Economics 102
Spring 2014
Answers to Homework #4
Due: April 9, 2014

Directions: The homework will be collected in a box before the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section you are registered, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. Please show your work eligibly and neatly; otherwise you will not receive full credit. Good luck!

1. **Marginal Productivity**

Suppose that the aggregate production function in a simple economy is given by $Y = 2A\sqrt{KL}$ where Y is real GDP, A is the level of technology, K is the number of units of capital, and L is the number of units of labor. Given this aggregate production function, the marginal productivity of Labor and the marginal productivity of Capital are given by $L = A\sqrt{\frac{K}{L}}$, $MPK = A\sqrt{\frac{L}{K}}$.Furthermore, suppose that the capital stock (K) in this economy is constant and given as K= 16; and that thelevel of technology is constant and given asA=1.

a. Find an equation that will express this economy's labor productivity. How does labor productivity change if the level of capital in this economy changes? Explain your answer. How does labor productivity change if the level of technology, A, in this economy changes? Explain your answer.
b. In a graph with labor measured on the horizontal axis and real GDP, Y, measured on the vertical axis draw the aggregate production function for this economy.
c. Suppose that you are told the number of units of labor in this economy is 64. Given this information, calculate the value of
   i. Labor productivity
   ii. Capital productivity
   iii. MPL
   iv. MPK
d. How do these variables (that you found in part (c)) change if the number of units of labor in this economy is 16? On the graph in (b) indicate the production points for L = 64 and L = 16 as well as the corresponding values of real GDP.
e. How do your answers to parts (b) and (c) change (solve questions (b) and (c) again) if the new level of capital stock is constant and given as K=4 while the level of labor in is given as 64 units.
f. How doyour answersto parts (b) and (c) change (solve questions (b) and (c) again) if the value of A is 2 and the value of K is 16? For (b) assume that the level of labor is 64 units.

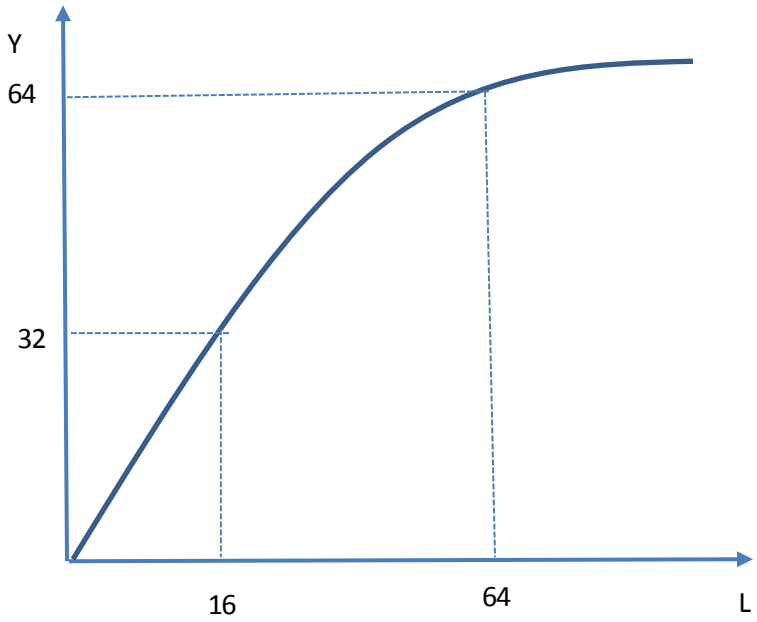
Answer:

1. **Marginal Productivity**

   a. The labor productivity is $\frac{Y}{L} = \frac{2A\sqrt{KL}}{L} = 2A\sqrt{\frac{K}{L}}$

We can easily see that if Capital K or technology A increases for a given amount of labor (that is, holding labor constant), then Labor productivity will increase.

b.



c. We can use the result from part (a)to find Labor productivity by plugging A=1, K=16 and L=64. Alternatively, we can find the production level first and then divided by the number of workers:

$$Y = 2A\sqrt{KL} = 2 \cdot 1 \cdot \sqrt{16 \cdot 64} = 64$$

Labor productivity $\frac{Y}{L} = \frac{64}{64} = 1$ units of output per unit of labor

Capital productivity $\frac{Y}{K} = \frac{64}{16} = 4$ units of output per unit of capital

Marginal productivity of Labor $MPL = A\sqrt{\frac{K}{L}} = 1\sqrt{\frac{16}{64}} = \frac{1}{2}$ additional unit of output from hiring one additional unit of labor when labor is equal to 64 units

Marginal Productivity of Capital $MPK = A\sqrt{\frac{L}{K}} = 1\sqrt{\frac{64}{16}} = 2$ additional units of output from hiring one additional unit of capital when capital is equal to 16 units

d. $Y = 2A\sqrt{KL} = 2 \cdot 1 \cdot \sqrt{16 \cdot 16} = 32$

Labor productivity $\frac{Y}{L} = \frac{32}{16} = 2$ units of output per unit of labor

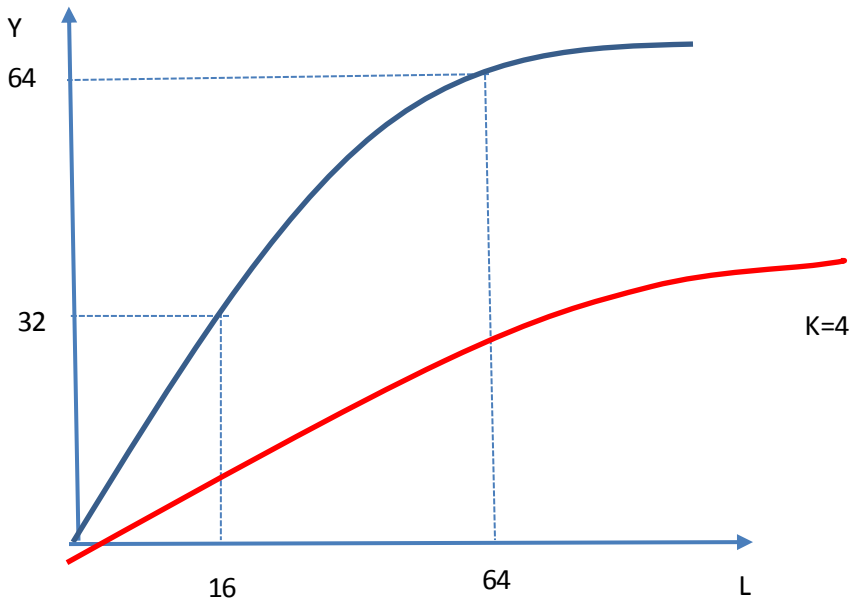
Capital productivity $\frac{Y}{K} = \frac{32}{16} = 2$units of output per unit of capital

Marginal productivity of Labor $L = A\sqrt{\frac{K}{L}} = 1\sqrt{\frac{16}{16}} = 1$additional unit of output from hiring one additional unit of labor when labor is equal to 16 units

Marginal Productivity of Capital $MPK = A\sqrt{\frac{L}{K}} = 1\sqrt{\frac{16}{16}} = 1$additional unit of output from hiring one additional unit of capitl when capital is equal to 16 units

Think about what the intuition is behind that result!

e. Part (b):



Part (c) for K= 4

$$Y = 2A\sqrt{KL} = 2 \cdot 1 \cdot \sqrt{4 \cdot 64} = 32$$

Labor productivity: $\frac{Y}{L} = \frac{32}{64} = 0.5$ units of output per unit of labor

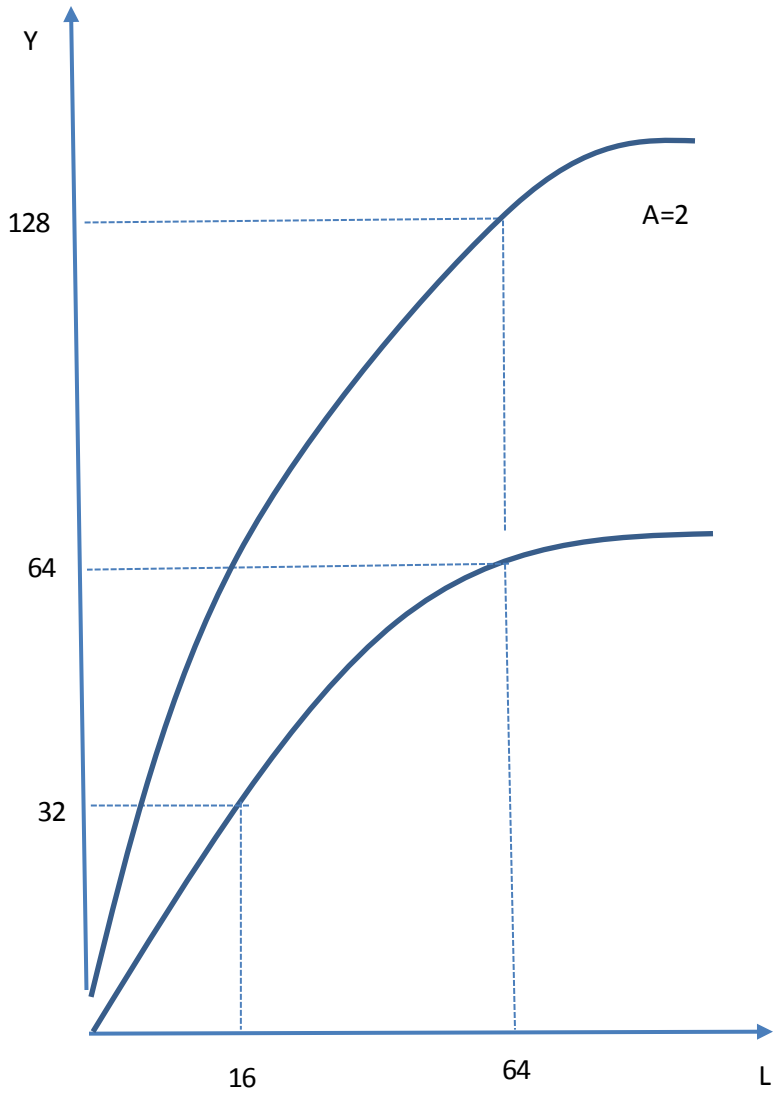
Capital productivity: $\frac{Y}{K} = \frac{32}{4} = 8$ units of output per unit of capital

Marginal productivity of Labor $L = A\sqrt{\frac{K}{L}} = 1\sqrt{\frac{4}{64}} = \frac{1}{4}$ additional unit of output per additional unit of labor

Marginal Productivity of Capital $MPK = A\sqrt{\frac{L}{K}} = 1\sqrt{\frac{64}{4}} = 4$ additional unit of output per additional unit of capital

Think about what the intuition is behind that result!

f. Part (b) Graph!



Part (c) for A =2

$$Y = 2A\sqrt{KL} = 2 \cdot 2 \cdot \sqrt{16 \cdot 64} = 128$$

Labor productivity $\frac{Y}{L} = \frac{128}{64} = 2$ units of output per unit of labor

Capital productivity $\frac{Y}{K} = \frac{128}{16} = 8$ units of output per unit of capital

Marginal productivity of Labor $L = A\sqrt{\frac{K}{L}} = 2\sqrt{\frac{16}{64}} = 1$ additional unit of output per additional unit of labor

Marginal Productivity of Capital $MPK = A\sqrt{\frac{L}{K}} = 2\sqrt{\frac{64}{16}} = 4$ additional units of output per additional unit of labor

Note that improvement in technology (Increase in A) leads to an increase in both labor and capital productivity. Think about what the intuition is behind that result!

2. Rule of 70

Fairuz has to save some money for her trip. She plans to travel for a few months around the world right after she graduates (2.5 years from today). She can choose one of these investments plans:

v. Plan A starts with an initial value of $300 and grows at a rate of 10 percent every month.
vi. Plan B starts with an initial vale of $2400 and grows at a rate 5 percent every month.

a. How long will it take her to save $2400 if Fairuz chooses plan A?
b. How long will it take her to save $4800 if Fairuz chooses plan B?
c. Fairuz believes that the total cost of her trip is going to be $9600. Given this information, which plan should she choose? Will she have this amount saved by her graduation day?
d. Suppose now that Fairuz realizes that the trip is going to much more expensive. She realizes that she may need to wait for some time after her graduation to make the trip since she must first save up the necessary amount before embarking on her trip. If the total cost of the trip is $19,200, how long will she need to wait after her graduation if she waits to take the trip until she has saved up the full amount? Which plan will she choose if she wants to minimize the length of time she must wait before taking the trip?
e. If the total cost of the trip is $38,400, how long will she need to wait after her graduation if she waits to take the trip until she has saved up the full amount? Which plan will she choose if she wants to minimize the length of time she must wait before taking the trip?

Answer:

2. Rule of 70

a. Use the rule of 70 to answer this question. Given that the growth rate is 10%, the amount of money in plan A will be doubled every 7 months because $n = \frac{70}{R} = \frac{70}{10} = 7$ (Note that here we talk about months, since the growth rate is monthly).

Today	7 months	14 months	21 months
300	600	1200	2400

It will take 21 months.

b. The same idea as in part (a), but here $n = \frac{70}{R} = \frac{70}{5} = 14$

Today 14 months
2400 4800

c. It's useful to make a table for both plans, so we can compare them:

	Today	7 months	14 months	21 months	28 months	35 months	42 months	49 months	
A	300	600	1200	2400	4800	9600	19,200		
B	2400		4800		9600		19200		

We can see that plan B reaches $9600 (28 months) before plan A does (35 months). If Fairuz chooses plan B, then she will have enough money at the day of her graduation (28 months< 30 months (2.5 years)).

d. To answer this question, use the table from part (c). The two plans reach $19,200 at the same time (after 42 months).Fairuz will be indifferent between the two plans. She has to wait one more year after she graduates! 42-30=12 months=1 year.
e. If the cost is $38400, Fairuz will choose plan A. It will take 49 months! (If she chooses plan B, it will take 56 months to reach $38400. You should take the time to work out this answer.)

3. Long-run Economic Growth

For this question, you may need to use Excel or similar spreadsheet software. If you are not familiar with the software, there are lots of online resources and tutorials available. If you do not have such software installed on your computer, most computers in campus libraries have them available.

Imagine a simple economy in which there are only two factors of production: capital (K) and labor (L). The production function is given by

$Y = 2L^{0.7}K^{0.3}$.

The capital stock in this economy is constant at 100. Use Excel (or similar spreadsheet software) to generate the following table. Note that you need to generate output (Y), labor productivity (Y/L) and the Marginal Product of Labor (MPL=change in Y/change in L) for all values of labor between 1 and 100.

Note in this problem you will be asked to generate two large tables: you do not need to submit these tables with your homework, but you do need to submit the graphs drawn from these tables. The graphs need to be generated by the spreadsheet program and NOT hand drawn graphs of the data.

Labor (L)	Output (Y)	Labor Productivity	MPL
1			
2			
3			
4			
5			
…	…	…	…
96			
97			
98			
99			
100			

a) With the table you just generated, plot Y against L. (i.e. with Y on the y-axis and L on the x-axis)
b) How is the marginal product of labor reflected on the graph you just plotted? How does it change as the amount of labor increases?

c) Holding capital constant at 100, how does labor productivity change as labor decreases?

d) Suppose capital stock falls and now is at K=50. Intuitively, what do you think will happen to labor productivity and the marginal product of labor? Explain your answer.

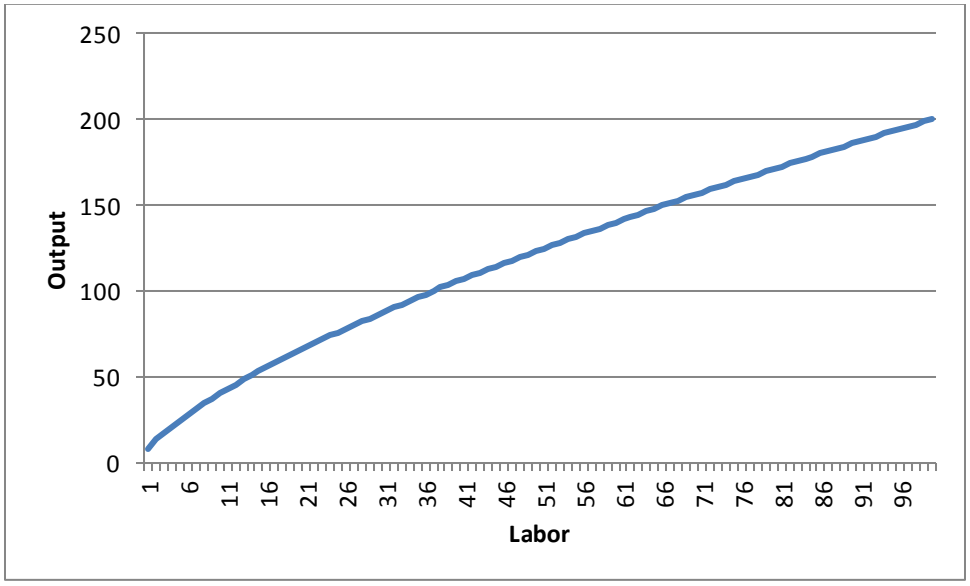
e) Using the new capital stock level given in (d), generate a new table. In a new graph, plot the following: Y against L when K=100, and Y against L when K=50. Provide a brief explanation of the plot you have generated from the spreadsheet. You should be able to plot the graph using the spreadsheet program: do not submit a hand drawn graph here. We want you to show us that you can get the spreadsheet program to generate this graph.

Answer:

3. Long-run Economic Growth

Labor (L)	Output (Y)	Labor Productivity	MPL
1	7.962143411	7.962143411	7.962143411
2	12.93454013	6.467270066	4.97239672
3	17.17967224	5.726557414	4.245132109
4	21.01222244	5.253055609	3.832550194
5	24.56456052	4.912912104	3.552338087
…	…	…	…
96	194.3658038	2.02464379	1.419475163
97	195.78085	2.018359278	1.415046135
98	197.1915264	2.012158432	1.410676376
99	198.5978908	2.006039301	1.406364496
100	200	2	1.402109153

a) With the table you just generated, plot Y against L. (i.e. with Y on the y-axis and L on the x-axis)



b) How is the marginal product of labor reflected on the graph you just plotted? How does it change as the amount of labor increases?

Marginal product of labor is the slope of the production function. As labor increases, MPL decreases.

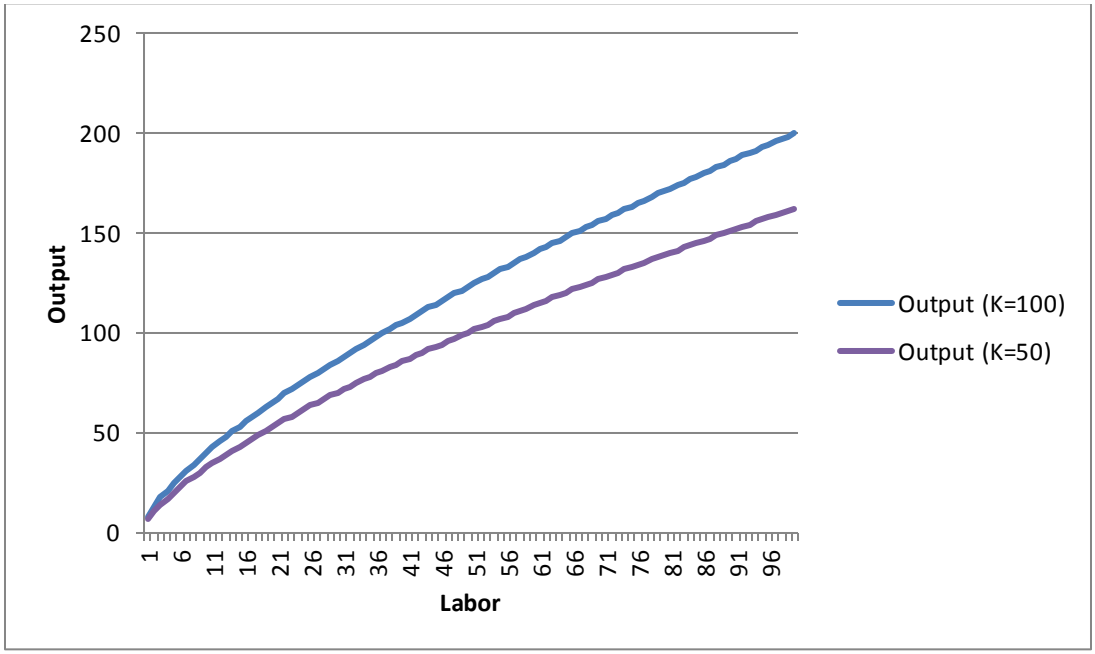
c) Holding capital constant at 100, how does labor productivity change as labor decreases?

Labor productivity increases as labor decreases when capital is fixed

d) Suppose capital stock falls and is K=50. Intuitively, what do you think will happen to labor productivity and the marginal product of labor? Explain.

Labor productivity and marginal product of labor will both be lower when the capital stock decreases. Intuitively, each worker will have less capital to work with.

e) Using the new capital stock level, generate a new table. In a new graph, plot the following: Y against L when K=100, and Y against L when K=50. Provide a brief explanation.



4. Discussion

Read the following article by Greg Mankiw and discuss the main ideas of the author. Do you agree or disagree with the author of this article? In your essay highlight the main points of the article and then provide either a rebuttal or a persuasive argument in favor of the author's position. (Try to keep your answer to a succinct one page maximum!) But, we expect you to use standard English, punctuation, and paragraphs in your answer-basically an intro with your quick summary, a discussion of your supporting arguments, and then a conclusion. No bullet point answers please!

When the Scientist Is Also a Philosopher[1]

Do you want to know a dirty little secret of economists who give policy advice? When we do so, we are often speaking not just as economic scientists, but also as political philosophers. Our recommendations are based not only on our understanding of how the world works, but also on our judgments about what makes a good society.

The necessity of political philosophy arises because most policies are good for some people and bad for others. For example, an increase in the minimum wage, as proposed by President Obama, may raise incomes for some low-wage workers, but it will cause some businesses to make smaller profits, some customers to pay more and some workers to lose their jobs.

Similarly, the Affordable Care Act has provided greater opportunity for some people to get health insurance, but it also caused cancellations for others who were previously happy with their insurance.

Evaluating the overall effect of these policies requires balancing competing interests.

To strike this balance, many economists think in terms of a "social welfare function" that aggregates individuals' well-being into a summary measure. This approach dates back to the utilitarian philosophers of the 19th century, such as Jeremy Bentham and John Stuart Mill. The utilitarians suggested that each person in society receives a certain amount of happiness, or "utility," from an allocation of society's resources. The job of policy makers, they argued, is to do their best to maximize the total utility of everyone in society. According to utilitarians, taking a dollar from Peter and giving it to Paul is justified if Peter's decrease in utility is smaller than Paul's increase, as would plausibly be the case if Peter is richer than Paul.

Philosophers have long debated the validity of utilitarianism as an ethical criterion. Some of the most famous thought experiments in this area involve what ethicists call trolley problems. Imagine that you are on a bridge and see a runaway trolley car below you, hurtling toward three children playing on the tracks. A fat man is standing next to you. You can push him off the bridge and into the path of the trolley, killing him but saving the children. What do you do?

If, like a true utilitarian, you have no trouble sacrificing the fat man, try another scenario. You are a doctor with four dying patients. One needs a new liver, one needs a new heart, and two need a new kidney. A perfectly healthy patient walks into your office for his annual checkup. Are you still willing to pursue the utilitarian course of action? At this point, almost everyone balks. Sometimes, respecting natural rights trumps maximizing utility.

Another problem with the utilitarian approach is that there is no objective way to compare one person's happiness with another's, especially when people have different preferences. Peter may work long hours at a dreary job to earn a high income because he gets a lot of utility from money. Paul may be forgoing a higher income for a job that requires fewer hours or offers more personal satisfaction because he doesn't care as much about money. In this case, equalizing incomes by moving a dollar from Peter to Paul could reduce total utility.

Perhaps the biggest problem with maximizing a social welfare function like utility is practical: We economists often have only a basic understanding of how most policies work. The economy is complex, and economic science is still a primitive body of knowledge. Because unintended consequences are the norm, what seems like a utility-maximizing policy can often backfire.

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[1] By Greg Mankiw, *New York Times*, March 22, 2014.
http://www.nytimes.com/2014/03/23/business/economic-view-when-the-scientist-is-also-a-philosopher.html?smid=pl-share&_r=0

So, what is the alternative? At the very least, a large dose of humility is in order. When evaluating policies, our elected leaders are wise to seek advice from economists. But if an economist is always confident in his judgments, or if he demonizes those who reach opposite conclusions, you know that he is not to be trusted.

In some ways, economics is like medicine two centuries ago. If you were ill at the beginning of the 19th century, a physician was your best bet, but his knowledge was so rudimentary that his remedies could easily make things worse rather than better. And so it is with economics today. That is why we economists should be sure to apply the principle "first, do no harm."

This principle suggests that when people have voluntarily agreed upon an economic arrangement to their mutual benefit, that arrangement should be respected. (The main exception is when there are adverse effects on third parties — what economists call "negative externalities.") As a result, when a policy is complex, hard to evaluate and disruptive of private transactions, there is good reason to be skeptical of it.

As I see it, the minimum wage and the Affordable Care Act are cases in point. Noble as they are in aspiration, they fail the do-no-harm test. An increase in the minimum wage would disrupt some deals that workers and employers have made voluntarily. The Affordable Care Act has disrupted many insurance arrangements that were acceptable to both the insurance company and the insured; these policies were canceled because they deviated from lawmakers' notion of the ideal.

To be sure, you can find economists favoring a higher minimum wage and the Affordable Care Act. They acknowledge that there are winners and losers but argue that, on the whole, these policies increase social welfare.

Perhaps they are right. But keep in mind that in making that judgment, they are relying on forecasts from a far-from-perfect science, as well as a healthy dose of their own political philosophy.

Answer:

4. Discussion

It is your own opinion!