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Reviewed work(s):

Source: *Industrial and Labor Relations Review*, Vol. 30, No. 2 (Jan., 1977), pp. 175-187

Published by: [Cornell University, School of Industrial & Labor Relations](#)

Stable URL: <http://www.jstor.org/stable/2522871>

Accessed: 24/02/2012 10:02

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# STATISTICAL THEORIES OF DISCRIMINATION IN LABOR MARKETS

DENNIS J. AIGNER and GLEN G. CAIN

ECONOMIC discrimination has been difficult to explain by means of standard neoclassical economic models that assume pervasive competition. Why, after all, should two groups of workers who have the same productivity receive different remuneration? The challenge to explain this phenomenon is posed most sharply by the marked differentials in wages and earnings between blacks and whites and between men and women—differentials that remain substantial despite diligent efforts to control for supply-side productivity traits.

This paper examines that issue from a

perspective suggested by Kenneth Arrow, John J. McCall, Edmund S. Phelps, Melvin W. Reder, and A. Michael Spence, all of whom focused on certain implications of employer uncertainty about the productivity of racial (or sex) groups of workers, particularly in the context of hiring and placement decisions.<sup>1</sup> This paper offers several models that clarify the meaning of economic “statistical discrimination,” simplify the theory, and yield plausible empirical implications. On the other hand, the paper also identifies several shortcomings of “statistical discrimination” models; shows that the often-cited Phelps model does not constitute economic discrimina-

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Economic discrimination in labor markets is conventionally defined as the presence of different pay for workers of the same ability. This paper analyzes that problem with the aid of a simple stochastic model in which employers hire, place, and pay workers on the basis of imperfect information about their abilities. The available information consists of both group membership (black, white; male, female) and information about individual performance on some fallible indicator of ability (e.g., a test). Several types of economic discrimination within the context of competitive market assumptions are examined by means of several models, and the empirical plausibility and implications of these models are discussed. The authors conclude that the statistical theories are unlikely to provide an important explanation of labor market discrimination under conventional neoclassical assumptions.

Dennis J. Aigner and Glen G. Cain are both Professors of Economics at the University of Wisconsin. They express their gratitude for the extensive comments of Arthur S. Goldberger. This research was supported in part by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison by the Office of Economic Opportunity pursuant to the Economic Opportunity Act of 1964 (GGC), and by NSF grant GS-30005 (DJA). The opinions expressed here are those of the authors.—EDITOR

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<sup>1</sup>Kenneth Arrow, “Models of Job Discrimination” and “Some Mathematical Models of Race in the Labor Market,” in Anthony H. Pascal, ed., *Racial Discrimination in Economic Life* (Lexington, Mass.: Lexington Books, D. C. Heath and Co., 1972), pp. 83–102 and 187–204; and “The Theory of Discrimination,” in Orley Ashenfelter and Albert Rees, eds., *Discrimination in Labor Markets* (Princeton, N. J.: Princeton University Press, 1973), pp. 3–33; John J. McCall, *Income Mobility, Racial Discrimination, and Economic Growth* (Lexington, Mass.: Lexington Books, D. C. Heath and Co., 1972); and “The Simple Mathematics of Information, Job Search, and Prejudices,” in Anthony H. Pascal, ed., *Racial Discrimination in Economic Life* (Lexington, Mass.: Lexington Books, D. C. Heath and Co., 1972), pp. 205–44; Edmund S. Phelps, “The Statistical Theory of Racism and Sexism,” *American Economic Review*, Vol. 62, No. 4 (September 1972), pp. 659–61; Melvin W. Reder, “Human Capital and Economic Discrimination,” in Ivar Berg, ed., *Human Resources and Economic Welfare; Essays in Honor of Eli Ginzberg* (New York: Columbia University Press, 1972), pp. 71–88; and A. Michael Spence, “Job Market Signaling,” *Quarterly Journal of Economics*, Vol. 87, No. 3 (August 1973), pp. 355–74 and *Market Signaling* (Cambridge, Mass.: Harvard University Press, 1974).

tion, statistical or otherwise; and concludes that these models probably do not explain most labor market discrimination.

### The Basic Model

We introduce the statistical model of discrimination with the version by Phelps, contained in an article with the imposing title, "The Statistical Theory of Racism and Sexism."<sup>2</sup> The essential features are as follows. In the hiring and placement of workers, employers base their decisions on some indicator of skill,  $y$ , (such as a performance test) that measures the true skill level,  $q$ . The terms "ability," "productivity," and "skill" will be used interchangeably herein. In practice,  $y$  would undoubtedly involve a number of measures, but the assumption here will be that a single test score is all that is measured by  $y$ . The measurement equation is

$$(1) \quad y = q + u,$$

where  $u$  is a normally distributed error term, independent of  $q$ , with zero mean and constant variance;  $q$  is also assumed to be normally distributed with a mean equal to  $\alpha$  and with a constant variance.

Employers can observe the test score,  $y$ , but they are interested in this only insofar as it gives them information about the unobservable variable,  $q$ . Thus, the immediate interest of the employer is the expected or predicted value of  $q$ , which we shall label  $\hat{q}$ .

The expected value of  $q$ , given  $y$  ( $E(q|y)$ ) is:

$$(2) \quad \hat{q} = E(q|y) = (1 - \gamma)\alpha + \gamma y,$$

where  $\alpha$  is the group mean of  $q$  (and  $y$ ) and

$$(3) \quad \gamma = \frac{\text{Var}(q)}{\text{Var}(q) + \text{Var}(u)} = \frac{\text{Cov}(q,y)}{\text{Var}(y)} \\ = \left[ \frac{\text{Cov}(q,y)^2}{\text{Var}(q)\text{Var}(y)} \right] = r^2,$$

where  $r^2$  is the squared coefficient of correlation between  $q$  and  $y$ . In classical test score theory,  $\gamma$  is the reliability of a test score,  $y$ , as a measure of the true score,  $q$ . Clearly,  $0 < \gamma < 1$ .

By normal distribution theory, Equation 2 is the least squares regression, expressing  $q$  in terms of a group effect  $[(1-\gamma)\alpha]$  and an individual effect ( $\gamma y$ ). It is useful to think of Equation 2 as a conditional expectation from a linear population regression function:

$$(4) \quad q = (1 - \gamma)\alpha + \gamma y + u'$$

where  $u'$  is the usual well-behaved error term. In principle, the regression is operational, because employers could measure the actual  $q$  of a worker on the basis of a *post hoc* evaluation of the worker's performance.

Now, consider two differentiated groups of workers, say whites and blacks, with possibly different means,  $\alpha^W$  and  $\alpha^B$ , and possibly different variances of  $q$  and  $u$ . (Although we use whites and blacks throughout, our discussion is equally applicable to males and females.) The employer is assumed to pay a worker an amount,  $\hat{q}$ , based on the specific information available for each group and individual (see Equation 2):

$$(5a) \quad \hat{q}^W = (1 - \gamma^W)\alpha^W + \gamma^W y^W$$

$$(5b) \quad \hat{q}^B = (1 - \gamma^B)\alpha^B + \gamma^B y^B.$$

The slope,  $\gamma$ , will generally differ for the two groups if the variances of  $q$  and  $u$  differ, as shown by Equation 3.<sup>3</sup>

The nature of the hiring and placement process requires that the employer make a subjective assessment of a worker's skill. We assume that this *assessment* of  $q$ , given  $y$ , will equal the *expectation* of  $q$ , conditional on  $y$ . This assumption is in keeping with wage-maximizing behavior by work-

<sup>3</sup>Any random error in  $y$  as a measure of  $q$  is represented by  $u$ . A *systematic* error in  $y$  as a measure of  $q$  for one or the other racial groups could also be introduced, but this would not add substantively to our analysis. For example, if blacks scored below whites by some constant amount for any  $q$  value, a negative intercept term could be added to Equation 1. However, a simple transformation in which this intercept difference was added to  $q^B$  would restore comparability in the  $q$  values for both groups according to a new set of equations like Equations 5a and 5b. This type of bias in the test instrument would not, by itself, affect the reliability of the instrument and is, therefore, inconsequential. The unreliability of  $y$  as a measure of  $q$ , however, is another matter, as we demonstrate later.

<sup>2</sup>*American Economic Review* (September 1972).

ers and profit-maximizing behavior by employers, since a job market function of employers is to assess (or predict) factor productivity, given the costs of available information, and to pay the factors of production accordingly. Employers who are inefficient in this function will tend to be weeded out by the "market mechanism" of competition. As Spence concludes in considering a similar model of employer behavior, "In an equilibrium the subjective distribution and the one implied in the market mechanism are identical," assuming that neither group of workers is completely isolated from employers.<sup>4</sup>

To anticipate a possible source of confusion, we should emphasize that the assumed correspondence between the employers' subjective conditional expectation of  $q$  and the conditional expectation of realized  $q$  in the market implies that the group means (the  $\alpha$ s) are estimated without bias. In particular, employers will not persist in believing that  $\alpha^W > \alpha^B$  if, in fact,  $\alpha^W = \alpha^B$ . If employers mistakenly believe  $\alpha^W > \alpha^B$ , then they will mistakenly overpay whites relative to blacks, and we may doubt that such mistaken behavior will persist in competitive markets. Indeed, as an explanation of discrimination against blacks, a theory of discrimination based on employers' mistakes is even harder to accept than the explanation based on employers' "tastes for discrimination," because the "tastes" are at least presumed to provide a source of "psychic gain" (utility) to the discriminator.<sup>5</sup> To

interpret the "statistical theory of discrimination" as a theory of "erroneous" or "mistaken" behavior by employers, as have some economists,<sup>6</sup> is therefore without foundation. Furthermore, Andrew I. Kohen errs by claiming that, "Phelps [1972] demonstrates that irrespective of the validity of using sex" as a proxy variable for productivity characteristics of the job applicant, "discrimination is the outcome."<sup>7</sup> Phelps demonstrates no such result.

### Definitions of Economic Discrimination

Economic discrimination is said to exist when workers do not receive pay or remuneration commensurate with their productivity—when, in short, equal productivity is not rewarded with equal pay. Our focus is on labor market discrimination, which means that we will generally assume that the worker's pre-labor market investments and endowments are given. We adopt the prevailing convention of defining productivity in terms of physical output or actual job performance, acknowledging, however, that this definition can be ambiguous. As others have pointed out, discrimination against a particular group of workers can always be explained away by attaching a cost to some characteristic of the group that is not directly related to their work abilities.

It is necessary to distinguish group discrimination from individual discrimination that is independent of group mem-

<sup>4</sup>Spence, "Job Market Signaling," pp. 360-61.

<sup>5</sup>It is more precise to say that the wage policy of employers whose subjective assessment of  $q$  persistently differs from the expected value of actual  $q$  would not be viable unless all current and potential employers made the same error. Otherwise, the forces of competition would lead to an expansion of output by employers who erred the least (or not at all) at the expense of those who erred the most. Thus, imposing a wedge between the subjective expectation of  $q$  and the actual expected value of  $q$  is analytically equivalent to imposing employers' "tastes for discrimination" as a wedge between the employers' subjective evaluation of the worth (or productivity) of a worker and his actual worth. As both Gary S. Becker and Arrow have made clear, variance in tastes for discrimination among employers will lead to a situation in which relatively non-

discriminating employers drive the discriminating employers out of business in the long-run competitive equilibrium. See Gary S. Becker, *The Economics of Discrimination* (Chicago: University of Chicago Press, 1971), 2d ed.; and Arrow, "Models of Job Discrimination" and "The Theory of Discrimination."

<sup>6</sup>Cynthia B. Lloyd, "The Division of Labor Between the Sexes: A Review," in Cynthia B. Lloyd, ed., *Sex, Discrimination, and the Division of Labor* (New York: Columbia University Press, 1975), pp. 1-24 and Harriet Zellner, "Discrimination Against Women, Occupational Segregation, and the Relative Wage," *American Economic Review*, Vol. 62, No. 2 (May 1972), pp. 157-60.

<sup>7</sup>Andrew I. Kohen, section entitled "Differentiation in the Market," (pp. 1256-62) in Hilda Kahne, "Economic Perspectives of the Roles of Women in the American Economy," *Journal of Economic Literature*, Vol. 13, No. 4 (December 1975), pp. 1249-92.

bership. Race or sex discrimination is a consequence of group discrimination; discrimination among individuals within a group, on the other hand, carries no presumption of group discrimination nor, therefore, of race or sex discrimination. Group discrimination in labor markets is evident when the average wage of a group is not proportional to its average productivity. On this basis, our findings reveal that even nondiscriminatory practices by employers may yield a discriminatory outcome: groups that have the same average ability may receive different average pay.

Within-group or individual discrimination is inevitable. The fact that within a group, all individual workers with the same true ability will not receive the same pay is clearly shown in Equation 4, in which  $q$  is not exactly predicted by  $y$ . To illustrate that this does not necessarily involve group discrimination, consider a case in which all college graduates are offered one wage, equal to their average productivity and higher than the wage offered to all high-school graduates. Although individual discrimination occurs within each schooling group (except in the unrealistic case of zero variance in ability within each group), no presumption of between-group discrimination is warranted. The distinction between these two types of discrimination has not always been clear in the literature.<sup>8</sup>

<sup>8</sup>For one example, see Francine B. Blau and Carol L. Jusenius, "Economists' Approaches to Sex Segregation in the Labor Market: An Appraisal," in Martha Blaxall and Barbara Reagan, eds., *Women and the Workplace* (Chicago: University of Chicago Press), pp. 181-99. In discussing sex discrimination, Blau and Jusenius claim that "stereotyping, the treatment of each individual member of a group as if he/she possessed the average characteristics of the group" is "appropriately defined as a form of discrimination even if the employers' perceptions of the average [group] . . . differential are correct" (p. 194). In another source Michael J. Piore, in discussing race discrimination, labels as statistical discrimination a situation in which job candidates are rejected because they do not possess traits that "tend to be statistically correlated with job performance" (p. 56). See Michael J. Piore, "Jobs and Training," in Samuel H. Beer and Richard E. Barringer, eds., *The State and the Poor* (Cambridge, Mass.: Winthrop Press, 1970), pp. 53-83. If the decision rules are correct on average, however, then Piore's assertion that the employer is "discrimina-

Perhaps not so obvious is the fact that group discrimination may be absent even though the wages,  $\hat{q}$ , of blacks and whites with the same ability,  $q_0$ , are not generally equal. Generally,  $E(\hat{q}^B|q_0) \neq E(\hat{q}^W|q_0)$ —expressions obtained by taking expectations conditional on  $q$  in Equation 2. Thus:  $E(\hat{q}|q) = (1 - \gamma)\alpha + \gamma E(y|q)$ . But  $E(y|q) = q$ ; so

$$(6) \quad E(\hat{q}|q) = (1 - \gamma)\alpha + \gamma q,$$

and  $\gamma$  and  $\alpha$  may differ for blacks and whites. However, there need not be any average difference in compensation between groups, because the individual inequalities of the above expectation over the range of  $q$  may be offsetting between whites and blacks. These points are demonstrated and clarified in our analysis of particular models.

### A Phelps Model

The implications of Phelps's model, outlined previously, depend on assumptions about the average abilities, the variances of ability, and the variances of measurement error for the two groups—blacks and whites. Phelps makes three assumptions, each of which we question at some point in our discussion. In most of his paper he assumes, first, that  $u^W$  and  $u^B$  have the same variances; second, that the variance of  $q^W$  is less than the variance of  $q^B$ ; and, third, the the average ability of blacks is lower than that of whites.<sup>9</sup>

tory" says no more than that the decision maker, like the rest of us in our decisions, lacks perfect knowledge! Finally, consider the following quotation by Lester C. Thurow in *Generating Inequality* (New York: Basic Books, Inc., 1975):

[Statistical discrimination] occurs whenever an individual is judged on the basis of the average characteristics of the group, or groups, to which he or she belongs rather than upon his or her own characteristics (p. 172).

The statement is meaningless unless the following questions are answered: Are the group characteristics correctly judged on average? Is a person's age or test score, for example, an "own" characteristic, or does it also refer to the group (population) with that particular value or score? Is statistical discrimination averted only when all relevant "own" characteristics are known?

<sup>9</sup>The lower mean value of  $q^B$  is represented in Phelps's paper by a dummy variable for race (1 if black) with a negative coefficient, but the presenta-

It is disconcerting that Phelps assumed a difference in average abilities at the outset, because discrimination is defined as differences in pay for workers of the same ability, or, equivalently, a difference in pay that is not related to a difference in ability. Sometimes, of course, the assumption of equal ability is facilitated by narrowing the race (or sex) comparisons to subgroups of workers of the same age, schooling, or experience. For expository reasons, we will initially assume equal average abilities for the two groups:  $\alpha^B = \alpha^W = \alpha$ . The case of unequal average abilities will be examined later.

The other two assumptions in the basic Phelps model,  $Var(u^B) = Var(u^W)$  and  $Var(q^B) > Var(q^W)$ , dictate that the slope,  $\gamma$ , of the  $q$ -on- $y$  regression in Equations 2 or 4 is steeper for blacks than for whites. This is clear from Equation 3. It means that the test score,  $y$ , is a more reliable predictor of  $q$  for blacks than for whites. Accepting this unusual result for the moment, let us examine its consequences.

As Phelps noted, "at some high test score and higher ones the black applicant is predicted by the employer to excel over any white applicant with the same or lower test scores."<sup>10</sup> Figure 1A shows this—as well as the corollary proposition that at low test scores the white worker is predicted to excel over a black worker with the same test score. Low-scoring whites being paid more than low-scoring blacks is offset by high-scoring whites being paid less than high-scoring blacks. In what sense, then, does this picture depict racial economic discrimination? Each worker is paid in accordance with his expected productivity, based on an unbiased predictor. Moreover, the two groups, which have (by assumption) the same mean ability, receive the same mean wages.

tion is not entirely clear. His equation (5') would appear to represent a single regression model for workers of both races, in which  $z$  is an additive term that is equivalent to a dummy variable (0 if white). However, the additivity of  $z$  gives the wrong impression, because the all-worker regression requires a  $zy$  interaction to capture Phelps's assumption that the slope of  $y$  on  $q$  is different for the two races.

<sup>10</sup>Phelps, "The Statistical Theory of Racism and Sexism," p. 661.

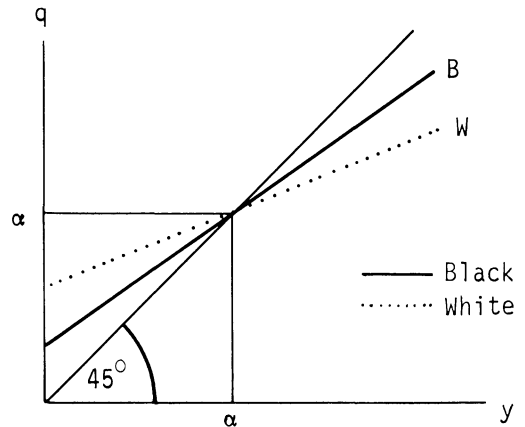


Figure 1A. Predictions of Productivity ( $q$ ) by Race and Test Score ( $y$ ), Assuming a Steeper Slope for Blacks.

The apparent definition of economic discrimination revealed by Figure 1A, and which we must ascribe to Phelps, is "different pay for the same  $y$  scores." But since  $y$  scores are intended only to indicate expected productivity, it is discrimination with respect to  $q$  and not  $y$  that is economically relevant.<sup>11</sup> Even a legal requirement that payments be equal for equal  $y$  scores would contribute nothing to the overall improvement of the status of blacks, since, as is clear in Figure 1A, what blacks would win at the lowest  $q$  values (relative to whites) they would lose at the highest  $q$  values.

In any event, the assumption that  $\gamma^B > \gamma^W$ —or that the  $y$  score is a more reliable indicator of  $q$  for blacks than whites—is

<sup>11</sup>Recall that the definition of economic discrimination as wage differences among workers with the same productivity implies pervasive *within*-group discrimination, given the conditional variance in  $q$ . Thus, some whites with a given test score,  $y_0$ , who are hired at a wage commensurate with  $E(q|y_0)$ , will have an actual  $q$  that is greater than  $E(q|y_0)$ ; others will turn out to have an actual  $q$  that is less than the expected value. We could fairly say that the former (positive residuals) are discriminated against and the latter (negative residuals) receive preferential treatment. Presumably, there is more of this sort of discrimination at the time of initial hirings than after the elapse of time, when the experience of workers and employers will narrow the conditional variance of  $q$ , given what will then be an augmented  $y$ . But, as stated earlier, a within-group conditional variance does not imply discrimination between groups.

unappealing. The Scholastic Aptitude Test has been found, for example, to be a *less* reliable indicator of college grades for blacks than for whites.<sup>12</sup> At the same time, we see no reason to assume the variance in true ability differs for the two races, although arguments can be made for a difference in either direction.<sup>13</sup> Moreover, an implication of the hypothesis that  $\gamma^B > \gamma^W$  is that the white-black differential in pay—reflecting a differential in expected  $q$ —narrows (and eventually becomes negative) as the  $y$  indicator increases. The bulk of the empirical evidence points, however, to the opposite result: if  $y$  is measured by years of school completed or by years of experience—two of the most important and commonly used indicators of productivity—the empirical relation between  $y$  and earnings (or wages) shows blacks faring worse relative to whites as  $y$  increases.<sup>14</sup>

A model that reflects this evidence and assumes that the testing process is less reliable for blacks is shown in Figure 1B. We will examine implications of this specification below, but the point here is that economic discrimination is no more evident in Figure 1B than it is in 1A. As be-

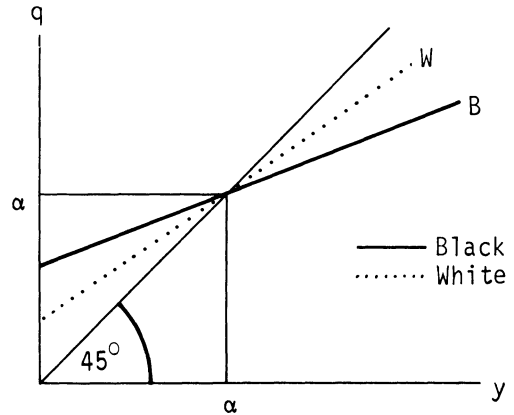


Figure 1B. Predictions of Productivity ( $q$ ) by Race and Test Score ( $y$ ), Assuming a Steeper Slope for Whites.

fore, each worker is paid according to his expected productivity, resulting in equal average wages for the two racial groups. The only difference shown by 1B is that whites with  $y$  scores above the mean receive higher wages than blacks, and the reverse is true for  $y$  scores below the mean.

#### An Alternative Model

Up to this point we have assumed explicitly that the employers know  $E(q|y)$  and implicitly that the dispersion of  $q|y$  is costless. This is equivalent to assuming that  $q$  enters the profit function linearly, or that the employer is risk-neutral with respect to  $q$ . It is more realistic to permit  $q$  to enter the profit function (or the "utility of profit" function) nonlinearly, which would allow the correct decision rule for hiring labor to involve higher moments of  $q$ . In the simple model adopted below, only the conditional variance of  $q$ , written  $Var(q|y) = Var(q)(1-\gamma)$ , is required to reflect risk aversion and to yield a theoretical explanation for economic discrimination.<sup>15</sup>

<sup>12</sup>Robert L. Linn, "Fair Test Use in Selection," *Review of Educational Research*, Vol. 43, No. 2 (1973), pp. 139-61.

<sup>13</sup>Blacks confront environmental restrictions on fulfilling their capacities, and this may lead to a smaller variance of  $q^B$ . On the other hand, perhaps whites face a more homogeneous set of environmental determinants of  $q$ , which would make the variance of  $q^W$  smaller. Any number of possibilities suggest themselves.

<sup>14</sup>For example, Finis Welch in "Education and Racial Discrimination," in Orley Ashenfelter and Albert Rees, eds., *Discrimination in Labor Markets* (Princeton, N.J.: Princeton University Press, 1973), pp. 43-81, remarks: "It is well known that, on balance, the ability of schooling to boost Negro earnings has been less than for whites, at least for males" (p. 43). Weiss supports this finding and also finds that "scholastic achievement" (as measured by test scores) was a better predictor of earnings for white males than for black males. See Randall D. Weiss, "The Effect of Education on the Earnings of Blacks and Whites," *Review of Economic Statistics*, Vol. 52, No. 2 (May 1970), pp. 150-59. Finally, a flatter age (experience)/wage profile is shown for black males and women than for white males in Robert E. Hall, "Why Is the Unemployment Rate So High at Full Employment?" *Brookings Papers on Economic Activity*, No. 3 (Washington, D.C.: The Brookings Institution, 1970), p. 394.

<sup>15</sup>The expression for the conditional variance of  $q$  is derived from normal distribution theory. It is an analogue of the expression for the residual variance in a simple linear regression as:  $(1-r^2) Var(\text{dependent variable})$ . Thus, in the population regression, Equation 4, we have  $Var(q|y) = Var(u') = Var(q) - \gamma^2 Var(y) = Var(q) - \left[ \frac{Var(q)}{Var(y)} \right]^2 Var(y)$ , using an expression for  $\gamma$  from Equation 3; so  $Var(q|y) = Var(q)(1-\gamma)$ .

To simplify the problem, assume that labor is the only factor of production, that output is fixed, and that prices and wage rates are exogenously determined. Thus, profits,  $\Pi$ , are solely a function of labor services. Given the number of workers required to maximize the utility-of-profits function,  $U(\Pi)$ , the employer need only choose the type of labor—here the assumed equally productive  $B$  or  $W$  groups—to maximize  $U(q)$ .

Several well-known utility functions result in a decision rule that depends on the variance of the argument. We adopt a function used by Michael Parkin,<sup>16</sup> which for our purposes may be written:

$$(7) \quad U(q) = a - be^{-cq} \quad b, c > 0,$$

whence

$$(8) \quad E[U(q|y)] = a - be^{-c E(q|y) + c^2/2 \text{Var}(q|y)},^{17}$$

where  $a$ ,  $b$ , and  $c$  are parameters of the utility function and  $e$  is the base of the natural logarithm.

It is easily seen that maximizing  $E[U(q|y)]$  is equivalent to maximizing its logarithm, which in turn is equivalent to maximizing  $[E(q|y) - k \text{Var}(q|y)]$ , where  $k = c/2$ . Let  $R = k \text{Var}(q|y)$ , which may be interpreted as a risk factor.

It follows that an employer with this utility function will attempt to hire from the group of workers that maximizes expected productivity,  $q$ , discounted for risk. This risk can arise from differing variances in the distribution of  $q$ , or  $u$ , or both. Substantively, the risk costs of variance in worker abilities may stem from variance in output within homogeneous jobs or from the costs of mistakes in assigning workers to heterogeneous job slots. For the sake of convenience we have adopted a conditional variance that does not depend on the level of  $y$ , so that the risk factor is constant over the range of test scores (see footnote 15).

The empirical question of whether the

conditional variance in  $q$ , given  $y$ , is larger or smaller for black or white workers is, therefore, crucial in determining the direction of discrimination. Assuming racial equality in  $\text{Var}(q)$ , this question hinges on the reliability of  $y$  as a predictor of  $q$ , namely  $\gamma$ , and we have already suggested that the tests are less reliable for blacks. Thus,  $\gamma^B < \gamma^W$  (as depicted in Figure 1B) follows from the assumption that  $\text{Var}(u^B) > \text{Var}(u^W)$ .<sup>18</sup>

Figure 2 shows the new  $y, q$  relationships incorporating the risk factors,  $R^B$  and  $R^W$ , and the assumptions,  $\gamma^B < \gamma^W$  and  $\alpha^B = \alpha^W = \alpha$ . The lines  $W$  and  $B$  are from Equation 2 in conjunction with the assumption that  $\alpha^W = \alpha^B$  and  $\gamma^B < \gamma^W$ . The parallel lines,  $W-R^W$  and  $B-R^B$  are the risk-discounted counterparts representing  $E(q|y) - k \text{Var}(q|y)$  or  $E(q|y) - R$ . To see that the figure reveals economic discrimination against blacks in the sense that they receive lower pay on average for the same expected ability, simply note the lower black value of  $(q-R)$ , given  $y = \alpha$ . The expected value of  $q$  for both whites and blacks equals  $\alpha$  for  $y = \alpha$ , and therefore the lower pay for blacks is entirely attributable to the larger  $R$  factor for blacks. In Figure 3, which graphs the conditional distribution of  $q$  for the point were  $y = \alpha = E(y)$  and where  $E(q) = \alpha$  for both races. The observed smaller conditional variance of  $q^W$  in Figure 3 (which would be the same at any value of  $y$ ) is, of course, precisely the source of the larger size of  $\gamma^W$ , given that  $\text{Var}(q^B) = \text{Var}(q^W)$ .

The risk discount borne by black workers in the form of a lower relative wage could be interpreted in terms of the extra search costs employers would have to bear to reduce the conditional variance of  $q^B$  to

<sup>18</sup>A lower slope for blacks would also result from the assumption that  $\text{Var}(u^B) = \text{Var}(u^W)$  and  $\text{Var}(q^B) < \text{Var}(q^W)$ . Indeed, the risk discount,  $\text{Var}(q|y)$ , is symmetric with respect to  $\text{Var}(u)$  and  $\text{Var}(q)$  since, by manipulations of relations in Footnote 15, we see

$$\text{Var}(q|y) = \frac{\text{Var}(q)\text{Var}(u)}{\text{Var}(q) + \text{Var}(u)}$$

In our comparisons between blacks and whites we cannot, however, interchange the terms "reliability" ( $= \gamma = r^2$ ) and "risk-discount" [ $= \text{Var}(q|y)$ ] unless we hold equal either  $\text{Var}(q)$  or  $\text{Var}(u)$  for the two groups.

<sup>16</sup>Michael Parkin, "Discount House Portfolio and Debt Selection," *Review of Economic Studies*, Vol. 37, No. 4 (October 1970), pp. 469-98.

<sup>17</sup>Since  $q$  is normally distributed,  $e^{-cq}$  is lognormal, and its expected value is  $e^{-cE(q) + (c^2/2)\text{Var}(q)}$ .



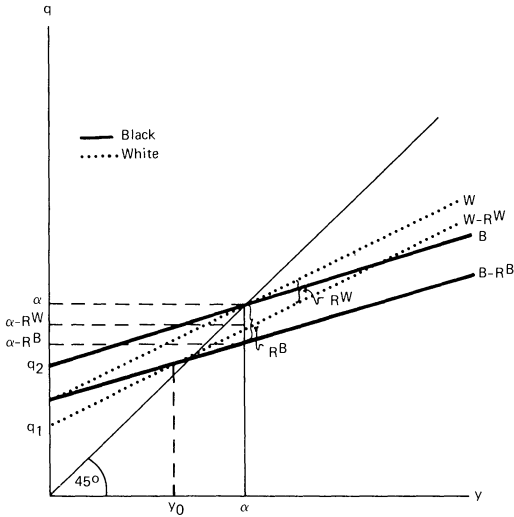


Figure 2. Predictions of Productivity ( $q$ ) by Race and Test Score ( $y$ ) with “Risk-Discounts” and Flatter Slope for Blacks.

equal  $Var(q^w|y)$ . However, the model does not require any ad hoc assumptions about the direct hiring costs being larger for blacks compared to whites, although cost differentials may exist. The geographic segregation of black workers away from white employers (firms), for example, may well impose extra search and informational costs upon black workers.<sup>19</sup>

Figure 2 represents a hypothetical model, of course, but it is consistent with our view of reality in two important respects. First, as a consequence of actual employer practices, economic discrimination against blacks, women, and other groups does exist, resulting in group differences in pay despite equal group abilities to perform on the job. However, if the definition of ability includes reliability in test-taking—on grounds, perhaps, that this aptitude conveys useful information to employers—then one could deny that *economic* discrimination exists. Our prefer-

<sup>19</sup>For an analysis of this particular disadvantage to black workers, see McCall, “The Simple Mathematics of Information, Job Search, and Prejudices,” and John F. Kain, “Housing Segregation, Black Employment, and Metropolitan Decentralization: A Retrospective View,” in George M. von Furstenberg, Bennett Harrison, and Ann R. Horowitz, eds., *Patterns of Racial Discrimination, Vol. 1: Housing* (Lexington, Mass.: Lexington Books, D. C. Heath and Co., 1974), pp. 5–20.

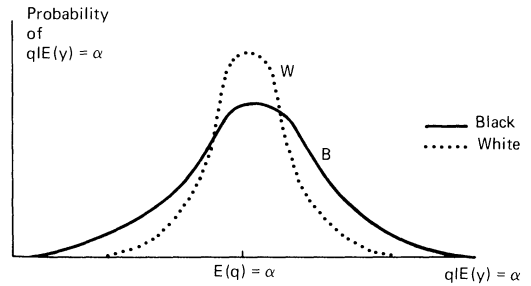


Figure 3. The Distribution of Productivity ( $q$ ), Given the Test Score  $y = E(y)$ , by Race.

$$\text{Note: } E(q|E(y)) = E(q) = E(q^B) = E(q^W) = E(y^B) = E(y^W).$$

ence is to retain the term “economic” in describing this type of discrimination, although such discrimination stems from inadequate test instruments rather than employers’ acting upon their tastes for discriminating against black or female workers.

A second realistic feature is that the differential wage or income advantage of white male workers increases as the indicator variable increases. In Figure 2, however, the wages of  $B$  workers exceed those of  $W$  workers with the same  $y$  scores for  $y < y_0$ , and only if the risk penalty were as large as  $q_2 - q_1$  would every  $W$  worker be paid more than every  $B$  worker for a given  $y$  score, over the whole range of  $y$ . We are not aware of any actual data revealing a smaller wage for  $W$ s compared to  $B$ s for low scores of productivity indicators. Furthermore, although we have not expressed the units in dollars, the empirical magnitudes of the differential ( $R^B - R^W$ ) borne by black workers—perhaps 10 to 30 percent for the same number of years of schooling completed—seem too large to be rationalized by risk aversion.

Indeed, three reasons are suggested for skepticism about the size of  $R$ .<sup>20</sup> First, large firms have some capacity to self-insure against risks of output variability or mistaken job assignments. In perfect capital markets, even small firms could “purchase” such insurance through various pooling devices. Second, dispersion in risk

<sup>20</sup>We are grateful to Orley Ashenfelter, H. Gregg Lewis, Donald A. Nichols, and Melvin W. Reder for helpful suggestions about this section.

aversion among employers leads to a situation in which those with the least aversion reduce the  $R$  discount by "bidding up" the wages of black workers, in the same manner as employers with the lowest "tastes for discrimination" serve to equalize the wages of black and white workers of the same productivity. Finally, a large  $R$  factor should activate a market for "test instruments" that are tailored to the separate groups to achieve more nearly equal reliability. (For example, just as tests in their native language have been prepared for foreign workers, so test developers could devise ways to communicate more clearly with members of minority groups.) The wage differential should not exceed the "signaling cost"—to use Spence's suggestive term.<sup>21</sup>

### Other Models of Discrimination

A second model that attempts to explain the wage differential comes from A. Michael Spence.<sup>22</sup> He provides a dynamic equilibrium analysis in which group differences in wages persist in competitive labor markets. In his model employers are uncertain about the workers'  $q$  values and base their wage offers ( $qs$ ) on the workers'  $y$  scores—which, like our  $y$  scores, do not represent productivity skills, per se. Workers attain  $y$  scores by investing their time and resources, and the Spence model assumes that the cost of  $y$  is negatively correlated with  $q$ . This assumption drives the wage system, under plausible conditions, to an equilibrium in which  $q$  is positively correlated with  $y$ , so that the employers' expectations are self-confirming and the workers' signaling behavior reproduces itself.<sup>23</sup>

In the Spence model, however, the level of  $y$  (call this  $y^*$ ) that distinguishes high- from low-ability workers (to use his simple case of just two levels of productivity) is arbitrary over a certain range. This leads

to multiple equilibria, and it then is a small step to show that there may be different equilibria for two distinguishable racial groups of workers, even though the two groups have identical productivity distributions and face the same signaling cost per unit of  $y$ . In particular, if the threshold level ( $y^*$ ) is higher for blacks than whites, but still low enough so that it pays high-ability blacks to acquire the  $y^*$  signal, then high-ability white workers will obviously earn a higher net wage—the employer's wage offer minus the costs of attaining  $y^*$ .

The question is, however, whether this type of discriminatory situation is stable. If workers know their own productivity (or, equivalently, their costs of attaining  $y^*$ ), then the high-ability black workers will know they are being underpaid (or, more accurately, overtaxed for  $y$  signals), relative to high-ability white workers. It is not difficult to construct examples of arrangements between individual high-ability black workers and employers, whereby the former agree (to the benefit of both parties) to accept lower wages in return for signaling with a lower  $y^*$ —eventually a  $y^*$  as low as that for white high-ability workers.<sup>24</sup> In equilibrium there would be common wage offers,  $y$  attainments, and, therefore, common net wages for similarly productive white and black

<sup>24</sup>A simple example of a breakdown in the Spence model of discrimination may be obtained from Glen G. Cain, Department of Economics, University of Wisconsin—handout entitled, "Criticisms and Counter-Examples of the Spence Model of 'Market Failures' in a World of Job Market Signaling," April 27, 1976. See also the suggestions of Stiglitz and Thurow about "trial-period" wage offers by workers as mechanisms by which a unique equilibrium (or, at least, a nondiscriminatory equilibrium) can be achieved. See Joseph E. Stiglitz, "Theories of Discrimination and Economic Policy," in George M. von Furstenberg, Ann R. Horowitz, and Bennett Harrison, eds., *Patterns of Racial Discrimination, Vol. II, Employment and Income* (Lexington, Mass.: Lexington Books, D. C. Heath and Co., 1974), pp. 5–26 and Thurow, *Generating Inequality*, p. 173. Still another source of possible attainment of a unique equilibrium—this time from actions initiated by employers—is suggested by Spence, *Market Signaling* in Appendix E (especially pp. 174–76), and this possibility is given credence and analyzed by John G. Riley, "Competitive Signaling," *Journal of Economic Theory*, Vol. 10 (April 1975), pp. 174–86.

<sup>21</sup>Spence, *Market Signaling*.

<sup>22</sup>Spence, *Market Signaling* and "Job Market Signaling."

<sup>23</sup>Indeed, the informational feedback loop that Spence describes provides an effective counterargument to the claim, mentioned earlier, that employers' "erroneous" or "mistaken" behavior will sustain discrimination.

workers—in other words there would be no economic discrimination.

Another model of economic discrimination is suggested by the work of the psychologist R. L. Thorndike (reported by Linn).<sup>25</sup> It incorporates reliability differences but without recourse to risk aversion. As we present it, this “selection-truncation model” contains one feature of Spence’s model—namely, a lower-bound threshold value of  $y$ , below which all  $y$  scores signal what is, in effect, a single  $q$  value. Above the lower-bound threshold, the  $y, q$  relation is positive and continuous as it was previously in our models. As in the preceding case, black workers have equal average ability but receive a lower average wage; here, however, employers are not practicing racial discrimination.

The crucial assumption of the selection-truncation model is that the hiring or selection decision is confined to the upper end of the  $y$  distribution. Given their advantage in test reliability, whites tend to be preferred on the basis of expected values of  $q$ , given  $y$ , even though their  $q$  distribution is actually identical to that of blacks. The potential preference for blacks at the lower end of the  $y$  distribution is nullified because workers with low  $y$  scores are not hired. Clearly, a higher average value of  $\hat{q}$  (or wage rate) for whites would emerge—evidence of economic discrimination in market outcomes—despite the fact that employers are not race-biased in their hiring process: that is, they hire workers solely on the basis of  $E(q|y)$ .

Figure 4 shows this result in an extreme form. Perfect reliability for  $W$ s and zero reliability for  $B$ s are assumed. The distribution of  $q$  is identical for  $W$ s and  $B$ s (as indicated on the vertical axis). Only values of  $q > \alpha$  are eligible for hire. (Assume that  $\alpha$  represents a comprehensive, legal minimum wage, here unrealistically set equal to the wage corresponding to the overall average value of productivity.) Given the costs of hiring and associated costs of making a mistake, all blacks, but only half the whites, would be unemployed or not in the labor force. The model has,

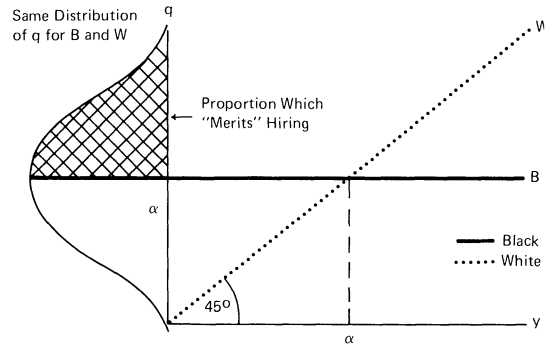


Figure 4. Predictions of Productivity ( $q$ ) by Race and Test Score ( $y$ ), Assuming Perfect and Zero Predictive Relations for Whites and Blacks, Respectively.

of course, greater relevance to, say, college admissions than to the labor market, but it may be at least suggestive of some economic situations. In fact, theories of the labor market that emphasize pervasive long-run wage rigidities, such as the “job competition” theory of Thurow, are fertile soil for the type of selection-truncation model of discrimination presented here.<sup>26</sup>

### Unequal Average Abilities

We return now to the model in Phelps’s paper in which the variances of  $q$  and  $u$  are equal for the two groups and the mean ability of blacks is less than the mean ability of whites. Here, the systematic effect of blackness,  $\alpha^B < \alpha^W$ , leads to a lower predicted value of  $q$  for blacks than whites, even if the  $y$  scores are equal, because  $y$  is, by assumption, a fallible indicator. Phelps remarks that  $\alpha^B < \alpha^W$  might reflect “disadvantageous social factors.”<sup>27</sup>

<sup>26</sup>Thurow, *Generating Inequality*, pp. 173–75.

<sup>27</sup>The three words in quotes are used by Phelps but are not written in a single phrase, although the expression fairly conveys his meaning. Without more information, however, the interpretation of this expression—and of  $\alpha^B < \alpha^W$ —could be ambiguous. Does the lower value of  $\alpha^B$  reflect a real deficiency in skills, as would be the case if the social factors were less schooling, less training, and poorer health? Or does  $\alpha^B < \alpha^W$  reflect merely a misconception or a false stereotype held by employers? In accordance with our earlier expressed preference for believing that employers will not persist in erroneous behavior, we assume the first interpretation.

<sup>25</sup>Linn, “Fair Test Use in Selection.”

The relevant  $y, q$  relation for  $W$ s and  $B$ s is shown in Figure 5. The  $B$  line is below and parallel to the  $W$  line; the equality of slopes is a consequence of the assumptions of equal variances of  $q, u$ , and, therefore,  $y$ . As noted earlier, competitive forces in the market lead employers to pay workers according to their expected productivity; thus, white workers will be preferred to (and get higher wages than) black workers with the same  $y$  score. Unlike Phelps, we do not believe that different pay for the same  $y$  score demonstrates economic discrimination. Indeed, were  $B$ s to get paid the same as  $W$ s when both had the same  $y$  scores, there would manifestly be discrimination against  $W$ s, since by Phelps's assumption the latter are more productive (i.e., they have a higher average  $q$ ).

One could argue, of course, that the very existence of different average ability,  $\alpha^B < \alpha^W$ , demonstrates a type of discrimination in which workers are not paid in accordance with their *innate* abilities, and we would agree. But we would generally view its source in premarket discrimination—discrimination in schooling or in the acquisition of other forms and amounts of human capital that workers possess when they enter the labor market. (In Spence's

terms, the discriminated groups are faced with higher costs of signaling.) Given these handicaps, however, the differential pay (or difference in employer demand) appears to be no more economically discriminatory than are the lower wages that would be paid to workers with less experience, other factors (like the  $y$  score) being equal. (Of course, "legal discrimination," or other definitions of discrimination, need not be bound by economic terms, nor are wage payments in accordance with expected productivity necessarily synonymous with good social policy.)

Finally, although Figure 5 demonstrates nondiscrimination by the "outcome" criterion of a proportional relation between average compensation for the groups and their respective average productivities, at every ability level a black will be paid less than his white counterpart. This is seen also in Equation 6,  $E(\hat{q}|q) = (1 - \gamma)\alpha + \gamma q$ . For the same ability ( $q$  value) and regression slope ( $\gamma$ ) but lower black mean ability,  $(1 - \gamma)\alpha^B < (1 - \gamma)\alpha^W$ .

This apparent paradox is resolved simply by recalling that here we *assume* less average ability for blacks *and* imperfect information. Thus, blackness is *assumed* to provide information that  $E[\hat{q}^B|y]$  ex ante is lower than  $E[\hat{q}^W|y]$  ex ante for all  $y$ . But the employer's ex ante expectation that a black worker will be less productive than a white worker, given the same  $y$  scores, must result in a black worker with a given  $q$  ability receiving a lower wage than a white worker with that same  $q$  ability, *on average*, because  $E(\hat{q}|q)$  and  $E(\hat{q}|y)$  involve exactly the same parameters. In the presence of *perfect* information ( $\gamma = 1$ ), the systematic difference in  $E(\hat{q}|y)$  or  $E(\hat{q}|q)$  for the two racial groups disappears, of course, and the lines for both color groups coincide with the 45° line.

In light of known premarket discrimination against blacks, the assumption by employers of unequal average abilities is realistic, and so is the assumption of imperfect information. The systematic inequality in  $E(\hat{q}|q)$  that results is, therefore, profoundly disturbing, and perhaps this inequality is what is referred to by the economists mentioned in footnote 8. One consolation is that this inequality should

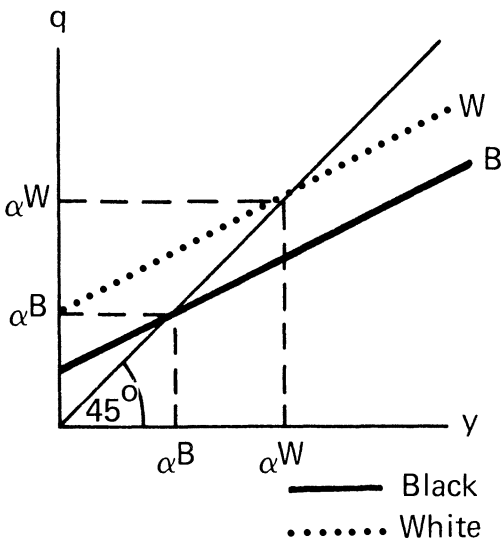


Figure 5. Prediction of Productivity ( $q$ ), by Race and Test Score ( $y$ ), Assuming the Slopes Are Equal.

decline as employers assimilate more knowledge over time, thereby reducing  $Var(u)$  and raising  $\gamma$ .

We end this discussion with the cautionary remark that assuming  $\alpha^B < \alpha^W$  along with  $\gamma^B = \gamma^W$  may not be very realistic. If the "raw-labor" abilities of blacks and whites are equal, a higher average ability of white workers must reflect an advantage in human capital acquisitions. Under these conditions, there is no basis for assuming that white workers have the same variance in  $q$ , and, therefore, no presumption that the  $\gamma$  values for the two groups are equal.

### Conclusions

Several economists have heretofore advanced statistical theories of discrimination in labor markets. Although the Phelps model does not, in our opinion, explain or describe racial or sex discrimination, it provides a useful point of departure for several models that do. The models focus on differential reliability in productivity indicators among identifiable groups of workers. On empirical grounds the differential is more plausibly introduced when blacks (or women) are assumed to have less reliable scores. When we combine this reliability differential with risk aversion by employers, our model depicts economic discrimination that is qualitatively consistent with empirical evidence. In another model the combination of lesser reliability for blacks (or women) on tests with truncation of lower-scoring applicants also reveals a kind of economic discrimination. Both examples call attention to the potential inequities that may stem from lower test reliabilities for minority groups.

We are reluctant, however, to claim too much for these models. In the model that uses risk aversion, there are grounds for questioning whether the size of the risk premium would be very large, and we know of no empirical support for a "cross-over" point at the lower end of the indicator scale, where blacks earn more than whites for comparable indicator scores. Obviously, we have made no thorough attempt to test the model, or even to give more satisfactory empirical definitions of

the  $y$  variable. The  $\hat{q}$  variable itself has been assumed to represent a wage rate throughout, implicitly relying on the proposition that wages measure productivity and that competition will, on average, match equal productive abilities with equal wages. Finally, one may argue that there is no discrimination when productivity is defined either to include the informational content of "signaling" or in terms of contributions to an employer's utility function that allows for risk aversion.

In Spence's model of market signaling, discrimination may result if the costs of signaling differ for different groups or if different groups have different initial signals. The former case represents a type of pre-labor market discrimination that results in the discriminated group having lower average abilities in the labor market. The latter case is not clearly indicative of sustained discrimination. A number of variations of the Spence model are currently being investigated by various economists, however, so final judgments should be withheld.

Other real world influences that affect economic discrimination have also been ignored. We have not dealt with monopoly or monopsony.<sup>28</sup> Tastes for discrimination by employers or their systematic subjective underevaluations of the abilities ( $q$  values) of the discriminated groups have been

<sup>28</sup>As the reader may know, however, the evidence for these anticompetitive sources for sustained economic discrimination is meager. Orley Ashenfelter produces evidence *against* a net antiblack discrimination effect of unions in "Racial Discrimination and Trade Unionism," *Journal of Political Economy*, Vol. 80, No. 3 (May/June 1972), pp. 435-64. Becker, *The Economics of Discrimination*, 2d ed., pp. 7-8, has disputed the claim made by Thurow, *Poverty and Discrimination* (Washington, D.C.: The Brookings Institution, 1969), that monopsony power by employers is an important explanation. And Armen A. Alchian and Reuben A. Kessel have argued that monopoly power in the product market is consistent with long-run economic discrimination only (or mainly) when there are constraints on the employers' ability to maximize money profits, as in regulated monopoly industries. (See "Competition, Monopoly, and the Pursuit of Pecuniary Gain," Universities-National Bureau Committee for Economic Research, *Aspects of Labor Economics: A Conference of the Committee* (Princeton, N.J.: Princeton University Press, 1962), pp. 156-75.

downplayed. While neither of these latter modes of behavior, by itself, is consistent with long-run economic discrimination in a competitive model, introducing additional factors may provide consistency. It would take a more extensive discussion to deal with Arrow's list of additional considerations, which includes capital market imperfections, wage rate rigidities, discontinuities in hiring decisions, and self-fulfilling

prophecies.<sup>29</sup> It is fair to say, however, that most of the explanations of discrimination that rely on noncompetitive, disequilibria, and "noneconomic" forces have been offered very tentatively and leave a number of unanswered questions.

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<sup>29</sup>Arrow, "The Theory of Discrimination," pp. 26-32. See Stiglitz, "Theories of Discrimination and Economic Policy" for insightful and sometimes skeptical comments about these considerations.