

Directions:

- The homework will be collected in a box **before** the lecture.
- Please place **your name, TA name** and **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time.
- **Show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. (Perfectly competitive market)

Assume that there is a perfectly competitive market. All individual firms in this market have the same cost function and this cost function is described by the following equation:

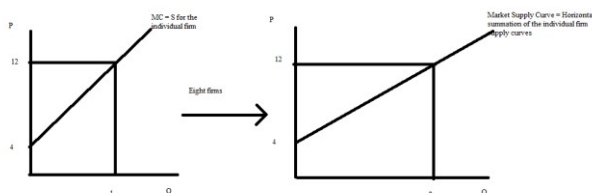
$$TC = 4q^2 + 4q + 36$$

Each firm's marginal cost is described by the following equation:

$$MC = 8q + 4$$

- a. Assume that there are 8 firms in the short-run. Derive the short run market supply function.

Method I: to find the market supply curve, you can add the individual firm supply curves together horizontally. Here is the image:



To use this method you must choose some arbitrary firm quantity (I chose $q = 1$), then compute the price for this quantity using the firm's MC curve. Then multiply this firm quantity by 8 to find the market quantity at this price. Now we have two points on the market supply curve: $(Q, P) = (0, 4)$ and $(8, 12)$ and we can write the market supply curve. Market supply curve is: $P = Q + 4$.

Method II: In a competitive market each firm equates P to MC to decide how much of the good to supply. Using this idea we can write:

$$P = 8q + 4$$

We can rewrite this equation in x-intercept form as:

$$q = (P - 4)/8$$

This tells us the individual firm supply curve. But, there are eight firms in the industry so to find the market supply curve we need to multiply this equation by 8. So, Q is total market supply and nq is the number of firms times the individual firm's supply curve. This gives us:

$$Q = nq = 8q = P - 4$$

The market supply curve in x-intercept form is $Q = P - 4$ or in y-intercept form it is $P = Q + 4$.

- b. Assume that the market demand is given by $P = 132 - Q_d$. There are still 8 firms in the market in the short-run. Calculate the market equilibrium price and quantity. Calculate the level of production for each firm. Then calculate the level of economic profit for each firm.

In the equilibrium, $Q_d = Q_s$

$$132 - P = P - 4$$

$$2P = 136, P = \$68 \text{ per unit}, Q = 64 \text{ units}$$

$$q = Q/8 = 8 \text{ units produced by each firm}$$

$$\text{Profit for each firm} = TR - TC = 68 \cdot 8 - (4 \cdot 8 \cdot 8 + 4 \cdot 8 + 36) = \$220$$

- c. Calculate the long run equilibrium price and quantity. How many firms are in the industry when the industry is in long run equilibrium? (hint: the number of firms can be a fractional amount.)

The long run equilibrium condition is $MC = ATC$ for each firm. So,

$$8q + 4 = 4q + 4 + 36/q$$

$$4q = 36/q$$

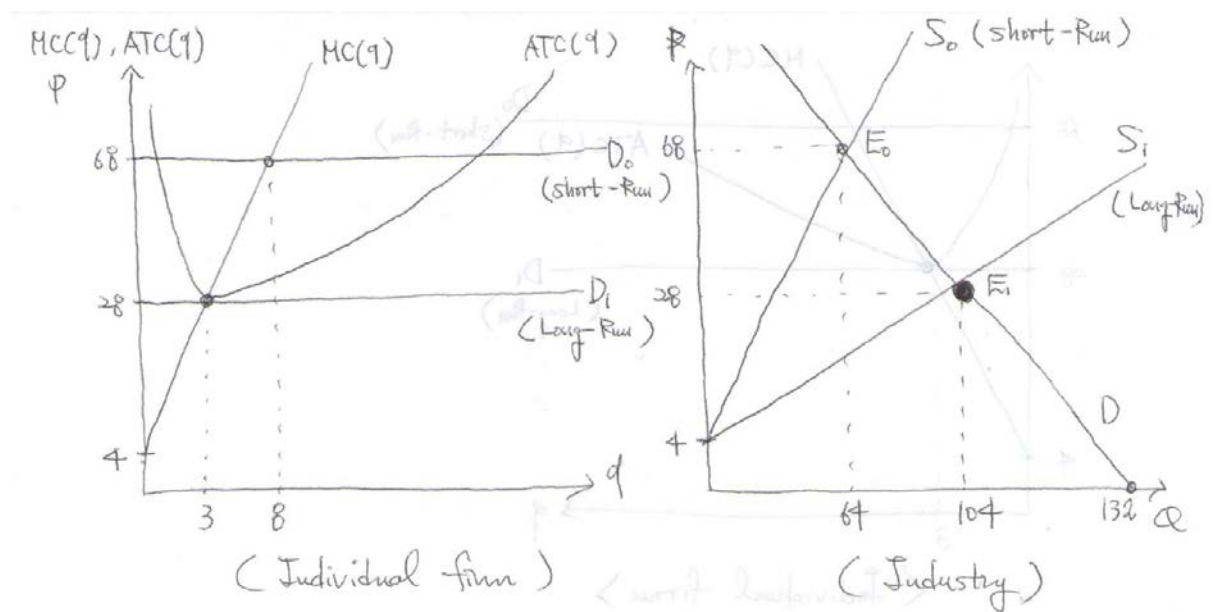
$$q^* = 3 \text{ units produced by each firm}$$

$$P^* = MC = 8 \cdot 3 + 4 = \$28 \text{ per unit}$$

$$Q^* = 104 \text{ units produced by the entire market}$$

$$n^* = 104/3 = 34.67 \text{ firms in the market}$$

- d. Graphically illustrate the individual firm's supply curve, the short run equilibrium and the long run equilibrium. Draw the demand and the supply curves for an individual firm in one graph, and draw the market demand and the supply curves in an adjoining graph. Clearly label both graphs for both the short run and the long run equilibriums.



2. (Monopoly)

Consider a market for smart phones. Assume that there is only one firm in the market. The market demand for smart phones is given by the equation:

$$Q_d = 60 - P$$

The monopoly firm's total cost and marginal cost curves are given by the following equations:

$$TC = 60 + \left(\frac{1}{2}\right)Q^2$$

$$MC = Q$$

- a. Derive an equation for the marginal revenue curve for this monopolist.

The marginal revenue curve for a monopolist with a linear demand curve will have the same y-intercept as the demand curve and a slope that is twice the demand curve's slope. Thus, $MR = 60 - 2Q$. To get this answer you first need to rewrite the demand curve in slope-intercept form:

$$P = 60 - Q$$

Then double the slope to -2 and use the same y-intercept:

$$MR = 60 - 2Q$$

b. Find the profit maximizing price and quantity for this monopolist. Calculate the monopolist's profit.

The profit maximizing rule for a monopolist is to produce that quantity where $MR = MC$ and then charge the price from the demand curve associated with this quantity. Thus,

$$MR = MC$$

$$60 - 2Q = Q$$

$$Q = 20 \text{ units of output}$$

$$P = 60 - P = 60 - 20 = \$40 \text{ per unit of output}$$

$$\text{Profit} = (P - ATC) * Q$$

$$P = \$40 \text{ per unit of output}$$

$$ATC = 60/Q + (1/2)Q = 3 + 10 = \$13 \text{ per unit of output}$$

$$\text{Profit} = (P - ATC) * Q = (40 - 13) * 20 = \$540$$

c. Suppose that this monopoly decides to produce the level of output that would be produced if this market were a perfectly competitive market. Determine what this level of output would be. Then, determine the price the monopoly would charge and the level of profits that the monopoly would earn given this production and pricing decision.

Now, we need to find where $P=MC$ because a perfectly competitive market produces that level of output where the price paid by consumers for the last unit is equal to the marginal cost of producing the last unit of output. Thus,

$$60 - Q = Q$$

$$Q = 30 \text{ units of output}$$

$$P = \$30 \text{ per unit of output}$$

$$\text{Profit} = (P - ATC) \cdot Q$$

$$ATC = 60/Q + (1/2)Q = 2 + 15 = \$17 \text{ per unit of output}$$

$$\text{Profit} = (P - ATC) \cdot Q = (30 - 17) \cdot 30 = \$390$$

3. (Price Discrimination)

Suppose that the market for Chipotle is a monopoly. And there are two different types of consumers, Group A and Group B, that eat at Chipotle. The demand curves for these two groups can be described by the following equations:

$$\text{Group A's demand curve: } P = 30 - 2Q$$

$$\text{Group B's demand curve: } P = 10 - Q$$

You are also told that Chipotle's total cost curve and marginal cost curves can be described by the following equations:

$$TC = 2Q$$

$$MC = 2$$

a. Suppose the monopolist cannot distinguish whether people are in group A or are in Group B. The monopolist will therefore set a single price for their good. Given this information, what is the profit maximizing price and quantity for the monopolist? Calculate the monopolist's profit.

To answer this question you must first find the market demand curve. The market demand is:

$$P = 30 - 2Q \text{ if } P \geq 10$$

$$P = 50/3 - (2/3)Q \text{ if } P < 10$$

Once you have the market demand curve, you need to find the monopolist's MR curve. Since the market demand curve has two linear segments, the MR curve will have two linear segments. Thus:

$$MR = 30 - 4Q \text{ if } P \geq 10$$

$$MR = 50/3 - (4/3)Q \text{ if } P < 10$$

Now, set $MR = MC$, to find the profit maximizing quantity for the single price monopolist:

$$MC = MR$$

(Case I: $P \geq 10$)

$$30 - 4Q = 2$$

$Q=7$ units of the good

$P=30-2Q=\$16$ per unit of the good

Profit = $(16-2)*7 = \$98.00$

(Case II: $P<10$)

$2 = 50/3 - (4/3)Q = 2$

$Q = 11$ units of the good

$P = \$28/3$ per unit of the good

Profit = $(28/3 - 2)*11 = \$242/3 = \80.67

The profit is greater when $P=\$16$ and $Q=7$. Therefore, the monopolist determines the price at $\$16$ and produces 7 units of the good. The profit is $\$98.0$.

b. Now suppose that Chipotle distributes coupons to its customers and only people in Group B actually collect the coupon. The firm now can distinguish between consumers in Group A and consumers in Group B because only people in group B will have the coupon. Now the firm can charge different prices to the two groups. What price should the firm charge Group A and what price should the firm charge Group B if the firm wants to profit maximize? What quantity of the good will be sold to Group A and what quantity to Group B if the firm wants to profit maximize? When the firm engages in third degree price discrimination, what is the firm's profit?

To find the answer to this set of questions we need to first have a plan. The firm will set each Group's MR curve equal to the MC curve to find the quantity of the good it will provide to the group. Then it will find the price to charge by substituting this quantity into the Group's demand curve. Once have the price and quantity for each group it is a simple matter to calculate the profit with this pricing scheme.

Group A: $MR_A = 30 - 4Q$

$MC = 2$

$30 - 4Q = 2$

$Q_A = 7$ units of output

$P_A = 30 - 2Q = 30 - 2(7) = \16 per unit of output

Profit from group A = $(P - ATC) * Q_A = (16 - 2)*7 = \98

$MR_B = 10 - 2Q$

$MC = 2$

$$10 - 2Q = 2$$

$$Q_B = 4 \text{ units of output}$$

$$P_B = 10 - Q = 10 - 4 = \$6 \text{ per unit of output}$$

$$\text{Profit from group B} = (P - ATC) Q_B = (6 - 2) * 4 = \$16$$

$$\text{Total profit for the third degree price discriminating monopolist} = 98 + 16 = \$114$$

c. Now the firm acquires personal information about each of its consumers. Chipotle now has perfect information about each consumer's exact demand for its product. This monopoly firm uses this information to charge a different price to each of its consumers. Given that the firm is now practicing perfect price discrimination, how many Chipotles will be sold in this market? What is the firm's profit?

When the firm perfectly price discriminates it produces that quantity where the price paid for the last unit is equal to the marginal cost of producing that last unit. Thus, we need to find where $P = MC$ for each of the two groups. We will use the demand curve for each group for P and the MC curve. The demand curve in this situation is also the firm's MR curve, since each unit is sold for a different price, the marginal revenue from each unit sold is going to be equal to its price. Thus, $P = MC$ for Group A:

$$30 - 2Q = 2$$

$$Q = 14 \text{ units sold to the consumers in Group A}$$

$$P = MC \text{ for Group B:}$$

$$10 - Q = 2$$

$$Q = 8 \text{ units sold to the consumers in Group B}$$

$$\text{Profit from group A} = TR - TC = \left\{ \left[\left(\frac{1}{2} \right) * 28 * 14 \right] + 2(14) \right\} - 2(14) = \$156$$

$$\text{Profit from group B} = TR - TC = \left\{ \left[\left(\frac{1}{2} \right) * 8 * 8 + 2(8) \right] - 2(8) \right\} = \$32$$

$$\text{Total profit for this perfect price discriminating monopolist} = 156 + 32 = \$188$$

4. (Game Theory)

Imagining two drivers, Driver I and Driver II, are driving on a rainy night in opposite directions. There is a narrow bridge in front of the two drivers. The bridge is so narrow that both drivers know that only one vehicle can cross the bridge at a time. The drivers can barely see what's on the other side of the bridge, but a gleam of light tells each of them there's another driver on the other side of the bridge facing the same dilemma.

Now let's suppose if both of the drivers choose to drive onto the bridge then both drivers will get stuck in the middle, which is the worst scenario for both drivers. Each driver will spend 5 minutes backing up to get off the bridge and arrange for who will travel across the bridge first. On the other hand if one of the drivers chooses to wait while the other one

chooses to drive onto the bridge, then the one who drives first across the bridge will gain two minutes on their drive while the driver who has to wait will only lose a minute waiting. If both drivers decide to wait then they will both lose ten minutes time trying to figure out what to do. Assume that both drivers seek to minimize their additional travel time. The detailed payoff matrix is listed below:

		Driver II	
		Travel Across Bridge	Wait
Driver I	Travel Across Bridge	$(-5,-5)$	$(2,-1)$
	Wait	$(-1,2)$	$(-10,-10)$

a) Is the choice of both drivers choosing to wait an equilibrium? Explain your answer.

No, because if driver II chooses Wait, the other driver will choose Travel Across Bridge instead of Wait. So (Wait,Wait) cannot be an equilibrium.

b) Is there any strictly dominant strategy for Driver I?

No, because if driver II chooses Travel Across Bridge then Driver I will choose Wait, however if Driver II chooses Wait, then Driver I will choose Travel Across Bridge. Travel Across Bridge or Wait could both be Driver I's potentially better action. Neither of these two strategies is a dominant strategy since Driver I's better action depends upon Driver II's choice of strategy.

c) What is your prediction for the equilibrium outcome of this game?

I would predict either (Wait, Travel Across Bridge) or (Travel Across Bridge, Wait). Because if Driver II chooses Wait, it's better for Driver I to choose Travel Across Bridge rather than Wait. Now, if Driver I chooses Travel Across Bridge, then Driver II has to choose Wait. Since you and the other driver are exactly the same in terms of the payoffs, the other possible equilibrium is also clear if you think about it for a moment or two.

Suppose you saw that the driver on the other side of the bridge is your uncle who is a well-known slow tempo person. He also sees you. So, effectively you are now Driver I and your Uncle is Driver II. You understand your Uncle is always going to wait in this kind of situation so the payoff matrix now becomes:

		Driver II: Your Uncle	
		Travel Across Bridge	Wait
Driver I: You	Travel Across Bridge	$(-5,-5)$	$(2,3)$
	Wait	$(-1,2)$	$(0,3)$

a) Is there a strictly dominant strategy for your uncle?

Yes, Wait is his dominant strategy. No matter what your choice of strategy, Travel Across Bridge or Wait, your Uncle will choose Wait.

b) What is your prediction for the equilibrium to this game now?

I will predict (Travel Across Bridge, Wait). Since you know your Uncle will always choose Wait, you can safely choose Travel Across Bridge.

5. (Externality)

Jimmy is the troubadour of the town who sings songs everyday in the central park. He puts out his hat and people pay him for the songs he sings. The marginal private benefit (folks like his voice!) to Jimmy of singing Q songs is $MPB = 120 - Q$. But, Jimmy finds that singing tires him out: his marginal cost equation for singing is given by the equation: $MPC=2Q$. Assume that there are no fixed costs for Jimmy when singing.

a. Given this information, how many songs will Jimmy sing each day? What price will people pay per song (assume that the price is in dollars)?

Jimmy will solve:

$$MPB=MPC$$

$$120 - Q = 2Q$$

$$Q = 40 \text{ songs per day}$$

$$P = 120 - Q = 120 - 40 = \$80 \text{ per song}$$

b. What is Jimmy's total cost of singing each day and how much does he earn in revenue from singing each day?

The total cost singing equals the area of the triangle under his marginal private cost function. Since he will sing 40 songs a day, total cost = $0.5(40*80) = \$1600$.

Jimmy earns revenue of \$80 per song and he sings 40 songs, so each day he earns total revenue of \$3200.

From the mayor's perspective, everyone in the park is happier when Jimmy sings. To be specific, for each song he sings, it creates 30 dollars benefit for the town. That is the marginal social benefit of Jimmy's singing can be written as: $MSB = 120 - Q + 30$.

c) How many songs would the mayor Like Jimmy to sing each day? What is the deadweight loss that occurs in this situation when Jimmy only considers his marginal private benefit and not the marginal social benefit?

The mayor solves: $MSB = MPC$

$$120 - Q + 30 = 2Q$$

$$Q = 50 \text{ songs per day}$$

The mayor wants Jimmy to sing 50 songs each day.

$$DWL = (1/2)(\$30 \text{ per song})(50 \text{ songs} - 40 \text{ songs}) = \$150$$

d) In order to achieve that many songs, the mayor would like send Jimmy a direct subsidy for each song he sings. What will the subsidy per song need to be in order to get Jimmy to sing the optimal number of songs per day?

Assume the subsidy level is t . Then for each song he sings he will get $120 - Q + t$. Jimmy then solves:

$$120 - Q + t = 2Q \text{ or } Q = (120 + t)/3$$

The mayor wants $Q = 50$, so Jimmy plugs that into his equation to find $t = \text{subsidy per song} = \30 per song .

6. (Public Good)

There are two major department stores at the mall: Sears and JC Penny. They both benefit from the mall security patrol. Assume that the quantity of the mall patrol is Q units and that we can write down the marginal benefits and the total benefits provided by mall security for each store as:

$$\text{Sears: } MPB_{\text{sears}} = 5 - 0.5Q$$

$$\text{Total Benefit}_{\text{sears}} = 5Q - 0.25Q^2$$

$$\text{JC penny: } MPB_{\text{JC}} = 2 - 0.25Q$$

$$\text{Total Benefit}_{\text{JC}} = 2Q - 0.125Q^2$$

The marginal cost of hiring one additional unit of mall patrol is always \$4. That is $MC = 4$. Answer the following questions:

a. Assume Sears is the first department store in the mall. How many units of mall patrol will Sears choose to hire? What is the net benefit (its total benefit minus its total cost) to Sears of hiring this amount of mall security?

$$\text{Sears solves: } MPB_{\text{sears}} = MC$$

$$5 - 0.5Q = 4$$

$$Q = 2 \text{ units of mall security}$$

We can also calculate its net benefit:

$$\text{Total Benefit} = 5*(2) - 0.25*(2)(2) = \$9$$

$$\text{Total Cost} = 4*(2) = \$8$$

$$\text{Net Benefit} = \text{Total Benefit} - \text{Total Cost} = \$1$$

b. Now JC Penny opens a store in the mall. Notice that the mall security patrols that Sears hired also benefit JC Penny. That is both stores can consume this mall security: mall security is not rival and also is not excludable (JC Penny can benefit from the mall security even when it does not pay for any mall security). Now, that JC Penny is in business at the mall, does JC Penny want to hire any more units of mall security patrols? Explain your answer. What is JC Penny's net benefit after its decision?

At $Q = 2$, JC Penny's marginal benefit of hiring an additional patrol is:

$$\text{MPB}_{\text{JC}} = 2 - 0.25(2) = 1$$

$$\text{MPB}_{\text{JC}} = 1 < \text{MC} = 4$$

Therefore JC Penny has no incentive to hire any more mall security patrols. In this market, JC Penny is a free rider.

Its net benefit is now:

$$\text{Total Benefit} = 2*(2) - 0.125*(2)(2) = \$3.50$$

$$\text{Total Cost} = (4)(0) = 0$$

$$\text{Net Benefit} = \text{Total Benefit} - \text{Total Cost} = \$3.50$$

c. Now the leasing manager for the mall is in charge of determining how many mall security units should be hired. The manager will make her decision based upon the total marginal benefits of hiring mall security units. How many units of mall security will she hire? Explain your answer.

The total marginal benefit of mall patrol is the vertical summation of the individual Marginal Benefit Curves for the two companies. Thus,

$$\text{MTB} = \text{MPB}_{\text{sears}} + \text{MPB}_{\text{JC}} = 7 - 0.75Q$$

The manager then solves: $\text{MTB} = \text{MC}$ to find the optimal number of mall security units to hire. Thus,

$$7 - 0.75Q = 4,$$

Q = 4 units of mall security

d. Suppose the manager charges Sears \$3 per unit of mall security patrols and charges JC Penny \$1 dollar per unit of mall security patrols. Will the two stores both accept this price?

At Q = 4 The marginal benefit for Sears is $5 - 0.5 \cdot (4) = \$3$ per unit of mall security patrol. Thus Sears will accept the offer. Similarly at Q = 4 the marginal benefit for JC Penny is $2 - 0.25 \cdot (4) = \$1$ per unit of mall security patrol. Thus, JC penny will also take the offer.

e. What is the net benefit to the manager of hiring the amount of mall security patrols on the basis of the total marginal benefits of these patrols? What's the net benefit for both of these two department stores? How does your result compare with parts (a) and (b)?

Notice that the manager can gather $3 + 1 = \$4$ for each mall security patrol that he hires. Comparing this net benefit with the marginal cost of providing an additional mall security patrol, $MC = 4$, the net benefit to the manager is zero.

For Sears its net benefit is now:

$$5 \cdot (4) - 0.25(4)(4) - 3 \cdot (4) = \$4$$

For JC Penny its net benefit is now:

$$2 \cdot (4) - 0.125 \cdot (4)(4) - 1 \cdot (4) = \$2$$

Comparing this result with (a) and (b), we find Sears is better off, while the free rider JC Penny is worse off in terms of net benefit. However the total net benefit which is $4 + 2 = \$6$ in the last case, is bigger than the results we have from parts (a) and (b), where the total benefit is $1 + 3.5 = \$4.50$.