Chapter 17: Consumption [1]

1 Exercise: Two-period Fisher Model of Consumption

Consider a consumer that lives for two periods, $t = 0$ and $t = 1$. This consumer wants to maximize utility over his or her lifetime, which is given by the function $U(c_0, c_1)$, where c_0 is consumption at time $t = 0$ and c_1 is consumption at time $t = 1$. With this lifetime utility function, assume that the consumer wants to uniformly smooth consumption across time. The consumer receives income y_0 at $t = 0$ and y_1 at $t = 1$, which is known ahead of time with certainty. The gross rate of return is $(1 + R)$, so \$1 saved at $t = 0$ yields $\$(1 + R)$ at $t = 1$; R is the real interest rate.
There are two consumers, Albert and Beatrice, who receive the following fixed income independent of R:

	y_0	y_1
Albert	\$100	\$100
Beatrice	\$0	\$210

a) You observe consumption levels:

	c_0	c_1
Albert	\$100	\$100
Beatrice	\$100	\$100

Solve for R. *(hint: use the budget constraint)*

b) Suppose that the interest rate increases. What will happen to c_0 and c_1 for Albert? Is he better or worse off as a result of the change in R?

c) Again, suppose that the interest rate increases. What will happen to c_0 and c_1 for Beatrice? Is she better or worse off as a result of the change in R?

2 Exercise: Consumption Function

Let's go though a few alternatives to the Keynesian consumption function, $C = \bar{C} + MPC(Y - T)$.

a) Define W = current wealth; R = years to retirement; Y = yearly income; T = remaining years of life. Write Modigliani's life-cycle consumption function and average propensity to consume.

b) Define Y^P = permanent income; Y^T = transitory income; α = fraction of Y^P consumed annually. Write Friedman's permanent income consumption function and average propensity to consume.

c) Assume: $\bar{C} = 0$, $T = 0.2Y$; $W = 0$, $T = R + 10$; $Y^P = 0.75Y$. You observe that $\frac{C}{Y} = 0.35$ in aggregate data for households. Solve for MPC, R, and α.

[1] Econ 302, Week 15 (last week!), 12/11/2009; UW-Madison. TAs Lihan Liu and Scott Swisher.

3 Exercise: Intertemporal Consumption (optional)

Consider a consumer that lives for two periods, $t = 0$ and $t = 1$. This consumer wants to maximize utility over his or her lifetime, which is given by the following function.
Lifetime utility:

$$U(c_0, c_1) = c_0^{\frac{1}{2}} + \beta c_1^{\frac{1}{2}} \tag{1}$$

where c_0 is consumption at time $t = 0$ and c_1 is consumption at time $t = 1$. $0 < \beta < 1$ is some constant less than one; the consumer is impatient, preferring to consume today (β is called the discount factor).
The consumer receives income y_0 at $t = 0$ and y_1 at $t = 1$, which is known ahead of time with certainty. The gross rate of return is $(1 + R)$, so \$1 saved at $t = 0$ yields \$$(1 + R)$ at $t = 1$; R is the real interest rate. This means that we can write c_1 in terms of y_0, y_1, and c_0; all the income that is left over at time $t = 1$ is consumed.
Budget constraint:

$$(1 + R)(y_0 - c_0) + y_1 = c_1$$

Since $(y_0 - c_0)$ is saved in the first period, $(1 + R)(y_0 - c_0)$ plus new income y_1 can be used for consumption in the second period. Because you want to maximize utility, you'll consume all of your income in the second period.

a) Let $\beta = 1 + R = 1$. Write out the utility function and budget constraint under this assumption. Argue that $c_0 = c_1 = \frac{y_0 + y_1}{2}$ (complete consumption smoothing) is best in terms of maximizing utility. How did you arrive at your answer? *(hint: think about what happens if $c_0 \neq c_1$)*

b) Consider the general case with no assumptions on β or $(1 + R)$. First, let's use the budget constraint to eliminate c_1 as something you have to choose. Write out $U(c_0)$, lifetime utility as a function of only c_0 and income. *(hint: substitute the budget constraint into the utility function for c_1)*

c) Maximize $U(c_0)$ with respect to c_0 and solve for the utility-maximizing (c_0^*, c_1^*) as a function of income. *(hint: set $U'(c_0) = 0$ and solve for c_0, then solve for c_1 using the budget constraint; you don't need to simplify)*

d) Compute partial derivatives $\frac{\partial c_0^*}{\partial \beta}$ and $\frac{\partial c_0^*}{\partial R}$. Can you sign them? Interpret. *(hint: again, either write c_0^* as a product and use the product rule or keep c_0^* as a fraction and use the quotient rule)*

e) A typical Keynesian consumption function of form $C_t = \bar{C} + MPC(Y_t - T_t)$ at time t has consumption today depending only on current disposable income. Using your previous results, discuss why this is incomplete if consumers are intertemporal utility-maximizers. Propose an alternative. *(hint: what did utility-maximizing consumption depend on in the previous parts?)*