profits is calculated to be much higher. That is, through model mispecification, the planner overestimates the costs of increasing capital ratios.

8 Conclusion

This paper studied how much providing credit lines to firms contributes to bank risk and its welfare implications. I explicitly modelled endogenous contract choice such that credit line contracts have a purpose in the economy, but may contribute to socially excessive risk taking by the bank. I show that term loans account for 80% of bank losses in crisis times and credit lines account for 20%. While term loans are always risky in level terms, increasing financial fragility in good times is driven by increasing credit line risk. In normative exercises, I show how regulators can implement the constrained efficient allocations, and how model mispecification can lead to inaccurate policy.

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A Proofs and derivations

Lemma 1

Proof. Suppose the splitting rule R^F is state-contingent, i.e., R^F differs on the realization of the liquidity shock. Denote R_0^F return to firm when liquidity shock is 0 and R_ℓ^F return to firm when liquidity shock is ℓ. Then, the bank's problem is

$$\max_{I,R_0^F,R_\ell^F} V_{CL} = Pp_H\left(R - R_\ell^F\right)I + (1-P)\,p_H\left(R - R_0^F\right)I - (I-k) - P\ell I$$

Firm incentive compatibility constraint (IC) are

$$R_0^F \geq \frac{B}{\Delta p}$$

$$R_\ell^F \geq \frac{B}{\Delta p}$$

Firm participation constraint (PC) is

$$Pp_H R_\ell^F I + (1-P)\,p_H R_0^F I \geq k$$

We want to investigate the contract in which the bank receives more in the shock state so that it is naturally encouraged to continue the project and less if it doesn't have to pay the liquidity shock. Therefore, $R_\ell^F < R_0^F$. Then, it follows that the first IC constraint is slack and the second IC constraint is binding. Since the PC binds, this gives

$$\begin{aligned}
& Pp_H R_\ell^F I + (1-P)\,p_H R_0^F I = k \\
\iff & Pp_H \frac{B}{\Delta p} I + (1-P)\,p_H R_0^F I = k \\
\iff & Pp_H \frac{B}{\Delta p} I + (1-P)\,p_H R_0^F I = k \\
\iff & P\frac{B}{\Delta p} + (1-P)\,R_0^F = \frac{k}{p_H I}
\end{aligned}$$

we see that to satisfy the PC constraint with equality, the bank has to choose R_0^F such that lower R_0^F leads to higher project size I. Since the bank's objective function is increasing in project size I and decreasing in R_0^F, it must be that the bank lowers R_0^F as much as possible and both IC binds. If both IC binds, it must be that the return to firm is not state-contingent. $\square$

Lemma 2

Proof. Since the liquidity shock takes values of either ℓ or 0, any transfer $\hat{\ell} < \ell$ is not sufficient to continue the project. Therefore contracts that offer such underinsurance will only be costly to the bank without changing any outcomes. Similarly for $\hat{\ell} > \ell$, the additional transfer beyond the continuation cost does not affect firm outcomes and only increases firm returns so will not be offered by the bank. Therefore the only values $\hat{\ell}$ can take are either 0 or ℓ in equilibrium. $\square$

Lemma 3

Proof. The optimal term loan contract solves,

$$\max_{I_{TL}, R_{TL}^F} V_{TL} = (1-P)\, p_H \left(R - R_{TL}^F\right) I_{TL} + P\chi I_{TL} - (I_{TL} - k)$$

Firm incentive compatibility constraint (IC) is

$$R_{TL}^F \geq \frac{B}{\Delta p}$$

Firm participation constraint (PC) is

$$(1-P)\, p_H R_{TL}^F I_{TL} \geq k$$

It must be that both IC and PC are binding. Proof is by contradiction in considering the alternative cases.

1. **Neither binds**: Bank can decrease R_f by ε and both constraints remain slack, but objective function increases. Contradiction.

2. **IC binds, but PC is slack**: Assume that

$$R_{TL}^F = \frac{B}{\Delta p}$$

$$(1-P)\, p_H R_{TL}^F I_{TL} \geq k$$

then objective becomes

$$\begin{aligned}
\max_{I_{TL}, R_{TL}^F} V_{TL} &= (1-P)\, p_H \left(R - R_{TL}^F\right) I_{TL} + P\chi I_{TL} - (I_{TL} - k) \\
&= (1-P)\, p_H \left(R - \frac{B}{\Delta p}\right) I_{TL} + P\chi I_{TL} - (I_{TL} - k) \\
&= \left[(1-P)\, p_H \left(R - \frac{B}{\Delta p}\right) - 1 + P\chi\right] I_{TL} + k
\end{aligned}$$

From Assumption 1, we know that

$$\begin{aligned}
& p_H \left(R - \frac{B}{\Delta p}\right) < min\left\{1 + P\ell, \frac{1 - P\chi}{1 - P}\right\} \\
\Longleftrightarrow & p_H \left(R - \frac{B}{\Delta p}\right) < \frac{1 - P\chi}{1 - P} \\
\Longleftrightarrow & (1-P)\, p_H \left(R - \frac{B}{\Delta p}\right) - 1 + P\chi < 0
\end{aligned}$$

Therefore, the optimal $I_{TL} = 0$, but this contradicts PC being slack.

3. **PC binds, but IC is slack**: Assume that

$$R_{TL}^F \geq \frac{B}{\Delta p}$$

$$(1-P)\, p_H R_{TL}^F I_{TL} = k$$

then,

$$R_{TL}^F = \frac{k}{(1-P)\, p_H I_{TL}}$$

therefore objective becomes

$$\begin{aligned}
\max_{I_{TL},R^F_{TL}} V_{TL} &= (1-P)\,p_H\left(R-R^F_{TL}\right)I_{TL}+P\chi I_{TL}-(I_{TL}-k)\\
&= (1-P)\,p_H\left(R-\frac{k}{(1-P)\,p_H I_{TL}}\right)I_{TL}+P\chi I_{TL}-(I_{TL}-k)\\
&= ((1-P)\,p_H R I_{TL}-k)+P\chi I_{TL}-(I_{TL}-k)\\
&= [(1-P)\,p_H R-1+P\chi]\,I_{TL}
\end{aligned}$$

From Assumption 1, we know that

$$\begin{aligned}
&min\left\{1+P\ell,\frac{1-P\chi}{1-P}\right\}<p_H R\\
\iff&\frac{1}{1-P}<p_H R\\
\iff&0<(1-P)\,p_H R-1
\end{aligned}$$

Therefore, the optimal $I_{TL}=\infty$, but then binding PC gives $R^F_{TL}=0$, which then contradicts IC being slack. Therefore, it must be that both constraints are binding, giving

$$R^F_{TL}=\frac{B}{\Delta p}$$

$$I_{TL}=\frac{k}{(1-P)\,p_H\frac{B}{\Delta p}}$$

Similary, the optimal credit line contract solves,

$$\max_{I_{CL},R^F_{CL}} V_{CL}=p_H\left(R-R^F_{CL}\right)I_{CL}-(I_{CL}-k)-P\ell I_{CL}$$

Firm incentive compatibility constraint (IC) is

$$R^F_{CL}\geq\frac{B}{\Delta p}$$

Firm participation constraint (PC) is

$$p_H R^F_{CL} I_{CL}\geq k$$

PIt must be that both IC and PC are binding. Proof is by contradiction in considering the alternative cases.

1. **Neither binds**: Bank can decrease R_f by epsilon and both constraints remain slack, but objective function increases. Contradiction.

2. **IC binds, but PC is slack**: Assume that

$$R_{CL}^F = \frac{B}{\Delta p}$$

$$p_H R_{CL}^F I_{CL} \geq k$$

then objective becomes

$$\begin{aligned}\max_{I_{CL},R_{CL}^F} V_{CL} &= p_H \left(R - R_{CL}^F\right) I_{CL} - (I_{CL} - k) - P\ell I_{CL} \\ &= p_H \left(R - \frac{B}{\Delta p}\right) I_{CL} - (I_{CL} - k) - P\ell I_{CL} \\ &= \left[p_H \left(R - \frac{B}{\Delta p}\right) - 1 - P\ell\right] I_{CL} + k\end{aligned}$$

From Assumption 1, we know that

$$\begin{aligned}&p_H \left(R - \frac{B}{\Delta p}\right) < min \left\{1 + P\ell, \frac{1 - P\chi}{1 - P}\right\} \\ \Longleftrightarrow &p_H \left(R - \frac{B}{\Delta p}\right) < 1 + P\ell \\ \Longleftrightarrow &p_H \left(R - \frac{B}{\Delta p}\right) - 1 - P\ell < 0\end{aligned}$$

Therefore, the optimal $I_{TL} = 0$, but this contradicts PC being slack.

3. **PC binds, but IC is slack**: Assume that

$$R_{CL}^F \geq \frac{B}{\Delta p}$$

$$p_H R_{CL}^F I_{CL} = k$$

then,

$$R_{CL}^{F} = \frac{k}{p_H I_{CL}}$$

therefore objective becomes

$$\begin{aligned}
\max_{I_{CL}, R_{CL}^{F}} V_{CL} &= p_H \left(R - R_{CL}^{F}\right) I_{CL} - (I_{CL} - k) - P\ell I_{CL} \\
&= p_H \left(R - \frac{k}{p_H I_{CL}}\right) I_{CL} - (I_{CL} - k) - P\ell I_{CL} \\
&= (p_H R - 1 - P\ell) I_{CL}
\end{aligned}$$

From Assumption 1, we know that

$$\begin{aligned}
& min\left\{1 + P\ell, \frac{1 - P\chi}{1 - P}\right\} < p_H R \\
\Longleftrightarrow & 1 + P\ell < p_H R \\
\Longleftrightarrow & 0 < p_H R - 1 - P\ell
\end{aligned}$$

Therefore, the optimal $I_{CL} = \infty$, but then binding PC gives $R_{CL}^{F} = 0$, which then contradicts IC being slack.

Therefore, it must be that both constraints are binding, giving

$$R_{CL}^{F} = \frac{B}{\Delta p}$$

$$I_{CL} = \frac{k}{p_H \frac{B}{\Delta p}}$$

□

Lemma 4

Proof. Simply comparing the two contracts gives,

$$
\begin{aligned}
& V_{CL}\left(I_{CL}, R_{CL}^F\right) > V_{TL}\left(I_{TL}, R_{TL}^F\right) \\
\Longleftrightarrow & p_H\left(R - R_{CL}^F\right) I_{CL} - \left(I_{CL} - k\right) - P\ell I_{CL} > (1-P)\, p_H\left(R - R_{TL}^F\right) I_{TL} + P\chi I_{TL} - \left(I_{TL} - k\right) \\
\Longleftrightarrow & \left[p_H\left(R - \frac{B}{\Delta p}\right) - 1 - P\ell\right] I_{CL} + k > \left[(1-P)\, p_H\left(R - \frac{B}{\Delta p}\right) - 1 + P\chi\right] I_{TL} + k \\
\Longleftrightarrow & \left[p_H\left(R - \frac{B}{\Delta p}\right) - 1 - P\ell\right] \frac{k}{p_H \frac{B}{\Delta p}} + k > \left[(1-P)\, p_H\left(R - \frac{B}{\Delta p}\right) - 1 + P\chi\right] \frac{k}{(1-P)\, p_H \frac{B}{\Delta p}} + k \\
\Longleftrightarrow & (1-P)\,\ell < 1 - \chi \\
\Longleftrightarrow & \ell < \frac{1-\chi}{1-P}
\end{aligned}
$$

$\square$

Lemma 5

Proof. Using the optimal contract terms, we see that,

$$
I_{TL} = \frac{k}{(1-P)\, p_H \frac{B}{\Delta p}} > \frac{k}{p_H \frac{B}{\Delta p}} = I_{CL}
$$

and

$$
\begin{aligned}
r_{TL} &= \frac{p_H\left(R - R_{TL}^F\right) I_{TL}}{I_{TL} - k} \\
&< \frac{p_H\left(R - \frac{B}{\Delta p}\right)}{1 - p_H \frac{B}{\Delta p}} \\
&= r_{CL}
\end{aligned}
$$

similarly,

$$
\frac{\partial\left(I_{TL} - k\right)}{\partial k} = \frac{1}{(1-P)\, p_H \frac{B}{\Delta p}} - 1 > 0
$$

$$\frac{\partial\left(I_{CL}-k\right)}{\partial k}=\frac{1}{p_H\frac{B}{\Delta p}}-1>0$$

$$\frac{\partial\left(I_{TL}-k\right)}{\partial B}=\frac{-\left(1-P\right)p_H\frac{1}{\Delta p}k}{\left(\left(1-P\right)p_H\frac{B}{\Delta p}\right)^2}<0$$

$$\frac{\partial\left(I_{CL}-k\right)}{\partial B}=\frac{-p_H\frac{1}{\Delta p}k}{\left(p_H\frac{B}{\Delta p}\right)^2}<0$$

and

$$\begin{aligned}\frac{\partial r_{TL}}{\partial B}&=\frac{-\frac{p_H}{\Delta p}\left[1-\left(1-P\right)p_H\frac{B}{\Delta p}\right]+p_H\left(R-\frac{B}{\Delta p}\right)\left[\left(1-P\right)\frac{p_H}{\Delta p}\right]}{\left(1-\left(1-P\right)p_H\frac{B}{\Delta p}\right)^2}\\&=\frac{\frac{p_H}{\Delta p}\left[\left(1-P\right)p_H R-1\right]}{\left(1-\left(1-P\right)p_H\frac{B}{\Delta p}\right)^2}>0\end{aligned}$$

$$\begin{aligned}\frac{\partial r_{CL}}{\partial B}&=\frac{-\frac{p_H}{\Delta p}\left[1-p_H\frac{B}{\Delta p}\right]+p_H\left(R-\frac{B}{\Delta p}\right)\left[\frac{p_H}{\Delta p}\right]}{\left(1-p_H\frac{B}{\Delta p}\right)^2}\\&=\frac{\frac{p_H}{\Delta p}\left[p_H R-1\right]}{\left(1-p_H\frac{B}{\Delta p}\right)^2}>0\end{aligned}$$

□

Lemma 6

Proof. We argue by monotonicity. By Lemma 4, we show for any given combination of (k, B, ℓ, R), only the ℓ type determines this cutoff. For the extensive margin on whether to receive a loan or not, we check NPV of project to the bank.

$$
\begin{aligned}
& V_{TL} > 0 \\
\Longleftrightarrow & (1-P)\, p_H \left(R - R_{TL}^F\right) I_{TL} - (I_{TL} - k) > 0 \\
\Longleftrightarrow & (1-P)\, p_H \left(R - \frac{B}{\Delta p}\right) \frac{k}{(1-P)\, p_H \frac{B}{\Delta p}} - \frac{k}{(1-P)\, p_H \frac{B}{\Delta p}} + k > 0 \\
\Longleftrightarrow & (1-P)\, p_H \left(R - \frac{B}{\Delta p}\right) \frac{1}{(1-P)\, p_H \frac{B}{\Delta p}} - \frac{1}{(1-P)\, p_H \frac{B}{\Delta p}} + 1 > 0 \\
\Longleftrightarrow & (1-P)\, p_H \left(R - \frac{B}{\Delta p}\right) > 1 - (1-P)\, p_H \frac{B}{\Delta p} \\
\Longleftrightarrow & (1-P)\, p_H R - (1-P)\, p_H \frac{B}{\Delta p} > 1 - (1-P)\, p_H \frac{B}{\Delta p} \\
\Longleftrightarrow & (1-P)\, p_H R > 1 \\
\Longleftrightarrow & R > \frac{1}{(1-P)\, p_H}
\end{aligned}
$$

NPV valuation of a term loan project does not depend on ℓ since it is off-the-equilibrium path. For credit line contracts, ℓ does feature into the calculation and therefore gives an indifference relationship between R and ℓ.

$$
\begin{aligned}
& V_{CL} > 0 \\
\Longleftrightarrow & p_H \left(R - \frac{B}{\Delta p}\right) \frac{k}{p_H \frac{B}{\Delta p}} - \left(\frac{k}{p_H \frac{B}{\Delta p}} - k\right) - P\ell \frac{k}{p_H \frac{B}{\Delta p}} > 0 \\
\Longleftrightarrow & p_H \left(R - \frac{B}{\Delta p}\right) \frac{k}{p_H \frac{B}{\Delta p}} - \frac{k}{p_H \frac{B}{\Delta p}} + k - P\ell \frac{k}{p_H \frac{B}{\Delta p}} > 0 \\
\Longleftrightarrow & p_H \left(R - \frac{B}{\Delta p}\right) \frac{1}{p_H \frac{B}{\Delta p}} - \frac{1}{p_H \frac{B}{\Delta p}} + 1 - P\ell \frac{1}{p_H \frac{B}{\Delta p}} > 0 \\
\Longleftrightarrow & p_H R \frac{1}{p_H \frac{B}{\Delta p}} - p_H \frac{B}{\Delta p} \frac{1}{p_H \frac{B}{\Delta p}} - \frac{1}{p_H \frac{B}{\Delta p}} + 1 - P\ell \frac{1}{p_H \frac{B}{\Delta p}} > 0 \\
\Longleftrightarrow & \frac{R}{\frac{B}{\Delta p}} - \frac{1}{p_H \frac{B}{\Delta p}} - P\ell \frac{1}{p_H \frac{B}{\Delta p}} > 0 \\
\Longleftrightarrow & R - \frac{1}{p_H} - \frac{P\ell}{p_H} > 0 \\
\Longleftrightarrow & p_H R - 1 - P\ell > 0 \\
\Longleftrightarrow & R > \frac{1 + P\ell}{p_H}
\end{aligned}
$$

□

Proposition 1

Proof. Recall credit line contracts are optimal iff $(1 - P)\,\ell < 1$. Assume further that $p_H R > \ell > p_H \left(R - \frac{B}{\Delta p}\right)$, and that the firm and bank decided on a wait-and-see policy. If the firm draws a liquidity shock ℓ, the project is still NPV positive since $p_H R > \ell$. The total surplus is still positive and it is efficient to pay ℓ and continue. However, if the bank gives a second loan of size ℓ to continue the project, the per-unit return from the project to the bank is $p_H \left(R - R^F_{CL}\right) = p_H \left(R - \frac{B}{\Delta p}\right)$, according to the optimal contract. Since, $\ell > p_H \left(R - \frac{B}{\Delta p}\right)$, the bank gets a negative return and thus will not want to continue. Therefore, a wait-and-see policy would not work. A natural question is why the firm and bank would not renogiate and continue if there is still positive surplus. The bank's return $p_H \left(R - R^F_{CL}\right)$ is already the maximum the bank can extract from the project without violating the firm's IC (recall the IC is binding in the optimal contract), therefore any surplus the firm tries to give the bank to induce it to continue will not be credible.

Similary, this type of renegotiation might be desired by the firm since the term loan contract costs less. The expected return to the investor from issuing another term loan to cover the realization of $\ell \cdot I_{TL}$ is

$$V_{TL2} = \max_{R^F_{TL2}} p_H \cdot \left(R - R^F_{TL2}\right) I_{TL} - \ell \cdot I_{TL}$$

subject to firm IC

$$R^F_{TL2} \geq \frac{B}{\Delta p}$$

Since IC is binding, the objective function is linear in I_{TL}, but $\ell > p_H \left(R - \frac{B}{\Delta p}\right)$ means bank will not agree to renegotiate. $\square$

Theorem 2

Proof. The proof is via Implicit Function Theorem. The FOC of the bank/planner is,

$$\begin{aligned}
[\ell^*] : 0 = & \left[(1 - P^*) R_{TL} + P^* \chi I_{TL} + R_{CL} - P^* \mathcal{L} - R_D (L_{TL} + L_{CL})\right] \cdot \frac{\partial P^*}{\partial \ell^*} \\
& + \int_0^{P^*} \left[(1 - P) \frac{\partial R_{TL}}{\partial \ell^*} + P\chi \frac{\partial I_{TL}}{\partial \ell^*} + \frac{\partial R_{CL}}{\partial \ell^*} - P \frac{\partial \mathcal{L}}{\partial \ell^*} - R_D \left(\frac{\partial L_{TL}}{\partial \ell^*} + \frac{\partial L_{CL}}{\partial \ell^*}\right)\right] dG(P) \\
& - \theta \left[\xi \left[(1 - P^*) R_{TL} + P^* \chi I_{TL} + R_{CL}\right] - P^* \mathcal{L} - R_D (L_{TL} + L_{CL})\right] \cdot \frac{\partial P^*}{\partial \ell^*} \\
& + \theta \int_{P^*}^{1} \left[\xi \left[(1 - P) \frac{\partial R_{TL}}{\partial \ell^*} + P\chi \frac{\partial I_{TL}}{\partial \ell^*} + \frac{\partial R_{CL}}{\partial \ell^*}\right] - P \frac{\partial \mathcal{L}}{\partial \ell^*} - R_D \left(\frac{\partial L_{TL}}{\partial \ell^*} + \frac{\partial L_{CL}}{\partial \ell^*}\right)\right] dG(P)
\end{aligned}$$

Define the FOC as $g(\ell^*, \theta)$. Then,

$$\begin{aligned}
\frac{\partial g(\ell^*, \theta)}{\partial (\ell^*)} = & (1 - \theta) \left[\frac{\partial (1 - P^*) R_{TL}}{\partial \ell^*} + \frac{\partial P^* I_{TL}}{\partial \ell^*} + \frac{\partial R_{CL}}{\partial \ell^*} - \frac{\partial P^*}{\partial \ell^*} \mathcal{L} + P^* \frac{\partial \mathcal{L}}{\partial \ell^*} - R_D \left(\frac{\partial L_{TL}}{\partial \ell^*} + \frac{\partial L_{CL}}{\partial \ell^*}\right)\right] \cdot \frac{\partial P^*}{\partial \ell^*} \\
& + (1 - \theta) \left[(1 - P^*) R_{TL} + P^* \chi I_{TL} + R_{CL} - P^* \mathcal{L} - R_D (L_{TL} + L_{CL})\right] \cdot \frac{\partial^2 P^*}{\partial (\ell^*)^2} \\
& + (1 - \theta) \left[(1 - P^*) \frac{\partial R_{TL}}{\partial \ell^*} + P^* \chi \frac{\partial I_{TL}}{\partial \ell^*} + \frac{\partial R_{CL}}{\partial \ell^*} - P^* \frac{\partial \mathcal{L}}{\partial \ell^*} - R_D \left(\frac{\partial L_{TL}}{\partial \ell^*} + \frac{\partial L_{CL}}{\partial \ell^*}\right)\right] \cdot \frac{\partial P^*}{\partial \ell^*} \\
& + \int_0^{P^*} \left[(1 - P) \frac{\partial^2 R_{TL}}{\partial (\ell^*)^2} + P\chi \frac{\partial^2 I_{TL}}{\partial (\ell^*)^2} + \frac{\partial^2 R_{CL}}{\partial (\ell^*)^2} - P \frac{\partial^2 \mathcal{L}}{\partial (\ell^*)^2} - R_D \left(\frac{\partial^2 L_{TL}}{\partial (\ell^*)^2} + \frac{\partial^2 L_{CL}}{\partial (\ell^*)^2}\right)\right] dG(P) \\
& + \theta \int_{P^*}^{1} \left[(1 - P) \frac{\partial^2 R_{TL}}{\partial (\ell^*)^2} + P\chi \frac{\partial^2 I_{TL}}{\partial (\ell^*)^2} + \frac{\partial^2 R_{CL}}{\partial (\ell^*)^2} - P \frac{\partial^2 \mathcal{L}}{\partial (\ell^*)^2} - R_D \left(\frac{\partial^2 L_{TL}}{\partial (\ell^*)^2} + \frac{\partial^2 L_{CL}}{\partial (\ell^*)^2}\right)\right] dG(P)
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial g\left(\ell^*,\theta\right)}{\partial \theta} &= -\left[\left(1-P^*\right)R_{TL}+P^*\chi I_{TL}+R_{CL}-P^*\mathcal{L}-R_D\left(L_{TL}+L_{CL}\right)\right]\cdot\frac{\partial P^*}{\partial \ell^*} \\
&+\int_{P^*}^{1}\left[\left(1-P\right)\frac{\partial R_{TL}}{\partial \ell^*}+P\chi\frac{\partial I_{TL}}{\partial \ell^*}+\frac{\partial R_{CL}}{\partial \ell^*}-P\frac{\partial \mathcal{L}}{\partial \ell^*}-R_D\left(\frac{\partial L_{TL}}{\partial \ell^*}+\frac{\partial L_{CL}}{\partial \ell^*}\right)\right]dG\left(P\right)
\end{aligned}
$$

With functional form assumptions, we can show that

$$
\frac{\partial g\left(\ell^{SP},\theta=1\right)}{\partial\left(\ell^*\right)}<0
$$

$$
\frac{\partial g\left(\ell^{SP},\theta=1\right)}{\partial \theta}>0
$$

such that

$$
\frac{\partial \ell^*}{\partial \theta}\left(\ell^{SP},\theta=1\right)=\frac{-\frac{\partial g}{\partial \theta}\left(\ell^{SP},\theta=1\right)}{\frac{\partial g}{\partial \ell}\left(\ell^{SP},\theta=1\right)}>0
$$

Procedure for $\frac{\partial R^*}{\partial \theta}$ is equivalent. □

Theorem 3

Proof. The proof is again via Implicit Function Theorem.

$$
\begin{aligned}
\frac{\partial g\left(\ell^*,\xi\right)}{\partial\left(\ell^*\right)} &= \left(1-\xi\right)\left[-\frac{\partial P^*}{\partial \ell^*}R_{TL}+\left(1-P^*\right)\frac{\partial R_{TL}}{\partial \ell^*}+\frac{\partial P^*}{\partial \ell^*}\chi I_{TL}+P^*\chi\frac{\partial I_{TL}}{\partial \ell^*}+\frac{\partial R_{CL}}{\partial \ell^*}\right]\cdot\frac{\partial P^*}{\partial \ell^*} \\
&+\left(1-\xi\right)\left[\left(1-P^*\right)R_{TL}+P^*\chi I_{TL}+R_{CL}\right]\cdot\frac{\partial^2 P^*}{\partial\left(\ell^*\right)^2} \\
&+\left(1-\xi\right)\left[\left(1-P^*\right)\frac{\partial R_{TL}}{\partial \ell^*}+P^*\chi\frac{\partial I_{TL}}{\partial \ell^*}+\frac{\partial R_{CL}}{\partial \ell^*}\right]\cdot\frac{\partial P^*}{\partial \ell^*} \\
&+\int_{0}^{P^*}\left[\left(1-P\right)\frac{\partial^2 R_{TL}}{\partial\left(\ell^*\right)^2}+P\chi\frac{\partial^2 I_{TL}}{\partial\left(\ell^*\right)^2}+\frac{\partial^2 R_{CL}}{\partial\left(\ell^*\right)^2}\right]dG\left(P\right) \\
&+\xi\int_{P^*}^{1}\left[\left(1-P\right)\frac{\partial^2 R_{TL}}{\partial\left(\ell^*\right)^2}+P\chi\frac{\partial^2 I_{TL}}{\partial\left(\ell^*\right)^2}+\frac{\partial^2 R_{CL}}{\partial\left(\ell^*\right)^2}\right]dG\left(P\right) \\
&-\bar{P}\frac{\partial^2 \mathcal{L}}{\partial\left(\ell^*\right)^2}-R_D\left(\frac{\partial^2 L_{TL}}{\partial\left(\ell^*\right)^2}+\frac{\partial^2 L_{CL}}{\partial\left(\ell^*\right)^2}\right)
\end{aligned}
$$

$$
\frac{\partial g\left(\ell^*,\xi\right)}{\partial \xi}=-\left[\left(1-P^*\right)R_{TL}+P^*\chi L_{TL}+R_{CL}\right]\cdot\frac{\partial P^*}{\partial \ell^*}+\int_{P^*}^{1}\left[\left(1-P\right)\frac{\partial R_{TL}}{\partial \ell^*}+P\chi\frac{\partial I_{TL}}{\partial \ell^*}+\frac{\partial R_{CL}}{\partial \ell^*}\right]dG\left(P\right)
$$

With functional form assumptions, we can show that

$$\frac{\partial g\left(\ell_{SP},\xi=1\right)}{\partial\left(\ell^{*}\right)}<0$$

$$\frac{\partial g\left(\ell_{SP},\xi=1\right)}{\partial\xi}>0$$

such that

$$\frac{\partial\ell^{*}}{\partial\xi}\left(\ell_{SP},\xi=1\right)=\frac{-\frac{\partial g}{\partial\xi}\left(\ell_{SP},\xi=1\right)}{\frac{\partial g}{\partial\ell}\left(\ell_{SP},\xi=1\right)}>0$$

Procedure for $\frac{\partial R^{*}}{\partial\xi}$ is equivalent. □

Model with outside investment opportunity

Suppose in addition to the environment in the main body, there is an investment opportunity for the bank in the intermediate period, but the bank is constrained in its lending in that period. Specifically, the bank receives an endowment of y at the beginning of subperiod 2, and has an investment opportunity of $R_2>1$. For the firm, those that receive a liquidity shock ℓI and pay it to continue will earn ℓI back at the end of period 2 - the liquidity shock is purely a liquidity issue, not one that lowers the project value. Then, for a term loan only contract, the bank's problem is

$$\max_{I,R_F} V_{TL}=(1-P)\,p_H\left(RI-R^F\right)-(I-k)+R_2I_2$$

$$R^F\geq\frac{B}{\Delta p}$$

$$(1-P)\,p_H R^F I\geq k$$

$$I_2\leq y$$

where we see that since the bank has no obligations to pay the first firm's liquidity cost, it may put y_2 fully into R_2. Note that the solution to the problem does not change - the IC and PC are the same and are still binding. Then, for a credit line contract, the bank's problem is

$$
\begin{aligned}
\max_{I,R^F} V_{CL} &= p_H\left(RI - R^F\right) - (I-k) + P\ell I + (1-P)\,R_2 y + PR_2\,(y - \ell I)\\
&= p_H\left(RI - R^F\right) - (I-k) + P\ell I + R_2\,(y - P\ell I)\\
&= p_H\left(RI - R^F\right) - (I-k) + R_2 y - P\ell I\,(R_2 - 1)
\end{aligned}
$$

$$R^F \geq \frac{B}{\Delta p}$$

$$p_H R^F I \geq k$$

$$\mathbb{1}_{shock}\ell I + I_2 \leq y$$

Now the bank has a non-trivial budget constraint in the intermediate period. Note the bank pays ℓI to the firm, but also knows it will receive this same ℓI amount back (just like a credit line loan). However, the bank is unable to put all its endowment y into R_2. This is equivalent to a negative cash flow shock to the bank compared to the term loan contract.

B Data and estimation

Table B.1: Estimation of term loan only model

Parameter		Value	Target	Data	Term loan only model
			Panel A: Micro firm parameters		
Firm size distribution k	x_m	1100	Total loans to bottom 84% firms	277	303
Firm size distribution k	σ	1.6	Total loans to top 16% firms	539	560
Moral hazard B	B_0	0.8	Average interest rate to bottom 84% firms	$415bp$	$489bp$
Moral hazard B	B_1	-0.2	Average interest rate to top 16% firms	$350bp$	$472bp$
Liquidity cost ℓ	ℓ_0	–	–	–	–
Liquidity cost ℓ	ℓ_1	–	–	–	–
			Panel B: Macro bank parameters		
Project quality distribution	$\bar{R}$	1.07	Bank leverage	0.92	0.90
Salvage rate	χ	0.5	TL loss in COVID	13%	14%
Shock distribution	α	1.4	Bank default rate	1%	0.94%
Shock distribution	L	0.01	Average excess drawdowns	3%	3%
Household endowment	ω	–	–	–	–
			Panel C: External calibration		
Discount factor	β	0.99	Risk-free rate	1%	1%
Household storage technology	R_f	1.01	Average deposit rate	–	–
Project success rates	(p_H, p_L)	$(1, 0)$	Normalized	–	–
Resource loss	ξ	0.2	FDIC liquidation cost	–	–
Capital requirement	ϕ^{CR}	0.08	Basel II baseline	–	–

Notes: Moments are in $ billions, unless otherwise noted.