Economics 101
Fall 2013
Answers to Homework 5
Due Tuesday, November 19, 2013

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Question 1: Public program evaluation using Income and Substitution effect

In homework 4 of this semester question 5 was a question where you worked through some indifference curve analysis. Recalling that question, let us know interpret good X as units of bundles of food and good Y as units of bundles of luxury goods. Continue to assume that the consumer has their preferences for these two goods defined by the utility function $u(X, Y) = XY$. And, just like last week we can assume that the marginal utility of consuming X is given by $Mu_x = Y$ and the marginal utility of consuming Y is given by $Mu_y = X$.
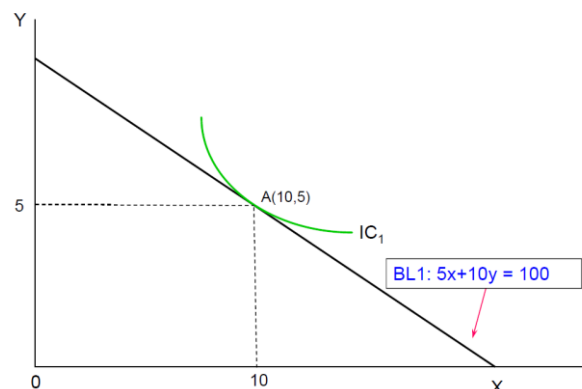
Use this information to answer the following questions.

   a) Suppose initially that price of a unit of a food bundle is $5 and the price of a bundle of luxurious goods is $10. Determine the optimal combinations of food bundles and luxurious goods bundles for this individual if her income available to spend on these two types of goods is equal to $100. Graphically illustrate the optimal bundle (let's call this combination of X and Y bundle A) in a well labeled graph; makes sure you indicate where the optimal consumption bundle A is located. Show the work you did to find your answer.

The equation for budget line is given by 5X+10Y = 100. The slope of the budget line is -0.5 (to see this, rewrite the equation for the budget line in slope intercept form, Y = 10 – 0.5X). Using the tangent condition that must hold at the optimal consumption bundle, one is able to show that

$$\frac{Mu_x}{Mu_y} = \frac{y}{x} = \frac{p_x}{p_y} = \frac{5}{10} \; \rightarrow \; x = 2y.$$

Substitute the above equation into the budget line equation, we obtain that $x = 10 \; and \; y = 5$.

   b) Suppose that the individual's income and the prices of both types of goods increase proportionately: for example, assume that income and the prices of both types of goods all triple. Compared to your answer in (a), how does this change in income and prices affect the optimal consumption bundle?
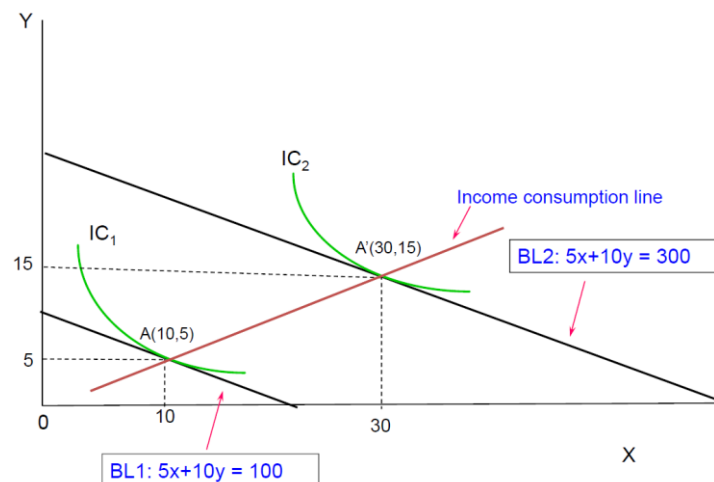
Because everything increases by a factor of three (income is now 3($100) or $300, the price of good X is now 3($5) or $15 and the price of good Y is now 3($10) or $30), the budget line remains the same. Note

from the question that with the new prices and income, the budget equation is now given by $15X+30Y = 300$. This equation is identical to $5X+10Y = 100$. So, we conclude that there is no change in the budget line, and hence no change in the optimal consumption bundle. This example shows you an important principle in that a proportional rise in the nominal value of price and income (i.e. inflation) doesn't alter you real feasible set of consumption, and hence your optimal consumption point.

    c) Suppose that income increases by three times the initial amount while the prices of good X and good Y stay at their initial level. Given these changes, what will happen to the new optimal consumption bundle? Given your result, can one of these two goods be a *Giffen goods* (hint: you might find it helpful to read in your text about Giffen goods-page 284)? Justify your reason. (Note: this question should be answered based solely on the result that you find in this question. No need to answer the next question and then return to this question.)

Now, we have a new budget line given the tripling of income. The equation of new budget line is given by $5X+10Y = 300$. The price ratio remains the same, with the value equal to $5/10$: this implies that the new budget line is parallel to the initial budget line. To obtain the new optimal consumption bundle, equate the price ratio to the marginal rate of substitution in the same way that we did for question (a). This gives us the equation $x = 2y$. Substituting the condition back into our new budget line equation, we would then find that $x = 30$, and $y = 15$.

The figure below illustrates the two budget lines and the optimal consumption bundle (point A for budget line 1 and point B for budget line 2) for each budget line. The curve that goes through both points is the income consumption curve (ICC). Notice from the figure that the consumption level of both goods increases in the same proportion as a rise in income. This implies that both goods are normal goods

Given that both goods are normal goods, it would then be <u>impossible</u> that either of these two goods is a Giffen goods. Giffen goods are a type of goods whose demand curve is an upward sloping line. Theoretically, this is possible, but it only happens in the case that the good is strongly inferior to the consumer. See page 284 in Krugman for further explanation.

    d) Return to the initial situation described in (a). Now, suppose that government imposes a $5 per unit tax on each food bundle purchased. Recalculate the optimal combination bundle for these two goods given this new tax. Then, determine the tax revenue that the government will raise from this individual.

With the \$5 per unit tax on each bundle of food, the new price of a food bundle will be \$10. Therefore, the new budget line equation is $10X + 10Y = 100$.

To solve for new optimal consumption bundle, we use the tangent condition. From the tangency condition, we find that
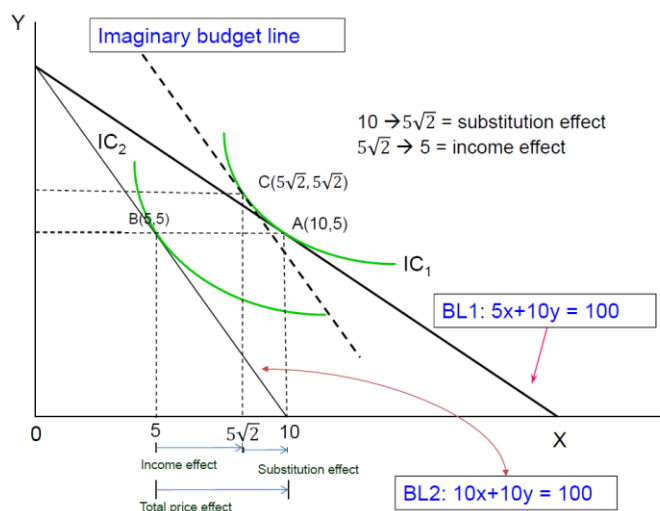
$\frac{Mu_x}{Mu_y} = \frac{y}{x} = \frac{p_x}{p_y} = \frac{10}{10} = 1$, which implies $y = x$.

Substitute the condition above into the new budget line equation: this gives us $y = x = 5$ as the optimal consumption bundle given the per unit tax on good X.

Correspondingly, the tax revenue from this individual is then equal to $tax * unit\ of\ X\ consumed = 5 * 5 = \$25$.

e) Draw a new graph of this individual's budget line and optimal consumption bundle from (a) as well as the new budget line and optimal consumption bundle once the tax per unit on good X is implemented by the government (you found this in (d)). Label the new budget line BL2 and the new optimal consumption bundle with the tax point B. Explain in your own words and with a step-by-step procedure how you can isolate the substitution effect from the income effect. For this problem, it might also be a good idea to calculate the bundle "C" on the imaginary budget line.

The initial optimal consumption bundle is A(10,5). This bundle-A yields us 50 units of satisfaction (U = XY = (10)*(5) = 50 units of satisfaction). We know from the figure below that "C" has to satisfy two conditions, i.e. the tangency condition where the budget line 3 (the imaginary budget line) is just tangent to indifference curve 1 and that the bundle yields us the same level of satisfaction as if it were at bundle-A. To get the numerical solution for bundle-C, we solve for (x,y) such that (i) $\frac{y}{x} = 1$ and (ii) $xy = 50$. The optimal consumption bundle for this individual when they are confronted by budget line 3 while holding utility constant and equal to 50 is given by $x = y = 5\sqrt{2}$.

As the price of the food bundle rises due to the implementation of the per unit tax on good X, the consumer tends to substitute away from the more expensive good (good X) and towards the good that is relatively cheaper (good Y). This is because both food and luxurious goods are substitutable. However, when the price of food increases due to the tax, this also implicitly changes the *real income* of the consumer. Therefore, the movement form A toward B is not solely due to the substitution effect, but in fact partly includes the income effect (the income effect is measured as the change in the consumption of good X as the individual moves from point C to point B).

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To isolate the substitution effect, one compensates the consumer with some amount of income transfer to make sure that he/she is as well off as before. Mechanically, this can be depicted as a parallel shift of BL2 to BL3 where BL3 is also tangent to the original IC curve (IC1). Point C in the figure above is the consumption bundle that arises under the imaginary budget line. Notice that both A and C are located on the same indifference curve. Thus, movement along the indifference curve is then considered as pure substitution effect (the substitution effect is measured as the change in the consumption of good X as the individual moves from point A to point C). Moving from C to B is then considered the income effect.

f) Given the new price of good X which includes the tax per unit on good X, how much money would the government need to give this individual to bring her back to her original level of satisfaction that she had before the tax on good X was implemented? If the amount of income compensation the government must make to the individual exceeds the total revenue the government collects from this individual with the tax on good X, what does this imply to you in terms of the welfare loss/gain of tax policy?

At the new price ratio, note that the income that would be needed to make the consumer as satisfied as they were before the tax was imposed is the value of the consumer's expenditure at bundle-C. We know from the previous question that bundle-C is $(5\sqrt{2}, 5\sqrt{2})$. Thus, the cost of this bundle can be calculated as $10 \times 5\sqrt{2} + 10 \times 5\sqrt{2} = 100\sqrt{2}$, which exceeds current income of consumer by $41 (an approximation). Recall that the tax revenue that the government raises from this individual from imposing the tax is $25. Thus, net tax revenue after rebating $41 to the consumer is negative, i.e. $25 − $41 = -$16. Therefore, tax policy creates a welfare loss of -$16 for this individual if measured in monetary terms.

Question 2: Production and cost analysis

Suppose that q is the number of patients that a hospital can serve per day. The number of patients served per day depends on two things: first, the number of doctors hired to practice in each day (L), and second the number of diagnostic machines (K) installed in the medical laboratory. The table below summarizes the production and cost functions for the hospital.

K	L	q	FC	VC	TC	AFC	AVC	ATC	MC	MPL
125	0				625	--	--	--	--	--
125	1									1
125		3								0.25
125	324	18								
125	625				1250					
125	1600	40								15/975
125	1936	44								
125	4624	68			5249					

a) Fill in the number for each cell in the table that has been left blanked. Make sure that the numbers that you fill in are consistent with one another.

K	L	q	FC	VC	TC	AFC	AVC	ATC	MC	MPL
125	0	0	625	0	625	--	--	--	--	--
125	1	1	625	1	626	625	1	626	1	1
125	9	3	625	9	634	208.33	3	211.3333	4	0.25
125	324	18	625	324	949	34.722	18	52.72222	21	0.04762
125	625	**25**	625	**625**	**1250**	**25**	**25**	**50**	43	**0.02326**
125	1600	40	625	1600	2225	15.625	40	55.625	65	15/975
125	1936	44	625	1936	2561	14.205	44	58.20455	84	0.0119
125	4624	68	625	4624	5249	9.1912	68	77.19118	112	0.00893

Given all the numbers that you've filled in, answer the following questions.

b) Is the hospital now operating under short-run or long-run production technology? Explain your answer.

The hospital is operating in the short-run because they have a fixed input, K, that creates a fixed cost of $625 no matter what level of output the hospital produces.

c) What is the price of a unit of capital and the price of a doctor?

Price of capital = FC/K = 625/125 = $5.

Price of doctor = VC/L = (1250 – 625)/625 = $1.

d) Find the range of output that minimizes average total cost.

ATC-minimizing output is the level of output such that MC = ATC. According to the table, ATC is greater than MC when q = 25. But when q = 40, it turns out that ATC is now smaller than MC. Since MC = ATC at the minimal point of average total cost, the range of output that minimizes average total cost has to be between 25 and 40 units of output.

e) Does the law of diminishing marginal productivity of labor hold given the technology of this hospital? Explain your answer.

Yes, because the marginal productivity of labor declines as more units of labor are utilized in production.

f) Suppose that the market price of medical service that each hospital can charge to each patient is $65. How many doctors will you hire if you wish to maximize the economic profit of the hospital? How much profit does the hospital earn when it profit maximizes? Explain your answer.

According to the profit-maximizing theory, the hospital should produce at that level of quantity where MR = MC (note that in this example, the price the hospital can charge each patient is also the marginal revenue the hospital receives when it treats each patient). Based on the table, MC = 65 when q = 40. Thus, the hospital should produce at this level, i.e. q = 40 (the hospital should see 40 patients a day). At the level of production, the hospital would need 1,600 doctors. Total revenue at q = 40 is 65*40 = $2,600 while total cost of operation is $2,225. When the hospital profit maximizes, its profits for the day are equal to $375.

g) Suppose that the price the hospital can charge each patient falls to $21 per patient. Should the hospital continue to operate in the industry in the short-run given this price decrease? Explain your answer.

Note first that price = $21: in this example, the price the hospital can charge per patient will also be the marginal revenue that the hospital will earn with each patient that is seen. The hospital should choose q = 18, where MR = MC. When the hospital produces at q = 18, the hospital earns profit equal to $21*18 – $949 = -$571. While it incurs negative profit, leaving the business would instead incur an even bigger loss in terms of the fixed cost that has been paid up front, i.e. -$625. Therefore, the best option that the hospital has in the short-run given this price change is to continue to operate at q = 18 and stay in the industry.

Question 3: Perfect competition and monopoly

Answer the following questions using the table below.

Firm 1	Quantity produced	Price per unit	Total Revenue	Marginal revenue	Total cost	Marginal cost	profit	Marginal profit	ATC
	0	$100	0		100				
	1	$100	100		130				
	2	$100			170				
	3	$100			220				
	4	$100			280				
	5	$100			350				
	6	$100			430				
	7	$100			520				
	8	$100			620				
	9	$100			730				
Firm 2	Quantity produced	Price per unit	Total Revenue	Marginal revenue	Total cost	Marginal cost	profit	Marginal profit	ATC
	0	200			100				
	1	180			130				
	2	170			170				
	3	160			220				
	4	150			280				
	5	140			350				
	6	130			430				
	7	120			520				
	8	100			620				

a) Complete the table for the two firms above.

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Firm 1	Quantity produced	Price per unit	Total Revenue	Marginal revenue	Total cost	Marginal cost	profit	ATC
	0	$100	0	100	100		-100	
	1	$100	100	100	130	30	-30	130
	2	$100	200	100	170	40	30	85
	3	$100	300	100	220	50	80	73.33
	4	$100	400	100	280	60	120	70
	5	$100	500	100	350	70	150	70
	6	$100	600	100	430	80	170	71.67
	7	$100	700	100	520	90	180	74.28
	8	$100	800	100	620	100	180	77.5
	9	$100	900	100	730	110	170	81.11
Firm 2	Quantity produced	Price per unit	Total Revenue	Marginal revenue	Total cost	Marginal cost	profit	ATC
	0	200	0		100		-100	
	1	180	180	180	130	30	50	130
	2	170	340	160	170	40	170	85
	3	160	480	140	220	50	260	73.33
	4	150	600	120	280	60	320	70
	5	140	700	100	350	70	350	70
	6	130	780	80	430	80	350	71.67
	7	120	840	60	520	90	320	74.28
	8	100	800	- 40	620	100	180	77.5

b) What is the profit-maximizing output for each firm? (Hint: remember that profit maximization requires MR = MC)

For firm 1, the profit maximizing output is 8 units. For firm 2, the profit maximizing output is 6 units.

c) What is the relationship between price, marginal revenue and marginal cost at the profit maximizing level of output for each firm?
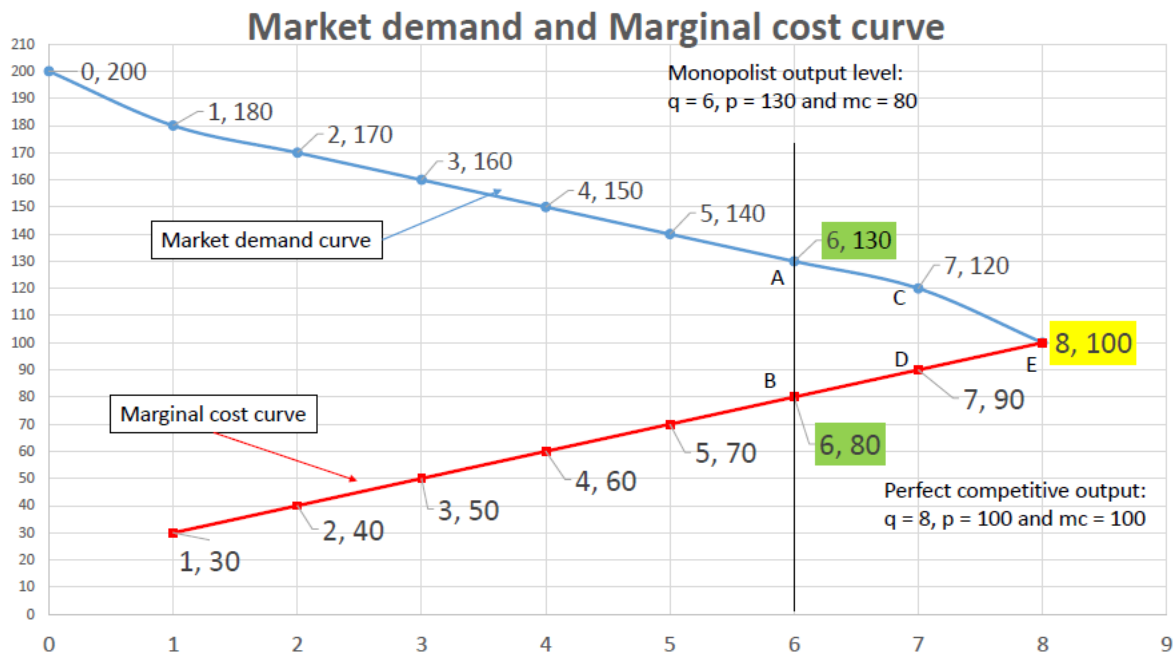
For firm 1, P = MR = MC at the profit maximizing level of output. For firm 2, P > MR at the profit maximizing level of output.

d) Which firm is in a competitive market, which is a monopolist and how do you know? Explain your answer.

Firm 1 is in a competitive market since P = MC and the firm is a price taking firm. Firm 2 is a monopolist since P > MR and the firm is a price searcher (that is, its output affects the price the monopolist charges).

e) What is the dollar amount of the dead weight loss for the monopolist? Explain your answer.

From the perspective of social optimality, it is socially optimal when markets are perfectly competitive since the market quantity of the good is that quantity where the marginal benefit of consuming an additional good (as measured by price) is equal to the marginal cost of producing that additional good (as measured by MC). In contrast, when the market is a monopoly, the monopoly leads to a higher price for the good than with perfect competition and a smaller quantity of the good than with perfect competition. The figure below compares the two outcomes that arise under 1) perfectly competitive and 2) monopoly. Note first that, under the monopoly environment, the firm chooses the profit maximizing level of output (where MR = MC), q = 6, and sets its price equal to 130 (p=130; this is the price from the demand curve for the monopolist when q = 6). In the case of the monopolist the price is set above its marginal cost (MC = 80 when q = 6). In contrast, given the same set up in the problem but a perfectly competitive industry (basically if we could get this monopoly to "act" as if it was in a perfectly competitive industry), the market price would have been 100 where P = MC = 100 and the market quantity would have been 8 units (where the demand curve intersects the supply curve, which in this case is the MC curve). So, the dead weight loss for the monopolist can be measured as the area of the geometric shape ACEDB that represents the difference in price due to the presence of a monopoly and the difference in quantity due to the presence of a monopoly. To calculate the area of the geometric shape ACEDB, we notice the following: area of ACEDB = area of ACDB + area of CED. The area of the trapezoid ACDB is ½ (50+30)*(7-6) = 40; meanwhile the area of triangle CED is ½ (120-90)*(8-7) = 15. Thus, the total area of ACEDB = 40 + 15 = 55. We can then conclude that the dead weight loss of monopoly is $55.

Market demand and Marginal cost curve

Question 4: Monopoly

Suppose there is a monopoly that mines diamonds. The marginal revenue and marginal cost functions for this monopolist are given by the following equations:

MC $= 2 + 2q$

MR $= 10 - 2q$

(a) Given the above information, write the monopolist's demand curve for diamonds in y-intercept form? Assume this demand curve is linear.

Since the marginal revenue function for a monopolist with a linear demand curve has twice the slope of the demand curve and the same y-intercept as the demand curve we can write the demand curve in slope-intercept form as

=> P = 10 - Q is the demand.

(b) Given the above information, what is the profit-maximizing level of output for this monopolist?

Profit-maximization occurs at that quantity where MC = MR. So we get: 8 = 4q => q = 2 is the profit maximizing quantity for the monopolist.

(c) What price should the monopolist charge when it produces the profit maximizing quantity? Explain your answer.

In order to find the price charged to consumers, we plug q = 2 into the demand curve. This allow us to determine the price consumers are willing to pay when they consume 2 units of the good. P = 10 - 2 => P = 8.

(d) Suppose you know that the monopolist's total costs can be calculated using the equation TC $= 2 + 2q + q^2$. Given this information, write equations for TFC, TVC, and ATC.

TC $= 2 + 2q + q^2$. Since fixed cost is the same regardless of quantity: TFC = 2. TVC is the total cost associated with changes in the level of output, so: TVC $= 2q + q^2$. Lastly, ATC = TC / q => ATC $= 2/q + 2 + q$.

(e) Given the above information and the work you have done, now calculate the monopolist's economic profit. How much profit/loss does the firm earn? Show how you calculated this answer.

The revenue for the monopolist is the optimal price times the optimal quantity => TR = 8*2 = $16.

The total cost at the profit maximizing quantity is: TC = 2 + 2*2 + 4 = 10.

So, profit is given as: TR - TC = $6. So, yes the firm makes a positive economic profit.

(f) Suppose the monopolist's fixed costs change and they are now equal to $8. How would this change the firm's TC equation? What is the economic profit for the monopolist given this change? Would your answer change if the fixed costs were to change to $10? Explain your answer fully.
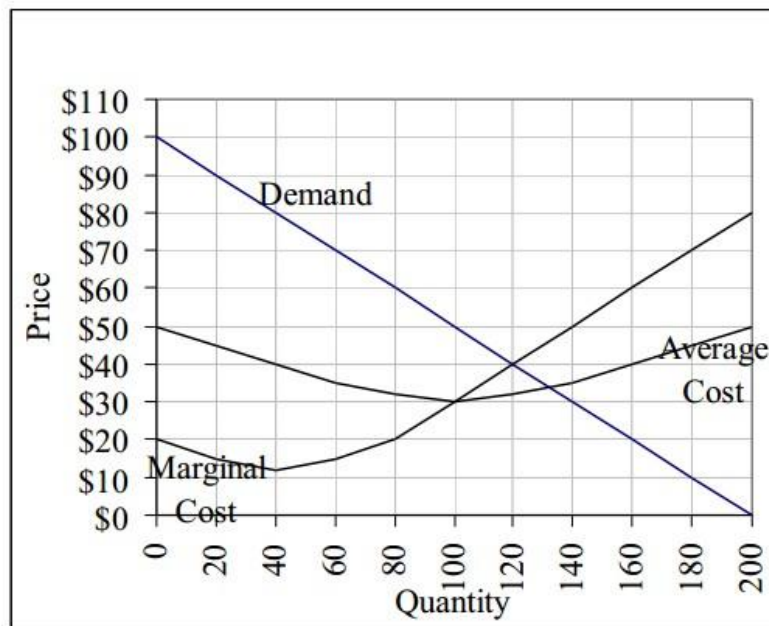
If the fixed costs were instead 8 (10), the marginal cost (the cost of producing an additional unit) would not change so the same optimal quantity would be produced => TR = $16 and TC = 8 + 4 + 4 = $16 (TC = 10 + 4 + 4 = $18). Economic profit would be 0 (-$2).

    (g) What does this tells us about a monopolist's profit and the costs of obtaining a monopoly?

It tells us two things: first, monopolies may be expensive to acquire since the creation of entry barriers is likely to be expensive; secondly, it tells us that certain types of monopolies may have very high fixed costs for reasons unrelated to the market structure. For example, a utility company may have large fixed costs and so earn negative economic profit even though they are the only firm in the market.

Question 5: Monopoly

Use the following graph for a monopoly to answer this question.

    a)   What will be the price and quantity sold for the monopoly? How much is the monopoly earning in profits? Explain your answer fully using a step-by-step approach. (Hint: Based on the graph, you won't be able to determine the exact value of average total cost at the profit-maximizing output. To calculate the total profit at that level of output, just use an approximate value of average total cost that you think is reasonable.)

First we must draw the Marginal Revenue curve, which bisects the x-axis halfway between the origin and where the linear demand curve crosses the x-axis. Next, we find the profit maximizing quantity where MR=MC, which corresponds to a quantity of 80 units. Finally, we go up to the demand curve to find the price the monopoly will charge, which is $60 per unit of the good. Profits = (price – average total cost) * quantity = ($60 - $32)*80 = $2240. (The purpose of this question is to show you how to construct the MR curve when we know that demand curve is the straight line. Based on what we see from the graph, we believe that ATC should be around $32. As we've mentioned in the hint, if you use any values of ATC that is close to $32, your answer is correct.)

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b) Based on your answer to part (a), explain in words what you think will happen in the long run.

This monopoly is earning positive profits. We know firms want to enter the industry (since there are positive profits), but because there are barriers to entry, new firms can't enter. The monopoly can continue charging this price and earning this profit in the long run.

c) Is the market efficient with the monopoly? If so, explain how you know this. If not, state what quantity would be economically efficient and why that quantity would be economically efficient.

No. In order to be economic efficient, we must be where the marginal benefit of the last unit equals the marginal cost of making the last unit. At a quantity of 80, the marginal benefit of the last one is $60 (from the demand curve), the price someone is willing to pay for the product. The marginal cost of making the 80th unit is only $20 (from the MC curve). The last unit was worth $60 to someone and only cost $20 to make, so we should be producing a higher quantity. The quantity of 120 is efficient. The demand curve shows someone is willing to pay $40 for the 120th unit and the MC curve shows the marginal cost of making the 120th unit is $40. Thus, at 120 units the MB = MC and we are efficient.

Question 6: Perfect competition in short run and long run

Suppose a firm sells its product in a perfectly competitive market where all the firms in the market charge a price of $40 per unit for the good. The firm's total cost is TC $= 40 + 8q - 2q^2 + q^3$ and the firm's MC $= 8 - 4q + 3q^2$.

a) How much output should the firm produce in the short run? Explain your answer.

A firm in a perfectly competitive market is a price taking firm. This implies that P = MR for the firm. To find the profit maximizing level of output for the firm we need to set P = MC. Thus, $40 = 8 - 4q + 3q^2$. This can be simplified to $0 = 32 + 4q - 3q^2$. Hmm, do you remember from your math class how to solve this? Try factoring or the quadratic equation (you may have to go to the internet to retrieve that formula!). Thus, $0 = (8 + 3q)(4 - q)$. Then, set $0 = 8 + 3q$ and $0 = 4 - q$ and solve for the two solutions for this equation. One solution will provide you with a negative quantity and we can conclude that this solution is not the correct one since the firm is not able to produce a negative quantity of the good. The other solution will be q = 4 units.

b) What price should the firm charge in the short run? Explain your answer.

Since this is a perfectly competitive market then all the firms in the market charge the same price as the market price. The market price is the $40 that each firm charges for a unit of the good.

c) What are the firm's short-run profits? Explain your answer.

To calculate profit, calculate total revenue (TR) and total cost (TC). TR = P*q = ($40 per unit)(4 units) = $160. TC $= 40 + 8*4 - 2 * 4^2 + 4^3 = $104, so the profit is $160 – $104 = $56.
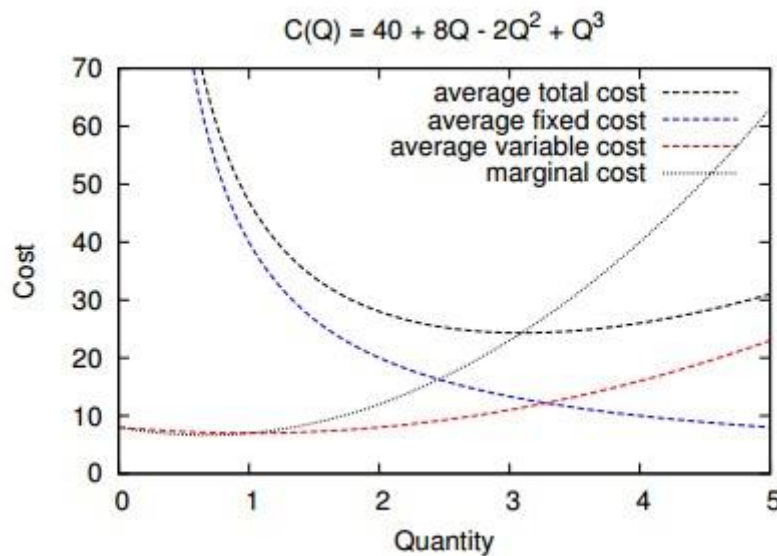
d) Given your answers in (a) through (c), what do you predict will happen in this market in the long run? Explain your answer.

In (c) you found that profits for this perfectly competitive firm were positive in the short run. This tells us that this firm is earning profits in excess of its opportunity cost and we should expect entry of new firms to

occur in the long run until the market price falls to that level where each firm in the industry will earn zero economic profit.

e) Draw a graph that represents this firm's average variable cost, average total cost and marginal cost for q between 0 and 5. Draw all curves in the same figure and as exactly as possible. How can you determine the long run market price for this perfectly competitive firm's product graphically?

Because other firms will copy your production process in the long run, there will be market entry until profit is zero, and MC = ATC for this representative firm in this perfectly competitive market.

$$C(Q) = 40 + 8Q - 2Q^2 + Q^3$$

Question 7: Perfect competition

Assume that the taxi industry in the town of New City is perfectly competitive. Also assume that the marginal cost of a taxi ride is constant and equal to $5 per trip (this assumption will make the math far simpler for this question) and that each taxi is capable of making 20 trips a day. We will let the demand function for taxi rides each day be $Q = 1,100 - 20P$.

a) What is the perfectly competitive price of a taxi ride?

At the perfectly competitive equilibrium, P = MC. Therefore, the competitive price of a taxi ride is $5.

b) How many rides will the citizens of New City make every day?

Substituting the P into the demand function, we find that the equilibrium numbers of taxi rides every day is $1,100 - 20*5 = 1,000$ taxi rides per day.

c) How many taxis will operate in New City?

Given that each taxi is capable of making 20 trips a day, the number of taxis needed in

New City is $1,000/20 = 50$ taxis.