Economics 102
Spring 2018
Answers to Homework #4
Due 4/5/17

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered**, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. **Please show your work**. Good luck!

Please remember to
- Staple your homework before submitting it.
- Do work that is at a professional level: you are creating your "brand" when you submit this homework!
- Not submit messy, illegible, sloppy work.

1. Aggregate Production Function

Consider the aggregate production function for Dane County:

$$Y = 2K^{1/2}L^{1/2}$$

where Y is real GDP, K is units of capital, and L is units of labor. 2 denotes the state of the technology in Dane County. Labor and capital are the only inputs used in Dane County to produce real GDP. Initially K is equal to **16** units. Answer the following questions based on the above information **(you are required to utilize Excel when answering this question).**

Direction for this problem: You don't need to submit the whole simulated data tables with your homework, but you do need to submit the graphs drawn from theses tables. **The graphs must be generated by using Excel, NOT by your hands.**

Scenario 1: Baseline Model

a. Fill in the following table (you will need to expand it from the truncated form provided here when using Excel). Round all your answers to the nearest hundredth. In your answer you may present the table for L from 1 to 10 units and from 90 to 100 units (that is, you can omit part of the table in the homework you turn in).

L	K	Y	MPL	Y/L
1	16	8.00	4.00	8.00
2	16	11.31	2.83	5.66
3	16	13.86	2.31	4.62
4	16	16.00	2.00	4.00

5	16	17.89	1.79	3.58
6	16	19.60	1.63	3.27
7	16	21.17	1.51	3.02
8	16	22.63	1.41	2.83
9	16	24.00	1.33	2.67
10	16	25.30	1.26	2.53
90	16	75.89	0.42	0.84
91	16	76.32	0.42	0.84
92	16	76.73	0.42	0.83
93	16	77.15	0.41	0.83
94	16	77.56	0.41	0.83
95	16	77.97	0.41	0.82
96	16	78.38	0.41	0.82
97	16	78.79	0.41	0.81
98	16	79.20	0.40	0.81
99	16	79.60	0.40	0.80
100	16	80.00	0.40	0.80

b. Describe verbally what happens to the marginal product of labor as the level of labor usage increases in Dane County. Explain the intuition behind this change in the marginal product of labor.

Holding constant the level of capital and technology, as the level of labor usage increases the marginal product of labor decreases: that is, the addition to total output from hiring an additional unit of labor gets smaller and smaller. This is not surprising given that we are holding capital and technology constant: as more and more labor is hired, the labor has less capital per worker to work with and a fixed amount of technology: this means that the additional workers will not be as productive as were the workers hired earlier who had access to more capital per worker.

c. As the amount of labor increases, what happens to labor productivity? Explain why labor productivity exhibits this pattern.

As labor usage increases given a fixed level of capital and technology, labor productivity decreases. This makes sense since we know that output is increasing as labor increases, but output is increasing at a diminishing rate. Since we are increasing labor by a unit at a time, but output is not increasing at a constant rate but rather is increasing at a diminishing rate this implies that Y/L will get smaller as L gets larger.

Scenario 2: Change in capital (K)
d. Now suppose an American multinational technology company decides to invest in Dane County, and as a result of this decision, the amount of capital stock (K) increases to 25 units.
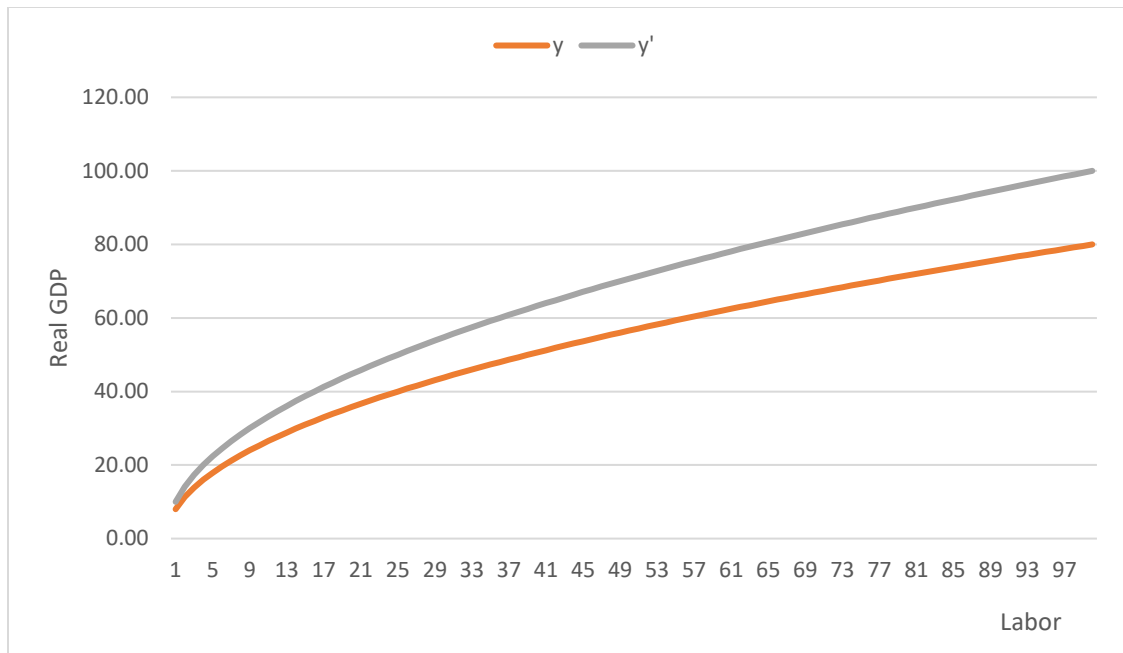
We also assume that the level of technology is held constant. Describe in words how this change in capital will cause the aggregate production function to change.

Holding everything else constant, an increase in capital should cause the aggregate production function to shift up at every level of labor usage.

e. Given the change in capital described in (d), fill in the following table (you will need to expand it from the truncated form provided here).

	L	K	Y	MPL	Y/L
1	25.00	10.00	5.00	10.00	
2	25.00	14.14	3.54	7.07	
3	25.00	17.32	2.89	5.77	
4	25.00	20.00	2.50	5.00	
5	25.00	22.36	2.24	4.47	
6	25.00	24.49	2.04	4.08	
7	25.00	26.46	1.89	3.78	
8	25.00	28.28	1.77	3.54	
9	25.00	30.00	1.67	3.33	
10	25.00	31.62	1.58	3.16	
90	25.00	94.87	0.53	1.05	
91	25.00	95.39	0.52	1.05	
92	25.00	95.92	0.52	1.04	
93	25.00	96.44	0.52	1.04	
94	25.00	96.95	0.52	1.03	
95	25.00	97.47	0.51	1.03	
96	25.00	97.98	0.51	1.02	
97	25.00	98.49	0.51	1.02	
98	25.00	98.99	0.51	1.01	
99	25.00	99.50	0.50	1.01	
100	25.00	100.00	0.50	1.00	

g. Use Excel to graph the original aggregate production function and the new aggregate production function in a graph with L on the horizontal axis and Y on the vertical axis. Does the graph support your prediction in (d)?

Scenario 3: Change in technology (A)

h. Now suppose this American multinational technology company decides not to invest capital in Dane County, but brings Artificial Intelligence and Machine Learning techniques to improve the operation of local factories in Dane County. As a result, the state of the technology in Dane County increases from 2 to 3. Assume that K is still at its initial level. Describe in words how this change in technology will cause the aggregate production function to change.
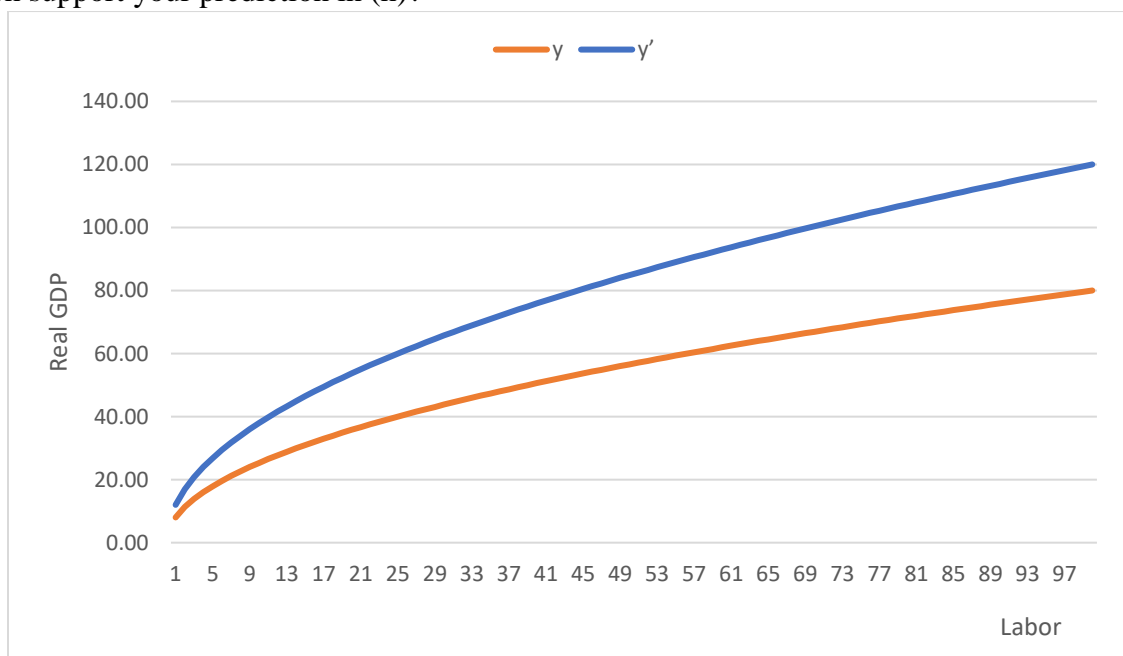
Holding everything else constant, an increase in technology should cause the aggregate production function to shift up at every level of labor usage.

i. Given the change in technology described in (h), fill in the following table (you will need to expand it from the truncated form provided here).

	L	K		Y	MPL	Y/L
1	16.00	12.00	4.00	12.00		
2	16.00	16.97	2.83	8.49		
3	16.00	20.78	2.31	6.93		
4	16.00	24.00	2.00	6.00		
5	16.00	26.83	1.79	5.37		
6	16.00	29.39	1.63	4.90		
7	16.00	31.75	1.51	4.54		
8	16.00	33.94	1.41	4.24		
9	16.00	36.00	1.33	4.00		
10	16.00	37.95	1.26	3.79		

4

j. Use Excel to graph the original aggregate production function and the new aggregate production function in a graph with L on the horizontal axis and Y on the vertical axis. Does the graph support your prediction in (h)?

2. Labor and Capital Productivity

Use the graph below of an economy's aggregate production function to answer the following set of questions. Assume this economy uses only capital (K) and labor (L) to produce real GDP.

Real GDP

a. Suppose this economy is initially producing at point D but then moves to point B. Explain verbally the change in the economy that results from this movement. Explain what caused this economy to move from D to B given the above graph.

b. Given the change described in (a), what happened to labor productivity? Explain your answer.

c. Now suppose that the economy is initially at point D and then something changes in this economy so that the economy produces at point E. Describe verbally what changed and then comment on how this movement from point D to point E affects labor productivity.

d. Given the change in (c), describe what happened to capital productivity as you moved from point D to point E.

a. When the economy moves from D to B the level of real GDP increases from Y4 to Y2. The level of real GDP increases because this economy has increased its use of capital from K to K' while holding its level of labor and technology constant.

b. Labor productivity increases as this economy moves from point D to point B. Recall that labor productivity is defined as Y/L. It is obvious that Y2/L1 is greater than Y4/L1 since Y2 > Y4. As Y increases due to the availability of more capital, this causes each of the units of labor to have higher productivity.

c. When the economy moves from point D to point E we know from inspection of the graph that labor is increasing while technology and capital are staying constant for this economy. Looking at the graph we see that capital is not changing because both points are on the same aggregate

production function. We see that labor is increasing from L1 to L2. As the level of labor increases while holding the level of capital constant, we find that the labor is less productive because they have less capital per unit of labor to work with. Hence, labor productivity decreases as this economy moves from point D to point E. We can see that by drawing the rays from the origin that pass through point D and point E. Notice that the ray that goes through point D has a steeper slope than the ray that goes through point E: this tells us that labor productivity has fallen when this economy moves from point D to point E.

d. When the economy moves from point D to point E, output increases from Y4 to Y3 as labor increases while technology and capital stay constant for this economy. Recall that capital productivity is defined as Y/K. It is obvious that Y3/L1 is greater than Y4/L1 since Y3 > Y4. The intuition here is that with more labor hired each unit of capital has more labor to work with and this makes each unit of capital more productive.

3. Rule of 70
The following table gives the annual growth rate of real GDP for 4 countries in 2016. Assume there is no population growth in any of the four countries.

	China	Japan	Russian Federation	Brazil
Real GDP Annual growth rate	6.1 %	1.1%	-0.4%	-4.4%

Data Source: World Bank national accounts data. (https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG)

a. Which countries can we apply the Rule of 70 to determine how long it will take their real GDP to double? Explain your answer.

a. We can apply the Rule of 70 to China and Japan. We cannot apply the rule of 70 to the Russian Federation and Brazil because they do not have a positive growth rate.

b. For the countries you specified in part (a), approximately how many years will it take their real GDPs to double? Round your answer to the nearest whole year where necessary.)

b. China's real GDP will double in 70/6.1 = 11.48 years, so in 2028. Japan's real GDP will double in 70/1.1 = 63.63 years, so in 2080.

4. Loanable Funds Market

Consider the loanable funds market for an economy. Initially the government of this economy is running a balanced budget. You are told that the demand for loanable funds curve is linear and that at an interest rate of 1%, $5,300 worth of loanable funds are demanded and at an interest rate of 5%, $3,300 worth of loanable funds are demanded. You are also told that the supply of loanable funds curve is linear and when interest rates are at 10%, $1,500 worth of funds are

supplied and when the interest rate is 2.5%, there are no funds supplied. Assume that this economy is initially a closed economy.

a. Given the above information, write an equation for the supply of loanable funds curve where r is the interest rate and Q is the quantity of loanable funds supplied. (If the interest rate is 3%, then the r in the equation would be 3). Assume that the interest rate is measured on the vertical axis and thus, provide your equation in slope-intercept form.

b. Given the above information write an equation for the demand for loanable funds curve where r is the interest rate and Q is the quantity of loanable funds demanded. (If the interest rate is 3%, then the r in the equation would be 3). Assume that the interest rate is measured on the vertical axis and thus, provide your equation in slope-intercept form.

c. Given the above information, what is the equilibrium interest rate and the equilibrium quantity of loanable funds?

Scenario 1: Start with the initial situation. Then, suppose the government increases government spending by $3400 while raising taxes by $2000. Answer the following questions and show the work you did to find your answers.
I. What will be the:
    a. equilibrium level of interest rate?
    b. equilibrium level of loanable funds?
    c. equilibrium level of private saving?
    d. equilibrium level of private investment spending?
II. How much does private investment spending change compared to the level of private investment spending at the initial equilibrium?

Scenario 2: Start with the initial situation. Then, suppose the government increases government spending by $2000 while raising taxes by $3400. Answer the following questions and show the work you did to find your answers.
I. What will be the:
    a. equilibrium level of interest rate?
    b. equilibrium level of loanable funds?
    c. equilibrium level of private saving?
    d. equilibrium level of private investment spending?
II. How much does private investment spending change compared to the level of private investment spending at the initial equilibrium?

Scenario 3: Start with the initial situation. Then, suppose this economy opens to international trade and the resulting trade surplus is $1,050. Answer the following questions and show the work you did to find your answers.

I. What will be the:
   a. equilibrium level of interest rate?
   b. equilibrium level of loanable funds?
   c. equilibrium level of private saving?
   d. equilibrium level of private investment spending?

II. How much does private investment spending change compared to the level of private investment spending at the initial equilibrium?

a. Supply of loanable funds from households: $Q = 200r - 500$ or $r = (1/200)Q + (5/2)$
The supply curve takes the X-intercept form: $Q = a*r + c$
The reciprocal of the slope of the supply curve, or a: $a = (1500 - 0)/(10 - 2.5) = 200$
X-Intercept or c: $1500 = 200*10 + c$, $c = -500$

b. Demand for loanable funds from businesses: $Q = 5,800 - 500r$ or $r = (58/5) - (1/500)Q$
The demand curve takes the X-intercept form: $Q = a*r + c$
The reciprocal of the slope of the demand curve, or a: $a = (5300 - 3300)/(1 - 5) = -500$
X-Intercept or c: $5300 = -500*1 + c$, $c = 5800$

c. The equilibrium interest and investment spending can be found by equating the supply and demand curves in the loanable funds market:
$Qs = Qd = 5,800 - 500r* = 200r* - 500$
$700r* = 6,300$
$r* = 9\%$
$Q* = I = 5,800 - 500(9) = 1,300$
Or, using the slope-intercept forms:
$(1/200)Q + (5/2) = (58/5) - (1/500)Q$
$(5/2)Q + 1250 = 5800 - Q$
$(7/2)Q = 4550$
$Q = 1,300$
$r = (1/200)Q + (5/2) = (1/200)(1300) + (5/2) = 9\%$
Or, $r = (58/5) - (1/500)Q = (58/5) - (1/500)(1300) = (58/5) - (13/5) = 9\%$
The equilibrium interest rate is 9% and equilibrium investment spending is $1,300.

Scenario 1:
I. The government budget deficit of $1,400 (3400-2000) can be modeled as an increase of $1,400 in the demand of loanable funds at every interest rate level. The new demand curve of loanable funds is given by:
$Qd' = 5,800 - 500r + 1,400 = 7,200 - 500r$
a., b. The new equilibrium interest rate and the equilibrium quantity of loanable funds can be found by equating the supply and demand of loanable funds:
$Qs = Qd' = 7,200 - 500r*' = 200r*' - 500$

700r*' = 7,700

r*' = 11%

Q*' = 7,200 – 500(11) = 1,700

The new equilibrium interest rate is 11% and the equilibrium quantity of loanable funds is $1,700.

c. The equilibrium level of private saving can be found by plugging in the equilibrium interest rate into the supply function for loanable funds:

Sprivate = 200(11)-500 = 1700

d. The equilibrium level of private investment spending can be found by plugging in the equilibrium interest rate into the original demand function for loanable funds:

Iprivate = 5800-500(11) = 300

II. Δ Iprivate = 300 – 1,300 = -1,000

The equilibrium level of private investment spending is $300 in this new scenario. Private investment spending decreased by $1,000 compared to the initial equilibrium when the government implemented its deficit spending and taxation package.

Scenario 2: The government budget surplus of $1,400 (3400-2000) can be modeled as an increase of $1,400 in the supply of loanable funds at every interest rate level. The new supply curve of loanable funds is given by:

Qd' = 200r - 500 + 1,400 = 200r + 900

a., b. The new equilibrium interest rate and the equilibrium quantity of loanable funds can be found by equating the supply and demand of loanable funds:

Qs = Qd' = 5,800 – 500r*' = 200r*' +900

700r*' = 4,900

r*' = 7%

Q*' = 5,800 - 500(7) = 2,300

The new equilibrium interest rate is 7% and the equilibrium quantity of loanable funds is $2,300.

c. The equilibrium level of private saving can be found by plugging in the equilibrium interest rate into the original supply function for loanable funds:

Iprivate = 200(7) – 500 = 900

d. The equilibrium level of private investment spending can be found by plugging in the equilibrium interest rate into the demand function for loanable funds:

Iprivate = 5800-500(7) = 2300

II. Δ = 2300 – 1,300 = 1,000

The new equilibrium level of private investment spending is $2300. It increased by $1,000 compared to the initial equilibrium.

Scenario 3: The trade surplus of $1,050 can be modeled as a decrease of $1,050 in the supply of loanable funds at every interest rate level. The new supply curve of loanable funds is given by:
$Qs' = 200r – 500 - 1,050 = 200r - 1,550$
a., b. The new equilibrium interest rate and the equilibrium level of loanable funds can be found by equating the supply and demand equations in the loanable funds market:
$Qs' = Qd = 5,800 - 500r*'' = 200r*'' - 1,550$
$700r*'' = 7,350$
$r*'' = 10.5\%$
$Q*'' = 5,800 – 500(10.5) = 550$
The new equilibrium interest rate is 10.5% and the equilibrium level of loanable funds is $550.

c. The new equilibrium level of private saving can be found by plugging in the equilibrium interest rate into the original supply function of loanable funds:
$Sp' = 200(10.5) – 500 = 1,600$
The new equilibrium level of private saving is $1,600.

d. The equilibrium private investment spending can be found by plugging in the equilibrium interest rate into the private demand function for loanable funds:
$Iprivate' = 5,800 – 500(10.5) = 550$

II. $\Delta = 550 – 1,300 = -750$
The equilibrium level of private investment spending is $550. It decreased by $750 compared to the initial equilibrium.

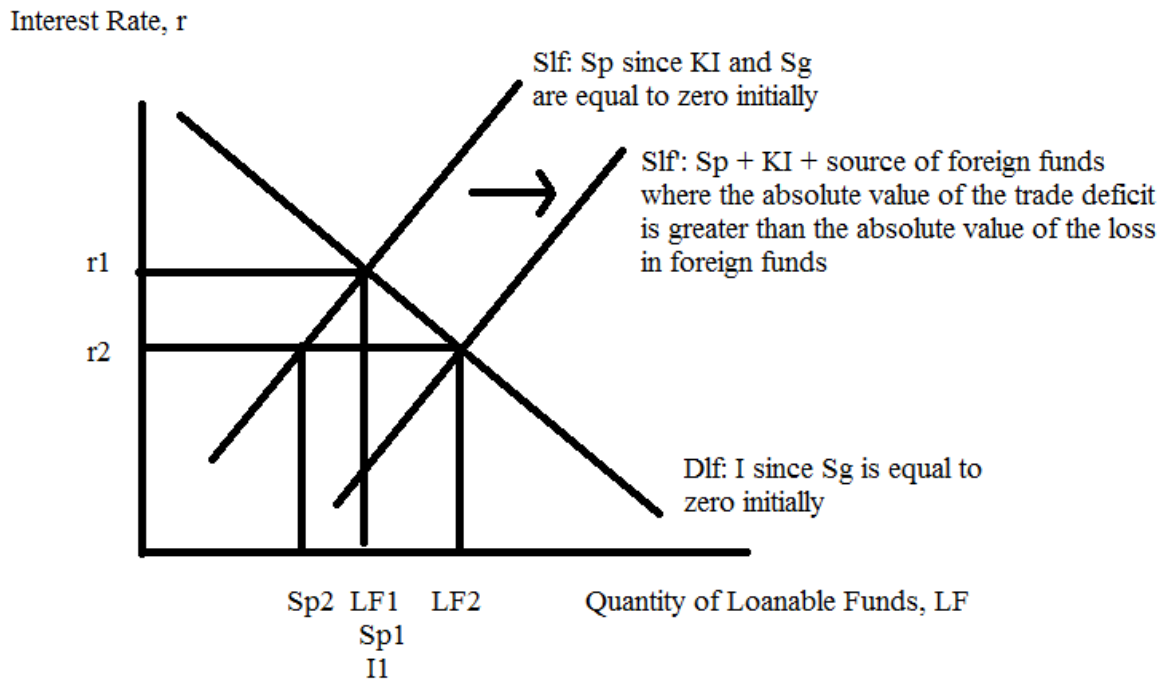## 5. Loanable Funds Market with Trade Deficit and Surplus

Suppose that the loanable funds market is initially in equilibrium and the country has zero net exports and is currently operating with a balanced budget. Provide your analysis about the market equilibrium for each of the following scenarios.

a. Holding everything else constant, what do you predict about the equilibrium interest rate, the equilibrium level of loanable funds and the equilibrium level of private saving if the country runs a trade deficit and at the same time loses funds that a major foreign company is providing as a source of funds to this economy? Explain your answer verbally and provide a graph of the loanable funds market to support your analysis. Make sure your graph is clearly labeled.

b. Holding everything else constant, what do you predict about the equilibrium interest rate, the level of private investment, the level of private saving, the level of consumption spending and the equilibrium level of loanable funds if the country runs a trade surplus and at the same time increases the size of the government deficit? Explain your answer verbally and provide a graph of the loanable funds market to support your analysis.(Just show the old and new equilibrium level of interest rate and loanable funds in your graph) Make sure your graph is clearly labeled.
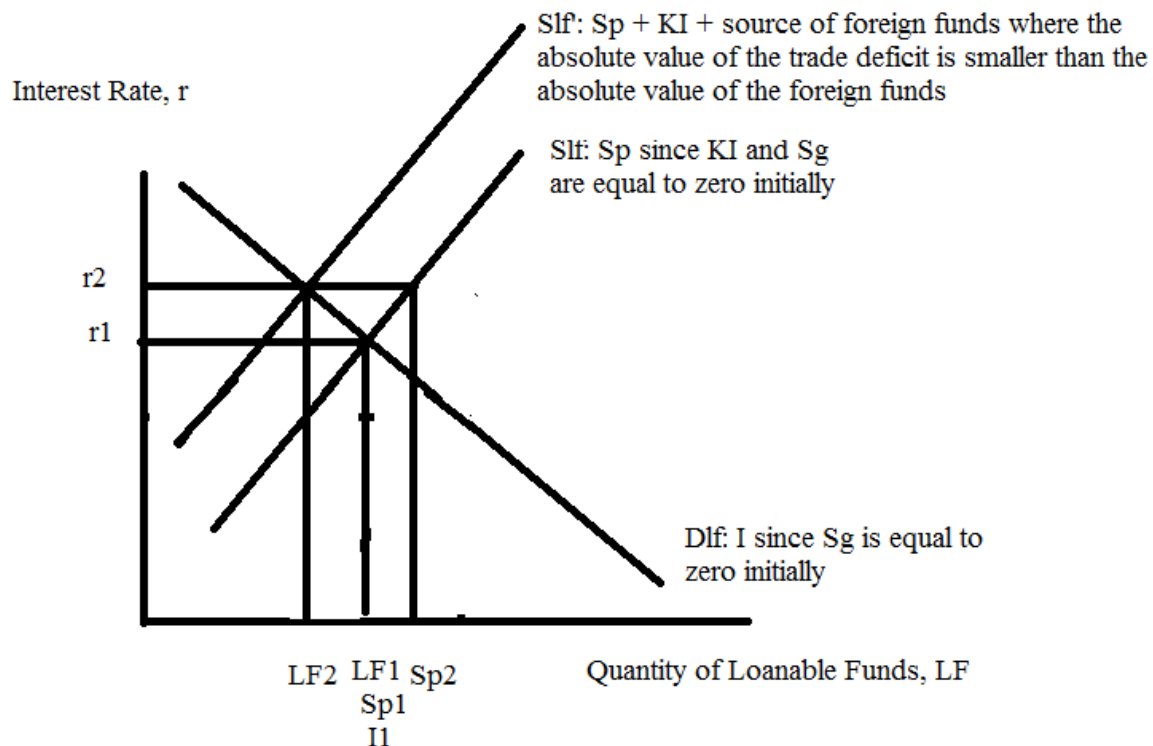
a.The equilibrium interest rate and equilibrium level of loanable funds are indeterminate because the outcome depends on the relative size of the trade deficit versus the amount of funds that the foreign company is reducing as a source of funds to this economy. In the graphs below the initial supply of loanable funds is given as Sp since there are no capital inflows initially (the balance of trade is zero) and the demand for loanable funds is given as I since the government is initially running a balanced budget.

Case 1: The size of trade deficit is larger than the size of the reduction in these foreign funds:

Interest Rate, r

Slf: Sp since KI and Sg are equal to zero initially

Slf': Sp + KI + source of foreign funds where the absolute value of the trade deficit is greater than the absolute value of the loss in foreign funds

r1

r2

Dlf: I since Sg is equal to zero initially

Sp2 LF1   LF2   Quantity of Loanable Funds, LF
      Sp1
      I1

If the size of the trade deficit is larger than the size of the reductions in these foreign funds, the net supply of loanable funds increases and consequently the equilibrium interest rate falls, and equilibrium level of loanable funds rise. The level of private saving would decrease relative to their initial levels.

Case 2: The size of the trade deficit is smaller than the size of the reduction in these foreign funds:

Interest Rate, r

Slf': Sp + KI + source of foreign funds where the absolute value of the trade deficit is smaller than the absolute value of the foreign funds

Slf: Sp since KI and Sg are equal to zero initially

r2

r1

Dlf: I since Sg is equal to zero initially

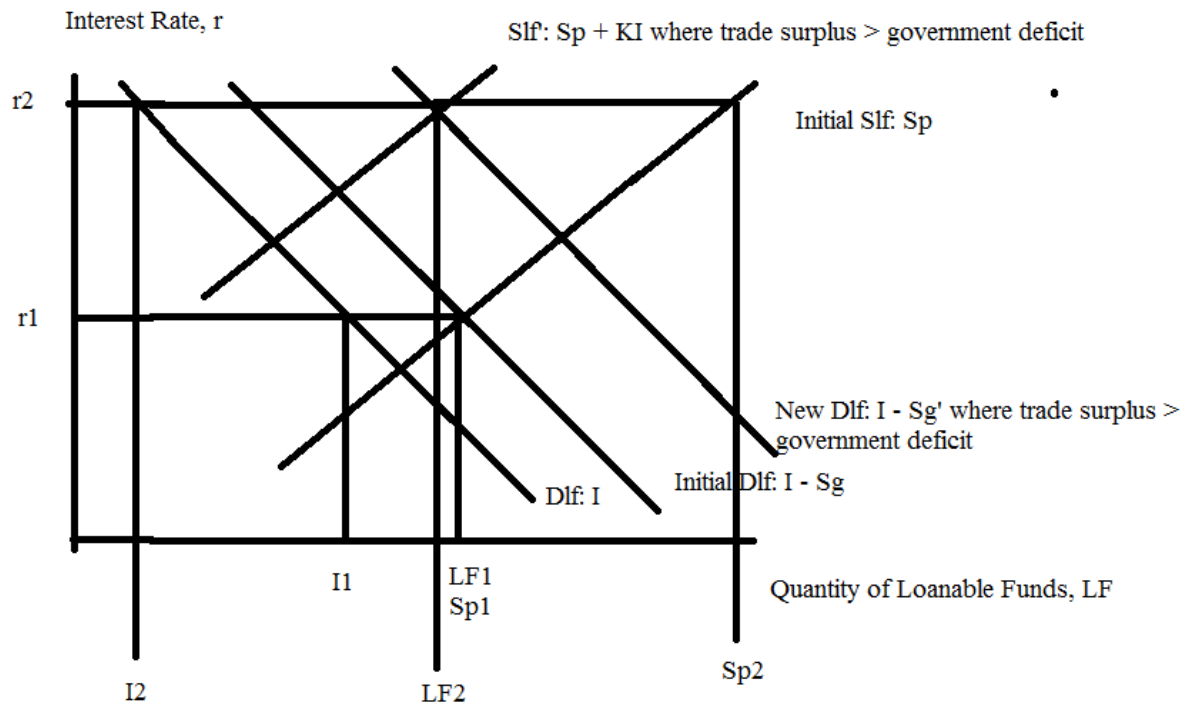LF2 LF1 Sp2   Quantity of Loanable Funds, LF
     Sp1
     I1

If the size of the trade deficit is smaller than the size of the reduction in these foreign funds, the net supply of loanable funds decreases and consequently the equilibrium interest rate rises, and the equilibrium level of loanable funds falls. The level of private saving would increase relative to their initial levels.

Case 3: The size of the trade deficit is equal to the size of the reduction in these foreign funds:

If the size of the trade deficit is equal to the size of the reduction in these foreign funds, the net supply of loanable funds does not change and consequently the equilibrium interest rate and the equilibrium level of loanable funds stay constant. There would be no change in the level of private saving relative to their initial levels.
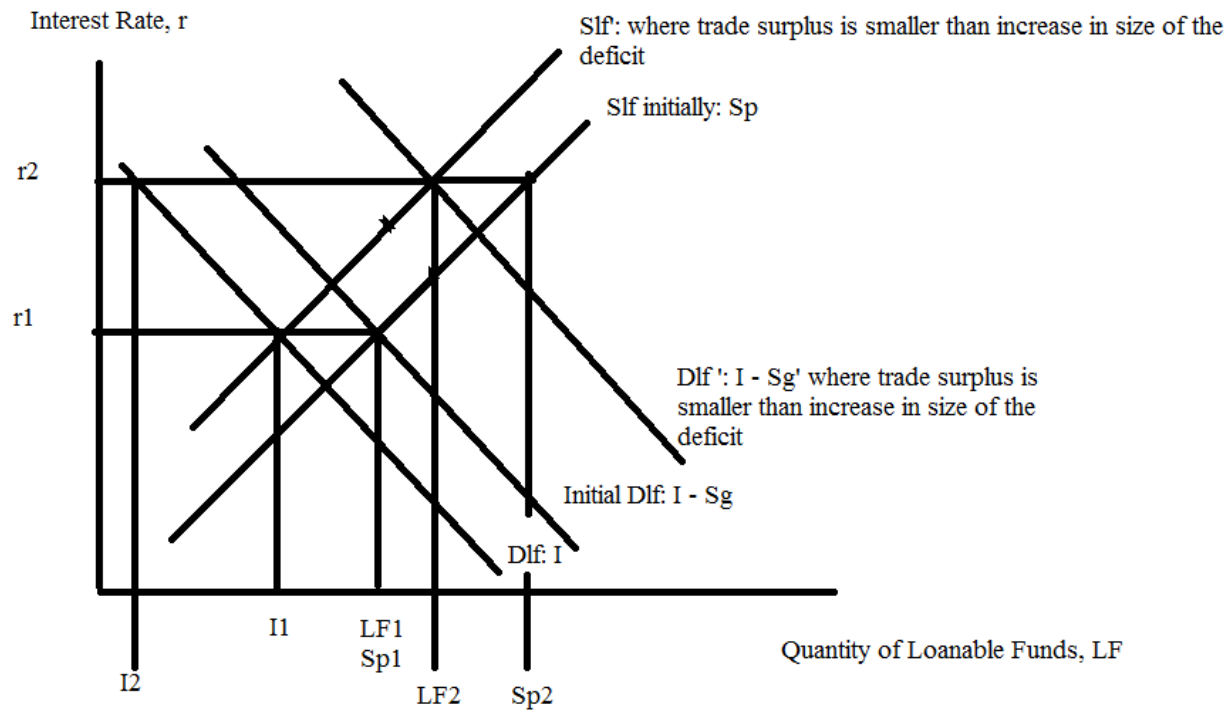
b. The equilibrium interest rate rises unambiguously but the equilibrium level of loanable funds is indeterminate because the outcome depends on the relative size of the trade surplus versus the government deficit. We cannot predict with certainty what happens to the level of private investment spending, consumption spending or private saving relative to their initial levels.

Case 1: The size of the trade surplus is larger than the size of the government deficit:

Interest Rate, r

Slf: Sp + KI where trade surplus > government deficit

r2

Initial Slf: Sp

r1

New Dlf: I - Sg' where trade surplus > government deficit

Dlf: I

Initial Dlf: I - Sg

I1   LF1
     Sp1
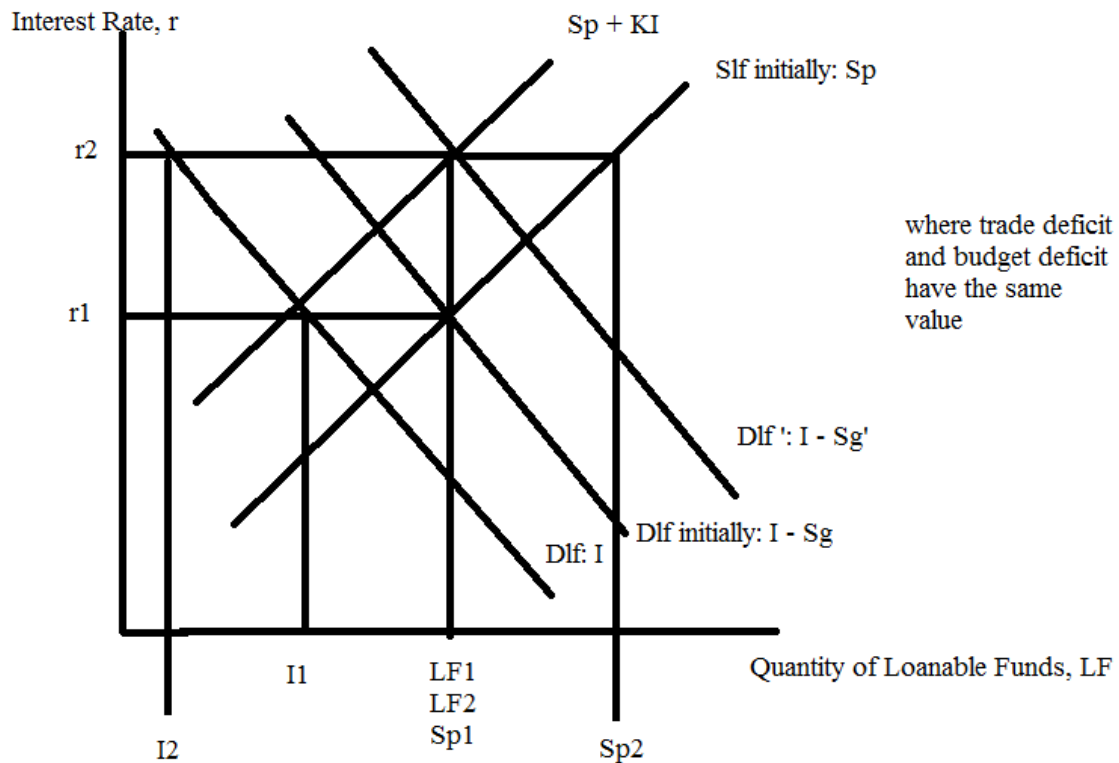
Quantity of Loanable Funds, LF

I2   LF2

Sp2

If the size of the trade surplus is larger than the size of the government deficit, the equilibrium level of loanable funds falls (LF1 versus LF2). As the interest rate rises (r1 to r2), this causes private investment spending to fall (I1 versus I2) and private saving to rise (Sp1 versus Sp2). As private saving rises, this leads to a decrease in consumption spending.

Case 2: The size of the trade surplus is smaller than the size of the government deficit:

14

Interest Rate, r

Slf': where trade surplus is smaller than increase in size of the deficit

Slf initially: Sp

r2

r1

Dlf ': I - Sg' where trade surplus is smaller than increase in size of the deficit

Initial Dlf: I - Sg

Dlf: I

I1   LF1
      Sp1

I2

LF2   Sp2

Quantity of Loanable Funds, LF

If the size of the trade surplus is smaller than the size of the government deficit, the equilibrium level of loanable funds rises. As the interest rate rises, the level of private investment spending falls and the level of private saving rises. As the level of private saving rises, the level of consumption spending falls.

Case 3: The size of the trade surplus is equal to the size of the government deficit:

Interest Rate, r

Sp + KI

Slf initially: Sp

r2

where trade deficit
and budget deficit
have the same
value

r1

Dlf ': I - Sg'

Dlf initially: I - Sg

Dlf: I

Quantity of Loanable Funds, LF

I1   LF1
             LF2
             Sp1   Sp2

I2

If the size of the trade surplus is equal to the size of the government deficit, the equilibrium level of loanable funds is unchanged. As the interest rate rises, the level of private investment spending decreases and the level of private saving increases. As the level of private saving increases, the level of consumption spending falls.