

Economics 101

Spring 2015

Answers to Homework #5

Due Thursday, May 7, 2015

Directions: The homework will be collected in a box **before** the lecture. Please place your name on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. Monopoly

Madison Gas and Electric (MG&E) is a monopoly firm that provides electricity in Madison. Its total cost and marginal cost functions are given by the following equations where Q is quantity measured in megawatts:

$$TC = 4Q^2 + 400Q + 10000$$

$$MC = 8Q + 400$$

The demand for electricity is given by the following equation where P is the price per megawatt:

$$Q = 2000 - P/2$$

- a. Find MG&E’s marginal revenue function. That is, write an equation for MG&E’s MR function.

To find a marginal revenue, first rewrite the demand curve in P intercept form as follows: $P = 4000 - 2Q$. The marginal revenue is an equation that has the same intercept and twice the slope as the inverse demand function. Thus:

$$MR = 4000 - 4Q.$$

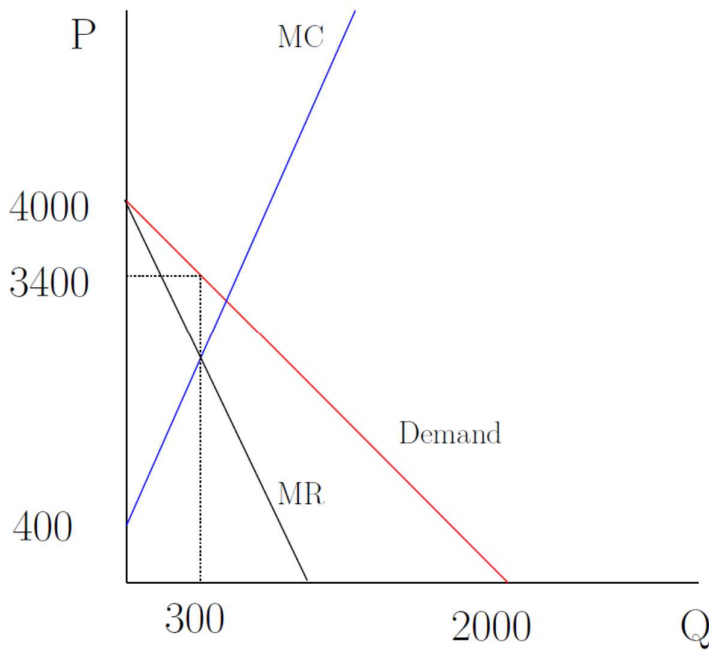
- b. Find the profit maximizing price and quantity. Show your work as well as your reasoning for finding these two answers.

The profit maximizing quantity should satisfy: $MR = MC$

$$4000 - 4Q = 8Q + 400 \leftrightarrow 3600 = 12Q.$$

Thus, MG&E will set $Q = 300$ megawatts. Given this quantity, the price is determined by the demand equation.

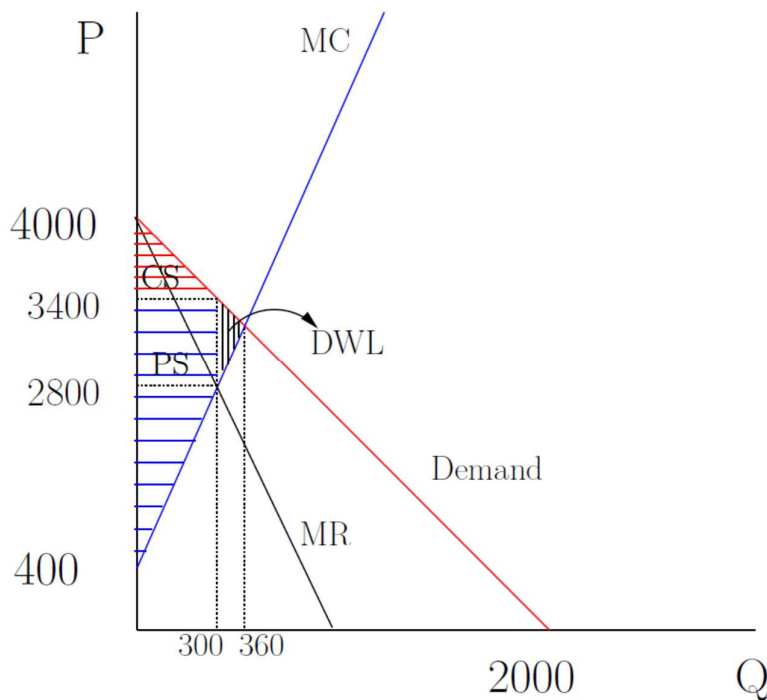
By plugging $Q = 300$ into $P = 4000 - 2Q$, we obtain $P = \$3400$ per megawatt.



The figure above illustrates this argument. Red line is the demand curve and black line is the MR curve. The optimal quantity satisfies $MR = MC$ and the optimal price is determined by demand at that quantity.

c. Calculate the values of consumer surplus, producer surplus and DWL under monopoly.

To find CS, PS and DWL, first draw a graph.



CS is an area below the demand curve and above the price consumers pay. This area corresponds to the red shaded area in the graph above and equals \$90,000. Here's the calculation: $CS = (1/2)(\$4000 \text{ per megawatt} - \$3400 \text{ per megawatt})(300 \text{ megawatts}) = \$90,000$.

PS is an area above the MC curve and below the price that producers receive for the product. Notice that this area is the sum of a triangle plus a rectangle. This area corresponds to the blue shaded area in the graph above. PS is equal to \$540,000. Here's the calculation: $PS = (1/2)(\$2800 \text{ per megawatt} - \$400 \text{ per megawatt})(300 \text{ megawatts}) + (\$3400 \text{ per megawatt} - \$2800 \text{ per megawatt})(300 \text{ megawatts}) = \$540,000$.

DWL is the black shaded triangle above with an area of \$18,000. Here's the calculation: $DWL = (1/2)(\$3400 \text{ per megawatt} - \$2800 \text{ per megawatt})(360 \text{ megawatts} - 300 \text{ megawatts}) = \$18,000$.

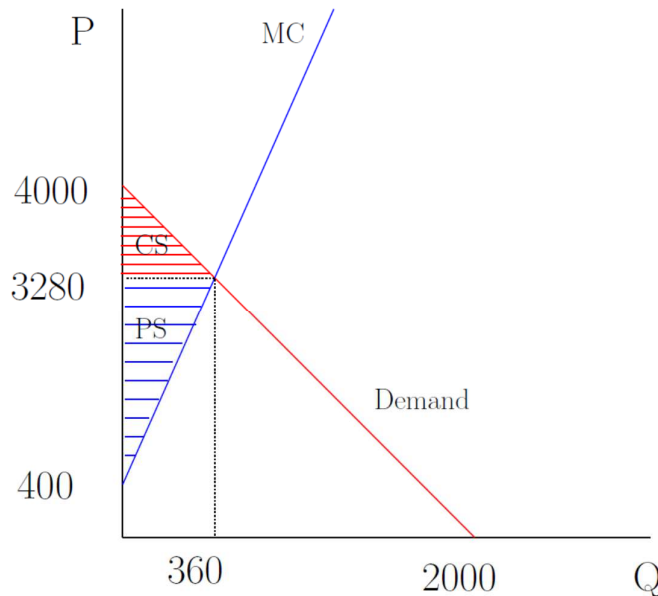
Suppose that the governor of Madison decides to impose MC regulation on MG&E. That is MG&E has to set a price that is equal to its marginal cost under this new regulation.

- d. Find the equilibrium price and quantity under this new regulation. Calculate consumer surplus and producer surplus. Show your work and your reasoning.

Under MC regulation: $MC = P = \text{demand}$ in equilibrium. Thus,

$$8Q + 400 = P = 4000 - 2Q$$

should hold in equilibrium. By solving the equality above, we find that $Q = 360$ megawatts and $P = \$3280$ per megawatt. To find CS and PS, draw a graph first.



By similar logic as in part (c), we find that consumer surplus is equal to \$129,600 and producer surplus is \$518,400. Here are the calculations:

$$CS = (1/2)(\$4000 \text{ per megawatt} - \$3280 \text{ per megawatt})(360 \text{ megawatts}) = \$129,600$$

$$PS = (1/2)(\$3280 \text{ per megawatt} - \$400 \text{ per megawatt})(360 \text{ megawatts}) = \$518,400$$

2. Price discrimination

Microsoft is a dominant firm in Operating Systems (OS) market. Microsoft knows that there are two types of users in the market: business users and personal users. Their demand curves are given by the following equations where Q is the quantity of operating systems and P is the price per system:

$$\text{Business user: } Q = 200 - P/2$$

$$\text{Private user: } Q = 150 - P/3$$

Once Microsoft develops its OS, production of an additional unit does not cost anything to Microsoft, i.e. the marginal cost of OS is 0. Assume for now that Microsoft can price discriminate, i.e. it can charge different types of consumers different prices.

- a. Derive the profit maximizing prices (there should be two different prices) for business and personal users. What is Microsoft's total profit given these prices (to find this you will need to determine how many units of OS Microsoft produces)?

The profit maximizing quantity satisfies $MR = MC$. To obtain MR, first rewrite demand equations in P intercept formula:

$$\text{Business user: } P = 400 - 2Q$$

$$\text{Private user: } P = 450 - 3Q$$

Thus, MR curves are

$$\text{MR for business user: } MR = 400 - 4Q$$

$$\text{MR for private user: } MR = 450 - 6Q$$

To find the profit maximizing quantities set $MR = MC$ for the different markets: thus,

MR = MC for business market

$$400 - 4Q_{business} = 0$$

$$Q_{business} = 100 \text{ units of OS}$$

MR = MC for private user market

$$450 - 6Q_{private} = 0$$

$$Q_{private} = 75 \text{ units of OS}$$

Thus, the profit maximizing quantities are 100 and 75 for business and personal users, respectively.

By plugging these numbers into the demand equation, we find that the profit maximizing price for business users is \$200 per OS unit and the profit maximizing price for private users is \$225 per OS unit. At the optimal prices, the profit from selling to business users is:

$$\text{Profit for Business Users} = TR - TC$$

$$\text{Profit for Business Users} = (\$200 \text{ per unit})(100 \text{ units}) - (\$0 \text{ per unit})(100 \text{ units}) = \$20,000$$

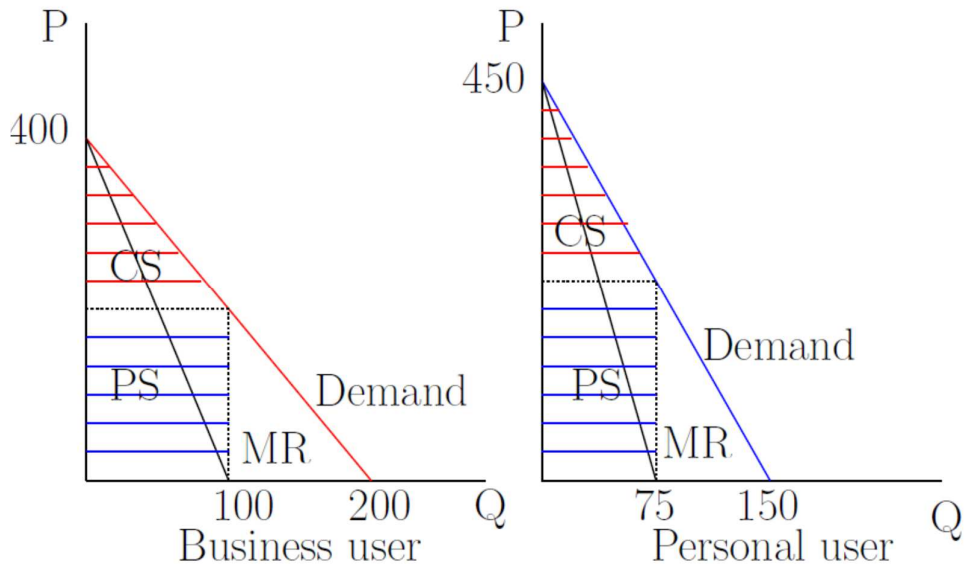
and the profit from personal users is:

$$\text{Profit for Personal Users} = TR - TC = (\$225 \text{ per unit})(75 \text{ units}) - (\$0 \text{ per unit})(75 \text{ units}) = \$16,875$$

Thus, the total profit is the sum of Profit for Business Users plus Profit for Personal Users or \$36,875.

- b. Derive the sum of consumer surpluses and producer surpluses in equilibrium when Microsoft price discriminates by charging different prices to these two consumer groups. You will find it helpful to draw graphs illustrating the markets you are analyzing.

The graph below illustrates the solution.



The graph on the left depicts the demand for OS by business users and the right one depicts the demand for OS by personal users. CS (a red shaded area) for business and personal users is \$10,000 and \$8,437.50, respectively. PS from business users is \$20,000 and from personal users is \$16,875. Notice that in this case producer surplus is equal to profits.

- c. Suppose that consumers are able to trade among themselves, i.e. personal users are able to sell their OS to business users and vice versa. If this is the case, can the prices you derived in part (a) be the equilibrium prices? Make a verbal argument to support your reasoning.

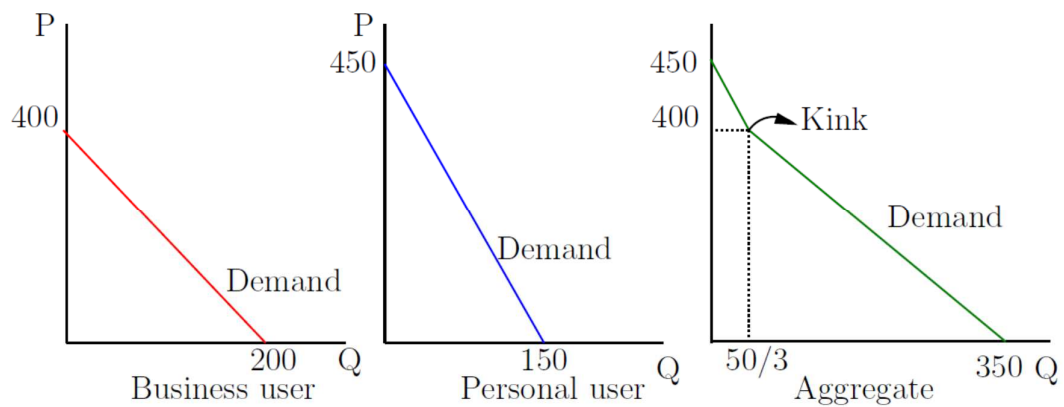
Since the price paid by business users is lower than the price paid by personal users, business users have an incentive to buy OS from Microsoft and sell it to personal users at a price lower than the price offered by Microsoft. As a result, personal users will not buy OS from Microsoft. Thus, the prices derived in part (a) cannot be an equilibrium prices if different types of users are allowed to trade.

Assume now that the Federal Trade Commission (FTC) does not allow Microsoft to price discriminate.

- d. Derive the market demand function for OS given the assumption that Microsoft no longer price discriminates.

The market demand curve is the horizontal sum of the individual demand curves; in this case there

are two individual demand curves. The following graph illustrates these demand curves:



The graph depicts the demand by business users, the second graph depicts the demand by personal users, and the third graph depicts the market demand curve. When finding the "kink point" of the market demand curve you need to consider a price of \$400 per unit. At that price personal users demand 50/3 units of OS while business users demand 0 units of OS. Thus, the kink point for the market demand curve occurs at (50/3, \$400). Mathematically, the market demand is given by the following equation

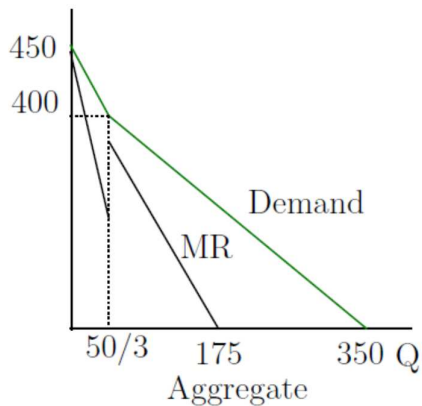
$$P = \begin{cases} 420 - 6Q/5 & \text{if } Q > 50/3 \\ 450 - 3Q & \text{if } 0 \leq Q \leq 50/3 \end{cases}$$

e. Derive the marginal revenue function that corresponds to the aggregate demand curve.

Since the aggregate demand curve has two parts, the marginal revenue also has two parts. For the first region ($0 \leq Q \leq 50/3$) MR is given by $MR = 450 - 6Q$ and for the second region MR is given by $MR = 420 - 12Q/5$. Thus, MR is:

$$MR = \begin{cases} 420 - 12Q/5 & \text{if } Q > 50/3 \\ 450 - 6Q & \text{if } 0 \leq Q \leq 50/3 \end{cases}$$

The following graph illustrates the marginal revenue curve (a black line).



As you can see, MR jumps at $Q = 50/3$.

- f. Find the profit maximizing price and quantity when price discrimination is not allowed. Calculate the value of consumer surplus. Show your work and your reasoning for these calculations! You will definitely find it helpful to draw a graph to see the area of CS!

At the profit maximizing quantity $MR = MC$. Thus,

$$420 - 12Q/5 = 0$$

$$2100 = 12Q$$

$$Q = 175 \text{ units of OS}$$

To find the price, substitute this quantity into the relevant demand curve: thus,

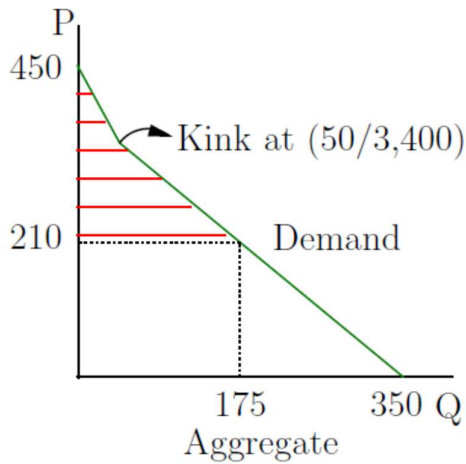
$$P = 420 - 6Q/5$$

$$P = 420 - 6(175)/5$$

$$P = \$210 \text{ per unit of OS}$$

To find CS recognize that this is a weird shape (see graph). CS consists of two triangles plus a narrow rectangle. Here is the calculation:

$$CS = (1/2)(\$450 \text{ per unit} - \$400 \text{ per unit})(50/3 \text{ units}) + (\$400 \text{ per unit} - \$210 \text{ per unit})(50/3 \text{ units}) + (1/2)(\$400 \text{ per unit} - \$210 \text{ per unit})(175 \text{ units} - 50/3 \text{ units}) = \$55875/3 = \$18,625. \text{ The shaded area in the graph below depicts CS.}$$



- g. Assume that the objective of the FTC is to maximize consumer surplus. Should FTC prohibit price discrimination? Explain your reasoning.

The sum of two consumer surpluses under price discrimination is \$18,437.50 which is slightly smaller than \$18,625, a consumer surplus without price discrimination. Therefore, the FTC should prohibit price discrimination.

3. Game Theory

Coke and Pepsi are two big firms competing in a beverage market. Since Coke and Pepsi are substitute goods, the demand for Coke increases as the price of Pepsi increases and the demand for Pepsi increases as the price of Coke increases. As a result, their demand curves are given by

$$Q_{Coke} = 100 - P_{Coke} + P_{Pepsi}/2,$$

$$Q_{Pepsi} = 100 - P_{Pepsi} + P_{Coke}/2.$$

The total cost of producing q quantity of Coke or Pepsi is given by $20q$.

- a. What are Coke and Pepsi's profits when they both set the price equal to \$80? Show how you found your answer.

When $P_{Coke} = P_{Pepsi} = \$80$, $Q_{Coke} = 100 - 80 + 80/2 = 60$. Thus, Coke's profit is $P_{Coke} * Q_{Coke} - 20 * Q_{Coke} = 80 * 60 - 20 * 60 = \$3,600$. By the same argument, Pepsi's profit is also \$3,600.

- b. What are Coke and Pepsi's profits when they both set the price equal to \$100?

When $P_{Coke} = P_{Pepsi} = \$100$, $Q_{Coke} = 100 - 100 + 100/2 = 50$. Thus, Coke's profit is $P_{Coke} * Q_{Coke} - 20 * Q_{Coke} = 100 * 50 - 20 * 50 = \$4,000$. By the same argument, Pepsi's profit is

also \$4,000.

- c. What are Coke and Pepsi's profits when Coke charges \$80 and Pepsi charges \$100 for its product?

Demand for Coke is $Q_{Coke} = 100 - 80 + 100/2 = 70$. Thus, Coke's profit is

$P_{Coke} * Q_{Coke} - 20 * Q_{Coke} = 80 * 70 - 20 * 70 = \$4,200$. Similarly, demand for Pepsi is

$Q_{Pepsi} = 100 - 100 + 80/2 = 40$. Thus, Pepsi's profit is

$P_{Coke} * Q_{Coke} - 20 * Q_{Coke} = 100 * 40 - 20 * 40 = \$3,200$.

Now assume that the two firms set the prices simultaneously. They can either set $P=80$ or $P=100$. Let π_{Coke} be Coke's profit and π_{Pepsi} be Pepsi's profit.

- d. Using your solution from previous parts, complete the table below.

		Pepsi	
		$P_{Pepsi} = 80$	$P_{Pepsi} = 100$
Coke	$P_{Coke} = 80$	$(\pi_{Coke}, \pi_{Pepsi}) =$ (\$3600, \$3600)	$(\pi_{Coke}, \pi_{Pepsi}) =$ (\$4200, \$3200)
	$P_{Coke} = 100$	$(\pi_{Coke}, \pi_{Pepsi}) =$ (\$3200, \$4200)	$(\pi_{Coke}, \pi_{Pepsi}) =$ (\$4000, \$4000)

- e. Does the game you mapped out in (d) have an equilibrium? Explain your answer.

When $P_{Pepsi} = \$80$, $\pi_{Coke} = \$3,600$ when $P_{Coke} = \$80$, and $\pi_{Coke} = \$3,200$ when $P_{Coke} = \$100$. Thus, in this case, charging $P_{Coke} = \$80$ is better for Coke than charging $P_{Coke} = \$100$. When $P_{Pepsi} = \$100$, $\pi_{Coke} = \$4,200$ when $P_{Coke} = \$80$, and $\pi_{Coke} = \$4,000$ when $P_{Coke} = \$100$. In this case, charging $P_{Coke} = \$80$ is better for Coke. As a result, no matter what price Pepsi chooses, charging $P_{Coke} = \$80$ always gives higher profits for Coke. By applying the same logic, we are able to show that no matter what price Coke charges, Pepsi attains higher profits when $P_{Pepsi} = \$80$. Therefore, $(P_{Coke}, P_{Pepsi}) = (\$80, \$80)$ is the rational prediction for this game.

4. Public Goods

- a. Find one example of a public good from the real world and argue why it can be classified as a public good.

A good is a public good if it is non-excludable and non-rival. A good is non-excludable if it is impossible to exclude someone from consumption of the good even if they have not paid for it. A

good is non-rival if one's consumption of the good does not prevent others from consuming the same good. Your example must satisfy these two properties. Some examples that would fit here: streetlights; lighthouses; a non-congested, non-toll road.

Suppose there are three types of consumers with different demand curves for public goods in the market. The following equations provide the demand curves for each group of consumers of this public good where P is the price per unit of the public good and Q is the quantity of units of the public good:

$$\text{Group One: } P = 100 - Q$$

$$\text{Group Two: } P = 50 - Q/2$$

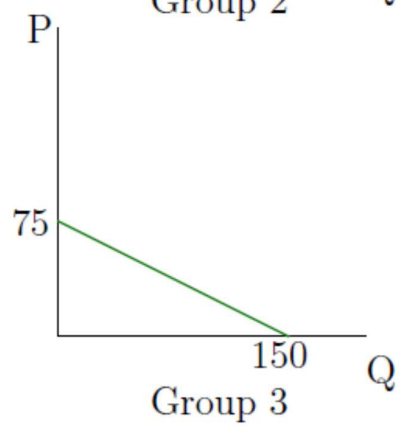
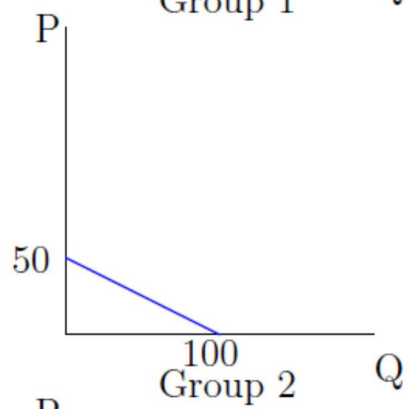
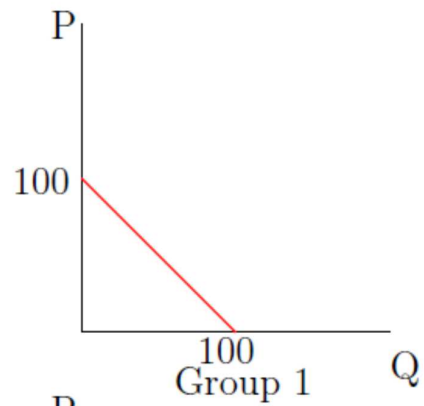
$$\text{Group Three: } P = 75 - Q/2.$$

The cost of producing public goods in this example is constant and is given by the following equations where TC is total cost and MC is marginal cost:

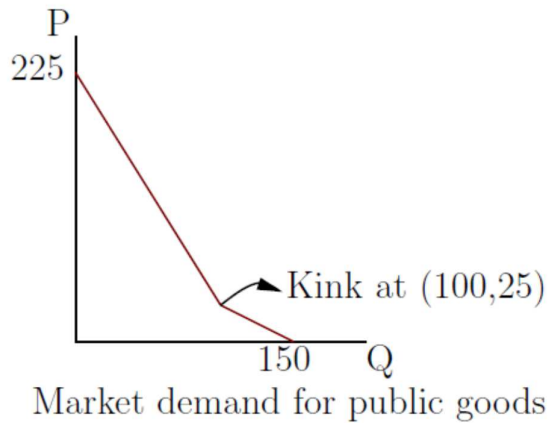
$$TC = 50Q$$

$$MC = 50$$

- b. By vertically summing the three demand curves, derive the market demand curve for the public good.



The first graph depicts the demand for Group One, the second graph depicts the demand for Group Two, and the third graph depicts the demand for Group Three. The graph below illustrates the market demand curve. Notice that it has a kink at $Q = 100$ and coincides with type 3's demand curve for $Q > 100$.



Mathematically, the aggregate demand curve is

$$P = \begin{cases} 75 - Q/2 & \text{if } Q > 100 \\ 225 - 2Q & \text{if } 0 \leq Q \leq 100 \end{cases}$$

- c. Find the socially optimal level of the public good. Also, find the total price of each unit of the public good. Show your work.

At the socially optimal level, market demand = MC holds. Thus,

$$225 - 2Q = 50$$

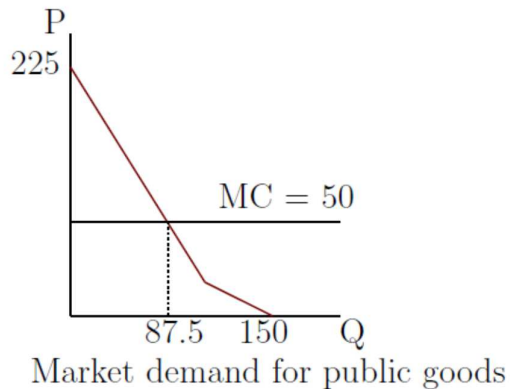
$$2Q = 175$$

$$Q_{\text{socially optimal}} = 87.5 \text{ units}$$

From the graph below we are able to conclude that the optimal level is 87.5 units of the public good.

The price per unit of the public good can be calculated as follows:

$$P = 225 - 2Q = 225 - 2(87.5) = \$50 \text{ per unit of the public good}$$



- d. In order to achieve the socially optimal level of a public good in the market, what price should each group of consumers pay? Show how you found your answer.

The demand curves are:

$$\text{Group 1: } P = 100 - Q$$

$$\text{Group 2: } P = 50 - Q/2$$

$$\text{Group 3: } P = 75 - Q/2$$

When $Q = 87.5$, Group 1 will pay $P = \$12.5$ per unit of the public good; Group 2 will pay $\$6.25$ per unit of the public good; and Group 3 will pay $\$31.25$ per unit of the public good. Together these three groups will contribute a total payment of $\$50$, which is the cost of a unit of the public good.

5. Externality

George Smith gains benefits from hosting a loud party. His marginal private benefit (MPB) and marginal private cost (MPC) of hosting T hours of a party are given by the following equations:

$$\text{MPB} = 30 - T$$

$$\text{MPC} = 2T$$

Both MPB and MPC are measured in dollars per party hour. George's neighbor hates loud parties and thus, he incurs a cost of $\$15$ per hour for each hour that George has a party.

- a. If there is no regulation of loud parties in the community, how long will George Smith's loud party last? Show how you found your answer.

George Smith optimally chooses T that equates MPB and MPC. Therefore,

$$30 - T = 2T$$

holds at the optimal level. Thus, $T = 10$ hours of loud party.

- b. Is the answer you obtained in part (a) a socially optimal level of loud party? Explain your

answer.

When George Smith chooses T, he ignores the negative externality of his activity. That is, he does not consider the cost of his activity on his neighbor. He ends up consuming and producing more loud party than is socially optimal since he does not take into account the negative externality generated by his behavior. Thus, it cannot be a socially optimal level.

- c. Since holding a loud party imposes a cost both on George Smith and his neighbor, the marginal social cost (MSC) is given by the sum of MPC and the external cost. Derive the MSC and find the socially optimal level of T. Show your work.

Since his neighbor incurs a cost \$15 per hour of loud party, the marginal negative externality of ten hours of loud party is \$150. By definition,

$MSC = MPC + \text{marginal negative externality}$. In this example, George's neighbor incurs a cost of \$15 per hour, which implies marginal negative externality cost per unit of \$15. Thus, $MSC = 2T + 15$.

At the socially optimal level, $MPB = MSC$. Therefore,

$$30 - T = 2T + 15$$

T socially optimal = 5 hours of loud party