

Economics 102
Fall 2017
Answers to Homework #4
Due 11/14/2017

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered**, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please remember to

- Staple your homework before submitting it.
- Do work that is at a professional level: you are creating your “brand” when you submit this homework!
- Do not submit messy, illegible, sloppy work.
- Show your work to get full credit.

1. Consider the aggregate production functions for Wisconsin and Minnesota:

Production Function for Wisconsin	
$Y_{WI} = \alpha K_{WI}^{1/2} L_{WI}^{1/2}$	
Y _{WI} :	Real GDP of Wisconsin
α:	The state of the technology
K _{WI} :	Units of capital
L _{WI} :	Units of labor (in this case, the number of workers)
Production Function for Minnesota	
$Y_{MN} = \beta K_{MN}^{1/3} L_{MN}^{2/3}$	
Y _{MN} :	Real GDP of Wisconsin
β:	The state of the technology
K _{MN} :	Units of capital
L _{MN} :	Units of labor (in this case, the number of workers)

Labor and capital are the only inputs used in both states to produce real GDP. Initially, K is equal to 25 units in both states. In Wisconsin, the state of the technology is represented by $\alpha = 2.5$ and in Minnesota the technology is $\beta = 2.5$. Answer the following questions based on the above information (**you are required to utilize Excel when answering this question**).

Direction for this problem: You don't need to submit the whole simulated data tables with your homework, but you do need to submit the graphs drawn from these tables. **The graphs must be generated by using Excel, NOT by your hands.**

a. Fill in the following table (you will need to expand it from the truncated form provided here when using Excel). Round all your answers to the nearest hundredth.

Marginal Product of Labor (MPL): The marginal product of a factor of production is generally defined as the change in output associated with a change in that factor, holding other inputs (in this case, K which is capital) into production constant. The marginal product of labor is then the change in output (Y) per unit change in labor (L).

$$\text{Marginal Product of Labor} = \Delta Y / \Delta L$$

Labor Productivity (LP): The labor productivity is a measure of economic performance that compares the amount of goods and services produced (output) with the number of workers in this example to produce those goods and services.

$$\text{Labor Productivity} = Y / L$$

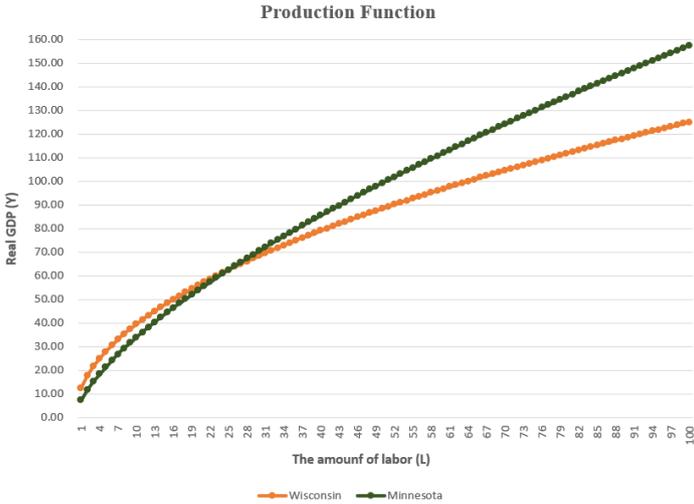
Wisconsin ($\alpha = 2.5$)				
L	K	Y	MPL	LP
1	25	12.50	---	12.50
2	25	17.68	5.18	8.84
3	25	21.65	3.97	7.22
4	25	25.00	3.35	6.25
5	25	27.95	2.95	5.59
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96	25	122.47	0.64	1.28
97	25	123.11	0.64	1.27
98	25	123.74	0.63	1.26
99	25	124.37	0.63	1.26
100	25	125.00	0.63	1.25

Minnesota ($\beta = 2.5$)				
L	K	Y	MPL	LP
1	25	7.31	---	7.31
2	25	11.60	4.29	5.80
3	25	15.21	3.60	5.07
4	25	18.42	3.21	4.61
5	25	21.37	2.95	4.27
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96	25	153.26	1.07	1.60
97	25	154.32	1.06	1.59
98	25	155.38	1.06	1.59
99	25	156.44	1.06	1.58
100	25	157.49	1.05	1.57

b. Use Excel to describe both states' real GDP (= value of each state's production function) **on the same graph**: measure L on the horizontal axis and Y on the vertical axis. Briefly compare the two states' real GDP level depending on the amount labor.

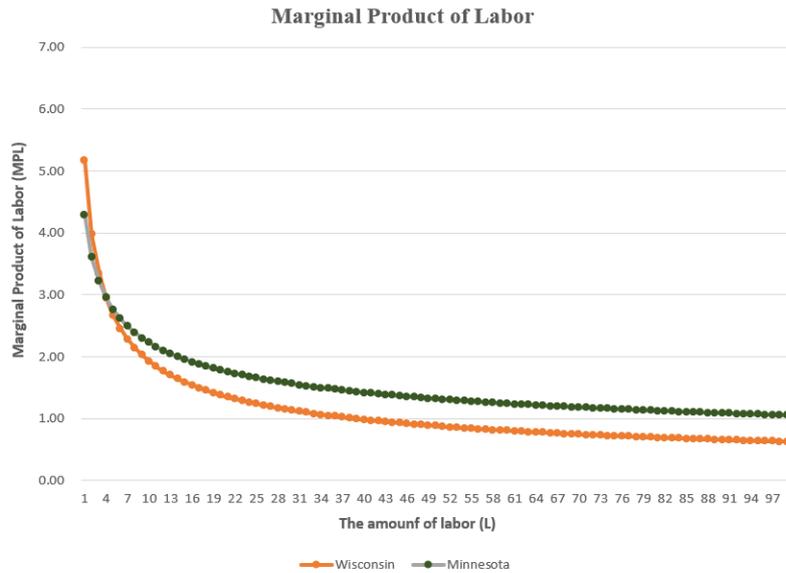
Answer: (Orange Line = Wisconsin and Dark Green Line = Minnesota)



For the given information, the technological level and the amount of capital accumulated in each state are same. For the low levels of labor inputs, the real GDP (= value of production function) of Wisconsin is higher than the real GDP of Minnesota. However, for high levels of labor inputs, the real GDP of Minnesota is higher than the real GDP of Wisconsin.

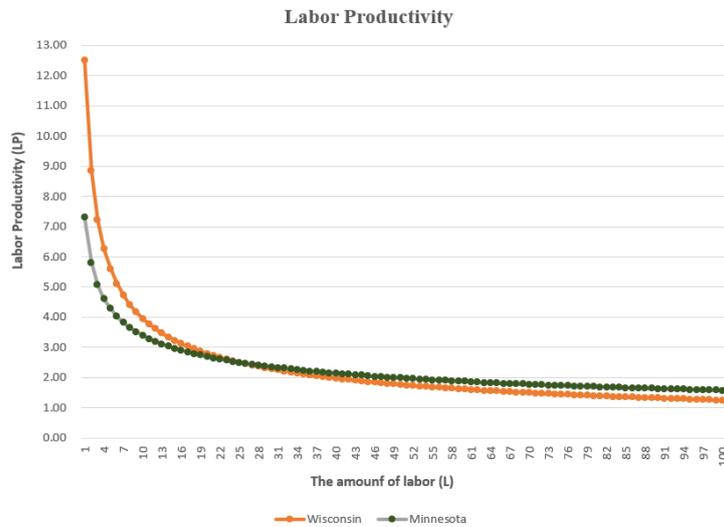
c. Use Excel to describe both states' marginal product of labor **on the same graph**: measure L on the horizontal axis and the marginal product of labor on the vertical axis.

Answer: (Orange Line = Wisconsin / Dark Green Line = Minnesota)



d. Use Excel to describe both states' labor productivity on the same graph: measure L on the horizontal axis and the labor productivity on the vertical axis.

Answer: (Orange Line = Wisconsin / Dark Green Line = Minnesota)



e. Based on the results from (c) and (d), explain the different real GDP patterns of the two states by using your economic intuition.

Answer:

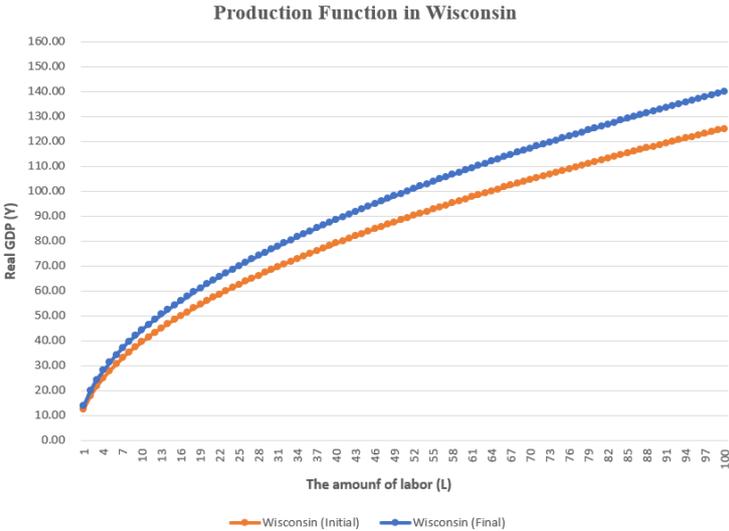
As the amount of labor rises, Minnesota has higher values for both the marginal product of labor and labor productivity. In this context, let's consider the definition regarding the marginal product of labor. It can be interpreted as the marginal contribution to real GDP that occurs when one more unit of labor is utilized. Therefore, the fact that Minnesota has higher values for both the marginal product of labor and labor productivity as the amount of labor rises implies that the

real GDP of Minnesota will increase faster than that of Wisconsin when each state adds an additional unit of labor. That is why the real GDP of Minnesota finally caught up with the real GDP of Wisconsin and then surpassed the real GDP of Wisconsin.

[Additional Comments] In this problem, we supposed that there are no changes in the technology and the level of capital so the only factor which can influence real GDP is the level of labor input. Let's look at each state's production function in more detail. The relative weight on the labor input is bigger in Minnesota (where it is $\frac{2}{3}$) than in Wisconsin (where it is $\frac{1}{2}$). This means that the effect of an increase in the amount of labor used on real GDP is more significant in Minnesota than in Wisconsin.

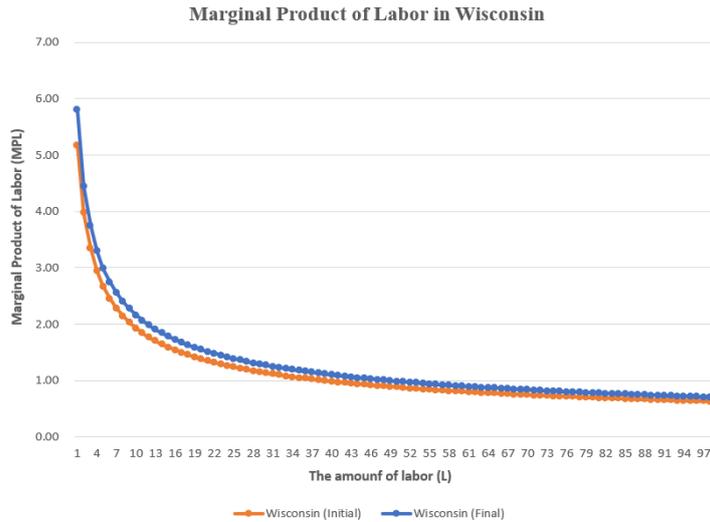
f. Suppose that the technology of Wisconsin increases from 2.5 to 3.5 while the amount of capital decreases from 25 units to 16 units. Use Excel to describe the initial real GDP of Wisconsin (which is the same as the answer from (b)) and the final real GDP of Wisconsin (considering the changes mentioned in this problem) **on the same graph**: measure L on the horizontal axis and Y on the vertical axis.

Answer: (Orange Line = Wisconsin Initial / Blue Line = Wisconsin Final)



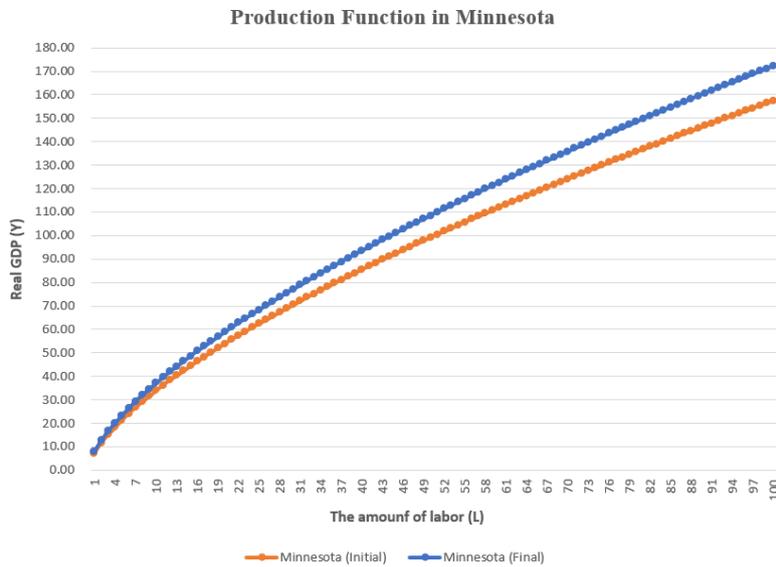
g. Based on (f), use Excel to describe the initial marginal product of labor in Wisconsin (which is the same as the answer from (c)) and the final marginal product of labor in Wisconsin (considering the changes mentioned in (f)) **on the same graph**: measure L on the horizontal axis and the marginal product of labor on the vertical axis.

Answer: (Orange Line = Wisconsin Initial / Blue Line = Wisconsin Final)

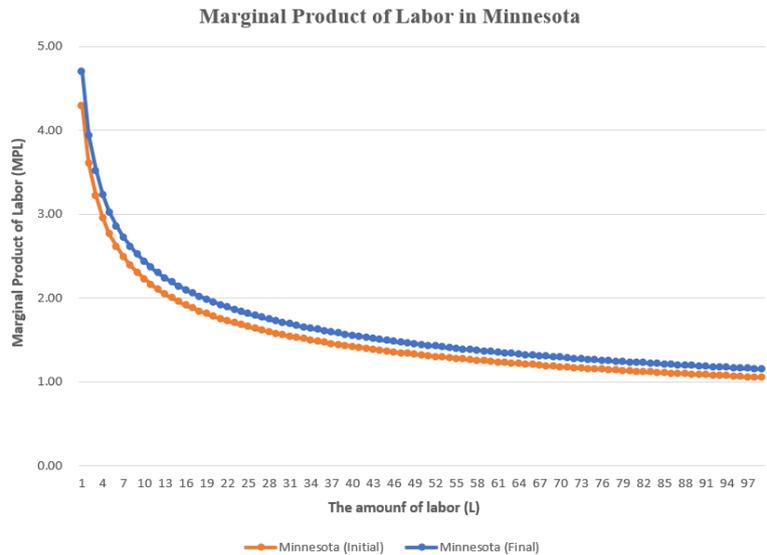


h. Suppose that the technology of Minnesota decreases from 2.5 to 2 while the amount of capital increases from 25 units to 64 units. Use Excel to describe the initial real GDP of Minnesota (which is same as the answer from (b)) and the final real GDP of Minnesota (considering the changes mentioned in this problem) **on the same graph**: measure L on the horizontal axis and Y on the vertical axis.

Answer: (Orange Line = Minnesota Initial / Blue Line = Minnesota Final)



i. Based on (h), use Excel to describe the initial marginal product of labor in Minnesota (which is the same as the answer from (c)) and the final marginal product of labor in Minnesota (considering the changes mentioned in (h)) **on the same graph**: measure L on the horizontal axis and the marginal product of labor on the vertical axis.



j. From the results of the previous four questions ((f) to (i)) we can check whether there is economic growth or not in each state. If real GDP increases regardless of the amount of labor, we can say there is economic growth. Now, explain the change in real GDP in each state. From the two states' scenarios, what kind of similarity can you find with regard to the economic growth?

Answer: In terms of the technology and the amount of capital used, the two states are faced with exactly the opposite situation. However, we can check that both states experience economic growth. One commonality between the two cases is that the marginal product of labor is decreasing irrespective of the amount of labor in each state.

2. Suppose we have the following data from two hypothetical towns: Gryffindor and Slytherin.

Year	Gryffindor		Slytherin	
	Real GDP	Population	Real GDP	Population
2019	\$50,000	200	\$40,000	150
2020	\$51,400	202	\$42,240	155

Express your answers in percentage terms to *two* places past the decimal.

a. Calculate the growth rate of real GDP for each town.

Answer:

$$\text{The growth rate of real GDP} = \frac{\text{Real GDP}_{2020} - \text{Real GDP}_{2019}}{\text{Real GDP}_{2019}} \times 100 (\%)$$

The growth rate of real GDP in Gryffindor = 2.80 %

The growth rate of real GDP in Slytherin = 5.60 %

b. At these rates of growth, approximately how many years will pass before each town's real GDP doubles?

Answer:

Consider the rule of 70 here. The rule of 70 is a way to estimate the number of years it takes for a certain variable to double.

For Gryffindor, $70/2.8 = 25$ years.

For Slytherin, $70/5.6 = 12.5$ years.

c. Find each town's real GDP per capita in each of the two years. Calculate the growth rate of real GDP per capita in each town.

Answer:

$$\text{Real GDP per capita} = \frac{\text{Real GDP}}{\text{Population}}$$

Year	Gryffindor			Slytherin		
	Real GDP	Population	Real GDP Per capita	Real GDP	Population	Real GDP Per capita
2019	\$50,000	200	\$250	\$40,000	150	\$266.67
2020	\$51,400	202	\$254.46	\$42,240	155	\$272.52

The growth rate of real GDP per capita

$$= \frac{\text{Real GDP per capita}_{2020} - \text{Real GDP per capita}_{2019}}{\text{Real GDP per capita}_{2019}} \times 100 (\%)$$

The growth rate of real GDP per capita in Gryffindor = 1.78 %

The growth rate of real GDP per capita in Slytherin = 2.19 %

d. At these rates of growth, approximately how many years will pass before each town's real GDP per capita doubles?

Answer:

For Gryffindor, $70/1.78 = 39.28$ years.

For Slytherin, $70/2.19 = 31.91$ years.

e. Suppose each town's growth rate of real GDP is constant over time. In this situation, which town has the larger amount of real GDP in 2045 and what is the difference between the two towns in terms of real GDP in 2045? Give the absolute value of the difference.

Answer:

Considering the answers from (b), Gryffindor will experience a doubling of real GDP between 2020 and 2045. Slytherin will experience the doubling of real GDP twice over the time period 2020 to 2045. As a result, in 2045, each town's real GDP is:

$$2045 \text{ Real GDP in Gryffindor} = \$51,400 \times 2 = \$102,800$$

$$2045 \text{ Real GDP in Slytherin} = \$42,240 \times 2 \times 2 = \$168,960$$

Therefore, Slytherin will have a larger amount of real GDP in 2045.

$$\text{The difference in 2045 real GDP between two towns} = \$168,960 - \$102,800 = \$66,160$$

3. For the following questions it may be helpful to draw the aggregate production function for each situation described.

- a. Suppose that while holding its capital and labor constant, an economy experiences an increase in technology. What happens to real GDP? What happens to labor productivity?
- b. Now suppose the economy increases the amount of labor it employs, while holding capital and technology constant. What happens to real GDP? What happens to labor productivity?
- c. Suppose the economy simultaneously experiences an increase in technology and a decrease in the amount of labor it employs. What happens to real GDP? What happens to labor productivity?
- d. Suppose an economy simultaneously experiences an increase in capital and an increase in the amount of labor it employs. What happens to real GDP? What happens to labor productivity? What happens to capital productivity?
- e. Suppose an economy experiences an increase in technology, but at the same time also decreases the amount of labor and the amount of capital it employs. What happens to real GDP? What happens to labor productivity?

SOLUTION:

a. Real GDP increases since technology has increased, and labor productivity increases since now the same amount of labor is producing more. If you drew a graph with labor on the x axis, you can see that the slope of the line from the origin through the original production point is flatter than the new production point, which shows that labor productivity has increased.

b. Real GDP increases, but labor productivity decreases. This is because of the diminishing returns to labor – increasing labor increases real GDP, but not proportionally. Drawing a graph could also show you this visually, since the slope of the line through the original production point will be steeper than the line through the new production point.

c. We cannot tell what happens to real GDP since the two changes move in opposite directions and we don't know which is bigger. We also cannot know what happens to labor productivity: it

may increase, decrease, or remain unchanged.

d. Real GDP increases, since both effects move in the same direction of increasing real GDP. But, we cannot know with certainty what happens to labor productivity or capital productivity: both are indeterminate given the information we have been given.

e. We cannot tell what happens to real GDP, since the increase in technology works to increase real GDP, but the decreases in the amount of capital and the amount of labor employed work to decrease real GDP, and we don't know anything about the size of each decrease. We also cannot know with certainty what happens to labor productivity or capital productivity: both are indeterminate given the information we have been given.

4. The following table gives the annual growth rate of real GDP for 4 countries in 2017. Assume there is no population growth in any of the four countries.

	Country A	Country B	Country C	Country D
Real GDP Annual growth rate	6%	5%	15%	-3%

a. To which countries can we apply the Rule of 70 to determine how long it will take their real GDP to double?

b. For the countries you specified in part (a), in which years will their real GDPs have doubled? (Round to the nearest year where necessary.)

SOLUTION:

a. We can apply the Rule of 70 to Countries A, B, and C. We cannot apply the rule of 70 to Country D because it does not have a positive growth rate.

b. Country A's real GDP will double in $70/6 = 11.67$ years, so in 2029. Country B's real GDP will double in $70/5 = 14$ years, so in 2031. Country C's real GDP will double in $70/15 = 4.67$ years, so in 2022.

5. Suppose that the loanable funds market is initially in equilibrium and the country has zero net exports and is currently operating with a balanced budget. Provide your analysis about the market equilibrium for each of the following scenarios.

a. Holding everything else constant, what do you predict about the equilibrium interest rate, the level of private investment and the equilibrium level of private saving if the country runs a trade surplus? Explain your answer verbally and provide a graph of the loanable funds market to support your analysis. Make sure your graph is clearly labeled.

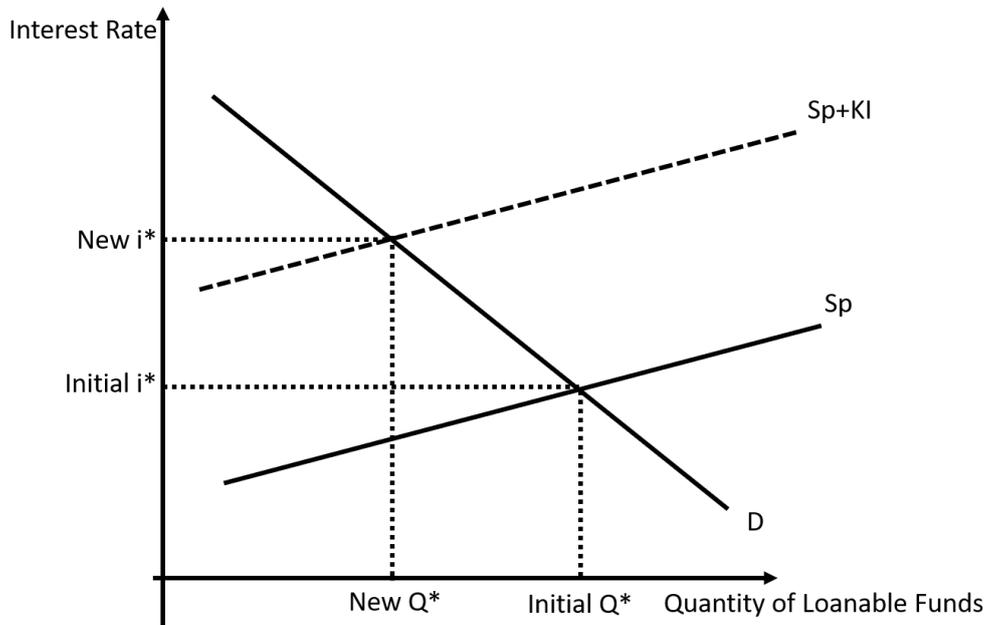
b. Holding everything else constant, what do you predict about the equilibrium interest rate, the level of private investment and the equilibrium level of private investment spending if the government increases the size of its surplus? Explain your answer verbally and provide a graph of the loanable funds market to support your analysis. Make sure your graph is clearly labeled.

c. Holding everything else constant, what do you predict about the equilibrium interest rate, the level of private investment and the equilibrium level of loanable funds if the country runs a trade deficit and at the same time loses funds that a major foreign company is providing as a source of funds to this economy? Explain your answer verbally and provide a graph of the loanable funds market to support your analysis. Make sure your graph is clearly labeled.

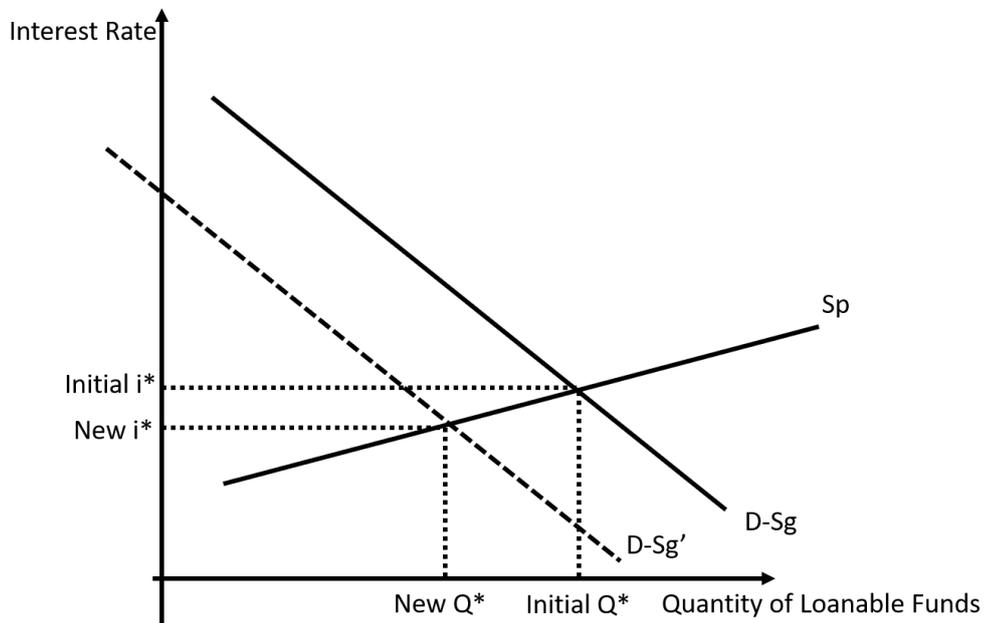
d. Holding everything else constant, what do you predict about the equilibrium interest rate, the level of private investment, the level of consumption spending and the equilibrium level of loanable funds if the country runs a trade surplus and at the same time increases the size of government deficit? Explain your answer verbally and provide a graph of the loanable funds market to support your analysis. Make sure your graph is clearly labeled.

SOLUTION:

a. If the country runs a trade surplus this will shift the supply of loanable funds curve to the left because when a country runs a trade surplus ($X - M > 0$) this implies that the country has capital inflows that are negative ($M - X < 0$). Negative KI inflows will cause the supply of loanable funds curve to shift to the left relative to its initial position. We can predict that the equilibrium interest rate will increase and that the equilibrium level of private investment will decrease when the country runs a trade surplus. We can also predict that the level of private savings will increase due to the increase in the interest rate. Below is a diagram to illustrate this answer.



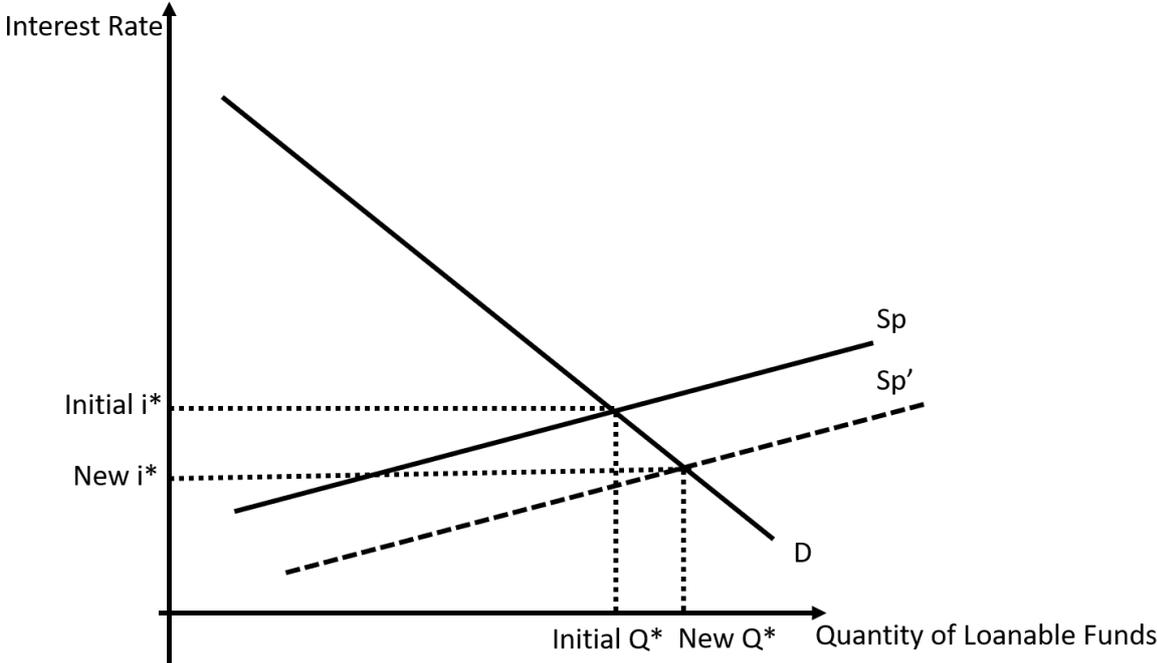
b. An increase in the size of government surplus can be modeled as a leftward shift of the demand curve for loanable funds because the increase in government saving can be used to offset the private demand in loanable funds. This will result in the equilibrium interest rate in the market falling and as the interest rate falls the level of private investment spending will increase. Below is a diagram that illustrates a potential outcome in this. The level of private saving will decrease as the interest rate falls.



c. The equilibrium interest rate and equilibrium level of loanable funds are indeterminate because the outcome depends on the relative size of trade deficit versus the amount of funds that

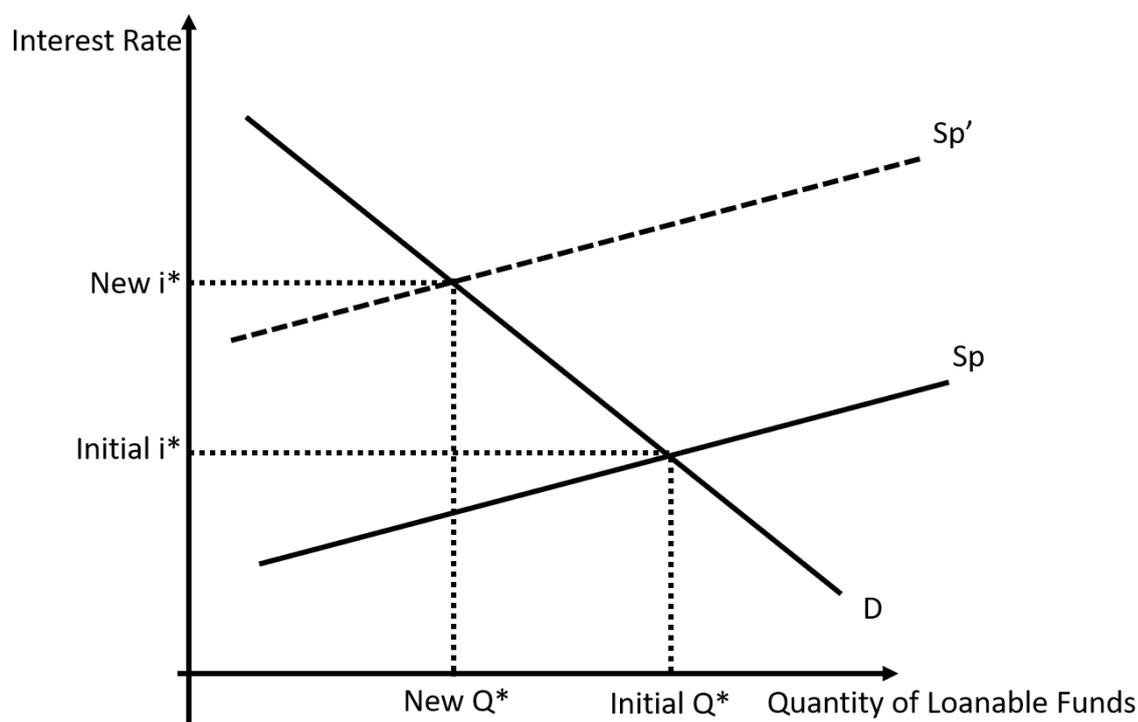
the foreign company is reducing as a source of funds to this economy.

Case 1: The size of trade deficit is larger than the size of the reduction in these foreign funds:



If the size of the trade deficit is larger than the size of the reductions in these foreign funds, the net supply of loanable funds increases and consequently the equilibrium interest rate falls, and equilibrium level of loanable funds rise. The level of private investment spending would increase and the level of private saving would decrease relative to their initial levels.

Case 2: The size of the trade deficit is smaller than the size of the reduction in these foreign funds:



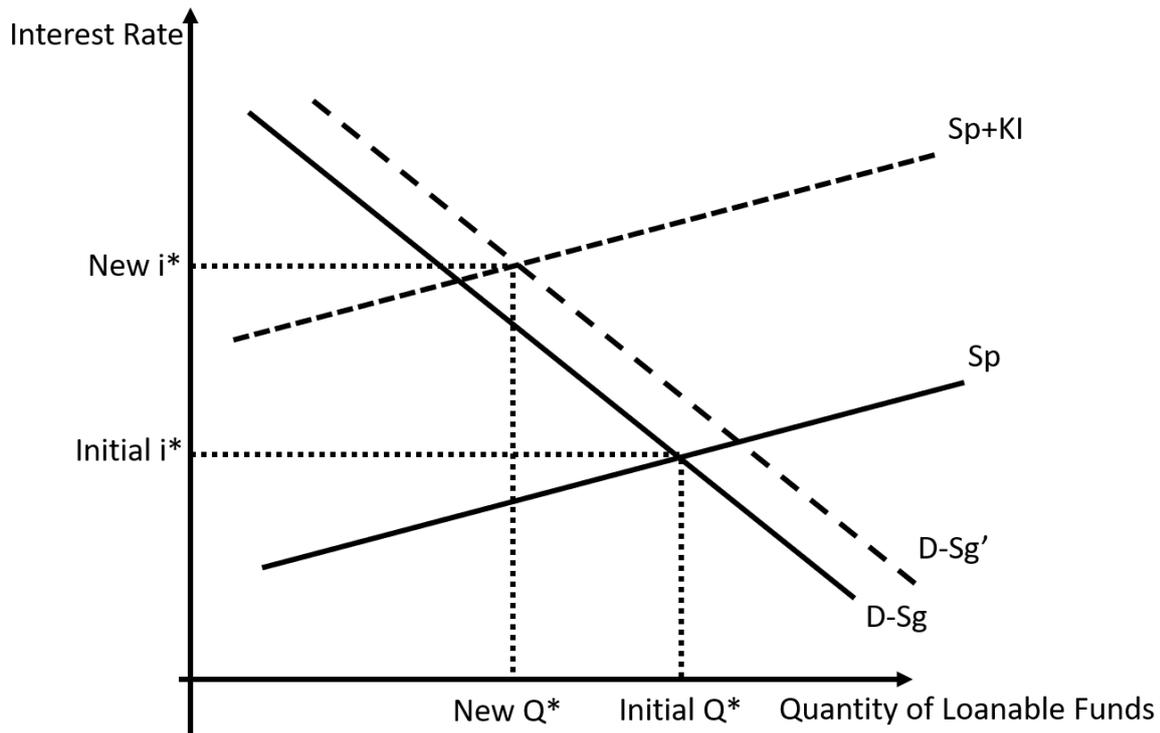
If the size of the trade deficit is smaller than the size of the reduction in these foreign funds, the net supply of loanable funds decreases and consequently the equilibrium interest rate rises, and the equilibrium level of loanable funds fall. The level of private investment spending would decrease and the level of private saving would increase relative to their initial levels.

Case 3: The size of the trade deficit is equal to the size of the reduction in these foreign funds:

If the size of the trade deficit is equal to the size of the reduction in these foreign funds, the net supply of loanable funds does not change and consequently the equilibrium interest rate and equilibrium level of loanable funds stay constant. There would be no change in the level of private investment spending or private saving relative to their initial levels.

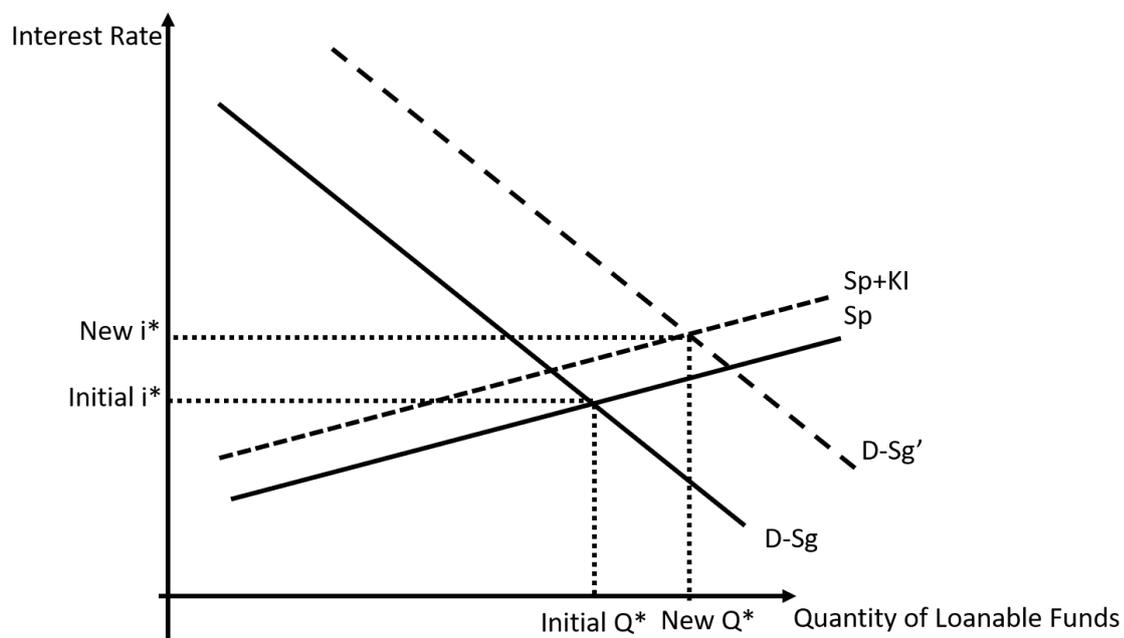
d. The equilibrium interest rate rises unambiguously but the equilibrium level of loanable funds is indeterminate because the outcome depends on the relative size of trade surplus versus government deficit. We cannot predict with certainty what happens to the level of private investment spending or private saving relative to their initial levels.

Case 1: The size of trade surplus is larger than the size of government deficit:



If the size of trade surplus is larger than the size of government deficit, the equilibrium level of loanable funds falls. As the interest rate rises, this causes private investment spending to fall and private saving to rise. As private saving rises, this leads to a decrease in consumption spending.

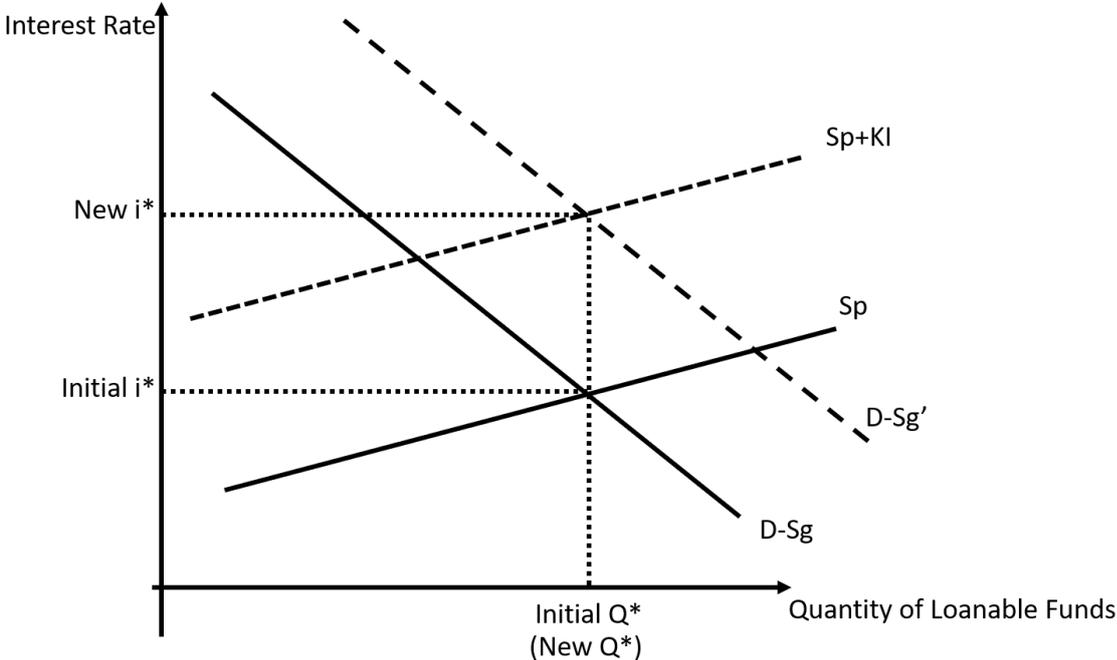
Case 2: The size of trade surplus is smaller than the size of government deficit:



If the size of trade surplus is smaller than the size of government deficit, the equilibrium level of loanable funds rises. As the interest rate rises, the level of private investment spending falls and

the level of private saving rises. As the level of private saving rises, the level of consumption spending falls.

Case 3: The size of trade surplus is equal to the size of government deficit:



If the size of trade surplus is equal to the size of government deficit, the equilibrium level of loanable funds is unchanged. As the interest rate rises, the level of private investment spending decreases and the level of private saving increases. As the level of private saving increases, the level of consumption spending falls.

6. Consider an economy where the demand for loanable funds from businesses is given by the following equation where Q is the quantity of loanable funds and r is the interest rate:

$$\text{Demand for loanable funds from businesses: } Q = 5,800 - 500r$$

Suppose that the supply of loanable funds from households (private savings) is given by the following equation where Q is the quantity of loanable funds and r is the interest rate:

$$\text{Supply of loanable funds from households: } Q = 200r - 500$$

In both the demand and supply for loanable funds equations the interest rate is expressed as a percentage (thus, if the interest rate is 3%, then the r in the equation would be 3).

a. If the economy is closed to the international trade and that the government runs a balance budget, what is the equilibrium level of interest rate and investment spending?

b. Holding everything else constant, suppose the government runs a budget deficit of \$1,400. How will this change the equilibrium level of private investment spending? Be specific with your answer.

c. In the scenario specified in part (b), what is the equilibrium level of private saving? Are private saving and private investment spending equal to each other in this scenario? Show your work and how you came to your conclusion.

d. Suppose the economy rebalances the government budget (the budget is now balanced). When this economy opens to international trade, the resulting trade surplus is \$1,050. What will be the equilibrium level of interest rate and private saving in this economy given this information and holding everything else constant?

e. In the scenario specified in part (d), what is the equilibrium level of private investment spending? Are private saving and private investment spending equal to each other?

SOLUTION:

a. The equilibrium interest and investment spending can be found by equating the supply and demand curves in the loanable funds market:

$$Q_s = Q_d = 5,800 - 500r^* = 200r^* - 500$$

$$700r^* = 6,300$$

$$r^* = 9$$

$$Q^* = I = 5,800 - 500(9) = 1,300$$

The equilibrium interest rate is 9% and equilibrium investment spending is \$1,300.

b. The government budget deficit of \$1,400 can be modeled as an increase of \$1,400 in the demand of loanable funds at every interest rate level. The new demand curve of loanable funds is given by:

$$Q_d' = 5,800 - 500r + 1,400 = 7,200 - 500r$$

The new equilibrium interest and investment spending can be found by equating supply and demand of loanable funds:

$$Q_s = Q_d' = 7,200 - 500r^{*' } = 200r^{*' } - 500$$

$$700r^{*'} = 7,700$$

$$r^{*'} = 11$$

$$Q^{*'} = 7,200 - 500(11) = 1,700$$

The new equilibrium interest rate is 11% and the equilibrium level of investment spending is \$1,700.

The equilibrium level of private investment spending can be found by plugging in the equilibrium interest rate into the *private* demand function for loanable funds:

$$I_{\text{private}} = 5,800 - 500(11) = 300$$

$$\Delta = 300 - 1,300 = -1,000$$

The equilibrium level of private investment spending is \$300. It decreased by \$1,000 compared to the initial equilibrium.

c. The equilibrium level of private saving can be found by plugging in the equilibrium interest rate into the supply function for loanable funds:

$$S_p = 200(11) - 500 = 1,700$$

The equilibrium level of private saving is \$1,700: this does not equal the equilibrium private investment of \$300. The difference between the two is the level of government deficit.

d. The trade surplus of \$1,050 can be modeled as a decrease of \$1,050 in the supply of loanable funds at every interest rate level. The new supply curve of loanable funds is given by:

$$Q_s' = 200r - 500 - 1,050 = 200r - 1,550$$

The new equilibrium interest and level of investment spending can be found by equating the supply and demand equations in the loanable funds market:

$$Q_s' = Q_d = 5,800 - 500r^{*''} = 200r^{*''} - 1,550$$

$$700r^{*''} = 7,350$$

$$r^{*''} = 10.5$$

$$Q^{*''} = 5,800 - 500(10.5) = 550$$

The new equilibrium interest rate is 10.5% and the equilibrium level of investment spending is \$550.

The new equilibrium level of private saving can be found by plugging in the equilibrium interest rate into the original supply function of loanable funds:

$$S_p' = 200(10.5) - 500 = 1,600$$

The new equilibrium level of private saving is \$1,600.

e. The equilibrium private investment spending can be found by plugging in the equilibrium interest rate into the *private* demand function for loanable funds:

$$I_{\text{private}}' = 5,800 - 500(10.5) = 550$$

The equilibrium level of private investment spending is \$550 which does not equal the equilibrium private saving of \$1,600. The difference between the two is the level of the trade surplus.