

Economics 102
Summer 2016
Answers to Homework #1
Due Thursday, June 23, 2016

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered**, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please remember to

- Staple your homework before submitting it.
- Do work that is at a professional level: you are creating your “brand” when you submit this homework!
- Not submit messy, illegible, sloppy work.

1. This set of questions will help you review some basic algebra, the slope-intercept form, finding a solution given two linear equations, and finding a new equation based upon an initial equation that has undergone a change. Each question below is independent of the other questions in the set.

a. You are given two pairs of coordinates that lie on a linear relationship. The two pairs of coordinates are $(x, y) = (20, 15)$ and $(5, 6)$. You are asked to find the equation for the line that these two points lie on.

b. You are given two pairs of coordinates that lie on a linear relationship. The two pairs of coordinates are $(x, y) = (4, 8)$ and $(20, 10)$. You are asked to find the equation for the line that these two points lie on.

c. You are given two equations:

$$\text{Equation 1: } y = 100 + 2x$$

$$\text{Equation 2: } y = 52 - 2x$$

Find the (x, y) solution that represents the intersection of these two lines.

d. You are given two equations where P is the variable measured on the y -axis (this is like our renaming y to be “ P ”) and Q is the variable measured on the x -axis (this is like our renaming x to be “ Q ”):

$$\text{Equation 1: } P = 64 + 2Q$$

$$\text{Equation 2: } P = 196 - 4Q$$

i. Find the (Q, P) solution that represents the intersection of the given lines.

Now, you are also told that equation 1 has changed and now the Q value is 18 units bigger at every P value than it was initially.

ii. Write the equation that represents the new Equation 1’.

iii. Given the new Equation 1’ and Equation 2, find the (Q', P') solution that represents the intersection of these two lines.

Answer:

a. Start by finding the slope of the equation using the two points: slope = (change in y)/(change in x) = $(15 - 6)/(20 - 5) = 9/15 = 3/5$. Then, use the slope-intercept form, $y = mx + b$, to find the equation for the line. Thus, $y = (3/5)x + b$. Then, plugging in one of the given point-in this case, let's use (20, 15) we get $15 = (3/5)(20) + b$ or $b = 3$. The equation is therefore $y = (3/5)x + 3$.

b. Start by finding the slope of the equation using the two points: slope = (change in y)/(change in x) = $(8 - 10)/(4 - 20) = 1/8$. Then, use the slope-intercept form, $y = mx + b$, to find the equation for the line. Thus, $y = (1/8)x + b$. Then, plugging in one of the given points-in this case, let's use (4, 8) we get $8 = (1/8)(4) + b$ or $b = 7.5$. The equation is therefore $y = (1/8)x + 7.5$.

c. To find where these two lines intersect set the two equations equal to one another:

$$100 + 2x = 52 - 2x$$

$$4x = -48$$

$$x = -12$$

$$y = 100 + 2x = 100 + 2(-12) = 76$$

$$\text{Or, } y = 52 - 2x = 52 - 2(-12) = 76$$

d.

i. To find the point of intersection of the two given lines (Q, P) we need to set the two equations equal to one another.

$$64 + 2Q = 196 - 4Q$$

$$6Q = 132$$

$$Q = 22$$

$$P = 64 + 2Q = 64 + 2(22) = 108$$

$$\text{Or, } P = 196 - 4Q = 196 - 4(22) = 108$$

$$(Q, P) = (22, 108)$$

ii. We know that (0,64) was on the original line represented by Equation 1; the new Equation 1' would contain the point (18, 64) since the Q value at every P value has increased by 18 units. The slope of Equation 1' is the same as the slope of Equation 1. Thus, $P' = b' + 2Q'$ where b' is the y-intercept of the new Equation 1'. Use the point (18, 64) to find the value of b' . Thus, $64 = b' + 2(18)$ or $b' = 28$. The equation for Equation 1' is $P = 2Q + 28$.

iii. To find where Equation 1' and Equation 2 intersect set the two equations equal to one another:

$$28 + 2Q' = 196 - 4Q'$$

$$6Q' = 168$$

$$Q' = 28$$

$$P' = 28 + 2Q' = 28 + 2(28) = 84$$

$$\text{Or, } P' = 196 - 4Q' = 196 - 4(28) = 84$$

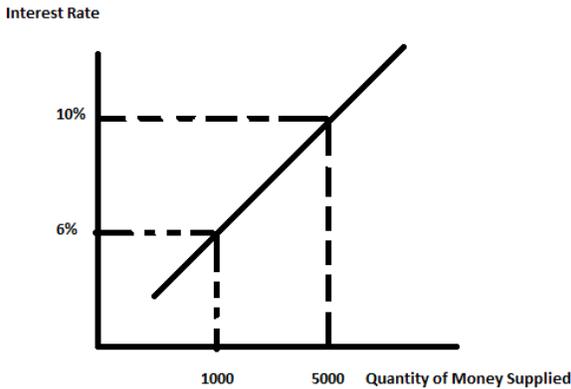
$$(Q', P') = (28, 84)$$

2. The price of money is called the interest rate. Suppose that when the interest rate is 6%, the supply of money is \$1000 and when the interest rate is 10% the supply of money is \$5000. Assume the relationship between the quantity of money supplied (Q) and the interest rate (r) is linear.

- a. Draw a graph representing the above information. In your graph measure Q on the horizontal axis and r on the vertical axis.
- b. Write an equation for this relationship in slope-intercept form.

Answer:

a.



b. We know that the two points $(Q, r) = (1000, 6)$ and $(5000, 10)$ both sit on this line. We also know the slope intercept form $y = mx + b$ and we can rewrite this general formula using our two variables as $r = mQ + b$ since the interest rate, r , is the variable measured on the vertical axis and quantity of money supplied, Q , is the variable measured on the horizontal axis. We then need to calculate the slope of this line using the two given points:

$$m = (6 - 10)/(1000 - 5000) = (-4/-4000) = (1/1000)$$

$$r = b + (1/1000)Q$$

To find the value of "b", substitute in the coordinates of a point that you know is on this line: $(Q, r) = (5000, 10)$

$$10 = b + (1/1000)(5000)$$

$$b = 5$$

$r = (1/1000)Q + 5$ is the equation for this relationship.

3. Angel's income in 2012 was \$60,000 and her income in 2013 was \$75,000. Her income in 2014 was \$60,000. Use this information to answer this next set of questions. For this set of questions assume there was no inflation during this three year period of time.

- a. What was the percentage change in Angel's income in 2013 relative to 2012?
- b. What was the percentage change in Angel's income in 2014 relative to 2013?
- c. Given that in both (a) and (b) you are measuring percentage changes and the numbers in both examples use \$60,000 and \$75,000, do you get the same answers? Explain your answer.

Answers:

a. Percentage change in Angel's income from 2012 to 2013 = $[(\text{New income} - \text{Initial income}) / (\text{Initial income})] * (100\%) = [(75,000 - 60,000) / (60,000)] * 100\% = (.25)(100\%) = 25\%$. Angel's income increased by 25% between 2012 and 2013.

b. Percentage change in Angel's income from 2013 to 2014 = $[(\text{New income} - \text{Initial income}) / (\text{Initial income})] * (100\%) = [(60,000 - 75,000) / (75,000)] * 100\% = (-.2)(100\%) = -20\%$. Angel's income decreased by 20% between 2013 and 2014.

c. Even though both (a) and (b) are measuring percentage changes and they both use \$60,000 and \$75,000 you do not get the same answers. That is because the base value, or the initial value, is different in the two questions. The choice of base matters in measuring percentage changes.

4. The following table provides data on the amount of labor Marco and Wenqi need in order to produce widgets (W) and gadgets (G). Assume that both Marco and Wenqi have linear production possibility frontiers and that the production of widgets and gadgets requires only labor as an input. In your answer measure widgets on the vertical axis and gadgets on the horizontal axis.

	Labor Needed to Produce One Gadget	Labor Needed to Produce One Widget
Marco	3 Hours of Labor	5 Hours of Labor
Wenqi	2 Hours of Labor	4 Hours of Labor

a. Assume that Marco and Wenqi each have 120 hours of labor they can devote to gadget and widget production. (Note: 120 is conveniently divisible by 2, 3, 4, and 5!) Fill in the following statements given this information. Assume both Marco and Wenqi produce at points on their PPFs.

- i. When Marco produces 10 gadgets, his widget production must equal _____.
- ii. When Marco produces 20 gadgets, his widget production must equal _____.
- iii. When Wenqi produces 8 gadgets, his widget production must equal _____.
- iv. When Wenqi produces 16 gadgets, his widget production must equal _____.

b. For Marco, the opportunity cost of producing an additional 2 gadgets is equal to _____.

c. For Wenqi, the opportunity cost of producing an additional 4 gadgets is equal to _____.

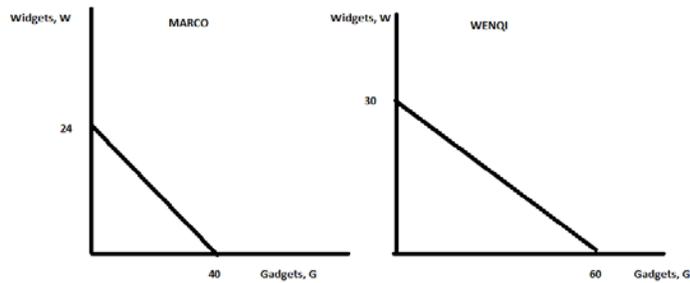
d. Marco has the comparative advantage in the production of _____ and Wenqi has the comparative advantage in the production of _____. Explain your answer.

e. Construct Marco and Wenqi's joint PPF measuring gadgets (G) on the horizontal axis and widgets (W) on the vertical axis.

f. The acceptable range of trading prices for 10 gadgets is _____. Depict this acceptable range of trading prices using the number line approach presented in class.

Answer:

a. First start by drawing two graphs: one representing Marco's PPF and the other representing Wenqi's PPF with both PPFs based upon each of these individuals having 120 hours of labor.



Given these two graphs it is relatively easy to write equations for the two PPFs:

Marco's PPF can be written as $W = 24 - (3/5)G$

Wenqi's PPF can be written as $W = 30 - (1/2)G$

Use these two equations to answer (a):

- i. If $G = 10$, then $W = 24 - (3/5)(10) = 18$ widgets
- ii. If $G = 20$, then $W = 24 - (3/5)(20) = 12$ widgets
- iii. If $G = 8$, then $W = 30 - (1/2)(8) = 26$ widgets
- iv. If $G = 16$, then $W = 30 - (1/2)(16) = 22$ widgets

b. The opportunity cost of Marco producing an additional 2 gadgets is 1.2 widgets: to see this answer start by assuming that G is initially 0. This implies $W = 24$. Then, if $G' = 2$, this implies $W' = 22.8$. Widget production decreases by 1.2 units when gadget production increases by 2 units.

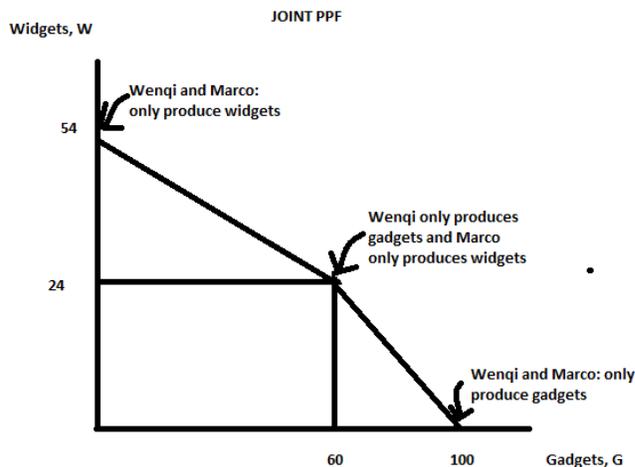
c. The opportunity cost of Wenqi producing an additional 4 gadgets is 2 widgets: to see this answer start by assuming that G is initially 0. This implies $W = 30$. Then $G' = 4$, and this implies $W' = 28$. Widget production decreases by 2 units when gadget production increases by 4 units.

d. Widgets; Gadgets

To see this: Marco's opportunity cost of producing 1 gadget is $(3/5)$ widget while Wenqi's opportunity cost of producing 1 gadget is $(1/2)$ widget: Marco's opportunity cost of producing gadgets is greater than Wenqi's opportunity cost of producing gadgets. That implies that Marco has the comparative advantage in the production of widgets.

Wenqi's opportunity cost of producing 1 widget is (2) gadgets while Marco's opportunity cost of producing 1 widget is $5/3$ gadgets: Wenqi's opportunity cost of producing widgets is greater than Marco's opportunity cost of producing widgets. That implies that Wenqi has the comparative advantage in the production of gadgets.

e.



Notice that the above figure does not show a straight line from $(0, 54)$ to $(100, 0)$ but instead it illustrates two line segments that meet at a kink. Initially at $(0, 54)$ both Marco and Wenqi produce only widgets. As you move down the curve, initially Marco continues to produce widgets since he has the comparative advantage in widget production while Wenqi begins to produce gadgets. Thus, when they start to produce gadgets, they will first assign the person to gadget production who has the comparative advantage in gadget production.

f. The acceptable range of trading prices for 10 gadgets will be between 5 widgets and 6 widgets.

The price of 1 gadget would lie between the opportunity costs of $1/2$ widget for Wenqi and $3/5$ widget for Marco. The price of 1 gadget is in this range since when Marco wants to buy gadgets using widgets, he would not pay a price that is higher than his opportunity cost of $3/5$ widget. Similarly, Wenqi will not sell the gadget for a price that is lower than his opportunity cost of $1/2$ widget. This trading range can be depicted using a number line.

