

Homework will be graded for both content and neatness. Sloppy or illegible work will not receive full credit. This homework requires the use of Microsoft Excel.

1) The following question is concerned with the effect of personal income taxes and minimum wage laws on labor market outcomes for workers. You are provided with an aggregate production function of the form $Y = AK^\alpha L^{1-\alpha}$, where $A = 9$, $\alpha = 0.5$, and $K = 25$ are fixed throughout the problem. The equation $L_S = 100[(1-t)w]^2$ describes labor supply, where t is level of taxes and w is the real wage. Therefore, we can write $(1-t)w$ as the after-tax real wage. Firms are assumed to hire labor until the marginal product of labor equals the real wage: $MPL = w$. This defines equilibrium in the labor market.

A) Compute the marginal product of labor (MPL). Recall that $MPL = \frac{\partial Y}{\partial L}$. It should only depend on L .

$$Y = (9)(25)^{0.5} (L)^{0.5} = 45L^{0.5}$$

$$MPL = \frac{\partial Y}{\partial L} = (45)(0.5)L^{-0.5} = 22.5L^{-0.5}$$

B) Set $MPL = w$ and solve for L . Call this labor demand, L_D . It should only depend on w .

$$w = MPL = 22.5L^{-0.5}$$

$$L^{0.5} = 22.5w^{-1}$$

$$L_D = 506.25w^{-2}$$

C) Let $t = 0$; find the equilibrium levels of w , L , and Y (the full employment level of output). What is the total factor payment received by labor, i.e. labor income?

$$L_S = L_D \Rightarrow 100[(1-t)w]^2 = 506.25w^{-2}$$

$$w^4 = \frac{506.25}{100} = 5.0625 \Rightarrow w_e = 1.5$$

$$L_e = 100 \cdot w_e^2 = 100 \cdot 1.5^2 = 225$$

$$Y_{FE} = 9 \cdot 25^{0.5} \cdot L_e^{0.5} = 45(225)^{0.5} = 675$$

$$\text{Labor Income} = w_e L_e = (1.5)(225) = 337.5$$

D) Repeat part (C) under the assumption that $t = 0.6$. Is labor better or worse off in terms of the after-tax real wage and after-tax labor income?

$$L_S = L_D \Rightarrow 100[(1 - 0.6)w]^2 = 506.25w^{-2} \Rightarrow 16w^2 = 506.25w^{-2}$$

$$w = \frac{506.25}{16} \Rightarrow w_e = 31.6406^{0.25} = 2.372$$

$$L_e = 16(2.372)^2 = 90 \Rightarrow Y_{FE} = 45(90)^{0.5} = 426.907$$

$$\text{Labor Income} = w_e L_e = (2.372)(90) = 213.454$$

$$w_{e, \text{after-tax}} = (1 - t)w_e = (0.4)(2.372) = 0.9488 \Rightarrow w_{e, \text{pre-tax}} > w_{e, \text{after-tax}}$$

$$\text{Labor Income}_{\text{after-tax}} = (1 - t)w_e L_e = (0.4)(213.454) = 85.382 \Rightarrow \text{Labor Income}_{\text{pre-tax}} > \text{Labor Income}_{\text{after-tax}}$$

The real wage increases, but the post-tax real wage is less than the pre-tax real wage. As both the real wage (after-tax) and labor income decrease after the tax on labor is imposed, labor is worse off. Fewer people are working due to the effect of the tax on labor supply, and those that are still in the labor force are receiving a lower after-tax wage.

E) Repeat part (C) with $t = 0$ and a minimum wage of $\underline{w} = 2$. Draw a rough graph showing the intersection of the labor demand and labor supply curves, the minimum wage (price floor), the equilibrium level of employment, and the equilibrium real wage. Is labor better or worse off in terms of the after-tax real wage and after-tax labor income?

Previous $w_e = 1.5 \Rightarrow \underline{w} = 2$ is a binding price floor $\Rightarrow L_S > L_D \Rightarrow$ surplus in the labor market

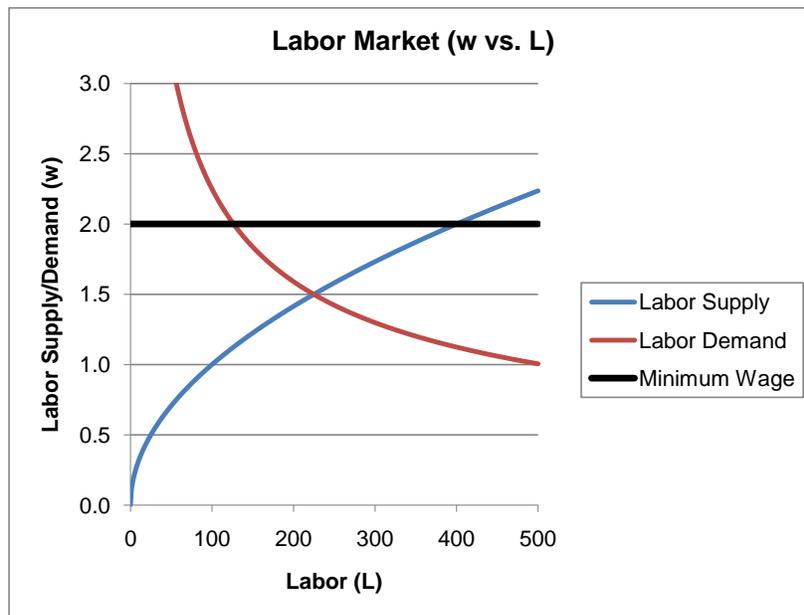
$$L_S(2) = 400$$

$$L_D(2) = 126.563 \Rightarrow L_e = L_D(2) = 126.563 \text{ (excess supply)} \Rightarrow w_e = 2$$

$$\text{Labor Income} = w_e L_e = (2)(126.563) = 253.125$$

The real wage has increased, but fewer workers can find employment under the binding price floor of $w=2$. As a result, total labor income has decreased, so workers as a whole are worse off under the

minimum wage. However, if you are able to find employment, your personal income has improved at the expense of your fellow workers who can't find a job. In that sense, the policy is arbitrarily redistributive.



F) Taking into account your answers to parts (C) through (E), should the government intervene in the labor market? Why or why not?

If the government imposes an income tax $t > 0$ or sets a minimum wage above the market-clearing wage without intervention, workers as a group are made worse off. However, if the government gives an earned income tax credit ($t < 0$) to workers, labor is better off. This analysis assumes perfect competition in the labor market, homogeneity across workers (all workers are the same and can't be differentiated by their level of human capital or skill), and doesn't account for regulations like workplace safety requirements that might decrease total employment slightly but make the remaining workers in the labor force much better off. Also note that output decreases under government intervention, which implies that labor market regulations can potentially cause a slowdown in economic activity and recession.

2) Let $Y = AK^{0.3}L^{0.5}N^{0.2}$ be a Cobb-Douglas production function that uses the level of technology (A), capital (K), labor (L), and land (N) to produce output (Y). Technological progress makes all other factors more productive, and land is used in combination with labor and capital. Time is represented by t and takes on values $t = 0, 1, 2, \dots, 20$. The next four equations describe how technology and the factors of production change over time; e is the exponential function. Excel is required for this problem.

$$A(t) = 2e^{0.03t} \quad (\text{technology})$$

$$K(t) = 10e^{0.05t} \quad (\text{capital})$$

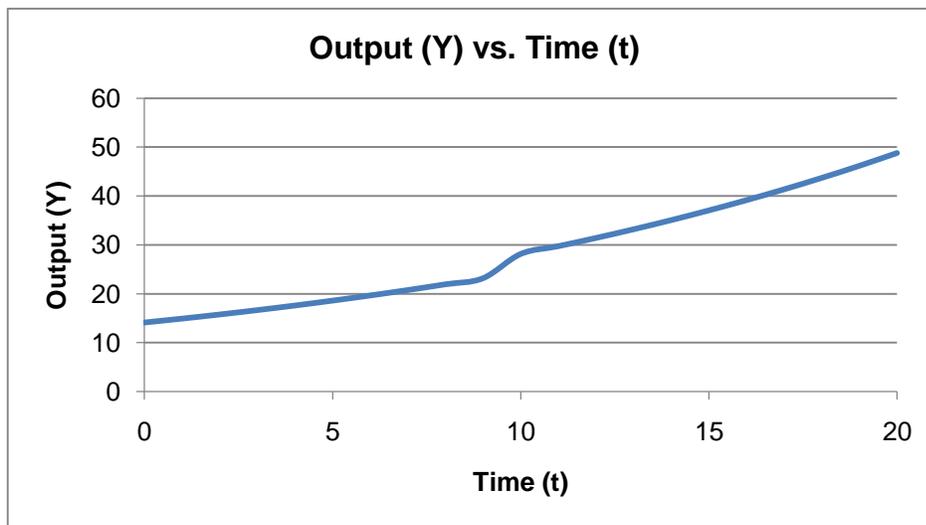
$$L(t) = 5e^{0.02t} \quad (\text{labor})$$

$$N(t) = \{ 10, \text{ if } 0 \leq t \leq 9; 20, \text{ if } t > 9 \quad (\text{land; land reclamation project completed at time } t=10)$$

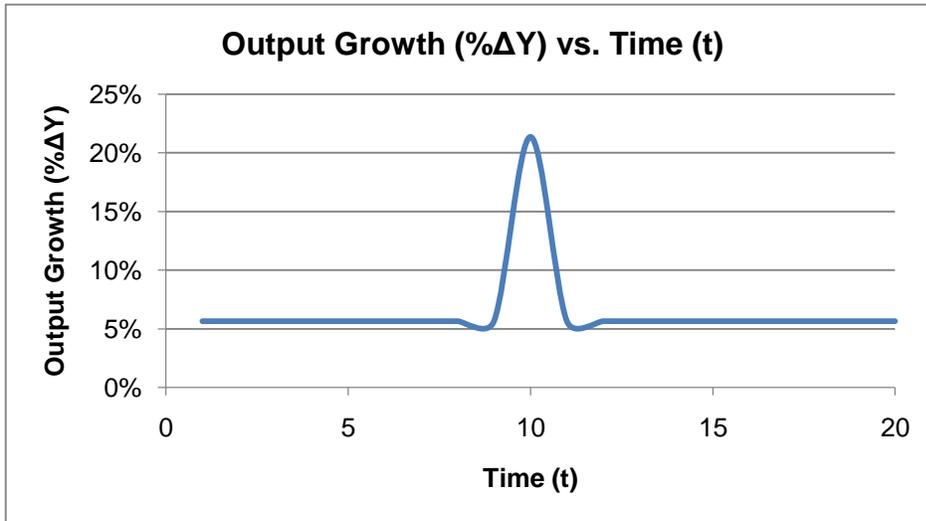
A) Compute Y , ΔY , and $\% \Delta Y$ for $t = 0, 1, 2, \dots, 20$. [HINT: for this question, see Excel file: Q2 tab. EXP(x) returns e to the power x in Excel.]

See Excel.

B) Graph Y and $\% \Delta Y$ for $t = 0, 1, 2, \dots, 20$. Talk about the graphs qualitatively.

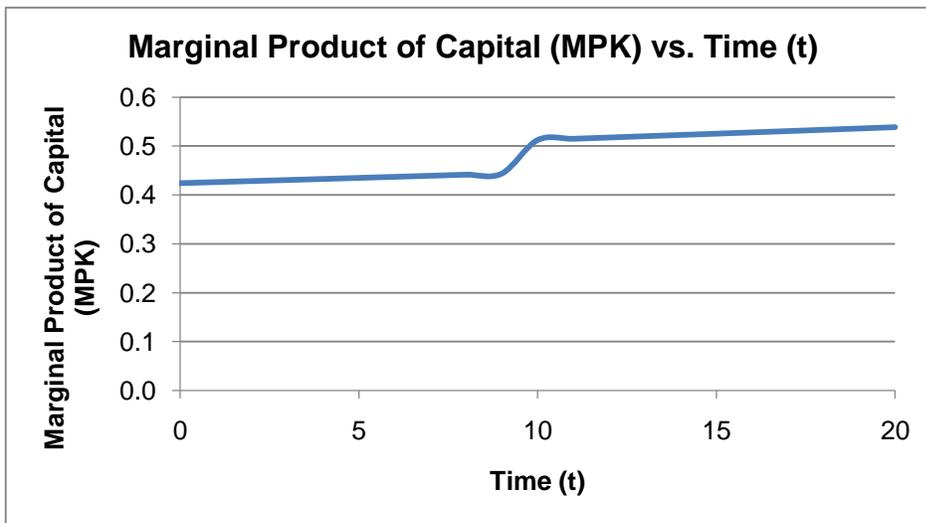


Output seems to be growing at a constant rate, with a discrete jump at $t = 10$ when the land reclamation project is completed. This project doesn't seem to affect the growth rate afterwards, however.



Output growth in terms of percentage change is constant around 6% except during the period when the land reclamation project is completed. Due to the discrete jump in output there, output growth is over 20% for $t = 10$. However, output growth returns to its constant rate after $t = 10$.

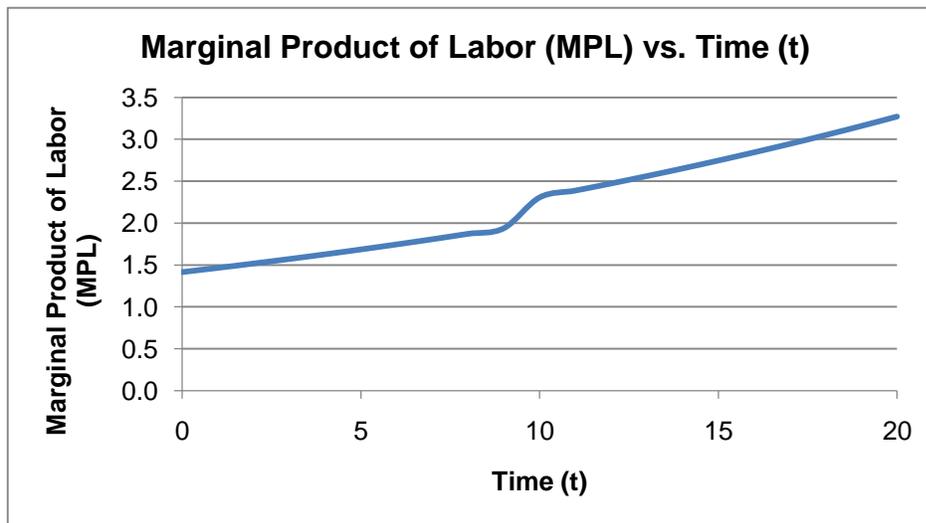
- C) What is the marginal product of capital (MPK) for this production function?
Graph it for $t = 0, 1, 2, \dots, 20$. Is it smooth (continuous)? Why or why not?



The marginal product of capital grows slowly both before and after the discrete jump at $t = 10$. The curve looks smooth because the graph connects the data points nicely, but really there is a non-smooth jump up when the land reclamation project is completed. Therefore, the graph is not continuous.

D) What is the marginal product of labor (MPL) for this production function?

Graph it for $t = 0, 1, 2, \dots, 20$. Is it smooth (continuous)? Why or why not?



Very similar to the marginal product of capital. However, the marginal product of labor is higher in terms of the level of the series. This is because labor enters into the production function with a higher exponent, 0.5 versus 0.3. This is carried over when the partial derivative is taken to get marginal products. Again, the graph is not continuous.

E) Is the level of output Y accelerating or decelerating during this time period? Why is this happening?

Technically, the slope of the graph of output versus time is changing, with the graph becoming steeper, so the level of output is accelerating (the rate of change, the growth rate, is increasing). However, if you use percentage change to measure the growth rate, the growth rate is constant across the entire period except for $t = 10$. This is because the factors of production are all growing exponentially due to the functional forms we used.

3) The following question refers to the model of the loanable funds market in Mankiw 7e chapter 3. You are provided with the following information about the structure of the economy:

$$C = 160 + 0.6Y_D \quad (\text{consumption function})$$

$$Y_D = Y - T \quad (\text{definition of disposable income})$$

$$Y = C + I(r) + G \quad (\text{equilibrium in the loanable funds market})$$

where Y_D stands for disposable income and $I(r)$ gives the level of investment as a function of the real interest rate. The economy is closed. Therefore, no international trade takes place and there are no international flows of savings or capital.

A) With $I(r) = 150$ a constant function that does not depend on the real interest rate, solve for the equilibrium levels of Y , Y_D , and C if $T = 100$ and $G = 150$ (we haven't fixed the level of output Y yet).

$$Y = 160 + 0.6(Y - 100) + 150 + 150 \Rightarrow 0.4Y = 400 \Rightarrow Y_e = 1000$$

$$Y_{D,e} = Y - T = 1000 - 100 = 900 \Rightarrow C_e = 160 + 0.6(900) = 700$$

B) Repeat part (A) provided that the government lowers taxes to $T = 75$. Did output increase or decrease relative to the old tax rate of $T = 100$? Intuitively, why did this happen?

$$Y = 160 + 0.6(Y - 75) + 150 + 150 \Rightarrow 0.4Y = 415 \Rightarrow Y_e = 1037.5$$

$$Y_{D,e} = Y - T = 1037.5 - 75 = 962.5 \Rightarrow C_e = 160 + 0.6(962.5) = 737.5$$

Output is allowed to vary, so a tax cut that increases disposable income can have a positive effect on the level of consumption and therefore output through the equation that defines equilibrium. Consumption goes up, so Y increases, which means that Y_D increases, again, and so on; i.e. Keynesian multiplier.

C) Now let's hold the level of output fixed, as in the long-run of the classical model. If $Y = 1200$ is held constant and $I(r) = 400 - 25r$, where a real interest rate of 2% is expressed as $r = 2$, solve for the equilibrium real interest rate. Assume that $T = 100$ and $G = 150$.

$$1200 = 160 + 0.6(1200 - 100) + 400 - 25r + 150 = 1370 - 25r \Rightarrow 25r = 170 \Rightarrow r_e = 6.8\%$$

D) Repeat part (C) provided that the government lowers taxes to $T = 50$. Did the real interest rate increase or decrease relative to the old tax rate of $T = 100$? Intuitively, why did this happen?

$$1200 = 160 + 0.6(1200 - 50) + 400 - 25r + 150 = 1400 - 25r \Rightarrow 25r = 200 \Rightarrow r_e = 8\%$$

A tax cut increases disposable income, which increases C , but Y is fixed. Therefore, I must decrease to achieve equilibrium in the loanable funds market. The real interest rate increases to achieve this.

E) Let $T = 100$. What level of government expenditure G is consistent with the equilibrium real interest rate you found in part (D)?

$$r = 8\% \Rightarrow I = 200 \Rightarrow 1200 = 1020 + G \Rightarrow G = 180$$

F) You are an economist on the Council of Economic Advisors (CEA), which advises President of the United States on economic policy. In the latest CEA report, President Obama wants to know which type of expansionary fiscal policy ($T \downarrow$ or $G \uparrow$) has the largest effect on the real interest rate in the long-run. Assuming that both T and G are changed by the same amount, what is your answer?

Solving for the equilibrium real interest rate and taking derivatives with respect to T and G ,

$$Y = 160 + 0.6(Y - T) + I(r) + G \Rightarrow I(r) = 0.4Y - 160 - G + 0.6T \Rightarrow 400 - 25r = 0.4Y - 160 - G + 0.6T$$

$$25r = 560 - 0.4Y + G - 0.6T$$

$$r_e = 22.4 - 0.016Y + 0.04G - 0.024T$$

$$\left| \frac{\partial r_e}{\partial T} \right| = |-0.024| = 0.024$$

$$\left| \frac{\partial r_e}{\partial G} \right| = |0.04| = 0.04$$

$$0.04 > 0.024 \Rightarrow G \text{ is more effective than } T$$

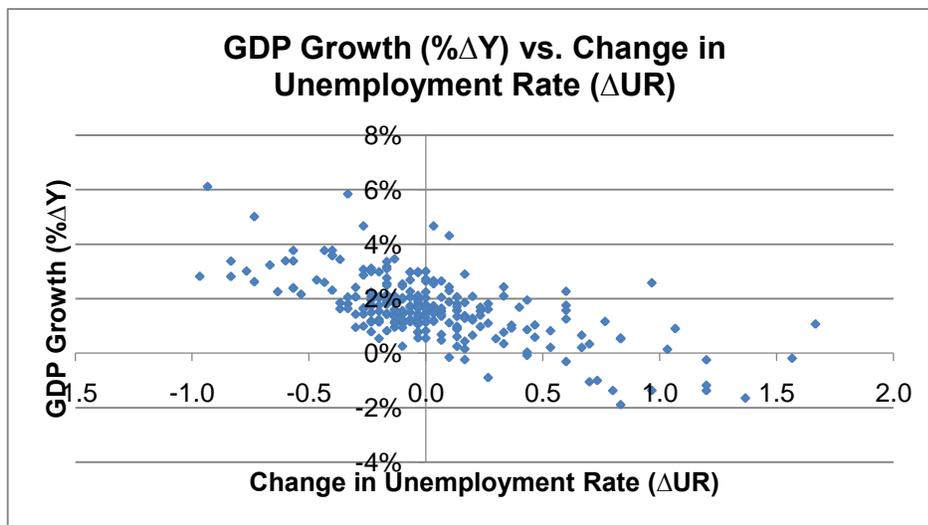
This is because G enters the right-hand side of the equilibrium condition directly, while T is multiplied by the marginal propensity to consume (MPC). If you change T and G by the same amount, the government expenditure will have a larger effect on the real interest rate in long-run equilibrium. This is like saying that the government expenditure multiplier is greater than the tax multiplier because consumers save some of their disposable income. Fiscal policy has no effect on the level of output here, however.

4) “Okun’s law” is an empirical relationship between the change in the unemployment rate and GDP growth. This question asks you to estimate and discuss the relationship using US data, which is provided in the Excel file (homework2fall2009tables.xls, tab Q4). Excel is required for this problem.

A) Compute $\% \Delta \text{GDP}$ (GDP growth) and ΔUR (change in the unemployment rate) for the data provided, from 1948 Q1 (first quarter) to 2009 Q2 (second quarter).

See Excel.

B) Graph $\% \Delta \text{GDP}$ versus ΔUR . In other words, graph $\% \Delta \text{GDP}$ on the vertical axis (y-axis) and ΔUR on the horizontal axis (x-axis). This should be a scatterplot. Label the graph and its axes.

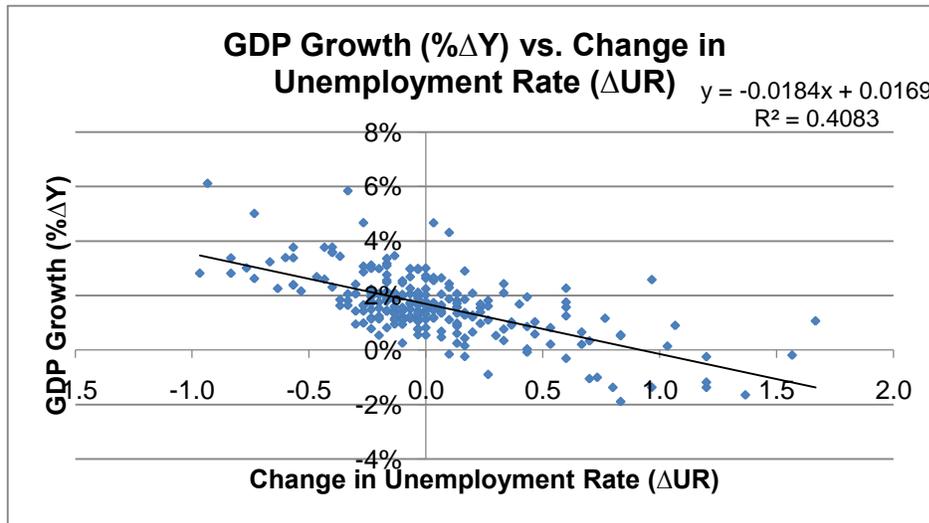


C) Visually inspect the graph. Do you notice any relationship between $\% \Delta \text{GDP}$ and ΔUR ? Is this relationship positive or negative? Describe this qualitatively. If we expect quarterly GDP growth above 3%, what is your prediction about the sign of ΔUR ?

The relationship between $\% \Delta \text{GDP}$ and ΔUR seems to be negative; when $\% \Delta \text{GDP}$ is positive and large, ΔUR is negative. You need twice as much positive growth to get a corresponding 1% decrease in the unemployment rate. If quarterly GDP growth is above 3%, we'd expect that $\Delta \text{UR} < 0$.

D) Add the best-fit linear trend line to the graph. To do this, right-click on your data points and select “add trendline” from the drop-down menu. Select the option “linear” and check the boxes for “display equation on chart” and “display R-squared value on chart”. R^2 is a measure of goodness-of-fit for the linear model, as in how closely your fitted line matches the data. A high value for R^2 is associated with a

model that fits the data well. However, if the data points are far away from the trend line, fit will be poor; therefore, we'd expect that R^2 is low. R^2 is between zero and one.



E) What is the slope of your linear trend line? Interpret.

$$\frac{dy}{dx} = -0.0184.$$

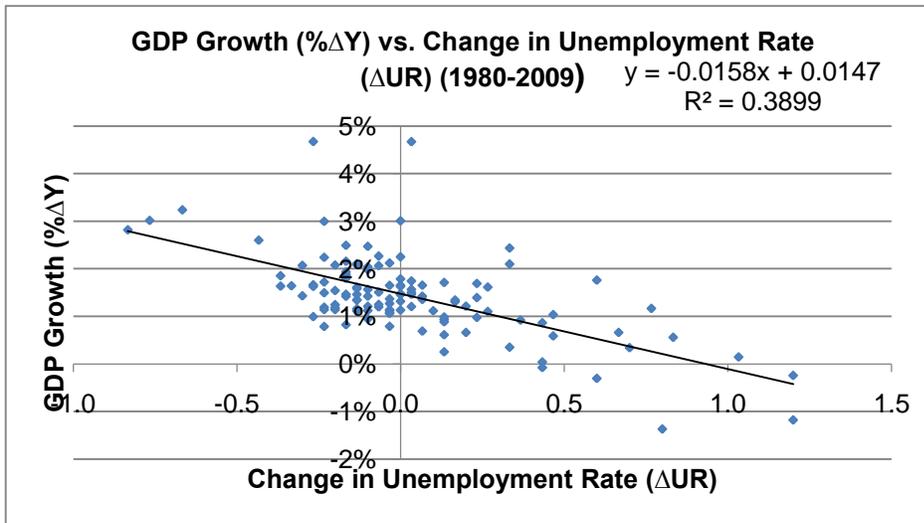
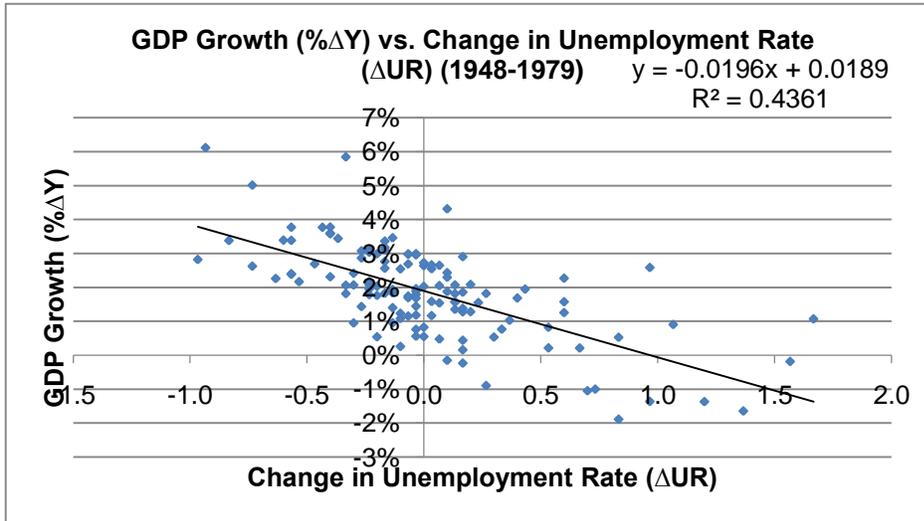
A 1% increase in the unemployment rate is associated with a -1.84% change in the GDP growth rate; the GDP growth rate decreases by approximately 2%. This is the typical statement of Okun's law that you read in a textbook.

$$\Delta UR \uparrow \Rightarrow \% \Delta Y \downarrow$$

F) Provided that the unemployment rate increased by 0.75% this quarter, what is your prediction for GDP growth using the fitted linear model?

$$\% \Delta Y = 0.0169 - 0.0184(0.75) = 0.0031 \Rightarrow \text{positive GDP growth of 0.31\%}$$

G) Let's split the sample at 1980 Q1. Repeat parts (B) through (D) for two periods: 1948 Q1 – 1979 Q4 and 1980 Q1 – 2009 Q2. For the two periods, is there a difference in the slope of the best-fit trend line?



$$\frac{dy}{dx} = -0.0196 \text{ (1948 - 1979)}$$

$$\frac{dy}{dx} = -0.0158 \text{ (1980 - 2009)}$$

There is a difference in the slope of the two best-fit trend lines. The sign is unchanged.

H) “The Great Moderation” is a phrase that refers to the increased macroeconomic stability in the last two decades of the 20th century and the first decade of the 21th century (until the financial crisis). This change is attributed to better central bank policy and productivity growth. Using your results from part (G), did the Great Moderation have an effect on the Okun’s law relationship between $\% \Delta \text{GDP}$ and ΔUR ? If there was an effect, did the relationship become weaker or stronger? Why do you think this happened?

The Great Moderation seems to have weakened the Okun’s law relationship; the slope has become smaller in magnitude and the R^2 value for the best-fit trend line is down as well. Better macroeconomic stabilization policy has kept recessions short, so there is less of a link between changes in unemployment and GDP growth rates. Additionally, countries have become more reliant on international trade, so their growth rates now depend more on the growth rates of their neighbors.

5) Refer to the article “How Did Economists Get It So Wrong?” by Paul Krugman in *The New York Times Magazine* when responding to this question (link on TA website).

NOTE: A bubble is roughly defined as an event where the actual price of an asset is greater than its “true” or fundamental value, what the asset is actually worth based on what it will likely pay out over time. For example, the housing bubble that occurred recently involved housing prices growing far in excess of family income, so eventually families would not be able to afford housing and housing prices would be forced downward. Usually we think of bubbles as temporary, so they “burst” eventually.

A) In what sense did economists “get it so wrong” according to Krugman? What were the consequences of this perceived error in judgment?

Economists failed to predict the financial crisis of 2007-2008 and the subsequent recession. The main consequence was a major decline in economic activity worldwide, coupled with increased unemployment.

B) Briefly compare and contrast what Krugman refers to as the “freshwater” and “saltwater” schools of macroeconomics in the United States. What advice would each school give in terms of fiscal and monetary policy? What do the two schools of thought agree on? Respond in a few paragraphs.

This is a short summary of what an answer should look like.

Freshwater: University of Chicago, University of Minnesota. Markets work well enough such that government intervention isn’t necessary. Monetary and fiscal policy are ineffective due to rational expectations and Ricardian equivalence. The Federal Reserve can pursue New Keynesian monetary policy because it doesn’t matter. Price and wage stickiness are not important. Financial markets perfectly incorporate all available information nearly instantaneously. Bubbles can’t happen.

Saltwater: Harvard, Yale, MIT, Princeton, Berkeley. Markets are subject to frictions, sticky prices, and sticky wages. Markets are not perfect. Bubbles can form in financial markets. Monetary and fiscal policy are effective, but only monetary policy is really necessary. The Federal Reserve should stabilize the macroeconomy through monetary policy and open market operations. Financial markets still work fairly well, though. Traders in financial markets are rational and free profit shouldn’t be available just by trading over a short period of time.

Agreement: Financial markets work well enough, and not much government intervention is required in that area. Economic agents are rational and operate according to rules defined in microeconomics. Both schools prefer monetary to fiscal policy. Both schools care about building up macroeconomics from microeconomic foundations (households maximize utility, firms maximize profit).

C) What does Krugman propose as an alternative to the two conventional schools of thought? Describe this third group. Were they more successful in terms of predicting the financial crisis? If so, did these predictions have an effect on economic policy-making? Why or why not?

Krugman talks about behavioral economics/finance (Shiller, Thaler, Shleifer, etc.). People in their models make mistakes, and suffer from some cognitive biases. They are not perfectly rational (bounded rationality). Bubbles can form very easily in their models. Some people operating under this school of thought predicted the financial crisis; these predictions did not have much of a policy impact, however. This is because the members of this school are not as influential as the New Keynesians when it comes to Federal Reserve policy. Behavioral economics is newer and their theories are not fully accepted.