Economics 101
Spring 2017
Answers to Homework #5
Due Thursday, May 4, 2017

Directions:

- The homework will be collected in a box **before** the lecture.
- Please place **your name**, **TA name** and **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time.
- **Show your work**. Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

Public Goods:

1. Provide an example of 1 good that is non-rival but excludable, 1 good that is non-excludable but rival; and one pure public good. For each good explain why this good meets the criterion. Your examples should be different from the ones provided in your textbook or in large lecture (we do want to add to the challenge here and make sure you think a bit about this!).

Answer:
Your answers will differ from our answers. But, here are some examples that meet the criterion.
Goods that are non-rival but excludable: An internet connection is a good that is non-rival but excludable. Amongst other possible goods, the list would include items like membership in clubs (for example, country clubs). A party, if not too crowded, can be an excludable but non-rival good as well.
Goods that are rival but not excludable include the beach, or the lake, or a public free road, if extremely crowded, can be rival but usually are not excludable.
Goods that are pure public gods could include the Beach, or the lake, or the free public road, if relatively empty, are pure public goods. Lighthouses, the parks in town, the view of the lake in the parks are all public goods as well.

2. Suppose there are three types of consumers with different demand curves for good public education, good public hospitals and good parks. Let's call this whole bundle of goods the Public Good. (Note that I am "stretching" the definition of public good because education and health are certainly not pure public goods, but for the sake of the interest in policies, let's leave these concepts as part of the public good that we are analyzing.) The following equations provide the demand curves for each group of consumers of this public good where P is the price per unit of the public good and Q is the quantity of units of the public good:

Group One: P = 200 – Q
Group Two: P = 40 – 3Q
Group Three: P = 50 – Q.

The cost of producing public goods in this example is constant and is given by the following equations where TC is total cost and MC is marginal cost:

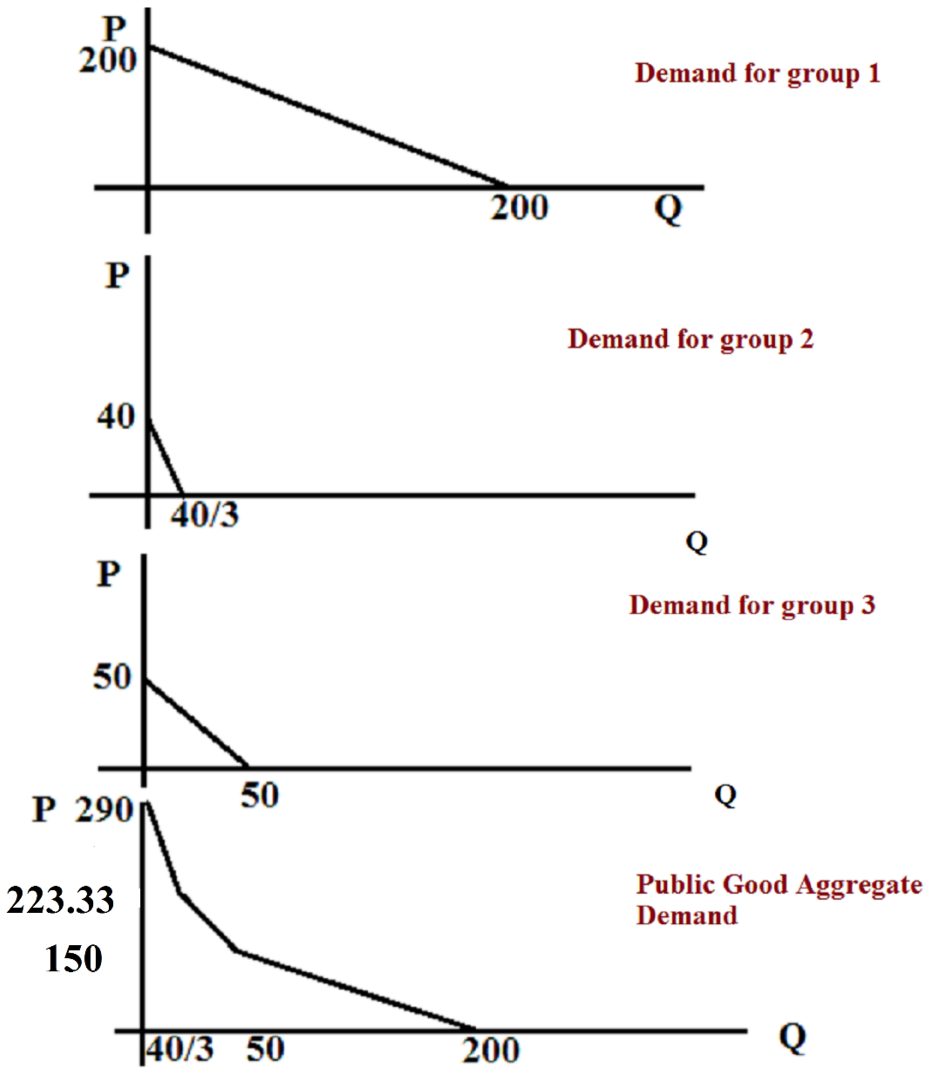
TC = 40Q

MC = 40

a. By vertically summing the three demand curves, derive the aggregate demand curve for the public good. Provide a set of four graphs stacked vertically to illustrate the work you are doing: the top graph should represent Group One, the second graph should represent Group Two, the third graph should represent Group Three, and the fourth graph should represent the aggregate or market demand for the public good. Make sure all your graphs are clearly and completely labeled.

Answer:

We must fix the quantity, Q, and add the values, P. For this problem think of the price as the marginal benefit that the consumers get from consuming the public good. In the public good case for a specific Q, the benefit is received by every citizen. Thus the social marginal benefit of Q is $MB_1 + MB_2 + MB_3 = P_1 + P_2 + P_3$. The demand of group one is defined only until $Q < 200$. The demand for group 2 will be defined only for $Q<40/3$ and the demand of group 3 will be defined only to $Q < 50$. This can be depicted in the following graph.



Note that the aggregation represents the vertical summation. In other words, whereas with the private good one fixes the Price and verifies the quantity that each consumer demands, in the case of the public good the analysis is inverted; the supply of public good, Q, - the number of parks, public hospitals, the number of sightseeing places in a town etc. - is fixed and the **total benefit is the sum of the private benefits.**

Thus from 0 to 40/3, the demand will be

$P_a = 290 - 5Q$

For Q between 40/3 and 50 the demand will be given by

$P_a = P_1 + P_3 = 250 - 2Q$

And for Q above 50 and below 200 we will have

$P_a = 200 - Q$

b. What is the Social Marginal Benefit of the Public Good?

Answer:
Thus the social marginal benefit of Q is $MB_1 + MB_2 + MB_3 = P_1 + P_2 + P_3$. That is, the social marginal benefit of a particular quantity of the public good is equal to the sum of the prices that each individual is willing to pay for that quantity of the public good.

c. Find the socially optimal level of the public good. Also, find the total price of each unit of the public good. Show your work.

Answer:
The aggregate demand obtained is our Marginal Social Benefit. The Marginal Social Cost is 40. Thus the Socially optimal level will be given by the intersection between the marginal social cost with the marginal social benefit. The social marginal cost is simply the marginal cost of provision of the public good, SMC = 40. At the same time the social marginal benefit is SMB = $P_a = P_1 + P_2 + P_3$ that was obtained in item b).
The challenge here is deciding which segment of the market demand curve for the public good we should use. But note that by inspection of the picture in b), we know that the intersection between P_a and the horizontal line with height of 40 will take place in a point on the third segment. Thus

200 – Q = 40 which implies Q = 160 units of the public good, which is ok!
Thus we find that the socially optimal level is 160 units of the public good and the price paid is $40 per unit of the public good which is the Marginal Social Cost.

For the sake of completeness let us present the algebraic solution:

Suppose that the optimal quantity is such that $Q < 40/3$ then:
290 – 5Q = 40 and thus Q = 50 units of the public good
Which is a contradiction with the assumption that $Q < 40/3$.
So, now let's assume that the optimal social quantity is in the interval (40/3, 50). Then:
250 – 2Q = 40 thus Q = 105 units of the public good
Which is again a contradiction since a quantity of 105 units of the public good lies outside the given interval. So, now let's assume that the optimal social quantity is such that $Q > 50$. Then:
200 – Q = 40 which implies Q = 160 units of the public good, which is ok!

Thus we find that the socially optimal level is 160 units of the public good and the price paid is $40 per unit of the public good which is the Marginal Social Cost.

d. Suppose that each consumer group has to pay an equal amount P per unit of public good. Based on P, the consumers must tell the government their optimal quantity of the public good. In the case where each consumer group offers their optimal bid for the socially optimal quantity of the public good, what level of the public good will each consumer group choose when deciding the optimal amount of the public good? Will there be consensus in this society on the optimal quantity of the public good?

Answer:
Each consumer group, in this case, must decide their optimal quantity of the public good. Thus, group one would equate their marginal benefit with marginal:
$MB_1 = 200 – Q_1 = P$ and thus $Q_1 = 200 - P$
$MB_2 = 40 – 3Q_2 = P$ and thus $Q_2 = (40 - P)/3$
$MB_3 = 50 – Q_3 = P$ and thus $Q_3 = 50 - P$

We can see that if consumers had to pay an equal amount, P, per unit of the public good, there would be no consensus over the optimal amount to be supplied! Some benefit more than the others from the public good so that there is no consensus in this society over the optimal amount of public good if all agents must contribute with the same amount per unit of the public good.

e. Suppose that the government recognizes the difficulty of finding a consensus on what the socially optimal amount of the public good is and therefore the government decides to set different prices for each group of consumers. If the government wishes to provide the aggregate socially optimal amount of the good, what prices would it charge each consumer group to get this outcome? Show how you found your prices for each group.

Answer:
Remember we want to set $Q_{so} = 160$. To reach a consensus though, we must charge different prices. Thus $P_1 = 200 – 160 = \$40$ per unit of the public good is the price paid by group 1. The other groups pay \$0 per unit of the public good since they do not want this much of the public good!
Note also that $P_1 + P_2 + P_3 = \$40 = MC$ and thus also equal to the optimal price found in the item c).

We find that to provide the optimal amount of the public good we would have to charge the individuals that benefit more from the good. Perhaps, it is important to note that it is assumed that individuals have the money to pay for this good. Of course that this is a unrealistic assumption.

f. Suppose that in this problem the MC was actually equal to: MC' = \$200 per unit of the public good. How would this alter the socially optimal amount of the good? What prices would each consumer group pay per unit of the public good with this change? Show your work!

Answer:
Given this change in the MC, we know through inspection of the graph we generated in (a) that MC' = \$200 must intersect the market demand for the public good in the middle segment or where $P = 250 - 2Q$. So, setting the marginal social cost equal to the marginal social benefit we get:
$200 = 250 - 2Q$
$2Q = 50$
$Q = 25$ units of the public good
Q = socially optimal quantity when MC' = 200 is equal to 25 units of the public good.

In this case only consumer groups 1 and 3 are going to provide the good because consumer group 2 is unwilling to demand a quantity greater than 40/3 units of the public good. The price paid by each of the other two groups would be such that they agree on the quantities. Thus
$200 – Q = P_1$ and $Q = 25$, thus $P_1 = \$175$ per unit of the public good
And
$50 – Q = P_2$ and $Q = 25$ so that $P_2 = \$25$ per unit of the public good

In order for us to achieve unanimity in society we need to charge different prices for each consumer group. In reality, though, it is very hard to do this because we are abstracting from the reality of individual budget constraints. The individuals who have the highest demand for public goods may be precisely those individuals least able to afford the public good.

Externalities:

3. Suppose that there are two people who live next door to one another. One person loves loud music and the other person loves flowers. The individual who loves loud music has a marginal private benefit curve for loud music given by the equation:

$MB_{music} = 10 – X$, where X is the amount of decibels of loud music

The individual who loves flowers has a marginal private benefit curve for flowers given by the equation:

$MB_{flowers}= 5 - F/2$, where F is the number of flowers

The flower lover receives no marginal benefit from the loud music and the loud music lover receives no marginal benefit from flowers.

The marginal private cost of providing loud music is equal to \$4 per decibel of music and the marginal private cost of growing flowers is equal to \$2 per flower.
When the music lover plays their music they create an externality on the flower lover that is equal to \$2 per decibel of music. When the flower lover grows flowers (growing flowers creates pollen, irritating the allergies of the music lover) this imposes an externality cost of \$1 per flower on the music lover.

a. If the music lover only considers her marginal private benefit and marginal private cost how much loud music will be produced?

Answer:
Set the marginal private benefit equal to the marginal private cost:
$10 - X = 4$
$X = 6$ decibels of music
The music lover would create loud music until the last decibel gave her \$4 in marginal benefit. This will occur at 6 decibels of music.

b. If the flower lover only considers his marginal private benefit and marginal private cost how many flowers will be produced?

Answer:
Set the marginal private benefit equal to the marginal private cost:
5 - F/2 = 2
F = 6 flowers
The flower lover would grow flowers until the last flower gave him $2 in marginal benefit. This will occur at 6 flowers.

c. Suppose that the externality from the loud music is internalized in this market. What is the socially optimal level of decibels?

Answer:
We need to equate the social marginal benefit of loud music to the social marginal cost of loud music. Thus,

Marginal private benefit of loud music = Marginal social benefit of loud music since the flower lover derives no marginal benefit from the loud music.
Marginal social cost of the loud music = Marginal private cost + externality cost = 4 + 2 = 6
10 – X = 6 and X = 4 units of loud music is the socially optimal amount of loud music.

d. Suppose that the externality from the flowers is internalized in this market. What is the socially optimal level of flowers?

Answer:
We need to equate the social marginal benefit of flowers to the social marginal cost of flowers. Thus,

Marginal private benefit of flowers = Marginal social benefit of flowers since the music lover derives no marginal benefit from the flowers.
Marginal social cost of the flowers = Marginal private cost + externality cost = 2 + 1 = 3
5 – F/2 = 3 and F = 4 flowers is the socially optimal amount of flowers.

e. How much would the flower lover need to compensate the music lover in order to get the music lover to produce the socially optimal amount of music?

Answer:
Since the externality cost of each decibel of loud music is $2, the flower lover would need to pay $2 per unit of decibel music not produced. So, if the optimal amount of loud music is two units less than the private optimum, then the flower lover would need to pay the music lover $4 to get the socially optimal amount of loud music.

f. How much would the music lover need to compensate the flower lover in order to get the flower lover to produce the socially optimal amount of flowers?

Answer:
Since the externality cost of each flower is $1, the music lover would need to pay $1 per flower not produced. So, if the optimal amount of flowers is two units less than the private optimum,

then the music lover would need to pay the flower lover $2 to get the socially optimal amount of flowers.

Game Theory:

4. Two countries that are the rulers of the Free Island A and Free Island B are disputing control over the country Smaller Island that has a lot of fish and game around. From the perspective of the rulers of both Free Island A and Free Island B, Smaller Island is the promised heaven. For the last couple of months, the two rulers have been threatening each other, saying that war is eminent. The value of the loss of their country's soldiers' lives is a loss of 6 utils to the country. As a result, every time the two countries are in battle each country loses 6 utils, assuming that every time there is a battle all the soldiers from each country die.

We are also told that if Free Island A takes control over Smaller Island and Free Island B does not invade Smaller Island, then Free Island A will receive a payoff of 10 utils while Free Island B will receive a payoff of 0 utils. If both Free Island A and Free Island B invade Smaller Island, then both countries' soldiers will die while the two invading countries share evenly the value of Smaller Island, which is 10 utils in all. If Free Island B invades Smaller Island and Free Island A does not invade Smaller Island, then Free Island B will receive a payoff of 10 utils while Free Island A will receive a payoff of 0 utils.

a. Describe the set of players in this game, the strategies available to each player, and the payoffs association with each combination of strategies facing the players. Draw a payoff matrix representing this game with Free Island A on the left side of the matrix and Free Island B on the top of the matrix.

Answer:
There are 2 players representing the countries. Free Island A and Free Island B.
The set of actions is $A_A = \{I_1, NI_1\}$ and $A_B = \{I_2, NI_2\}$ where A is action, I is invade and NI is not invade. This represents their possibilities of actions in the game.

A/B	I_B	NI_B
I_A NI_A	-1 , -1 0 , 10	10 , 0 0,0

b. Given the payoff matrix you created in (a), what is the best strategy for Free Island A? Explain your answer.

Answer:
For Free Island A to figure out its best action it must make an assumption about the strategy that Free Island B will use. If Free Island B chooses to "Invade", then Free Island A is better off "Not Invading". If Free Island B chooses to "Not Invade", then Free Island A is better off "Invading". Free Island A does not have a dominant strategy: Free Island A's best strategy is dependent upon the strategy that Free Island B chooses.

c. Given the payoff matrix you created in (a), what is the best strategy for Free Island B? Explain your answer.

Answer:
For Free Island B to figure out its best action it must make an assumption about the strategy that Free Island A will use. If Free Island A chooses to "Invade", then Free Island B is better off "Not Invading". If Free Island A chooses to "Not Invade", then Free Island B is better off "Invading". Free Island B does not have a dominant strategy: Free Island B's best strategy is dependent upon the strategy that Free Island A chooses.

d. Given the payoff matrix you created in (a) and your analysis in (b) and (c), what outcome(s) do you predict for this game?

Answer:
Either Free Island A will invade while Free Island B does not invade or Free Island B will invade while Free Island A does not invade.

e. Suppose Free Island A knows that Free Island B is governed by a crazy murderer and that this leader will always choose to "Invade". Given this information, what is the best strategy for Free Island A to pursue?

Answer:
Free Island A's best strategy to pursue if they know that Free Island B is going to "Invade", is "Not Invade".

f. If Free Island B reasons that Free Island A is going to always choose "Not Invade" due to Free Island B having a ruler only too happy to invade, then what is Free Island B's best strategy given this information? Assume that although Free Island B has a crazy leader, this leader can at times act quite rationally and can certainly do a bit of simple game theory!

Answer:
Free Island B will reason that Free Island A will choose "Not Invade" and given this choice the best strategy for Free Island B will be to always choose "Invade".

Price Discrimination:

5. The demand structure for a specific market has two groups of consumers. The demand for each group is given by the following equations:

Demand for Group One: $P = 10 - Q$

Demand for Group Two: $P = 5 - Q/3$

The marginal cost of producing the good is constant and equal to $1 per unit of output: that is, marginal cost can be written as:

$MC = 1$

For this problem, assume that there are no fixed costs.

a. Given the above information find the market demand for this product given the group demands.

Answer:
For prices greater than or equal to 5, the market demand curve is simply Group One's demand curve: that is,

$$P = 10 - Q \text{ when } 5 \leq P \leq 10$$

For prices less than or equal to 5, the market demand curve is:

$$P = 25/4 - (1/4)Q \text{ when } 0 \leq P \leq 5$$

b. Given this information and holding everything else constant, what is the socially optimal amount of the good? What price will this socially optimal amount of the good sell for in this market if the monopolist is only allowed to charge one price? Calculate the value of consumer and producer surplus when the socially optimal amount of the good is produced.

Answer:
To find the socially optimal amount of the good we need to set the market demand curve equal to the marginal cost curve. Here we assume that both the demand curve and the marginal cost curve include all the benefits and all the costs, respectively, that society faces with this good. So, since $MC = 1$ we will need to use the segment of the market demand curve $P = 25/4 - (1/4)Q$. Thus,

$1 = 25/4 - (1/4)Q$

$4 = 25 - Q$

Q socially optimal = 21 units of the good.

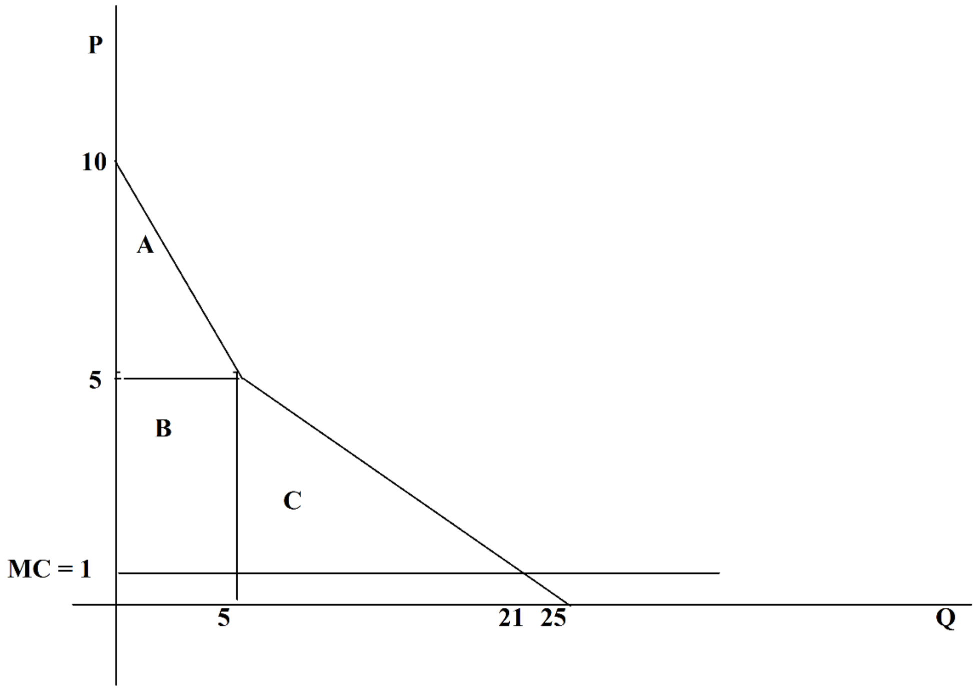
By letting A, B and C be the indicated areas in the figure below, we have

CS = A + B + C =

$(1/2)(10 - 5)(5) + (5 - 1)(5) + (1/2)((5 - 1)(21 - 5) = 12.50 + 20 + 32 = \$64.50$

PS = \$0 since MC is constant and there are no fixed costs.

These regions can be seen on the graph below.



c. Suppose this monopolist acts as a single price monopolist. What is the profit maximizing output for this monopolist and what is the price that they monopolist will charge (take the price out to three places past the decimal)? Calculate the single-price monopolist's profits, the value of CS if this market is served by a single-price monopolist, the value of PS if this market is served by a single-price monopolist, and the value of DWL under this arrangement. Show your work.

Answer:
To find the single-price monopolist's profit maximizing quantity and price we need the monopolist's marginal revenue curve: since the market demand curve has two segments there will be two marginal revenue curves. Thus,

For prices greater than or equal to 5, the MR = 10 – 2Q

For prices less than or equal to 5, the MR = (25/4) – (1/2)Q

Which is the relevant MR curve for the monopolist? Since MC = 1, we know it is likely the MR that goes with the lower segment of the demand curve. Thus,

(25/4) – (1/2)Q = 1

25 – 2Q = 4

21 = 2Q

Q = 10.5 units of the good

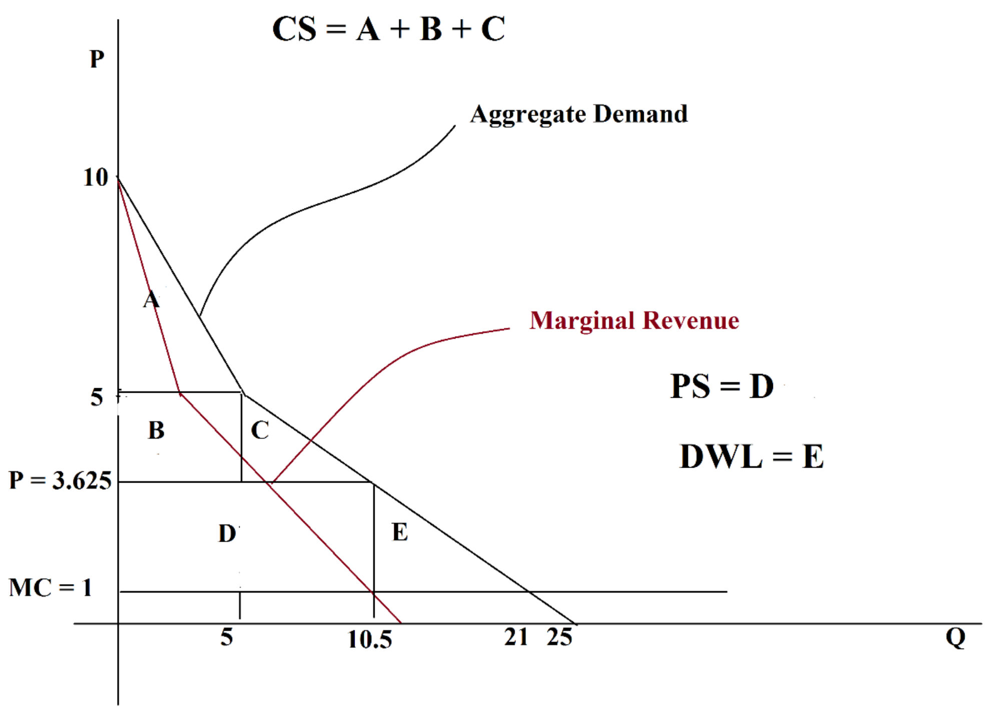
P = (25/4) – (1/4)Q = (25/4) – (1/4)(21/2) = $3.625 per unit of the good

TR for single-price monopolist = (3.625)(10.5) = $38.0625

TC for single-price monopolist = ATC*Q = 1(10.5) = $10.50

Profit for single-price monopolist = $27.5625

CS if firm acts as a single-price monopolist = (1/2)(10 – 5)(5) + (5 – 3.625)(5) + (1/2)(5 – 3.625)(10.5 – 5) = 12.5 + 6.875 + 3.78125 = $23.15625 ≈ $23.156
PS if firm acts as a single-price monopolist = (3.625-1)(10.5) = $27.5625 ≈ $ 27.563



d. Suppose that this monopolist is able to differentiate between these two groups at no cost. The monopolist decides to charge each group a different price: that is, the monopolist decides to practice third degree price discrimination. What price will Group One pay and what quantity of the good will Group One get? What price will Group Two pay and what quantity of the good will Group Two get? Calculate what this third degree monopolist's total profit level will be. Show how you found your answers.

Answer:
Group One: Equate MC = 1 to the MR curve for Group One
Marginal Revenue for Group One = 10 – 2Q
1 = 10 – 2Q
Quantity for Group One = 4.5 units of the good
Price for Group One is found by using Group One's demand curve and Q = 4.5: thus,
Price for Group One = 10 – Q =10 – 4.5 = $5.5 per unit of the good
TR from Group One = (5.5)(4.5) = $24.75
TC for Group One = ATC*Q = (1)(4.5) = $4.50 since MC = ATC = 1 since FC = 0 in this problem and MC is constant.
Profit from Group One = $20.25

We know that the profit maximizing quantity for the entire market is 10.5 units. So, if Group One is getting 4.5 units of the good, this suggests that Group Two should be getting 6 units of the good. Let's see if that is the case.

Group Two: Equate MC = 1 to the MR curve for Group Two
Marginal Revenue for Group Two = 5 – (2/3)Q
1 = 5 – (2/3)Q
(2/3)Q = 4
Quantity for Group Two = 6 units of the good
Price for Group Two is found by using Group Two's demand curve and Q = 6: thus,
Price for Group Two = 5 – (1/3)Q = 5 – (1/3)(6) = $3 per unit of the good
TR from Group Two = (3)(6) = $18
TC for Group Two = ATC*Q = (1)(6) = $6 since MTC = ATC = 1 since FC = 0 in this problem and MC is constant.
Profit from Group Two = $12

Total Profit from Group One and Group Two = $20.25 + $12 = $32.25

e. Compare your answer in (c) and (d). Is it an advantage to the monopolist to practice third degree price discrimination rather than be a single-price monopolist? Explain your answer.

Answer:
Yes, it pays to be a third degree price discriminator since it enhances the economic profit that the monopolist earns.