Economics 101
Fall 2013
Answers to Homework #6
Due Tuesday, Dec 10, 2013

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

1. **(Price Discrimination)**

Suppose Sundance Cinema is the only theater in Madison. The cinema owners know that there are three kinds of people in Madison: Students, Professors and Movie buffs. Their demand curves for movie areas follows:

Demand of Students: P = 10 – 2Qs
Demand of Professors: P = 20 – 2Qp
Demand of Movie Buffs: P = 30 – Qmb

The Total Cost for Sundance Cinema is given as TC =10 + 2Q and their Marginal Cost is given as MC = 2.
In this problem initially assume that Sundance Cinema can distinguish whether a person is a student, a professor or a movie buff.

a. Given the above information, at what price will they sell movie tickets to students?

Since their profit is maximized when MR = MC Sundance Cinema first needs to find the MR curve for the student market for movie tickets. We know that the demand of students is P = 10 – 2Qs and therefore the MR for students is given as MRs= 10 – 4Qs (remember for a linear demand curve, the MR will have the same y-intercept and twice the slope of the demand curve). Now set MRs = MC (i.e. 10 - 4Q = 2) to solve for Qs and Ps (the quantity of movie tickets students will buy and the price students will pay). Thus, Sundance Cinema will sell students Qs = 2 movie tickets and the price per movie ticket will be Ps = $6.

b. Given the above information, at what price will they sell movie tickets to professors? To movie buffs?

We follow the same procedure as we used in (a): thus, the MRp = 20 – 4Qp and the MRmb = 30 – 2Qmb. The MC for each segment of the market (professors and movie buffs is MC = 2). So, in the market selling to professors we have: MR = MC → 20 – 4Q = 2. They will sell at Qp = 4.5 and Pp = $11. In the market selling to movie buffs we have: MR = MC → 30 – 2Q = 2. They will sell at Qmb = 14 and Pmb = $16.

c. In this case, what is the profit of Sundance Cinema? (Hint: You should add the amount of tickets purchased by each group to find the total amount of movie tickets supplied.)

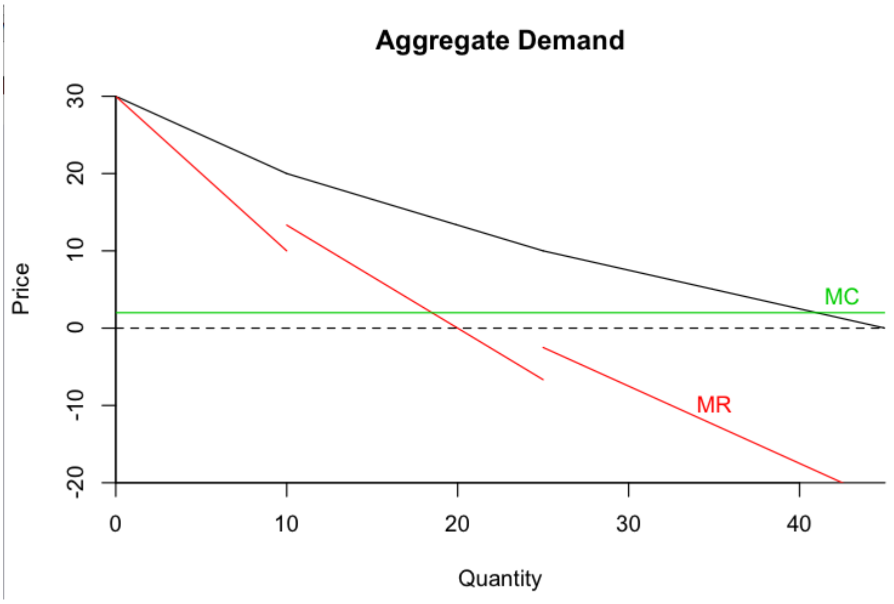
1) Revenue from movie tickets purchased by students : 2 X 6 = $12
2) Revenue from movie tickets purchased by professors : 4.5 X 11 = $49.5
3) Revenue from movie tickets purchased by movie buffs : 14 X 16 = $224

Total Revenue = 1) + 2) + 3) = $285.5.
Note that total amount of movie tickets supplied is the sum of 2 (amount of tickets purchased by students), 4.5 (amount of tickets purchased by professors) and 14 (amount of tickets purchased by movie buffs) or 20.5 movie tickets in all.
Total Cost = 10 + 2 (2 + 4.5 + 14) = $51.
Therefore, profit is 285.5 – 51 = $234.5.

d. Now, let's assume that the movie buffs really look like professors, and that Sundance Cinema cannot distinguish movie buffs from professors. In this case, they can set two prices: the normal price (for movie buffs and professors) and the student price. What will be the student discount (the gap between the normal price and the student price)?

The demand curve for professors and movie buffs is $P = 30 - Q$ (if $P > 20$), and $P = 80/3 - 2/3Q$ (if $P \leq 20$). To find this demand curve for professors and movie buffs you need to add the two demand curves together horizontally! This market demand curve for the professors and movie buffs will have a kink. That implies that there are two MR curves to consider: the MR for prices greater than $20 and the MR for prices less than or equal to $20. Thus, the MR curve in this case is $P = 30 - 2Q$ (if $Q < 10$), $P = 80/3 - 4/3Q$ (if $10 \leq Q$). To solve for the price and quantity this group (the professors and movie buffs) will have in this market we need to set MC = MR: in this case $2 = 80/3 - 4/3Q$. Then MR = MC when Q = 37/2 = 18.5 units and P = $43/3 = $14.33. Since there are no changes to the students' price and quantity, we have the price of a movie for a student is $6. Then, the student discount is therefore the difference between the price the professors and movie buffs pay and the price the students pay, or 43/3 – 6 = $25/3 = $8.33.

e. The mayor of Madison thinks price discrimination is socially inefficient so he prohibits it by law. Given this prohibition, will there be any change in the normal price of a movie ticket? With this prohibition on price discrimination, how many tickets will students buy? Is this policy more socially efficient then the result we got with price discrimination?



The market demand curve with the prohibition on price discrimination will now have two kink points: one occurring at a price of $20 per movie ticket and the other occurring at a price of $10 per movie ticket. The market demand is therefore $P = 30 - Q$ (if $P > 20$), P = 80/3 – 2/3Q (if 10 < P ≤ 20) and P = 22.5 – 1/2Q for P ≤ 10. Then MR curve in this case is P = 30 – 2Q (if Q > 10), P = 80/3 – 4/3Q (if 10 ≤ Q < 25), P = 45/2 – 1/2Q (if 25 ≤ Q). Then MR = MC when Q = 37/2 and P = 43/3 as d). Therefore, students will not buy any tickets (The ticket price is $43/3 which is greater than $10. Note that the student group will buy tickets when the price is less than $10.).

f. Are professors and movie buffs better or worse off under the policy described in (e) than the policy described in (d)? Is this policy prohibiting price discrimination more socially efficient than a policy that allows price discrimination?

Since professors and movie buffs face the same price in parts (d) and (e), they are indifferent between the two policies described in (d) and (e). However, this policy prohibiting price discrimination is worse than a policy of price discrimination for students, since the policy prohibiting price discrimination results in students going to zero movies since the price of movie tickets with the prohibition is now greater than $10 (remember students will not buy any tickets if the price is greater than or equal to $10). Sundance Cinema will lose their profits from tickets sold to students, while the profits from tickets sold to professors and movie buffs will remain the same given the two policies described in (d) and (e). A prohibition of price discrimination results in no one being better off and with both students and Sundance Cinema being worse off: therefore, this is a less efficient policy than one that allows price discrimination.

2. **(Common resources)**

Fisherwomen in Madison can choose between fishing in Lake Mendota or in Lake Monona. The total number of fish caught in each lake depends on the number of the fisherwomen who chose to fish in each lake.

Mendota

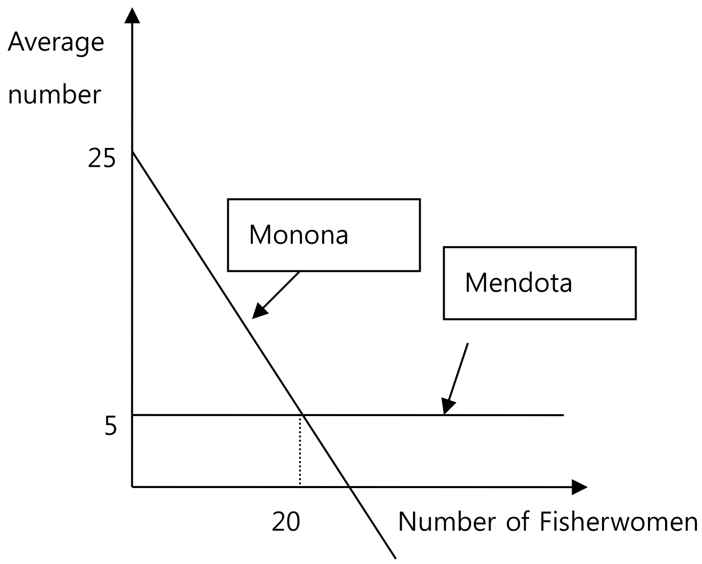
$A = 5L_A$, where A is the total number of fish caught in Lake Mendota if there were L_A fisherwomen who chose Mendota for fishing. For example, if 5 fisherwomen chose Mendota, then the total number of fish caught in Mendota is $A = 5 \cdot 5 = 25$. The marginal productivity of a fisherwoman fishing in Mendota is 5 $(MPL_A = 5)$, that is, each fisherwoman adds 5 fish to the total number of fish caught!

Monona

$B = 25L_B - L_B^2$, where B is the total number of fish caught in Lake Monona if there were L_B fisherwomen who chose Monona for fishing. For example, if 5 fisherwomen chose Monona, then the total number of fish caught in Monona is $B = 25 \cdot 5 - 5^2 = 100$. The marginal productivity of a fisherwoman fishing in Monona is $MPL_B = 25 - 2L_B$, Note that the marginal productivity of a fisherwoman fishing in Monona does depend on the number of fisherwomen fishing at this lake! For example, if $L_B = 1 \Rightarrow MPL_B = 25 - 2 = 23 \; but \; for \; L_B = 2 \Rightarrow MPL_B = 25 - 4 = 21$.

Finally suppose that there are **<u>50 fisherwomen</u>** in Madison, and fisherwomen in each lake divide equally the total number of fish caught in the same lake! For example, suppose that $A = 200 \; and \; L_A = 40, B = 150 \; and \; L_B = 10$, then each fisherwoman who chose Mendota will get $\frac{200}{40} = 5$ fish and those who chose Monona will get $\frac{150}{10} = 15$ fish.

a. Find the average number of fish caught in each lake? Draw them in the same graph (where you have the average number of fish on the Y axis and L (number of fisherwomen) on the X axis.



To find the average number of fish caught in each lake you need to divide the total number of fish caught in each lake by the number of fisherwomen at each lake. The total number of fish caught in Mendota is A and the total number of fish caught in Monona is B. The total number of fisherwomen at Mendota is L_A and the total number of fisherwomen at Monona is L_B. Thus,

Mendota: $\bar{A} = \frac{A}{L_A} = \frac{5L_A}{L_A} = 5$

Monona: $\bar{B} = \frac{B}{L_B} = \frac{25L_B - L_B^2}{L_B} = 25 - L_B$

b. Suppose one of the fisherwomen, Sia, chooses to fish in Mendota. Does her decision to fish in Mendota affect the number of fish that other fisherwomen who choose to fish in Mendota will catch? Suppose that Sia decides to fish in Monona. Does her decision to fish in Monona affect the number of fish that other fisherwomen who choose to fish in Monona will catch? Explain your answer.

Mendota: Sia's choice won't affect the number of fish caught by other fisherwomen fishing in Mendota since the average number of fish caught in Mendota is constant and equal to 5 fish.
Monona: Sia's choice will affect the number of fish caught by other fisherwomen fishing in Monona since the average number of fish caught in Monona depends on the number of fisherwomen fishing in the lake! For example, if there was only one fisherwoman who chose Monona, then she would catch an average 25 - 1 = 24 fish, but if Sia decides to do the fishing also in Monona then the two fisherwomen would each catch an average of 25 – 2 = 23 fish.

c. If each fisherwoman is free to choose between the two lakes, how many will choose Mendota? How many will choose Monona?

You can see from the graph you drew in (a) that the first 20 fisherwomen will choose fish in Monona because they can get more fish than they can get if they choose Mendota! Eventually the decision of these 20 fisherwomen decrease the average number of fish caught in Monona to 5 fish!
The answer would be $L_A = 30 \; and \; L_B = 20$
Analytically, we can find $L_A = 30 \; and \; L_B = 20$ by setting

$$\bar{A} = \bar{B}$$
$$5 = 25 - L_B \Rightarrow L_B = 20$$
And we know that the total number of fisherwomen is 50, so $L_A = 30$.

d. What is the total number of fish caught in each lake? What is the total number of fish caught in Madison? Do you think this is a socially optimal amount of fish to catch in Madison? Why or why not? In other words is that the maximum number of fish they can catch (in both lakes)? Compare the answer you found in (c) to what you would find if the number of fisherwomen in Mendota and Monona were respectively
$$L_A = 35 \;, \qquad L_B = 15$$

A=150 fish caught in Mendota
B=100 fish caught in Monona
Therefore there are a total of A+B=250 fish caught in Madison. This is not the socially optimal amount of fish to catch since none of the fisherwomen took into account how her choice affected the other fisherwomen. To see this consider as an example that 5 fisherwomen change their mind and instead of doing their fishing in Monona they move to Mendota. Let's see what happens to the number of fish caught in Madison with this change in the distribution of the fisherwomen:
$$L_A = 35 \;, \qquad L_B = 15$$
$$\Rightarrow A = 5 \cdot 35 = 175$$
$$y = 25 \cdot 15 - 15^2 = 150$$
So the total number of fish caught in both lakes is 150+175=325 fish!

e. Now suppose that the fisherwomen in Madison have decided in their yearly meeting that they want to be more efficient (increase the number of fish caught in both lakes), and that each fisherwoman will now get the average of the total number of fish caught in both lakes. How can they achieve this stated outcome? That is, how many fisherwomen will need to fish in each lake? What is the total number of fish caught now? How much fish will each fisherwomen catch on average with this policy change?

We can find that by setting the marginal productivity of a fisherwoman in Lake Mendota equal to the marginal productivity of a fisherwoman in Lake Monona, or
$$MPL_A = MPL_B$$
$$\Rightarrow 5 = 25 - 2L_B \Rightarrow L_B = 10$$
$$L_A = 40$$
Then:
$$A = 5 \cdot 40 = 200$$
$$B = 25 \cdot 10 - 10^2 = 150$$
A+B=200+150=350 fish caught in the two lakes.

f. Suppose now that the fisherwomen canceled their agreement in (e) and they are back to their previous arrangement where each fisherwoman makes her fishing choice individually and therefore gets the average number of fish caught in the lake she has chosen. The municipality is not satisfied with their decision and has decided to interfere. The municipality has decided that now fisherwomen who want to fish in Lake Monona must pay a fee of Z fish. Suppose the municipality wants to set the fee so that when each fisherwoman makes her fishing decision the overall outcome is the socially optimal allocation they achieved in (e). What level should the fee be set at in order to achieve this goal?

Again, the number of fisherwomen in each lake will be determined by the average number of fish in each lake but now we have to take into account that fisherwomen who choose to fish in Monona are going to get the average number of fish caught in Monona minus the fee! So, we need to set the following equation:

$$A = B\text{-}Z$$

$$5 = 25 - L_B - Z$$

Note that in the socially optimal equilibrium we have $L_B = 10$, so to find the fee that leads to that, we just need to plug $L_B = 10$ in the this equation.

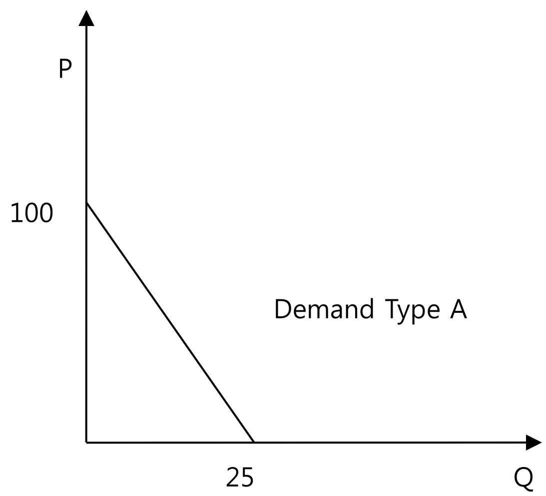
$$5 = 25 - 10 - Z$$

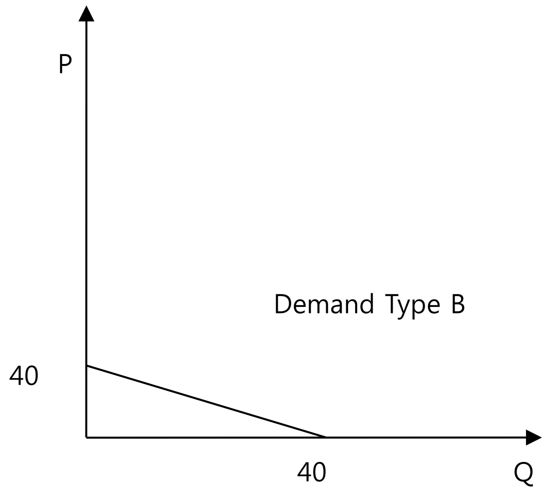
$$\Rightarrow Z = \$10$$

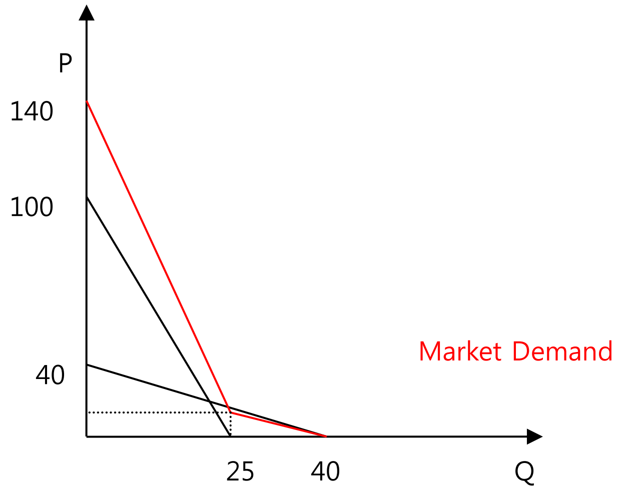
3. **(Public Good)**

The municipality of Madison is considering an increase in the number of bus stations in the city, but its tight budget doesn't allow Madison to simply erect another bus station. Suppose that the people of Madison are generally willing to cover the cost of this project but that within the population of Madison there are two types of people with regard to their degree of support for additional bus stations: Group A and Group B. both types support the idea and they are willing to help, but their demands for "new bus stations" are different:
Demand of Group A is $P^A = 100 - 4Q$
Demand of Group B is $P^B = 40 - Q$
where P is the price of people are willing to pay for a new bus station and Q is the number of bus stations demanded. Suppose that the marginal cost of construction of new bus stations is constant and equal to 20 (MC = 20).

a. In three separate graphs vertically oriented to one another draw the demand curve of Group A (in the top graph), the demand curve for Group B (in the middle graph) and the market demand in the bottom graph. Label each graph carefully and completely. Provide an equation(s) for the market demand curve as well as the range or domain for each portion of the market demand curve.







Market demand (sum vertically) : $P = 140 - 5Q \; for \; Q \leq 25$
$P = 40 - Q \; for \; Q > 25$

b. Suppose that no one in Madison acts as a free rider in this market. Given this assumption what will be the number of new bus stations in Madison? How much will each Group contribute to the construction of each bus station? Show how you found your answer.

To answer this question we need to find the intersection of MC and the market demand:

$$MC = P$$
$$20 = 140 - 5Q$$
$$\Rightarrow Q = 24 \text{ new bus stations}$$

$Plug\ Q = 24\ in\ the\ market\ demand\ to\ find\ the\ price$:
$P = 140 - 5 \cdot 24 \;\Rightarrow P = \$20$

To find how much each Group of demanders pays we go back to the demand of each group and find how much they are willing to pay for this number of bus stations:
Group A: $P^A = 100 - 4Q = 100 - 4 \cdot 24 \Rightarrow P^A = \$4$
Group B: $P^B = 40 - Q = 40 - 24 \Rightarrow P^B = \$16$
Notice that together the two groups contribute a total of \$20 per bus station which is exactly the price that must be paid per bus station.

c. How will your answer to (b) change if the marginal cost is $150? Or $5?

- MC = $150 : You can see from the market demand that the quantity demanded is zero for any price above $140! So, for this level of MC, we won't have any new bus stations!
- MC = $5: If you add the new MC to the graph you can see that the intersection between the MC and the market demand is in the "second part" of the market demand curve. But let's say you missed that and you tried to do the same as you did in part (b):

$$MC = P$$
$$5 = 140 - 5Q \Rightarrow Q = 27$$

Which can't be the solution since the market demand is equal to 140-Q only for quantities that are below 25!

Therefore we need to find the intersection between MC and the "second part" of the market demand:

$$MC = P$$
$$5 = 40 - Q \Rightarrow Q = 35$$

$Plug\ Q = 35\ in\ the\ market\ demand\ (second\ part)\ to\ find\ the\ price:$

$P = 40 - 35\ \Rightarrow P = \$5$

To find how much each group pays we go back to the demand for each group and find how much they are willing to pay for this number of bus stations:

Type A: $P^A = 100 - 4Q = 100 - 4 \cdot 35 \Rightarrow P^A < 0 \Rightarrow P^A = \$0$

Type B: $P^B = 40 - Q = 40 - 35 \Rightarrow P^B = \$5$

Which means that even though both groups were willing to pay for the new bus stations, Group A won't pay anything since they are not interested in having that many new bus stations! (The maximum number of new bus stations that Group A wants is 25)

d. Suppose now that the marginal cost has changed to a new, but constant level. We also know that when the market for new bus stations is in equilibrium the two groups pays the same price. Given this information, how many new bus stations will be constructed in Madison? How much will each group contribute for the construction of each new bus station? What is the implied marginal cost of new bus stations?

We need to find out what number of bus stations both groups are willing to pay the same price for: to do this, we just need to find the intersection of the two demand curves where $P^A\ equals\ P^B$:

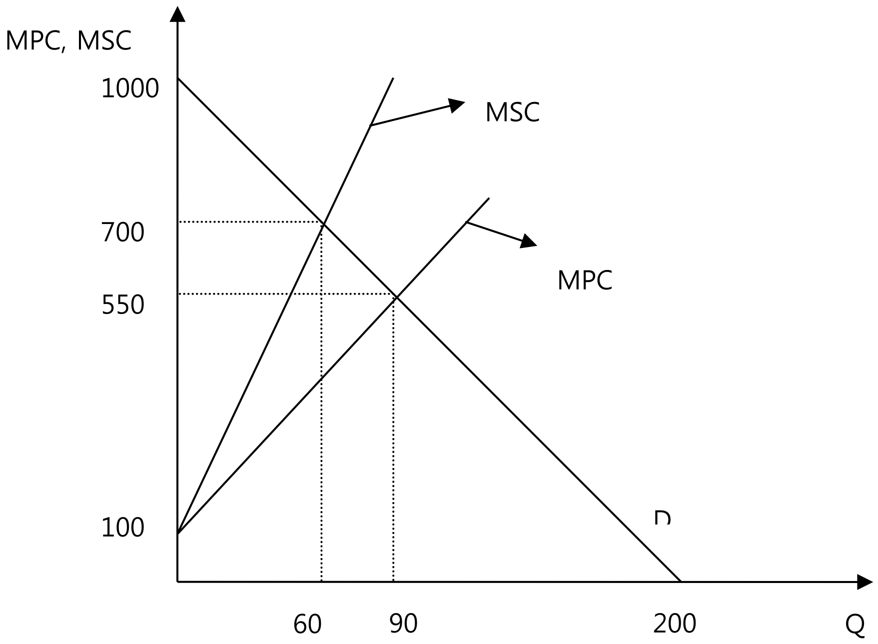
$$P^A = P^B$$
$$100 - 4Q = 40 - Q$$
$$\Rightarrow Q = 20$$

If we plug Q = 20 back into the demand curves, we get that each group is willing to pay $20! And the implied marginal cost must be such that the following is true: MC=$P^A + P^B = \$40$!

4. **(Externalities)**

Suppose that the demand for paper is given by the equation $P = 1000 - 5Q$ where P is the price per unit of paper and Q is the quantity of paper. Furthermore, suppose the private marginal cost of producing paper is 100+5Q (MPC=100+5Q). We also know that producing paper has an environmental cost of 5Q (paper production requires cutting down trees and this affects the overall environment)!

a. Draw a graph depicting the market demand for paper and the marginal private cost of producing paper. Label this graph clearly and completely.



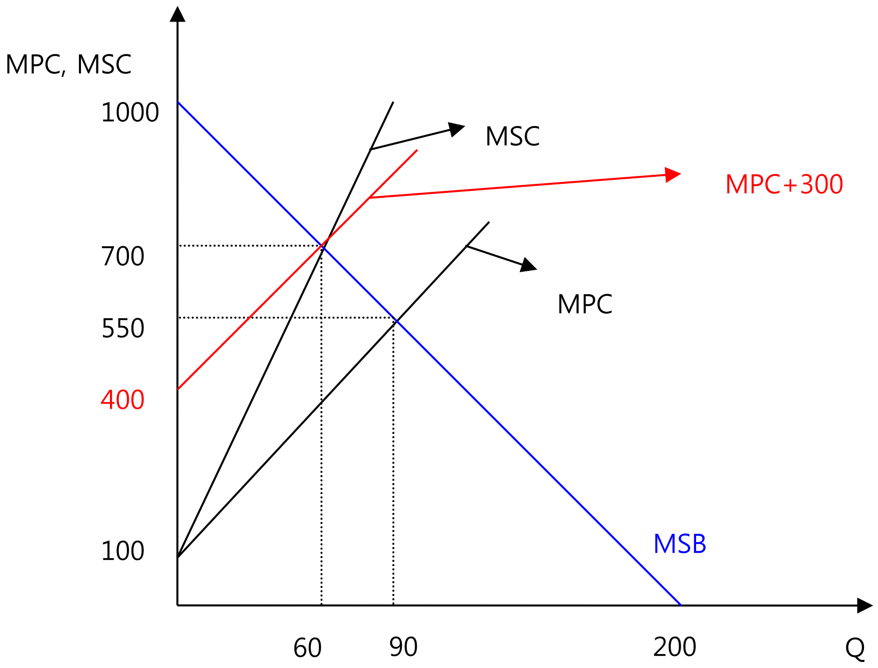
b. What is the output and the price of paper if the market for paper is unregulated and the environmental cost of paper production is not accounted for in the market? Is this the socially optimal amount of paper? Explain your answer.

In the unregulated paper market we can expect that the market will produce that quantity where the marginal private cost equals the marginal private benefit. In this problem we can also assume that the marginal private benefit is equal to the marginal social benefit (there are no externalities on the demand side of the market) and hence the marginal private benefit is just equal to the price of the good. Thus,

$$MPC = P$$
$$100 + 5Q = 1000 - 5Q$$
$$\Rightarrow Q = 90$$
$$\Rightarrow P = \$550$$

That is, the market if unregulated will produce 90 units of paper at a price of $550 per unit of paper.

c. What is the marginal social cost (MSC) of paper production? Draw the MSC of paper in the graph you provided in (a). Make sure you label this clearly and completely.



MSC is equal to the private marginal cost of producing paper plus the environmental cost of producing paper

$$MSC = MPC + 5Q = 100 + 10Q$$

d. What is the socially optimal output and price of paper given the information you have?

To find the socially optimal amount of paper we need to find where the MSC intersects the MSB. In this example remember that the MSB = MPB = P. So we can find the socially optimal output and price of paper by setting MSC = P:

$$MSC = P$$
$$100 + 10Q = 1000 - 5Q$$
$$\Rightarrow Q = 60$$
$$\Rightarrow P = \$700$$

That is, the socially optimal amount of output is 60 units of paper with a price of $700 per unit of paper.

e. Now suppose that the government wants this market to produce the socially optimal amount of output. What is the excise tax (T) or subsidy (S) per unit the government should impose on the producers of paper in order to achieve this socially optimal level of production?

The government should impose an excise tax (T) and not a subsidy (S)! The excise tax will force the firm to produce a smaller quantity than the quantity the firm produces in an unregulated market! Let's find the excise tax by setting the new private marginal cost (MC+T) equal to the demand:

$$MC + T = P$$
$$100 + 5Q + T = 1000 - 5Q$$

To find (T) that will force the firm to produce Q = 60, we can plug Q = 60 into the above equality:

$$100 + 5 \cdot 60 + T = 1000 - 5 \cdot 60$$
$$\Rightarrow T = \$300$$

Intuitively this should make sense to you: the socially optimal amount of the good is Q = 60, but the externality cost per unit of producing 60 units is given as 5Q or $300. By imposing an excise tax of $300 per unit you can push producers to produce the socially optimal amount of the good.

f. In a graph illustrate the MPC, the MSC, the MSB and the excise tax or subsidy you proposed in (e). In addition identify the market outcome as well as the socially optimal outcome in this market. Label your graph clearly and completely.

You can check the answer in the graph in part (d).

5. **(Game Theory)**

Professor Kelly has 100 chocolates that she plans to distribute in a game to be played by Moheb and Wooyoung. The game rules are as follows: Moheb and Wooyoung must select a number between 2 and 100: the person who selects the lower number will get the same number of chocolates as his number and the other person will get no chocolates. If Moheb and Wooyoung pick the same number then each of them will get half of that number of chocolates.

a. If Moheb and Wooyoung agree to cooperate in the selection of the numbers they choose, what is the maximum number of chocolates they can together get?

If both Moheb and Wooyoung select 100 as their number, then Professor Kelly will give each of them 50 chocolates. Then they will get 100 chocolates in all.

b. If Moheb and Wooyoung do not cooperate, and Moheb selects 100 as his number, will Wooyoung select 100 or 99 as his number? Is selecting 100 the dominant strategy for Wooyoung?

If Wooyoung selects 100 as his number, he will get 50 chocolates. However, if he selects 99 as his number, he will get 99 chocolates. Therefore, he will choose to select 99 as his number: selecting 100 as his number is not the dominant strategy for Wooyoung.

c. If Wooyoung selects a number *I* that is greater than 2, what is the best strategy for Moheb given that Moheb and Wooyoung are not cooperating?

Since Moheb and Wooyoung are assumed to not be cooperating, Moheb can only win chocolates if he selects a number that is lower than *I*. His optimal number selection that will yield Moheb the most chocolates is *I-1*.

d. Suppose you know that either Moheb or Wooyoung plan to select 2 as their number. Is it possible that the other individual will have a dominant strategy of selecting a number greater than 2? Explain your answer.

Given that your opponent selects 2 as their number then it is impossible that a strategy of selecting a number greater than 2 could be your dominant strategy. If your opponent selects 2 as their number, then selecting a number greater than 2 will give you nothing.

e. What will happen in this game if Moheb and Wooyoung do not cooperate? What will be the outcome?

Each of them will select 2 as their number since each player knows if he selects a number greater than 2 he may get nothing. Both of them will get 1 chocolate only. This is obviously not nearly as good an outcome as they can get if they can successfully cooperate: by cooperating they can get a total of 99 chocolates and if they share them evenly that amounts to 49.5 chocolates for each of them!

f. Based on previous results, complete the table below. Will they cooperate or not?

		Moheb	
		Cooperate	Do not cooperate
Wooyoung	Cooperate	(,)	(0 chocolates, 99 chocolates)
	Do not cooperate	(,)	(,)

		Moheb	
		Cooperate	Do not cooperate
Wooyoung	Cooperate	(50 chocolates, 50 chocolates)	(0 chocolates, 99 chocolates)
	Do not cooperate	(99 chocolates, 0 chocolates)	(1 chocolate,1 chocolate)

Since “Do not cooperate” is dominant strategy for both of them, they will not cooperate.