Economics 102
Spring 2017
Answers to Homework #1
Due February 9, 2017

Directions:

- The homework will be collected in a box **before** the lecture.
- Please place **your name**, **TA name** and **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time.
- **Show your work**. Good luck!

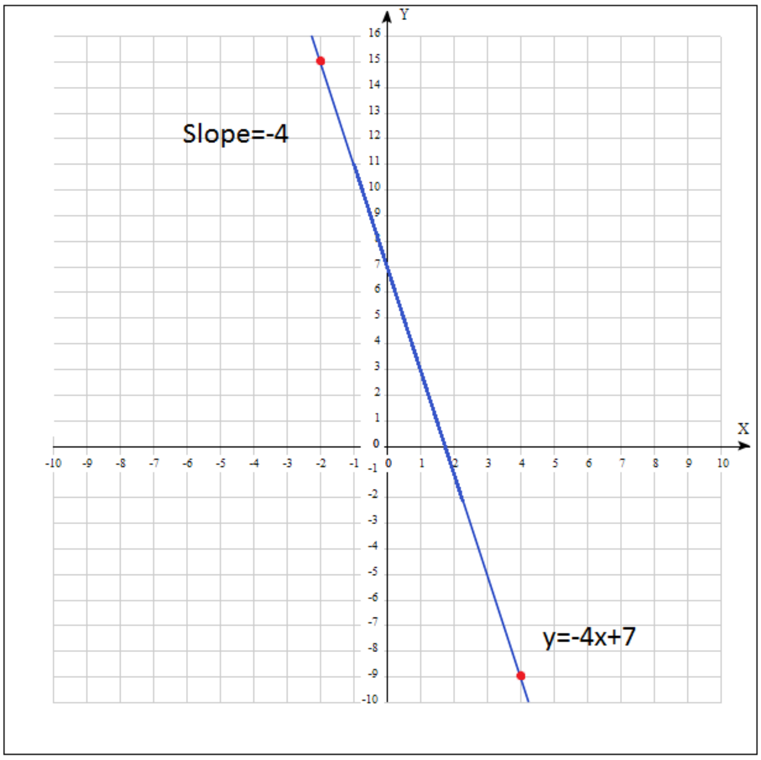
Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

<u>Part I: Math Review</u>

1. For this question use a linear equation that takes the form of y = ax + b.
   a) Write the equation of the straight line that passes through the points (x; y) = (4; -9) and (-2; 15). Graph this equation. What is the value of the slope of your equation? What is the value of the y-intercept?

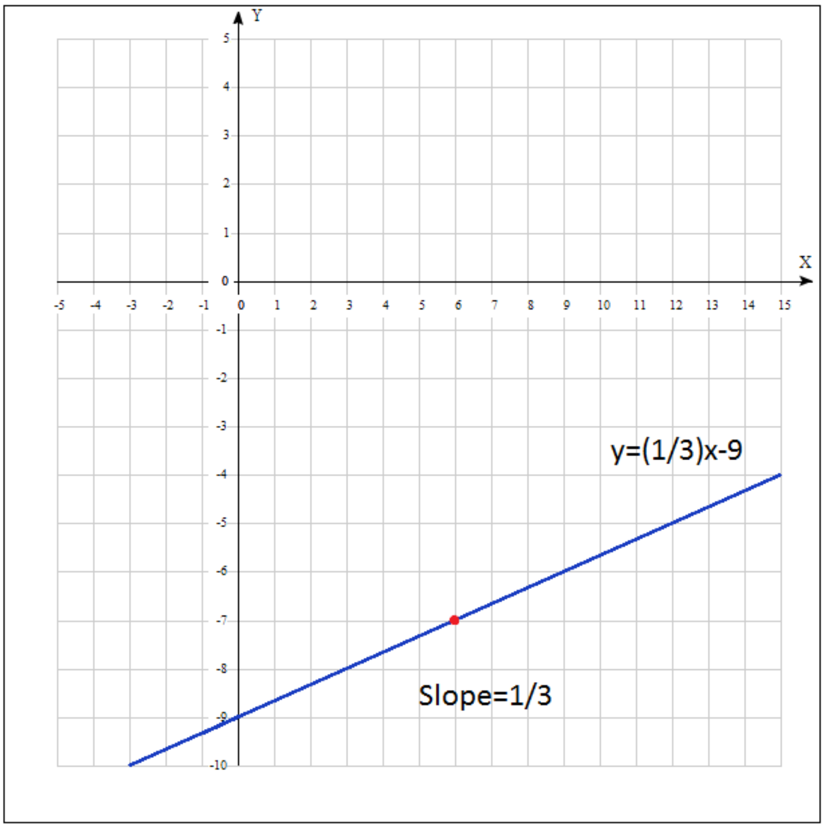
The line passes through (4; -9) and (-2; 15) and takes the form y = ax + b.
We substitute these points into y = ax + b and get -9 = 4a + b and 15 = -2a + b. Then we solve the system of two linear equations. From the first one we get b = -9 - 4a. Plug it in the second equation:
15 = -2a – 9 - 4a = -6a - 9. Hence, a = (-9 - 15)/6 = -4. Plug a = -4 back in to the equation b = -9 – 4a and get b = -9 - 4*(-4) = 16 – 9 = 7.

Alternatively, we could calculate "rise", Δy =15 - (-9) = 24, and "run", Δx = -2 – 4 = -6, in order to find the slope = a = Δy /Δx = 24/(-6) = -4. Then use one of the equations to find y-intercept, b: -9 = 4*(-4) + b. Thus, b = -9 + 16 = 7.
Overall, y = -4x + 7. The value of the slope is a = -4, the value of the y-intercept is b = 7.



b) Write the equation of the straight line that passes through the point (x; y) = (6; -7) and has slope equal to 1/3. Graph this equation. What is the value of the y-intercept?

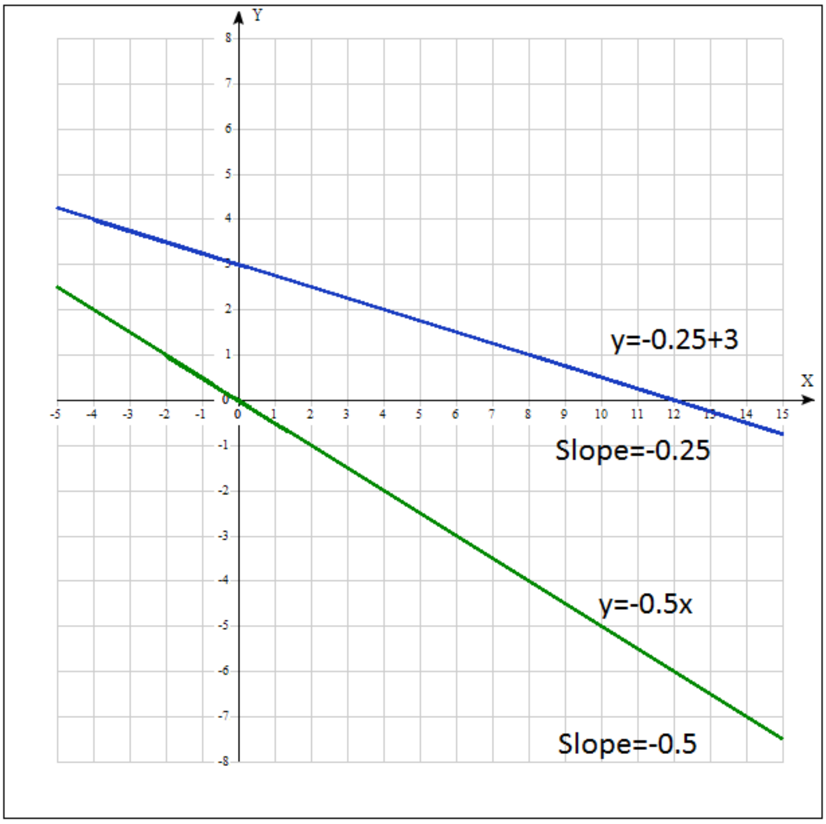
The slope is 1/3. Hence, a = 1/3. Then substitute the point (6; -7) and the slope a into the equation of the straight line: -7 = (1/3)*6 + b. Find b = -7 – 2 = -9.
The equation is y = 1/3 x - 9. The value of the y-intercept is b = -9.



c) Write the equation of the straight line, line #1, that fits the following description. If we take this straight line, line #1, and shift this line down by 3 units (that is, the y-intercept for line #2 is 3 units less than the y-intercept for the original line) and make this new line (line #2) twice as steep as line #1, then the new line #2 passes through the origin and for every 4 unit increase in the X variable, the Y variable decreases by 2 units. Graph the initial equation. What is the value of the slope of this initial equation (line #1)? What is the value of the y-intercept for line #1?

Let us find the equation for line #2. From the given information, we can calculate the slope of the line #2, a, as $\Delta y / \Delta x = -2/4 = -0.5$. Then the line passes through the origin, (0; 0). Hence, the y-intercept, b, is 0. The modified line is y = -0.5x.

Since we modified the initial equation by multiplying the slope by 2, in order to return from the equation for line #2 to the equation for line #1 we need to divide the slope by 2 and get a = -0.5/2 = -0.25. In addition, line #2's y-intercept is 3 units lower than line #1's y-intercept. Hence, line #1 should have a y-intercept 3 units greater that line #2, i.e. b = 0 + 3 = 3.
Thus, the equation for line #1 is y = -0.25x + 3. The value of the slope is a = -0.25, the value of the y-intercept is b = 3.



2. Consider the following equations:

Equation 1: y = -0.5x + 3
Equation 2: y = 1.5x - 5

a) Solve for the solution (x, y) for this system of equations. That is find the values of x and y where these two equations intersect with one another.

Set these two equations equal to each other: -0.5x + 3 = 1.5x - 5, so 2x = 8. Find x = 4. Plug x in one of the equations to find y: y = 1.5*4 – 5 = 1.
The answer is (x, y) = (4; 1).

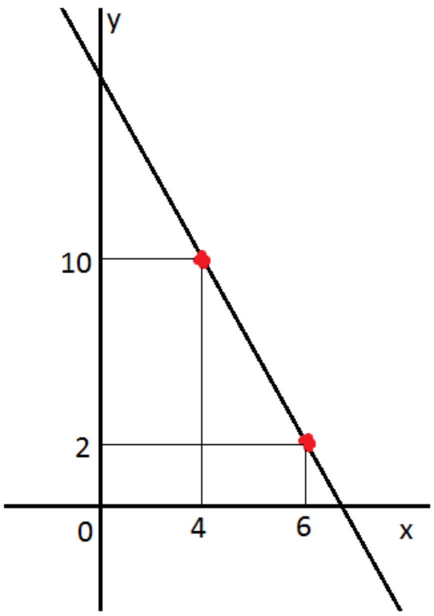
b) Now shift the Equation 2 upward by 10 units. Solve for the solution (x', y') where equation 1 and the new modified equation 2' intersect with one another.

When we shift Equation 2 up by 10 units, we get equation 2' as y' = 1.5x' – 5 + 10 = 1.5x' + 5.
Next, set these two equations equal to each other: -0.5x' + 3 = 1.5x' + 5, so 2x' = -2. Thus, x' = -1.
Plug x' in one of the equations to find y': y' = 1.5*(-1) + 5 = 3.5.
The answer is (x', y') = (-1; 3.5).

c) In addition to the shift of equation 2 described in (b) equation 1 has shifted to the right by 8 units. Using equation 2' and equation 1', find the solution (x", y") where equation 1' and equation 2' intersect with one another.

When we shift Equation 2 up by 10, we get equation 2' as y" = 1.5x" + 5.
When we shift Equation 1 to the right by 8, we get equation 1' as y" = -0.5(x" - 8) + 3 = -0.5x" + 7.
-0.5x" + 7 = 1.5x" + 5, so 2x" = 2. Thus, x" = 1. Plug x" in one of the equations to find y": y" = 1.5*1 + 5 = 6.5.
The answer is (x", y") = (1; 6.5).

3. Consider the following graph of the line.



a) Write the equation of this line. What is the value of the slope of your equation? What is the value of the y-intercept?

The slope = $\Delta y/\Delta x$ = (10 - 2)/(4 - 6) = 8/(-2) = -4.
Substitute one of the points and slope in y = ax + b. Let us use (4; 10). 10 = -4*4 + b, so b = 10 + 16 = 26 that is y-intercept.
The answer is y = -4x + 26.

b) Consider the area under the point (4; 10) as the area of the rectangle between the origin and the point (4; 10). This area is equal to 40. The area under the point (6; 2) is 12. What are the coordinates of the point under which the area is maximized if you consider only points located in the first quadrant?

Let's start with a simple solution. We know that the area is bigger at (x, y) = (4, 10) than (6, 2). We can also realize that at (x, y) = (26/4, 0), which is the x-intercept the area is equal to 0. So, logic would suggest that we at least test a couple of points that lie "higher than" (x, y) = (4, 10). So, what happens at (x, y) = (2, 18)? The area is 36. So, that suggest we are looking for a point on the line above (x, y) = (4, 10) and below the point (x, y) = (2, 18). So, try another point: (x, y) = (3, 14): the area here is 42…so continue this….and you should find that the point that maximizes the area is the midpoint of this line between the two intercepts (0, 26) and (26/4, 0) or the point (13/4, 13). Try this point out and see what the area is!

Here is a more advanced way to find the coordinates of the point under which the area is maximized. The area is equal to x*y = x(26 - 4x) = $-4x^2 + 26x$. It is a parabola with branches downwards. Hence, the maximum area is at the maximum of the parabola. The maximum of the parabola is calculated by using the formula: x = -b/(2a) where b is the coefficient of the x^2 term and a is the coefficient of the x term. So, x = -26/(2*(-4)) = 26/8 = 13/4, while y = 26-4*(13/4) =13.

The answer is (13/4; 13).

4. Consider Ashley who is taking a class at UW-Madison.

a) Ashley scored 82 out of 137 points for her first midterm. What is her score if that score is converted to a 100-point scale?

When we convert the point scale, the previous score ratio and the changed ratio should be the same. Let us denote her grade using 100-point scale as x. Then the following equation holds: 82/137 = x/100. Find x = 82/137*100 = 59.85.

b) Ashley scored 96 out of 137 points for her second midterm. What is the percentage change in her score from the first midterm to the second midterm?

The percentage change in value can be calculated as follows:
Percentage change in value = [(New Value – Initial Value)/(Initial Value)]*100% = [(96 – 82)/82]*100% = 17.07%.

c) For the course Ashley is taking the first midterm, second midterm and final are worth 25, 25 and 50 percent of the total grade, respectively. Each score is out of 137 possible points and you have already been given the information about her midterm grades. Ashley's total weighted score is 99.3 out of 137 points. What is her score for the final exam out of 137 points? If you were to measure her final exam score on a 100-point scale, what would be her grade? What is the percentage change in her score between the first midterm and her final if you use the 137-point scale? Will your answer change if you use the 100-point scale instead to calculate the percentage change in her score between the first midterm and her final? Explain your answer.

Let us denote her grade for final exam as y.
82*0.25 + 96*0.25 + y*0.5 = 99.3. Then, y = (99.3 - 82*0.25 - 96*0.25)/0.5 = 109.6 out of 137.

Let us denote her grade using the 100-point scale as z. Then the following equation holds: 109.6/137 = z/100. Find z = 109.6/137*100 = 80. The percentage change in her score between the first midterm and her final exam can be calculated as [(109.6 – 82)/(82)]*(100%) = [27.6/82]*100% = 33.6%. The percentage change in her score between the first midterm and her final exam using the 100 point scale is the same: [(80 – 59.85)/(59.85)]*100% = (20.15/59.85)*100 = 33.6%. The choice of scale does not impact the percentage change (you might get a slight difference due to rounding error).

Part II: Production Possibility Frontier

5. Consider the United States and Colombia. Suppose that both countries have one million workers who could either grow cut flowers or produce cars. One American worker can grow 1,200 cut flowers or produce 12 cars. One Colombian worker can grow 1,200 cut flowers or produce 4 cars.

   a) Given the above information, what is the opportunity cost for the U.S. to produce an additional million cars in terms of millions of cut flowers?

To produce an additional million cars the U.S. has to give up 1,200/12 = 100 million cut flowers. Therefore, the opportunity cost for the U.S. to produce one additional million cars is 100 million cut flowers.

   b) What is the opportunity cost for the U.S. to produce an additional million cut flowers in terms of million of cars?

To produce one more million of cut flowers the U.S. has to give up 12/1,200 = 1/100 = 0.01 million cars. Therefore, the opportunity cost for the U.S. to produce one additional million cut flowers is 0.01 million cars.

   c) What is the opportunity cost for Colombia to produce one additional million of cars in terms of million of cut flowers?

To produce an additional million cars Colombia has to give up 1,200/4 = 300 million cut flowers. Therefore, the opportunity cost for Colombia to produce an additional million cars is 300 million cut flowers.

   d) What is the opportunity cost for Colombia to produce one additional million of cut flowers in terms of million of cars?

To produce an additional million cut flowers Colombia has to give up 4/1,200 = 1/300 million of cars. Therefore, the opportunity cost for Colombia to produce one additional million cut flowers is 1/300 million of cars.

   e) Which country has the comparative advantage in growing cut flowers? Explain your answer.

Colombia has lower opportunity cost of producing cut flowers than the U.S. Hence, Colombia has the comparative advantage in growing cut flowers.

f) Which country has the comparative advantage in producing cars? Explain your answer.

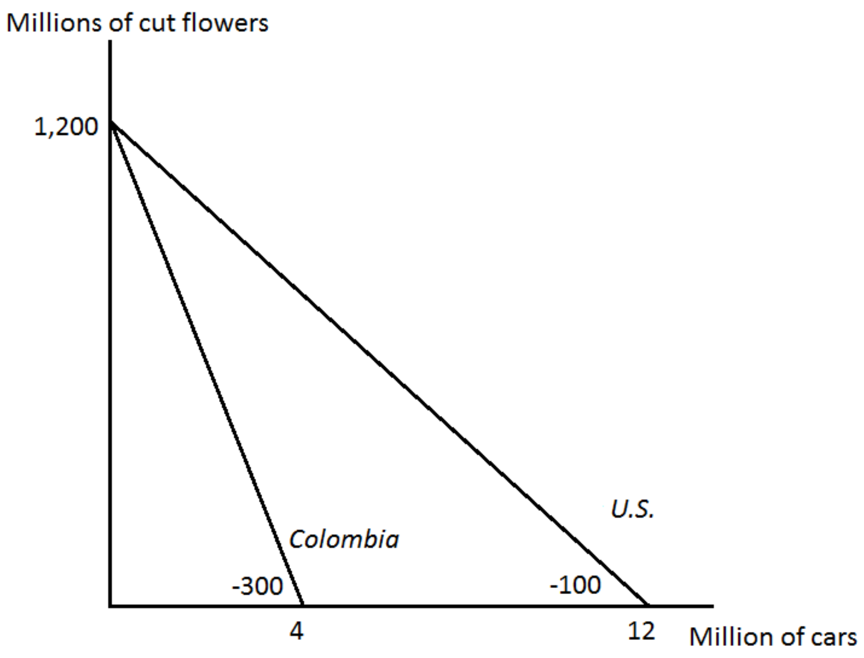
The U.S. has lower opportunity cost of producing cars than Colombia. Hence, the U.S. has the comparative advantage in producing cars.

g) Which country has the absolute advantage in producing cars?

Given the same resources, the U.S. is more productive in the manufacturing cars than Colombia. Therefore, the U.S. has an absolute advantage in producing cars. Note that given the same amount of resources, both countries could grow the same number of cut flowers. Thus, we could not make any conclusion about the absolute advantage in growing flowers.

h) Draw the Production Possibility Frontiers for the U.S. and Colombia. Label millions of cars on the x-axis and millions of cut flowers on the y-axis.

The U.S. could produce 1million*12 = 12 million of cars or 1million*1,200 = 1,200 million of cut flowers.
Colombia could produce 1million*4 = 4 million of cars or 1million*1,200 = 1,200 million of cut flowers.

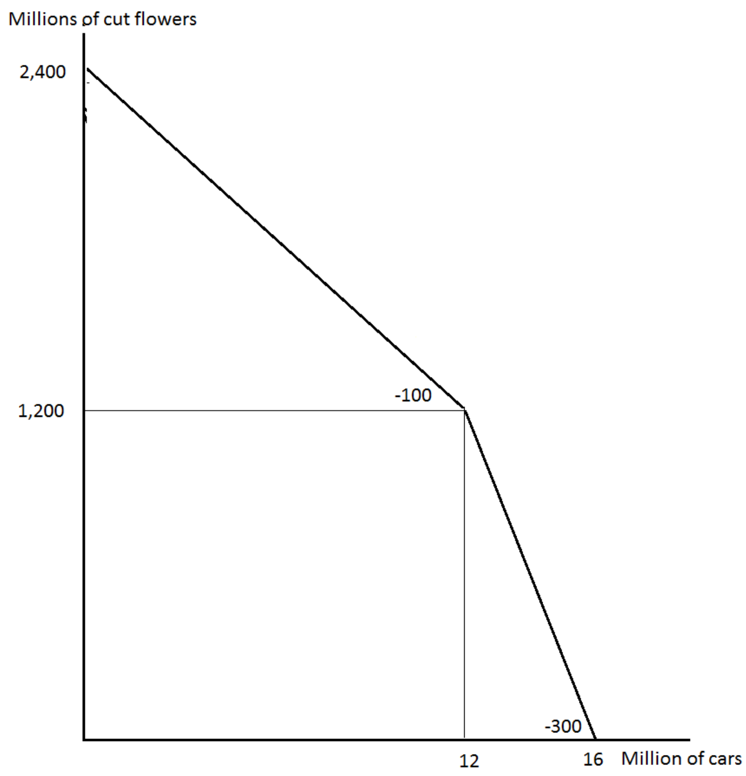


i) Now the U.S. and Colombia are engaged in trade with each other. Which country will export cut flowers? Which country will export cars?

When both countries specialize based on comparative advantage, Colombia will export cut flowers, while the U.S. will export cars.

j) Draw the joint PPF. Label millions of cars on the x-axis and millions of cut flowers on the y-axis. Write the equations for the joint PPF. Make sure you include the relevant ranges or domains for each equation you provide.

The maximum number of cars that can be produced by both countries is equal to 4 + 12 = 16 million cars. So the point (16; 0) is part of the joint PPF. The maximum number of cut flowers that can be produced by both countries is equal to 1,200 + 1,200 = 2,400 million cut flowers, so (0; 2,400) is also on the joint PPF. The kink point can be found where Colombia completely specializes in the production of one good, while the U.S. completely specializes in the production of the other good. Based on comparative advantage, Colombia will specialize in cut flowers and produce 1,200 million cut flowers, while the U.S. will specialize in the production of cars and produce 12 million cars. Therefore, the kink point is (12; 1,200).



Equation of the top segment of the joint PPF is F = 2,400 - 100C, where F is millions of cut flowers and C is millions of cars. This equation describes production when the countries produce from zero to 12 million cars.

Equation for the lower segment of the joint PPF takes form F = aC + b. We know that the slope, a, is equal to -300 and the line passes through (16; 0). Substitute known values in F = aC + b. We get 0 = -300*16 + b. Then b = 300*16 = 4,800. Hence, the equation is F = 4,800 - 300C. This equation describes production when the countries produce 12 million cars or more.

k) Assume that two countries produce 14 million cars. Which country produces the last million cars that are produced? How many million cut flowers will the two countries produce in all?

The first million cars are produced by the U.S, but the U.S. can produce maximum of 12 million cars. Hence, the last two million cars are produced by Colombia. Thus, Colombia produces the last million cars that are produced.

Since the U.S. completely specializes in the production of cars, it produces no cut flowers. If Colombia produces 2 additional million cars then it must give up producing 600 million cut flowers. Colombia can still produce 600 million cut flowers. Thus, both countries together produce a total of 600 million of cut flowers (all produced by Colombia) when they produce 14 million cars.

l) Illustrate the range of possible trading prices of 1 car in terms of cut flowers.

The U.S. has the comparative advantage in the production of cars. From the American perspective 1 car costs 100 cut flowers based on the opportunity cost producing a car in the United States. From the Colombian perspective 1 car costs 300 cut flowers based on the opportunity cost of producing a car in Colombia. Thus, for Colombia it is cheaper to buy cars from the U.S. than to produce cars domestically if the price of 1 car is lower than 300 cut flowers. The U.S. is happy to sell cars for any price greater than 100 cut flowers per car.
Overall, the range of possible prices of 1 car is from 100 to 300 cut flowers.