

Economics 102  
Summer 2015  
Answers to Homework #1  
Due Thursday, June 25, 2015

**Directions:** The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work (in fact, you WILL NOT receive full credit for the assignment unless you DO show your work).** Good luck!

**Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!**

1. This set of questions is meant as a quick review of some algebra and basic math skills. If you find this set difficult, you may need to do some math review in preparation for the class.

a. Suppose you are given two points  $(x, y) = (3, 5)$  and  $(10, 8)$ . Suppose these two points sit on a straight line. Write an equation in slope intercept form given these two points. Show your work. Don't worry if you get “ugly numbers”: keep your numbers as fractions, and not decimals!

b. Suppose you are given two points  $(x, y) = (4, 8)$  and  $(12, 24)$ . Suppose these two points sit on a straight line. Write an equation in slope intercept form given these two points and call this Line 1. Suppose you are then told that there is a Line 2 that has the following relationship to Line 1: for every  $x$  value, Line 2's  $y$ -value is twice the  $y$ -value found on Line 1. Sketch a graph of Line 1 and Line 2 and then write an equation for Line 2 given this information. As always, show your work. Graph these equations only in the first quadrant (that is,  $X$  is greater than or equal to 0 and  $Y$  is greater than or equal to 0).

c. Suppose you know that a straight line has slope of  $-5$  and also contains the point  $(x, y) = (10, 5)$ . Write an equation in slope-intercept form given this information. Show your work.

d. Suppose you know that a straight line has slope of  $\frac{1}{2}$  and also contains the point  $(x, y) = (34, 17)$ . Write an equation in slope-intercept form given this information. Show your work.

e. Suppose you are given the equation  $Y = 4X + 3$  and told that for every  $x$  value on this line, that the  $y$  value has increased by 10 units. Draw a graph illustrating these two lines given this information and then write an equation for the new line given this information. Show your work.

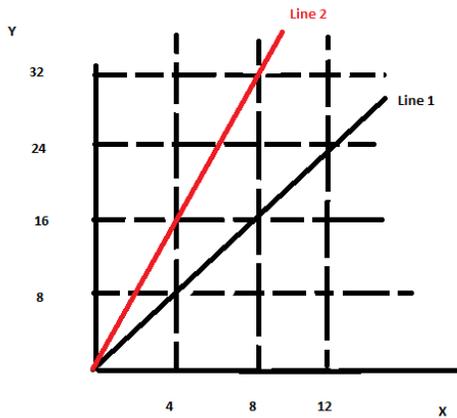
**Answers:**

a. Given this information we can calculate the slope of the line as  $\text{slope} = \text{rise/run} = (8 - 5)/(10 - 3) = 3/7$ . Then, we can write the equation in slope intercept form,  $y = mx + b$ , as  $Y = (3/7)X + b$ . To solve for  $b$  we can use one of the given points  $(x, y)$  in this equation: thus,  $8 = (3/7)(10) + b$

or  $b = 26/7$ . The equation in slope intercept form is therefore  $Y = (3/7)X + (26/7)$ . Note, that we can expect that the y-intercept is less than 5 if we draw a sketch of the line based on the given information:  $26/7 = 3$  and  $5/7$ , and this seems plausible!

b. Start by writing the equation for Line 1: you will first need to find the slope of this line and then use one of the given points and this slope to calculate the y-intercept. Thus, slope = rise/run =  $(24 - 8)/(12 - 4) = 2$ . Then,  $Y = 2X + b$  and using the point (4, 8) we get  $8 = 2(4) + b$  or b is equal to 0. Thus, the equation for Line 1 in slope-intercept form is  $Y = 2X$ .

Then graph Line 1 and Line 2: Line 1 you can graph using the equation you just found, and Line 2 can be graphed using the provided information. Thus,

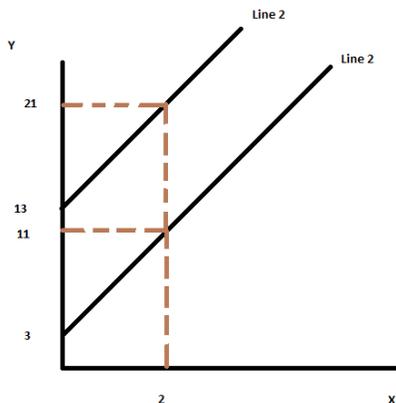


The hardest part of this problem is recognizing that this will NOT be a parallel shift since when you consider the point (0, 0) it sits on both lines (if you double zero, you still get zero). The equation for Line 2 can be written as  $Y = 4X$  since the slope of Line 2 is rise/run =  $16/4 = 4$ .

c. You are given the slope and one point: so,  $Y = b - 5X$ . Then, use the given point to solve for the y-intercept, b: thus,  $5 = b - 5(10)$  or  $b = 55$ . Thus, the equation in slope intercept form is  $Y = 55 - 5X$ .

d. You are given the slope and one point: so,  $Y = (1/2)X + b$ . Then, use the given point to solve for the y-intercept, b: thus,  $17 = (1/2)(34) + b$  or  $b = 0$ . Thus, the equation in slope intercept form is  $Y = (1/2)X$ .

e. Start by drawing the graph:



Note: I choose  $X = 2$  to get a second set of coordinates for Line 1 and then to use this information for getting  $(X, Y) = (2, 21)$  on Line 2. There is nothing special about  $X = 2$ , but it allows me to easily get a second point on the new line. It is relatively easy to take  $(0, 3)$  on Line 1 and get  $(0, 13)$  on Line 2.

Now, that I have a sketch I can easily write the new equation: Line 1 and Line 2 have the same slopes since they are parallel lines. So, the equation for Line 2 is simply  $Y = 4X + 13$ .

2. More math practice! Here is a second set of questions to work through for some basic math skill review.

a. You are given two equations:

$$\text{Equation 1: } Y = 10X + 20$$

$$\text{Equation 2: } Y = 10 - 2X$$

What are the  $(X, Y)$  values for where these two lines intersect? In your own words, explain what it means when these two lines intersect. As always, show your work. Don't be surprised if you get "ugly numbers" here: just keep your answers in fractions!

b. You are given two equations:

$$\text{Equation 1: } Y = 10X + 20$$

$$\text{Equation 2: } X = 2Y - 10$$

Are both of these equations in slope intercept form? Explain your answer. In the second equation, what does the numeric value "2" signify? In the second equation, what does the numeric value of  $(-10)$  signify? Use these two equations to find the values of  $X$  and  $Y$  that make both equations true. Expect "ugly numbers" and use fractions (no decimals, and no calculators here!).

c. Suppose you are given an equation  $Y = 10 + 4X$ . Calculate the area under this equation for all positive values of  $X$  greater than 2 and less than 4. Show your work and include a graph to illustrate your answer. Consider only the area found in the first quadrant. Assume that  $Y$  is measured as dollars per unit and  $X$  is measured as units: what type of units will the area you calculate be measured in given this information? Show your work!

Answers:

a. To find the (X, Y) values where these two lines intersect, you can set Equation 1 equal to Equation 2: thus,  $10X + 20 = 10 - 2X$  or  $12X = (-10)$  or  $X = (-5/6)$ . Then, use this X value in either equation to find the corresponding Y value: thus,  $Y = 10(-5/6) + 20$  or  $Y = (-50/6) + (120/6)$  or  $Y = 70/6$  or  $11 \text{ and } 2/3$ . Using the other equation:  $Y = 10 - 2(-5/6)$  or  $Y = 10 + 10/6$  or  $Y = 11 \text{ and } 2/3$  (these Y values have to be the same!). Thus, the (X, Y) values for where these two lines intersect are  $(X, Y) = (-5/6, 70/6)$ . The values you calculated represent where these two lines intersect; they are the x and y values for a point that “sits” on both lines.

b. No, Equation 2 is not in slope-intercept form. Equation 2 is in X-intercept form. This means that in Equation 2 the coefficient of the Y term, 2, signifies the reciprocal of the slope. Thus, the slope of Equation 2 would be  $(1/2)$  and not 2. The constant term in Equation 2 signifies the x-intercept and not the y-intercept. Thus,  $(-10)$  represents the x value when  $Y = 0$  for Equation 2. If we wanted to write Equation 2 in slope intercept form we would have  $Y = (1/2)X + 5$ . In this equation we see that the slope of Equation 2 is  $(1/2)$  and the y-intercept is 5.

To solve for where these two equations intersect we can either use a substitution method or we can rewrite one of the equations so that it is in the same form as the other equation. Here is the substitution method:

$Y = 10X + 20$  and we can replace X with  $(2Y - 10)$ . Thus,

$$Y = 10(2Y - 10) + 20$$

$$Y = 20Y - 100 + 20$$

$$19Y = 80$$

$$Y = 80/19 \text{ and } X = 2Y - 10 \text{ or } X = 2(80/19) - 10 \text{ which is } X = (-30/19)$$

So, (X, Y) where these two lines intersect is  $(-30/19, 80/19)$

Weird numbers, so let's verify with the second method. Here are the two equations in slope intercept form:

$$Y = 10X + 20$$

$$Y = (1/2)X + 5$$

$$\text{So, } 10X + 20 = (1/2)X + 5$$

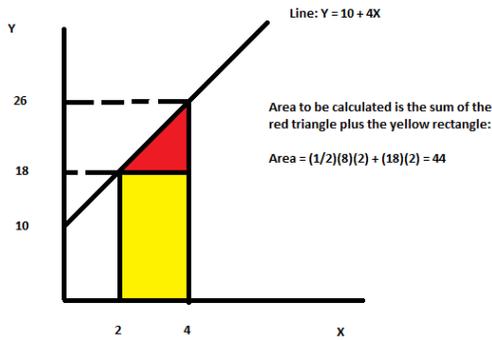
$$20X + 40 = X + 10$$

$$19X = -30$$

$$X = (-30/19)$$

And,  $Y = 10(-30/19) + 20$  or  $Y = (-300/19) + (380/19)$  or  $Y = 80/19$ . Using the other equation to find the Y value we get:  $Y = (1/2)(-30/19) + 5$  or  $Y = -15/19 + 95/19$  or  $Y = 80/19$ . So using this method we find the solution for this problem is  $(X, Y) = (-30/19, 80/19)$  which is exactly the same solution we found with the first method.

c. Start with the graph:



If the Y value is dollars per unit and the X value is units, then  $Y \cdot X$  is equal to (dollars/unit)(units) or just dollars. So, the area calculated is measured as \$44.

3. Bernie stays confused about percentages and he is struggling to figure out what he needs to do on his final exam in Chemistry in order to get the B he needs. Here is the information he has: he scored a 40 out of a possible 50 points on his first midterm in the class; he scored a 15 out of 25 points on the second midterm (it was tough!) and on the third midterm he got an 85 out of a 100 points. He knows that his final will have 50 points. And, he also knows that each midterm has equivalent weight to all the other midterms and that this weight is 20% of his final grade; he also knows that the final exam will be weighted as 40% of his final grade; and to get a B in the class he knows that his total weighted average must be at least an 84 on a 100 point scale. So, what score will Bernie need to make on that final exam if he is going to get a B in the class? Show your work! Here you will find it helpful to work in decimals instead of fractions: try to do this without a calculator though!

Answer:

To answer this question we need to do a lot of indexing of the midterm grades. To begin with let's calculate Bernie's score on each midterm based on a 100 point scale.

Midterm 1: 40/50 points is the same as 80/100 points. To see this recognize that to transform a 50 point test into a 100 point test requires us to multiply by a factor of 2. But, if we are going to multiply the denominator by 2, we must also multiply the numerator by 2 in order to keep the same value: hence,  $(40/50)(2/2) = 80/100$ . Effectively we are just rescaling the exam to 100 points.

Midterm 2: 15/25 points is the same as 60/100 points. Here the rescaling factor is 4 since  $25 \cdot 4 = 100$ .

Midterm 3: 85/100 points needs no rescaling since it is already on a 100 point scale.

So, Bernie's three midterms scored on a 100 point scale are respectively: 80, 60, and 85.

Now, we need to compute the final weighted grade:

Final weighted grade = (score on first midterm on 100 point scale)(weight of first midterm) + (score on second midterm on 100 point scale)(weight of second midterm) + (score on third

midterm on 100 point scale)(weight of third midterm) + (score on final exam on 100 point scale)(weight of final exam)

Thus, to get a B in the class Bernie has the following equation:

$$84 = 80(.2) + 60(.2) + 85(.2) + (\text{score on final exam on 100 point scale})(.4)$$

$$84 = 16 + 12 + 17 + (\text{score on final exam on 100 point scale})(.4)$$

$$39 = (\text{score on final exam on 100 point scale})(.4)$$

$$97.5 = \text{score on final exam on 100 point scale}$$

But, Bernie's final exam has only 50 points in all-so, we need to rescale this 97.5 out of 100 points to the number out of 50 points he needs to get. Thus, (score on final exam on 50 point scale)/50 = 97.5/100 or score on final exam on 50 point scale = 97.5/2 = 48.75. Whoa, Bernie is really going to have to perform on this final exam if he hopes to get the B in the class!

4. Suppose Bert and Ernie make cookies (C) and pies (P). The table below provides information about the amount of labor time that it takes them to make one cookie or one pie. Assume that there are no other resources involved in the making of cookies and pies and that both Bert and Ernie have linear production possibility frontiers.

	Number of Hours of Labor Needed to Make One Cookie	Number of Hours of Labor Needed to Make One Pie
Bert	½ Hour of Labor	1.5 Hours of Labor
Ernie	¾ Hour of Labor	1 Hour of Labor

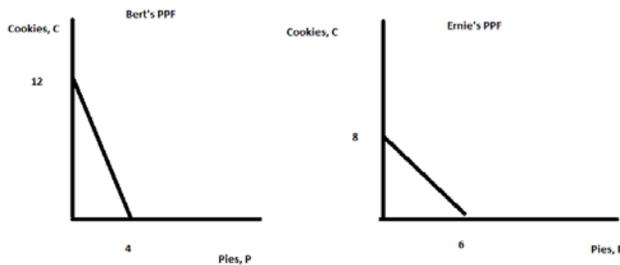
- Suppose that Bert and Ernie each have 6 hours of labor that they can use to produce Cookies and Pies. Draw a graph that represents Bert's PPF measuring cookies on the vertical axis and pies on the horizontal axis. Draw a second graph that represents Ernie's PPF measuring cookies on the vertical axis and pies on the horizontal axis. Then, write an equation for Bert's PPF in slope intercept form and an equation for Ernie's PPF in slope intercept form.
- Suppose that the number of hours of labor available to both Bert and Ernie changes so that they each have twelve hours of labor that they can use to produce cookies and pies. How will this affect the PPFs you drew in (a)? Write the new equations for Bert and Ernie's PPFs using this new level of labor.
- Suppose that Bert and Ernie decide to specialize and then trade with one another. What good should Bert produce? Explain your answer fully.
- Suppose that Bert and Ernie each have twelve hours of labor available to produce cookies and pies. Construct their joint PPF in a graph making sure to identify all intercepts and the coordinate values of all kink points.
- For each of the following combinations determine whether Bert and Ernie can produce the combination (feasible or infeasible) and whether the combination is efficient or inefficient. Complete the tables by entering your answers. Base your analysis on the joint PPF you constructed in (d).

Combination of (pies, cookies)	Feasible or Infeasible?	Efficient, inefficient, or Beyond the PPF?
(30, 4)		----
(12, 24)		
(15, 18)		
(21, 12)		---

- f. Given your work in this problem, provide a range of acceptable trading prices for 5 pies. Use the number line approach used in class to illustrate this trading range and include the directional arrows depicting how Bert views this transaction as well as how Ernie views this transaction.
- g. Given your work in this problem, provide a range of acceptable trading prices for 3 cookies. Use the number line approach used in class to illustrate this trading range and include the directional arrows depicting how Bert views this transaction as well as how Ernie views this transaction.

Answers:

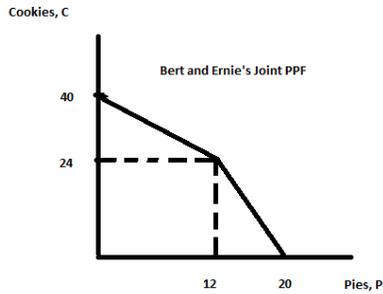
a.



Equation for Bert's PPF:  $C = 12 - 3P$

Equation for Ernie's PPF:  $C = 8 - (4/3)P$

- b. With twelve hours of labor available, the PPFs of Bert and Ernie will shift out from the origin. The new PPFs will be parallel to the initial PPFs (that is, the slope will be the same) but they will have different x and y intercepts. The equation for Bert's PPF will be  $C = 24 - 3P$  and the equation for Ernie's PPF will be  $C = 16 - (4/3)P$ .
- c. Bert should produce cookies since his opportunity cost of producing a cookie is lower than Ernie's opportunity cost of producing a cookie. Bert's o.c. of producing a cookie is  $1/3$  pie, while Ernie's o.c. of producing a cookie is  $3/4$  pie. It is cheaper for Bert to produce cookies than for Ernie to produce cookies.
- d.

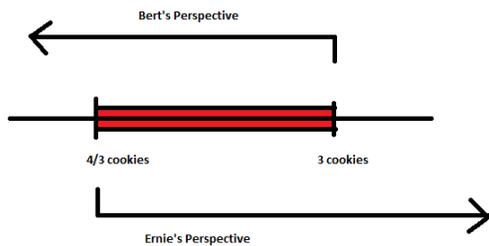


e.

Combination of (pies, cookies)	Feasible or Infeasible?	Efficient, inefficient, or beyond the PPF?
(30, 4)	Not feasible	this combination is beyond the joint PPF
(12, 24)	Feasible	Efficient: this combination lies on the PPF
(15, 18)	Not Feasible	this combination is beyond the joint PPF
(21, 12)	Not feasible	this combination is beyond the joint PPF

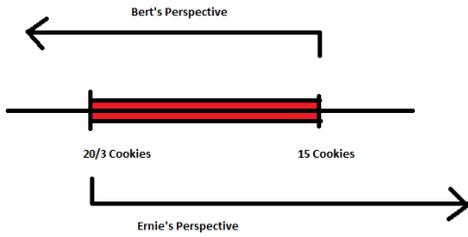
f. From our work in this problem we know that Bert's opportunity cost of producing 1 pie is 3 cookies and Ernie's opportunity cost of producing 1 pie is  $\frac{4}{3}$  cookies. So we know that Ernie has the comparative advantage in producing pies (so he will be willing to provide 1 pie for any price that is equal to or greater than  $\frac{4}{3}$  cookies) and that Bert will be willing to buy a pie from Ernie for any price that is less than or equal to 3 cookies. This is depicted in the figure below where the red horizontal line represents the trading range that is acceptable to both Bert and Ernie:

1 Pie:



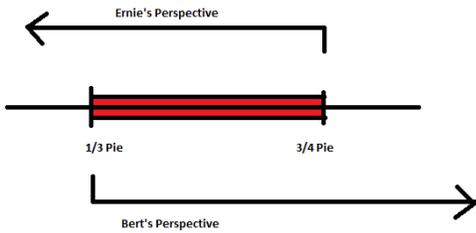
Now, we need to take this information and transform it to the trading range for 5 pies: so we multiply ( $\frac{4}{3}$  cookies) by 5 and we multiply (3 cookies) by 5 and get the following figure:

5 Pies:



g. From our work in this problem we know that Bert's opportunity cost of producing 1 cookie is  $1/3$  pie and Ernie's opportunity cost of producing 1 cookie is  $3/4$  pie. So we know that Bert has the comparative advantage in producing cookies (so he will be willing to provide 1 cookie for any price that is equal to or greater than  $1/3$  pie) and that Ernie will be willing to buy a cookie from Bert for any price that is less than or equal to  $3/4$  pie. This is depicted in the figure below where the red horizontal line represents the trading range that is acceptable to both Bert and Ernie:

1 Cookie



Now, we need to take this information and transform it to the trading range for 3 cookies: so we multiply ( $1/3$  pie) by 3 and we multiply ( $3/4$  pie) by 3 and get the following figure:

3 Cookies:

