

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. Suppose that Joe and Sue are playing tennis against one another and that both of these individuals are trying to decide on what their optimal strategies should be. Joe has a great topspin forehand and a strong approach shot (this is the shot a player makes as they come to the net) while Sue has a great backhand and a wicked drop shot (this is a shot where the ball barely cross the net and then hardly bounces off the surface of the court). Suppose that when Sue hits her backhand and Joe hits his forehand that Sue tends to win one point while Joe tends to lose one point. When Sue hits her backhand and Joe hits approach shot Sue tends to win the next two points while Joe loses the next two points. When Sue elects to play her drop shot and Joe adheres to his strong forehand Sue finds that she loses a point while Joe wins a point. And, finally, when Sue plays her drop shot and Joe plays his approach shot Sue tends to win a point for every two points that Joe wins (after all, with that approach shot Joe is closer to the net and has a better chance of getting to that drop shot).

a) Given the above information fill in the following payoff matrix where each entry indicates the number of points won (a positive numeric value) or points lost (a negative numeric value).

		Joe's Strategies	
		Forehand	Approach Shot
Sue's Strategies	Backhand		
	Drop shot		

b) Examine the payoff matrix you created in (a). Does Sue have a dominant strategy? Explain your answer.

c) Examine the payoff matrix you created in (a). Does Joe have a dominant strategy? Explain your answer.

d) Suppose Sue follows her dominant strategy, can you predict what Joe will do given the above information? Explain your answer.

e) You plan to watch Sue and Joe play each other this weekend. Describe the match you anticipate seeing. Who do you predict will win?

Answers:

a)

		Joe's Strategies	
		Forehand	Approach Shot
Sue's Strategies	Backhand	1; -1	2; -2
	Drop shot	-1; 1	1; 2

b) Sue has a dominant strategy of playing her backhand. To see this think about Sue looking at this table as two separate columns: a column where Joe plays “forehand” and a column where Joe plays “approach shot”. If Joe plays “forehand” Sue will win one point using her backhand strategy and lose one point playing her drop shot strategy: it is clearly in Sue’s best interest to play “backhand” if Joe plays “forehand”. If Joe plays “approach shot” Sue will win two points using her backhand strategy and win one point playing her drop shot strategy: it is clearly in Sue’s best interest to play “backhand” if Joe plays “approach shot”. So, Sue has a dominant strategy of “backhand” since this is her best strategy no matter what strategy Joe chooses.

c) Joe has no dominant strategy. To see this think about Joe looking at this table as two separate rows: a row where Sue plays “backhand” and a row here Sue plays “drop shot”. If Sue plays “backhand” Joe will lose one point using his forehand strategy and lose two points playing his approach shot strategy: it is clearly in Joe’s best interest to play “forehand” if Sue plays “backhand”. If Sue plays “drop shot” Joe will win one point if he plays forehand and he will win two points if he plays approach shot: it is clearly in Joe’s best interest to play “approach shot” if Sue plays “drop shot”. So, Joe has no dominant strategy since his best strategy changes when Sue changes her strategy.

d) If Sue follows her dominant strategy of “backhand” then the payoff matrix shows us that Joe is better off when he pursues his “forehand” strategy: he’s still losing points, but he’s losing fewer points than he would if he played his “approach shot” strategy.

e) At the match you can anticipate that Sue will be hitting a lot of backhands and that Joe will be hitting a lot of forehands. And, you will see Sue win the match.

2. Consider two firms in an industry consisting solely of these two firms. Microfibers and Big Fibers produce identical products. Both firms are trying to decide whether they want to advertise or not advertise. They know that when they advertise the other firm benefits from this advertisement since they are selling identical products.

Microfibers knows that if Big Fibers advertises then Microfibers will earn a profit of \$100,000 for the year if it also advertises and a profit of \$80,000 for the year if it does not advertise. Microfibers knows that if Big Fibers does not advertise then Microfibers will earn a profit of \$120,000 for the year if it advertises and a profit of \$50,000 for the year if it does not advertise.

Big Fibers knows that if Microfibers advertises then Big Fibers will earn a profit of \$80,000 for the year if it also advertises and a profit of \$60,000 for the year if it does not advertise. Big Fibers knows that if Microfibers does not advertise then Big Fibers will earn a profit of \$100,000 for the year if it advertises and a profit of \$60,000 for the year if it does not advertise.

a) Given the above information, construct a payoff matrix for this situation. Put Microfibers on the left hand side of the payoff matrix and Big Fibers on the top of the matrix. Make sure your payoff matrix identifies the strategies that each firm faces as well as the payoff from each combination of strategies.

b) Identify if these two firms have the dominant strategies and, if so, what these dominant strategies are. Explain your answer.

c) Given the above information, can you predict what these two firms will do? Explain your answer.

Answers:

a)

		Big Fibers	
		Advertise	Do not Advertise
Microfibers	Advertise	\$100,000; \$80,000	\$120,000; \$60,000
	Do not Advertise	\$80,000; \$100,000	\$50,000; \$60,000

b) Microfibers has a dominant strategy of “advertise”: no matter what Big Fibers does, Microfibers is better off “advertising”. Big Fibers has a dominant strategy of “advertise”: no matter what Microfibers does, Big Fibers is better off “advertising”.

c) Both firms will pursue their dominant strategies and both firms will therefore advertise.

3. Suppose there are two firms in a market and these two firms agree to form a cartel and divide up the market evenly. The two firms know the following:

Market Demand for the Product: $P = 100 - Q$

Marginal Cost of producing the good: $MC = 0$

Fixed Cost of production: $FC = 0$

a) What is the profit maximizing quantity and price for the cartel? Explain your answer and provide a graph of this market to illustrate your answer.

- b) What are the industry profits given your answer in (a)? Show your work.
- c) What is the level of production for each firm if both firms adhere to the cartel agreement? What are the profits for each firm? Explain your answer.
- d) Suppose one of the firms decides to cheat on the cartel agreement and sell the product for \$48 per unit. How many units can this firm sell at this price and what will be its profits when it follows this pricing strategy? Assume that the other firm does not drop its price and consumers know all prices, so the other firm sells zero units.
- e) Suppose that one of the firms drops its price as described in (d), but now the other firm matches this price decrease. If the two firms continue to split the market evenly, what will the profit for each firm equal now that both firms are selling the good for \$48 per unit? Explain your answer.
- f) Make a payoff matrix for these two firms with each firm having a choice of charging the profit maximizing price (see (a)) or the “cheating on cartel” price of \$48. Put Firm A on the left hand side of the payoff matrix and Firm B at the top of the matrix.
- g) Does each firm have a dominant strategy? Explain your answer.
- h) What do you predict will be the outcome of this game? Explain your answer.
- i) If you apply the above logic many times to successively lower prices (e.g. \$46, \$44, ...), what will the price eventually be?

Answers:

a) The cartel will maximize its total profits by producing that quantity where $MR = MC$ and then pricing this quantity off the demand curve. So, we first need to find the MR curve for the cartel: $MR = 100 - 2Q$. We can get this equation by remembering that for a downward sloping linear demand curve the MR curve shares the same y-intercept and has twice the slope as the demand curve. Then, set MR equal to MC:

$$100 - 2Q = 0$$

$$2Q = 100$$

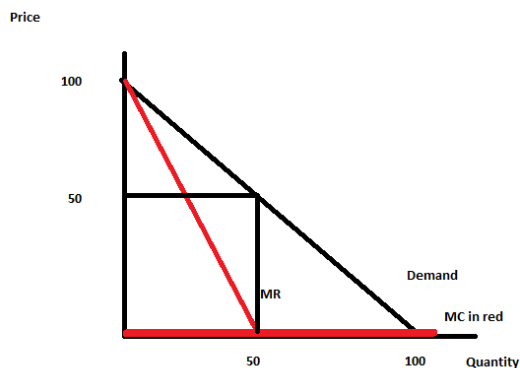
$$Q = 50 \text{ units}$$

So the cartel should produce a total of 50 units: one firm will produce 25 units and the other firm will produce another 25 units since they have agreed to split the market. To find the profit maximizing price, plug this quantity into the demand curve:

$$P = 100 - Q$$

$$P = 100 - 50 = \$50 \text{ per unit}$$

Here's a graph to illustrate this answer:



b) Profits = TR – TC

$$TR = P \cdot Q = (\$50 \text{ per unit})(50 \text{ units}) = \$2500$$

$$TC = ATC \cdot Q = MC \cdot Q \text{ since there are no FC}$$

$$TC = (\$0 \text{ per unit})(50 \text{ units}) = \$0$$

$$\text{Profits} = \$2500$$

c) Each firm will produce half of the total amount produced: so each firm will produce 25 units of the good. Since each firm is selling 25 units at a price of \$50 per unit, each firm will earn profits of \$1250 or half of the total profits earned by the cartel.

d) If the price is \$48 per unit, then 52 units of the good will be demanded (use the demand curve and the price of \$48 per unit to find this quantity). So, the cheating firm will be the only firm selling in this market: it will sell 52 units at a price of \$48 per unit. The cheating firm's profit will therefore be equal to $(\$48 \text{ per unit})(52 \text{ units}) - (\$0 \text{ per unit})(52 \text{ units}) = \2496 .

e) If both firms drop their price to \$48 per unit and split the market, then each firm will sell 26 units (half of the total of 52 units demanded at this price). Each firm will earn profit equal to $(\$48 \text{ per unit})(26 \text{ units}) - (\$0 \text{ per unit})(26 \text{ units}) = \1248 (or half of the \$2496).

f)

		firm B	
		P = \$50	P = \$48
Firm A	P = \$50	\$1250; \$1250	\$0; \$2496
	P = \$48	\$2496; \$0	\$1248; \$1248

g) Yes, each firm has a dominant strategy of cheating on the cartel price and charging \$48 per unit.

We can see this from Firm A's perspective by looking at the payoff matrix as two columns: if Firm B holds to a price of \$50, Firm A will earn more profit by charging a price of \$48 (\$1250 versus \$2496); if Firm B goes to a price of \$48, Firm A will earn more profit by charging a price of \$48 (\$0 versus \$1248). No matter what Firm B does, Firm A is better off charging a price of \$48.

We can see this from Firm B's perspective by looking at the payoff matrix as two rows: if Firm A holds to a price of \$50, Firm B will earn more profit by charging a price of \$48 (\$1250 versus \$2496); if Firm A goes to a price of \$48, Firm B will earn more profit by charging a price of \$48 (\$0 versus \$1248). No matter what Firm A does, Firm B is better off charging a price of \$48.

h) Firm A and Firm B will both pursue their dominant strategies and charge a price of \$48. They will make \$1248 in profit at each firm which is less than they would make if they would cooperate and adhere to the cartel agreement.

i) The same logic works for any price: if both firms charge a price above zero, charging a positive price below the current price dominates charging the current price, regardless of what the other firm does. If we apply this logic many times we see that the price must approach zero! This is known as the "Bertrand paradox": if firms are competing by choosing prices, even two firms are sufficient to drive price down to marginal cost. You can learn more about this if you take a later course in Industrial Organization.

4. Joe, Mary, and Louise live in the same community (they are the only residents) and they are debating installing some street lamps. Thankfully each of these individuals is willing to reveal their preferences and demand for street lamps, but the community is still trying to decide how many street lamps they should buy. Here is the relevant information that they have gathered:

Joe's demand for street lamps: $Q = 20 - 2P$

Mary's demand for street lamps: $Q = 20 - 4P$

Louise's demand for street lamps: $Q = 10 - P$

Marginal social cost of a street lamp: $MSC = \$3$

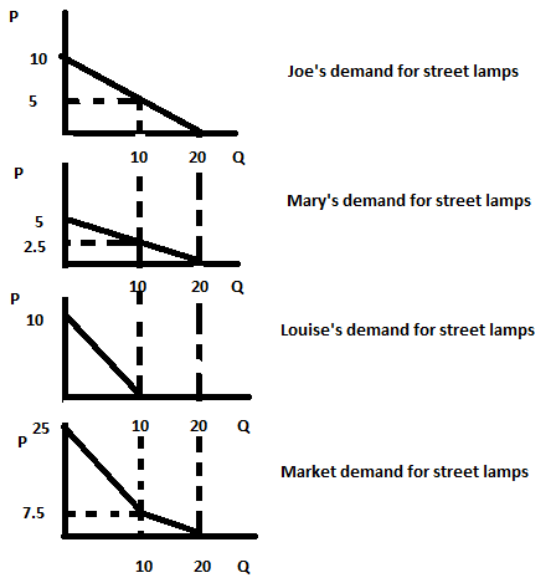
a) Given the above information draw an illustration of these three demand curves plus the market demand curve for street lamps. In your illustration provide four different graphs that are vertically stacked with the market demand curve the bottom graph in the stack. Make sure all your graphs are clearly and completely labeled. Describe verbally how you found the market demand curve.

b) Write the equation(s) for the market demand curve and provide a range or domain for any segments of the demand curve. Show how you found these equations.

c) What is the socially optimal amount of street lamps for this community? Explain how you found your answer. How much will Joe pay per street lamp? How much will Mary pay per street lamp? How much will Louise pay per street lamp?

Answers:

a)



To find the market demand curve, we need to hold the quantity constant and then add the prices each of these individuals are willing to pay for this quantity of the good. We do this because the street lamps are non-rival goods: that is, one person's use of the street lamp does not diminish another person's ability to also consume this street lamp. Thus, when the quantity is 10 units, Joe is willing to pay \$5 per street lamp, Mary is willing to pay \$2.5 per street lamp, and Louise is willing to pay \$0 per street lamp: this implies that the point $(Q, P) = (10, 7.5)$ is on the market demand curve. We can repeat this process for $Q = 20$ and $Q = 0$ to get the different end points of the market demand curve.

b) The market demand curve has two linear segments so we need two equations. For prices greater than or equal to 7.5 the market demand curve is $P = 25 - (7/4)Q$. For prices less than or equal to 7.5 we need to do a bit more work. First, the slope of this lower segment is equal to $\text{rise/run} = (-7.5/10) = (-3/4)$. Then, we know that the points $(Q, P) = (10, 7.5)$ and $(20, 0)$ sit on this lower segment. So, use the slope-intercept form and go to work!

$$y = mx + b$$

$$P = (-3/4)Q + b$$

$$0 = (-3/4)(20) + b$$

$$b = 15$$

So, the market demand equation for the lower segment is $P = 15 - (3/4)Q$.

c) To find the socially optimal amount of street lamps we need to equate the marginal social benefit curve (the market demand curve) to the marginal social cost curve. We know that MSC is given as $MSC = 3$. We need to use the equation for the lower segment of the market demand curve for our MSB: $MSB = 15 - (3/4)Q$. Thus,

$$3 = 15 - (3/4)Q \text{ where } Q \text{ is the socially optimal quantity of street lamps}$$

$$12 = (3/4)Q$$

$$Q = 16 \text{ street lamps} = \text{socially optimal quantity of street lamps}$$

$$\text{Using Joe's demand curve: } Q = 20 - 2P$$

$$16 = 20 - 2P$$

$$2P = 4$$

$P = \$2$ per street lamp is the amount that Joe will pay per street lamp.

Using Mary's demand curve: $Q = 20 - 4P$

$$16 = 20 - 4P$$

$$4P = 4$$

$P = \$1$ per street lamp is the amount that Mary will pay per street lamp.

Using Louise's demand curve: $Q = 10 - P$

$$16 = 10 - P$$

$P =$ a number that is less than zero: this is a nonsense answer. We can conclude that Louise will not contribute anything for the provision of the 16 street lamps.

Lest you be concerned about this: notice that Joe and Mary's contribution per street lamp sums to \$3 which is the MSC of providing an additional street lamp. So, we will be able to collect enough money to pay for the lamps and all three will enjoy having the socially optimal amount of street lamps in their community.

5. Consider the market for college education in the economy of Smallville. The market demand curve for a year of college education is given by $P = 60,000 - 2Q$ where P is the price per year of college and Q is the quantity of students attending college per year. This market demand curve expresses the marginal private benefit of going to college but does not include the social benefits derived from this education. The market supply curve for a year of college education is given by $P = Q$. This market supply curve expresses the marginal social cost of going to college. The social benefit of going to college for a year is equal to \$12,000 per year per student, in addition to the private benefit that goes to the student directly.

- Given the above description is there a negative or positive externality in this market? Explain your answer.
- Given the above description, is this a consumption or a production externality? Explain your answer.
- What quantity of students will attend college this year and what price will they pay given the above information? Show your work.
- Suppose that the described externality is internalized in this market. Write the new equations we will need in order to find the socially optimal amount of college education to provide this year. Explain how you got these equations.
- What is the socially optimal amount of college education to provide this year given the above information? What is the "right" (the one that corresponds to the socially optimal amount of the good) price for a year of college? Explain your answer.
- What is the deadweight loss that occurs when the externality is not internalized in this market? Show your work.

Answers:

a) This is a positive externality since the economy derives extra social benefits from the education of its students.

b) This is a consumption externality. We know this because the market supply curve is the MSC of producing this good while the market demand curve is the marginal private benefit (MPB) rather than the marginal social benefit (MSB) of consuming the good.

c) We can find the market solution by equating the market supply curve to the market demand curve: thus,

$$Q = 60,000 - 2Q$$

$$3Q = 60,000$$

$Q = 20,000$ college students this year will be the market outcome

$P = \$20,000$ per college student or

$$P = 60,000 - 2Q = 60,000 - 2(20,000) = \$20,000 \text{ per college student}$$

d) The market supply curve does not change since this curve expresses the MSC of providing the good: Market supply curve = marginal social cost = $MSC = Q$.

The market demand curve expresses only the marginal private benefits from consuming a college education for the year. We need to add in the social benefit which is \$12,000 per student per year. So, the new marginal social benefit curve (MSB) will be $P = 72,000 - 2Q$ (the MSB curve is effectively shifting up from the MPB by \$12,000 per student).

e) Using these two equations from (d) we get:

$$Q = 72,000 - 2Q \text{ where } Q \text{ is the socially optimal amount of the good}$$

$$3Q = 72,000$$

$Q = 24,000$ college students per year is the socially optimal amount of the good

Note: the market, left alone, under produces this good since the market fails to take into account the social benefits derived from the consumption of this good.

$$P = \$24,000 \text{ or}$$

$$P = 72,000 - 2Q = 72,000 - 2(24,000) = \$24,000$$

$$f) \text{ DWL} = (1/2)(\$32,000 \text{ per student per year} - \$20,000 \text{ per student per year})(24,000 \text{ students per year} - 20,000 \text{ students per year}) = \$24,000,000 = \$24 \text{ million}$$