

Economics 102
Spring 2018
Answers to Homework #1
Due 2/8/2018

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered**, because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please remember to

- Staple your homework before submitting it.
- Do work that is at a professional level: you are creating your “brand” when you submit this homework!
- Do not submit messy, illegible, sloppy work.
- Show your work to get full credit.

1. You are given two pairs of coordinates that have a linear relationship. The two pairs of coordinates are $(x, y) = (13, 48)$ and $(27, 90)$.

a. Find the expression of the line (Line 1) that goes through these two points in y-intercept form.

b. In this linear relationship, if the level of x decreases by 66, what is the resulting change in y?

c. There is another linear relationship represented by the following expression:

$$\text{(Line 2): } y = 97 - 5x$$

Find the (x, y) solution that represents the intersection of these two lines.

d. Now Line 1 is shifted in such way that for every x value, the y value is 21 units larger, at the same time Line 2 is shifted in such way that for every y value, the x value is 7 units smaller. Represent these shifts in a clearly labeled graph measuring X along the horizontal axis and Y along the vertical axis. In your graph represent both the initial lines (Line 1 and Line 2) and the new lines (Line 1' and Line 2'). Find the coordinates for the new intersection of these two new lines (Line 1' and Line 2').

SOLUTION:

a. Start by finding the slope of the equation using the two points: slope = (change in y)/(change in x) = $(48 - 102)/(13 - 27) = 3$. Then, use the slope-intercept form, $y = mx + b$, to find the equation for the line. Thus, $y = 3x + b$. Then, plugging in one of the given point-in this case, let's use $(27, 90)$ we get $90 = 3(27) + b$ or $b = 9$. The equation is therefore $y = 3x + 9$.

b. The change in y is given by the change in x multiplied by the slope: (change in y) = slope*(change in x) = $3*(-66) = -198$.

c. To find where these two lines intersect, set the two equations equal to one another:

$$3x + 9 = 97 - 5x$$

$$8x = 88$$

$$x = 88/8 = 11$$

$$y = 3x + 9 = 3(11) + 9 = 42$$

$$\text{Or, } y = 97 - 5(11) = 42$$

$$(x, y) = (11, 42)$$

d. Line 1 is shifted upward by 21 units and Line 2 is shifted leftward by 7 units.

The new expression for Line 1 is $y = 3x + 9 + 21 = 3x + 30$, the new expression for Line 2 is $y = 97 - 5(x + 7) = -5x + 62$.

To find where these two lines intersect, set the two equations equal to one another:

$$3x + 30 = -5x + 62$$

$$8x = 32$$

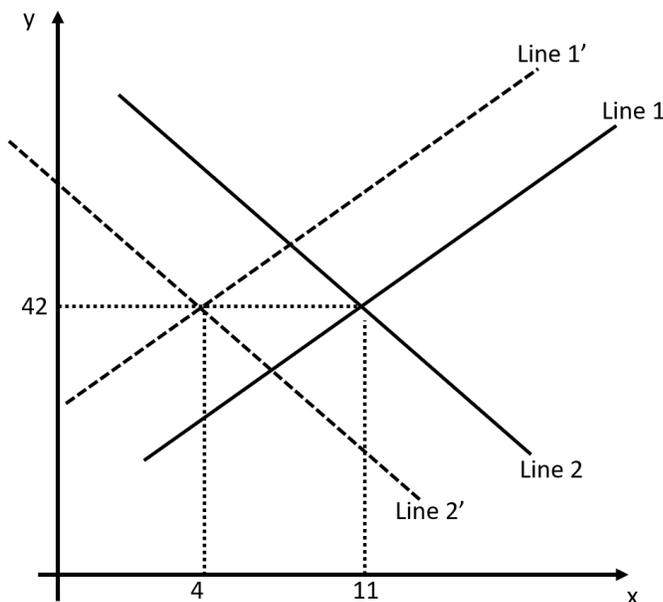
$$x = 32/8 = 4$$

$$y = 3x + 30 = 3(4) + 30 = 42$$

$$\text{Or, } y = -5x + 62 = -20 + 62 = 42$$

$$(x, y) = (4, 42)$$

Graph:



2. The price of money is called the interest rate. Suppose that when the interest rate is 4%, the demand for loans is \$23,000 and when the interest rate is 2% the demand for loans is \$28,000. Assume the relationship between the quantity of loans demanded (L) and the interest rate (r) is linear.

a. Write an equation for this relationship in L-intercept form.

b. What would the equation for this relationship be if it was written in r-intercept form?

c. What is the amount of loans demanded at the interest rate of 10%? What is the amount of loans demanded when the interest rate is 20%?

d. What is the level of interest rate above which no one would find it worthwhile to borrow money?

SOLUTION:

a. We know that the two points $(r, L) = (4, 23000)$ and $(2, 28000)$ both sit on this line. We also know the slope intercept form $y = mx + b$ and we can rewrite this general formula using our two variables as $L = mr + b$ since the quantity of loan supplied, L, is the variable measured on the vertical axis and the interest rate, r, is the variable measured on the horizontal axis. We then need to calculate the slope of this line using the two given points:

$$m = (\text{change in } L)/(\text{change in } r) = (23000 - 28000)/(4 - 2) = -5000/2 = -2500$$

To find the value of "b", substitute in the coordinates of a point that you know is on this line: $(r, L) = (4, 23000)$

$$23000 = -2500*4 + b$$

$$b = 23000 + 2500*4 = 33000$$

$L = -2500r + 33000$ is the equation for this relationship.

b. To write the equation in r-intercept form we simply need to solve the y-intercept form for "r".

Thus,

$$L = -2500r + 33000$$

$$2500r = -L + 33000$$

$$r = -(1/2500)L + 66/5$$

c. To find the amount of loans demanded when interest rate is 10%, plug in $r = 10$ into the expression of the demand function:

$$L = -2500r + 33000$$

$$L = -2500(10) + 33000 = 8000$$

The amount of loans demanded when the interest rate r is 10% is \$8,000.

To find the amount of loans demanded when the interest rate is 20%, plug in $r = 20$ into the expression of demand function:

$$L = -2500r + 33000$$

$$L = -2500(20) + 33000 = -17000 < 0$$

The fact that the amount of loans demanded (according to demand function) is negative suggests that the interest rate level is too high for people to demand any loans. Hence the amount of loans demanded when the interest rate is 20% is \$0.

d. To find the level of interest rate above which no one would find it worthwhile to borrow money is to find the “cutoff” interest rate at which the demand for loans is \$0:

$$0 = -2500r + 33000$$

$$2500r = 33000$$

$$r = 33000/2500 = 66/5 = 13.2$$

The level of interest rate above which no one would find it worthwhile to borrow money in this market is 13.2%.

3. On May 22, 2010, a developer named Laszlo Hanyecz bought two large pizzas worth \$40 using 10,000 units of a then-little-known digital currency called “bitcoin”. On Nov. 18, 2017, the trading price for one unit of bitcoin (BTC) reached \$10,000 for the first time.

a. What is the approximate percentage change in the trading price of bitcoin from May 22, 2010 to Nov. 18, 2017? Show your work in finding your answer for this question.

b. Suppose that the price for a large pizza rose by 25% from May 22, 2010 to Nov. 18, 2017. How many large pizzas can he buy with 10,000 units of bitcoin in Nov. 18, 2017?

SOLUTION:

a. The price for one unit of bitcoin in May 22, 2010 was $\$40/10,000 = \0.0004 . The price for one unit of bitcoin in Nov. 18, 2017 was \$10,000. The percentage change in trading price of bitcoin is given by $[(10000-0.0004)/0.0004]*100\% = 2,500,000,000\%$ or $2.5 * 10^9 \%$ (2.5 billion percent).

b. The price for a large pizza in May 22, 2010 was $\$40/2 = \20 . That in Nov. 18, 2017 the price of a large pizza was $\$20*1.25 = \25 . The number of large pizzas that could be bought with 10,000 units of bitcoin in Nov. 18, 2017 is given by: $10,000*\$10,000/\$25 = 4,000,000$ or $4 * 10^6$ (4 million)

4. Wenqi, Erika and Wentao make bagels (B) and cups of coffee (C) for the ECON 102 students during the week. They all have linear production possibility frontiers. Wenqi knows that he can make $(C, B) = (41, 15)$ and $(13, 29)$ or any other combination of the two goods that lie on the line containing these two points. Erika knows that the maximum number of bagels she can make is 52 and the maximum number of cups of coffee she can make is 78. Wentao knows that he must give up 3 bagels for every cup of coffee he makes. Wentao is currently producing 18 bagels and 17 cups of coffee.

a. Represent the production possibility frontiers for Wenqi, Erika and Wentao in three clearly labeled graphs. Please measure cups of coffee (C) on the horizontal axis and bagels (B) on the vertical axis.

b. Write out the B-intercept form expressions of the individual PPFs for Wenqi, Erika and Wentao.

c. Who has the absolute advantage in producing bagels (B) and cups of coffee (C)?

d. Who has the comparative advantage in producing bagels (B) and cups of coffee (C)?

e. Construct Wenqi, Erika, and Wentao's joint PPF in a clearly labeled graph. Please measure cups of coffee (C) on the horizontal axis and bagels (B) on the vertical axis. Identify the y-intercept, the x-intercept, and the coordinates of any "kink points" in your graph.

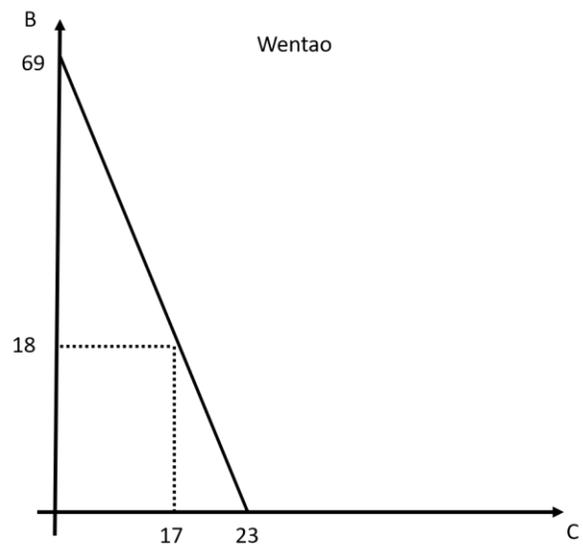
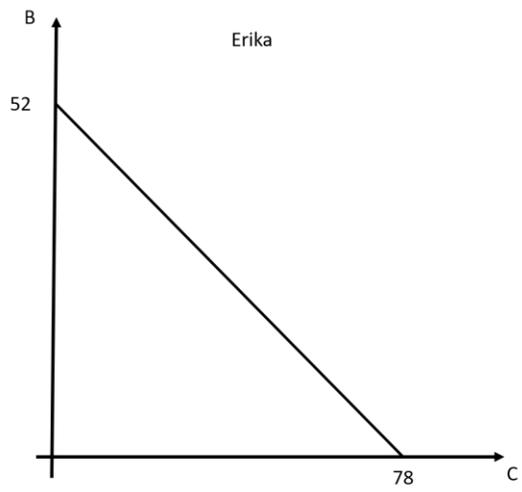
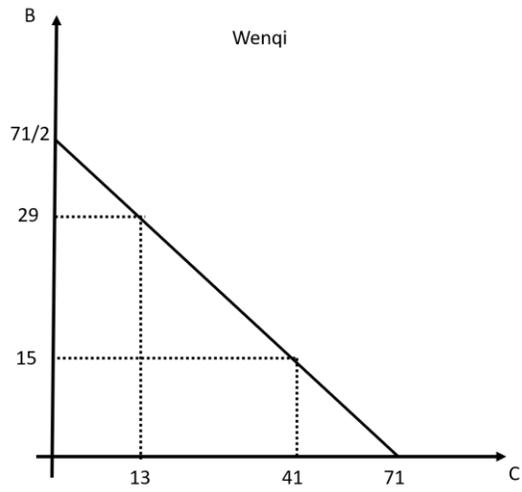
f. Write out the expression (that is, the equation) and the corresponding range for each segment of the joint PPF.

g. If the three-person economy of Wenqi, Erika and Wentao is making 100 bagels (B) efficiently, how many cups of coffee (C) is the economy currently making? How many bagels (B) are made by each person?

h. Use the number line approach to illustrate the acceptable range of trading prices for one cup of coffee (C) in terms of bagels (B) for Wenqi, Erika and Wentao.

SOLUTION:

a. Graphs of the individual PPFs:



b. Wenqi: The slope for Wenqi's PPF is $(15 - 29)/(41 - 13) = -1/2$. The y-intercept of Wenqi's PPF b satisfies $15 = (-1/2)*41 + b$, so $b = 15 + 41/2 = 71/2$, so the expression for Wenqi's PPF is $B = (-1/2)C + 71/2$.

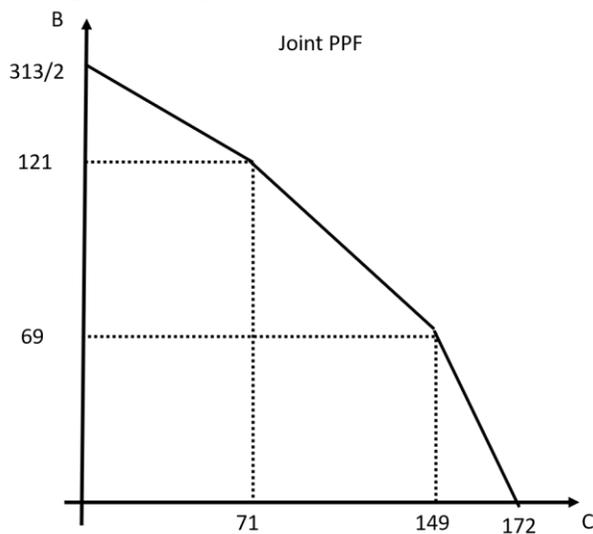
Erika: The slope for Erika's PPF is $(52 - 0)/(0 - 78) = -52/78 = -2/3$. The y-intercept of Erika's PPF can be read directly from the graph which is 52, so the expression for Erika's PPF is $B = (-2/3)C + 52$.

Wentao: The slope for Wentao's PPF is -3 because he must give up 3 bagels for every cup of coffee. The y-intercept of Wentao's PPF b' satisfies $18 = (-3)*17 + b'$, so $b' = 18 + 51 = 69$, so the expression for Wentao's PPF is $B = -3C + 69$.

c. Wentao has the absolute advantage in producing bagels (B) because he has the potential to make 69 bagels (compared to Wenqi's 71/2 and Erika's 52). Erika has the absolute advantage in producing cups of coffee (C) because she has the potential to make 78 cups of coffee (compared to Wenqi's 71 and Wentao's 23).

d. Wentao has the comparative advantage in producing bagels (B) because he gives up only 1/3 cups of coffee (C) to make one unit of bagel (compared to Wenqi's 2 and Wentao's 3/2). Wenqi has the comparative advantage in producing cups of coffee (C) because he gives up only 1/2 bagels to make one cup of coffee (compared to Erika's 2/3 and Wentao's 3).

e. Graph for the joint PPF:



f. In the top segment of the joint PPF, only Wenqi is producing cups of coffee (C) and Erika and Wentao are producing bagels (B). The expression for this segment is therefore:

$$B = (-1/2)C + 313/2 \text{ for } 0 \leq C \leq 71.$$

In the middle segment, both Wenqi and Erika are producing cups of coffee (C) and Wentao and Erika are producing bagels (B). We can look at the graph and see that the slope of the segment is $(121 - 69)/(71 - 149) = -52/78 = -2/3$. We then use one of our points that we know is on this segment to solve for the y-intercept of this equation. We know that $(C, B) = (71, 121)$ and $(149,$

69) both sit on this segment. Thus, $B = (-2/3)C + b$ and using one of our points: $121 = (-2/3)(71) + b$ and thus, $b = 121 + 142/3 = 505/3$. The equation for the middle segment is therefore; $B = - (2/3)C + 505/3$ for $71 \leq C \leq 149$.

In the bottom segment, all three are producing cups of coffee (C) and if any bagels are being produced they are produced by Wentao. The slope of this segment is $(69 - 0)/(172 - 149) = -3$. And we know that $(C, B) = (149, 69)$ and $(172, 0)$ both sit on this segment. Thus, the equation for this segment can be written as $B = (-3)C + b$ and using one of our points: $0 = (-3)(172) + b$ and thus, $b = 172/2$. The equation for the lowest segment of the joint PPF is therefore: $B = -3C + 173/2$ for $149 \leq C \leq 172$.

g. Since the economy is currently producing efficiently, it must be producing a combination of (C,B) that is on the joint PPF. Since $B = 100$ falls into the middle segment of the joint PPF, we can plug in $B = 100$ into the expression for the middle segment of PPF that is:

$B = - (2/3)C + 505/3$ for $71 \leq C \leq 149$ and solve for C:

$$100 = - (2/3)C + 505/3$$

$$(2/3)C = 205/3$$

$$C = 205/2 \text{ cups of coffee}$$

The economy is currently producing $205/2$ cups of coffee (C).

In the middle segment of the joint PPF, Wentao is devoted to making bagels (B) only and he is making 69 bagels (his maximum capability). Since Wenqi is devoted to making cups of coffee (C) only in this segment of the joint production possibility frontier, the remaining 31 bagels must have been made by Erika.

h. Here is the number line illustrating the range of trading prices for 1 cup of coffee (C) in terms of bagels (B) :

Trade takes place when price is above the production cost to the seller but below the production cost to the buyer.

