Economics 101
Spring 2018
Answers to Homework #4
Due Thursday, April 5

Directions:

- The home will be collected in a box **before** the lecture.
- Please place **your name, TA name, and section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Please **staple** your homework: we expect you to take care of this prior to coming to the large lecture. You do not need to turn in the homework questions, but your homework should be neat, orderly, and easy for the TAs to see the answers to each question.
- Late homework will **not** be accepted so make plans ahead of time.
- **Show your work**. Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional? For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you submit any work for someone else.

Part I – Elasticity

1. Consider the following market demand and market supply curves for Kimchi in Madison, where P is the price per unit of kimchi and Q is the quantity of kimchi.

Demand: $Q = 150 - 3P$
Supply: $Q = 2P$

a) Given the above information find the equilibrium price and quantity in this market.

Solution:
Set demand equal to supply, thus,
150 – 3P = 2P, we get
Pe = 30, Qe = 60.

b) Calculate the point elasticity of demand at this equilibrium. Is demand elastic or inelastic at this point? Explain your answer. Given this answer, is the total revenue of producers being maximized at this equilibrium? If not, will producers enhance their total revenue by increasing or by decreasing the price they charge for kimchi? Explain the reasoning behind your answer.

Solution:
Using the point elasticity formula to calculate the price elasticity of demand:
e = – (1 / slope) * (P / Q)

Note, in this equation, slope is "rise/run" assuming price is on the vertical axis, so before we calculate the elasticity, we should rewrite the demand in P-intercept form:
P = (–1/3) *Q + 50, with Pe = 30, Qe = 60 from part (a), we get
e = – (1 / (–1/3)) * (30 / 60) = 3/2 = 1.5

Since the value of point elasticity of demand is larger than one, we can conclude that demand is elastic at the equilibrium point. Then total revenue is not maximized at this equilibrium point and total revenue will increase if producers decrease the price of kimchi since demand is elastic.

c) Calculate the point elasticity of supply at this equilibrium. We know on kimchi's demand curve, the price elasticity of demand will decrease as the price of kimchi decreases (or quantity increases). Will this property hold for the price elasticity of supply for kimchi? (*Hint:* use the point elasticity formula to verify your answer.)

Solution:
Again, before we apply the point elasticity formula, we should rewrite the supply curve in P form:
P = Q/2,
Then use the point elasticity formula to calculate the price elasticity of supply:
e = (1/ slope) * (P / Q) = (1 / (1/2)) * (30 / 60) = 1
(Pay attention: for supply, we do not want a negative sign here, since supply curves have a positive slope.)

For a certain P (P > 0), with point elasticity formula, we find the price elasticity of supply:
e = (1/ slope) * (P / Q) = (1/ (1/2)) * (P / 2P) = 1
So, the price elasticity of supply for kimchi on the supply curve is a constant number (all are unit elastic points, you can ignore the origin here) and it's another case for a curve with all points on it to have same elasticity.

d) Suppose the government decides to impose an excise tax of $10 per unit on kimchi. Calculate the Consumer Tax Incidence (CTI) and Producer Tax Incidence (PTI) of this excise tax. Compare the ratio of the tax incidence (CTI/ PTI) with the ratio of elasticity (price elasticity of demand/ price elasticity of supply) at the initial equilibrium. Does our conclusion about the relationship between elasticity and tax incidence (in homework #3, problem 1, part (d)) still hold here?

Solution:
The tax has the effect of adding a $10 per unit cost to producers, shifting the supply curve upward. The original supply curve had a P - intercept of $0, so the new will have a P - intercept of $10. The slope is unchanged, so using this information we can determine that the new supply curve will be P = Q/2 + 10.
As before, set demand equal to supply, thus,
(–1/3) *Q + 50= Q/2 + 10, we get

Qt = 48, Pt = $34.
Here Pt is the price consumer paid after tax is imposed.
The price producers will receive equals the new equilibrium price minus the excise tax. Thus Pnet = $34– $10 = $24.

CTI = (Pt – Pe) * Qt = (34 – 30) * 48 = $192
PTI = (Pe – Pnet) * Qt = (30 – 24) * 48 = $288
CTI/PTI = 192/288 = 2/3.
From part (a) & (b), we know
price elasticity of demand/ price elasticity of supply = 1.5/1 = 3/2.

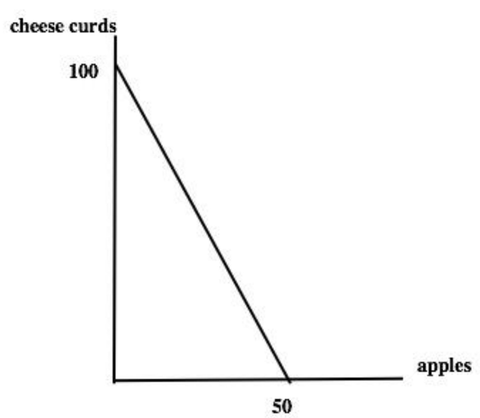
Then we can find that the ratio, CTI/PTI, is the reciprocal of the ratio of elasticity at the initial equilibrium point. And the conclusion still holds: The tax incidence will depend on the price elasticities of supply and demand and the tax falls mainly on the side of market with the lower (more inelastic) elasticity.

Part II – Consumer Theory

2) Ally's income is $100. She consumes cheese curds and apples. Suppose the price of cheese curds is $1 and price of one apple is $2.

a. Draw Ally's budget line on the graph. Put cheese curds on y-axis and apples on x-axis. What is the equation of the budget line in slope-intercept form? Let A stand for apples and C for cheese curds.

Solution: The equation of the budget constraint or budget line is found by using the base formula I = PxX + PyY. Thus, C + 2A = 100 and writing this in slope intercept form we get,
C = 100 – 2A. The y-intercept is 100, the x-intercept is 50, and the slope is -2.

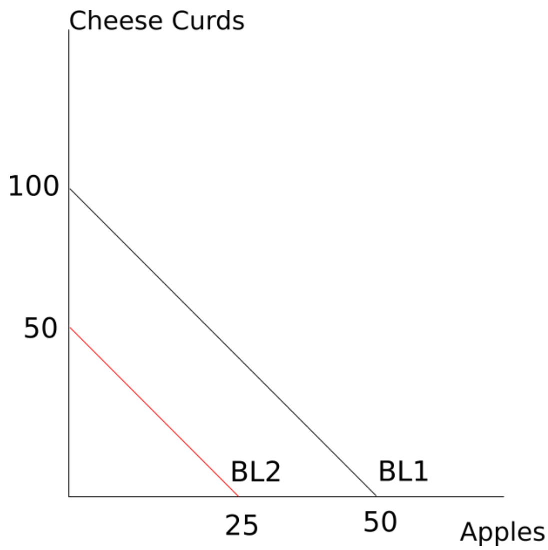


b. Ally decides she wants to have 25 servings of cheese curds and 40 apples every month. Is this bundle affordable for her? Does it exhaust her income? What if her choice is 15 servings of cheese curds and 40 apples?

Solution: The first bundle is not affordable: it costs 25 + 40*2 = $105, which is more than her income of $100. The second bundle is affordable but does not exhaust her income: 15 + 2*40 = $95 < $100.

c. Ally works as an Economics 101 TA. The department decides to decrease TAs' wages. Ally's new income decreased to $50, while the prices for the two goods stay at their initial level. What is the equation of this new budget line (BL1)? How does it compare to the budget line from part (a), (BL2)?

Solution: the new budget constraint or budget line is C + 2A = 50 or in slope intercept form C = 50 – 2A. The new budget line has the same slope as the original budget line but is now closer to the origin: the change in income holding prices constant results in a parallel shift of the budget line. BL1 on the graph stands for the budget line with income $100 and BL2 on the graph stands for the budget line with income $50.

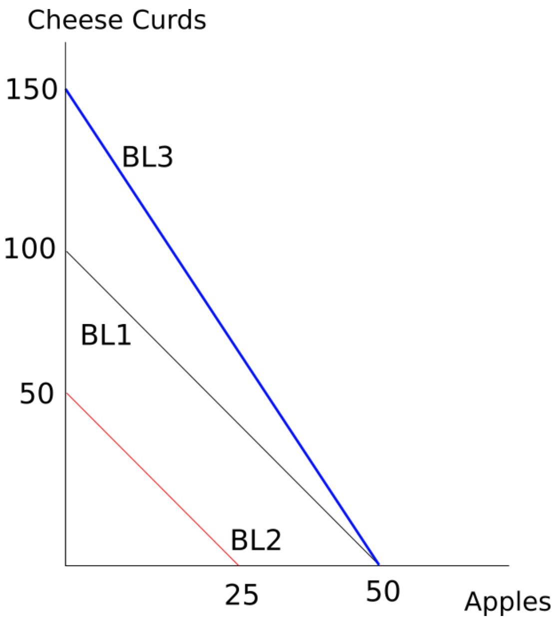


d. Think of a different scenario. Let the income level be at $100, but due to the financial crisis the prices for both goods doubled from their initial levels. What happened to Ally's budget line? How does it compare to the budget lines, BL1 and BL2, from parts (a) and (c)?

Solution: the new price of apples is $4 and the new price of a serving of cheese curds is $2. The new budget line is 2C + 4A = 100 or C = 50 – 2A. The new budget line is a parallel shift relative to the original budget line and this new budget line is exactly the same as the budget line in (c), (BL2).

e. Suppose now it is known that the prices of apples and cheese curds have changed so that Ally now consumes 60 servings of cheese curds and 30 apples with her income. The new price of one unit of cheese curds is $1 and the price of one apple is $3. What is Ally's new budget line? Draw a graph where you represent this new budget line, BL3, along with budget lines BL1 and BL2. Label your graph carefully and completely!

Solution: The income after her decision is 1*60 + 3*30 = $150, so the budget line is C + 3A = 150 or in slope intercept form C = 150 – 3A. The new budget constraint or budget line on the graph is BL3.



f. Let Ally's utility function be U = AC, and thus her marginal utilities are given by $MU_C = A$ and $MU_A = C$. Assume that the price of one serving of cheese curds at $1, price of one apple $2 and income at $100. Given this information and holding everything else constant, what is the consumption bundle, (A, C) that maximizes her utility?

Solution: From the solution to (a) we know that the budget constraint has an equation C = 100 – 2A. Recall that the optimal bundle that maximizes the individual's utility will equate marginal utilities per dollar:

$$Mux/Px = MUy/Py$$

This implies that we must have A = C/2. Using this equation along with the budget equation, we have two equations and two unknowns. Plugging A = C/2 into the budget line, we have C = 100 – C, so C = 50 and A = 25. Consuming the bundle (A, C) = (25, 50) will maximize Ally's utility given the constraints imposed by the income and the prices of the two goods.

3) Assume a graduate student consumes only two food items: coffee(X) and chocolates(Y). He spends his monthly stipend of $300 on these two food items. One bar of chocolate costs him $2 and one cup of coffee costs him $2. The utility function for this graduate student is given by the equation:

$$U = XY^2$$

This graduate student's marginal utility from consuming coffee (X) and his marginal utility from consuming chocolates (Y) are given by the following equations:

$$MU_X = Y^2$$

$$MU_Y = 2XY$$

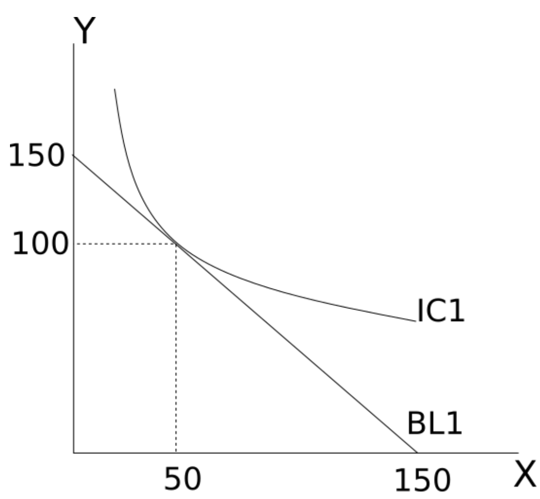
(a) Given the above information and holding everything else constant, graph the budget line (BL1) with chocolates measured on the y-axis and coffee measured on the x-axis. Under the current price levels, how many bars of chocolates and how many cups of coffee does the graduate student consume when this student maximizes their utility? What is the total utility at this optimal bundle?

The tangency condition for utility maximization is:

$$\frac{MU_X}{MU_Y} = \frac{Y^2}{2XY} = \frac{P_X}{P_Y} = \frac{2}{2} = 1$$

We have 2X = Y for the utility maximizing bundle. The student will spend all $300 on the two goods hence, $X\,p_X + Y\,p_Y = 2X + 2Y = 300$

Together, 3Y = 300 and Y = 100 and hence, X =50.
$U = XY^2 = 50 * 10{,}000 = 500{,}000.$

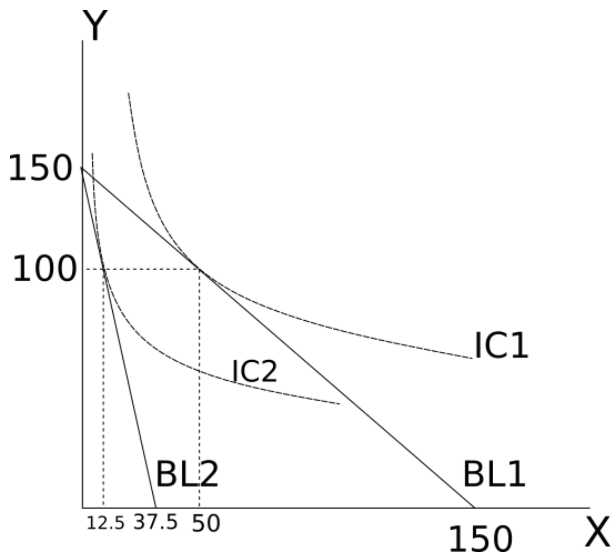


(b) Due to an environmental tax imposed on the usage of paper cups, the price of coffee has increased to $8 per cup. How does the budget line, BL2, change? Given this change and holding everything else constant, what is the optimal consumption bundle now? Show all your work for how you found this answer! **Assume in this problem that it is possible to consume partial cups of coffee.**

The budget line shifts in. Again, the tangency condition for utility maximization is:

$$\frac{MU_X}{MU_Y} = \frac{Y^2}{2XY} = \frac{P_X}{P_Y} = \frac{8}{2} = 4$$

We have Y = 8X for the utility maximizing bundle. $X\,p_X + Y\,p_Y = 8X + 2Y = 300$. Hence, 3Y = 300 and X = 12.5 and Y = 100.



(c) Find the income and substitution effects on the amount of coffee consumed due to this environmental tax. Show your work and provide your reasoning in finding this answer. (You will need a calculator for some of the computations.)

To find the income and substitution effects, one needs to find the intermediate point, at which the original indifference curve is tangent to the hypothetical budget line (referred to as BL3 in your class notes) parallel to the new budget line (referred to as BL2 in this problem and in your class notes). In other words, the intermediate point gives the utility level of 500,000 but with the new price ratio.

$XY^2 = 500{,}000 \;\; and \;\; \frac{MU_X}{MU_Y} = \frac{Y^2}{2XY} = \frac{P_X}{P_Y} = \frac{8}{2} = 4$

$XY^2 = 500{,}000 \;\; and \;\; Y = 8X$

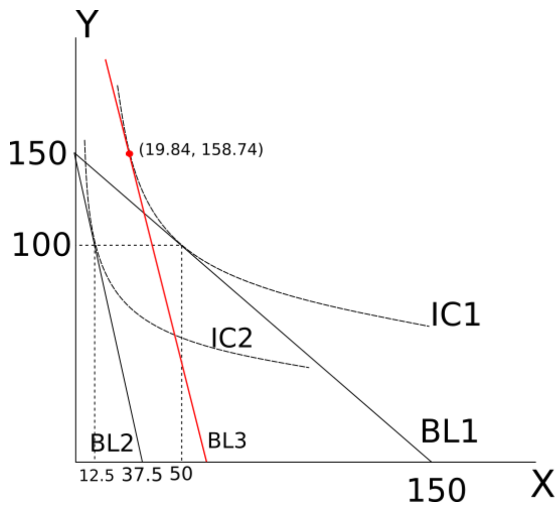
$64X^3 = 500{,}000$

$X^3 = 7812.5$

$X = 19.84$

X = 19.84 and Y = 158.74

The substitution effect on coffee consumed is -30.16 cups of coffee since consumption of coffee falls from 50 units to 19.84 units when we hold utility constant but consider how this individual responds to the change in the price of coffee from \$2 a cup to \$8 a cup (the individual substitutes away from the relatively more expensive coffee). The income effect is -7.34 cups of coffee since consumption of coffee falls from 19.84 units to 12.5 units when we compare the utility maximizing bundles on BL3 to BL2. Here the consumer is comparing his consumption with the adjusted or compensated income (BL3 where he gets the same level of utility as he had initially but faces the new prices) and the consumption he chooses when he faces the new prices and the given level of income (BL2).



(d) The economics department agrees that the reduction in the level of coffee consumption due to this environmental tax reduces the productivity of graduate students. The department approves a cash grant for this graduate student such that his utility is restored to the pre-environmental tax level of utility. What is the amount of the cash grant this student receives? What is the optimal consumption bundle for this student after receiving the cash grant? Show your work and your reasoning for this set of answers.

Receiving a cash grant pushes the new budget line outwards whilst keeping the slope fixed to the ratio of the new price levels. The tangency is still $\frac{MU_X}{MU_Y} = \frac{Y^2}{2XY} = \frac{P_X}{P_Y} = \frac{8}{2} = 4$ but the utility level is back to 500,000. The hypothetical budget line in the graph for part (c) and the optimal bundle the student chooses after receiving the cash grant will be exactly (19.84, 158.74). To be able to consume (19.84, 158.74) under the current price levels, the student needs an income of 8*19.84 + 2*158.74 = 476.20. His current income is \$300 so he needs additional income of \$176.20.

(e) Some of the faculty disagree with this plan described in (d). They instead propose a cash grant for this graduate student so that he can consume the exact bundle that he used to consume before the environmental tax. What is the amount of this cash grant? Would he stick to the original consumption bundle after receiving this cash grant? What is the new optimal consumption bundle given this new cash grant? Show how you found your answers and provide a cogent explanation behind your work.

The original bundle of consumption was (50,100). Under the new prices, to consume this original bundle the student needs: 8*50 + 2*100 = $600. The department needs to give the graduate student an additional income of $300. When the additional income is given, it is not optimal for the student to consume the original bundle of (50,100).

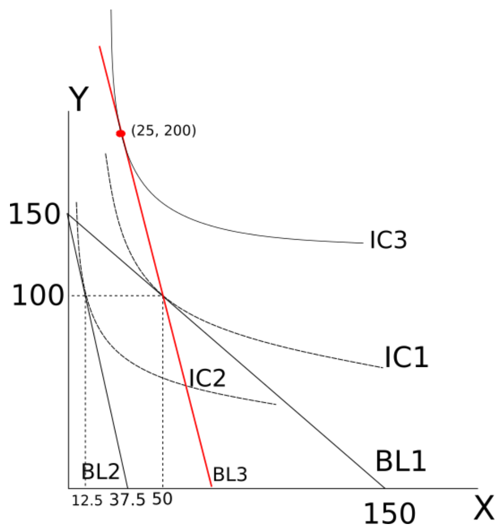
$\frac{MU_X}{MU_Y} = \frac{Y^2}{2XY} = \frac{P_X}{P_Y} = \frac{8}{2} = 4$ so, Y = 8X

The new budget line after the cash transfer is:

$$X\, p_X + Y\, p_Y = 8X + 2Y = 600$$

Hence, 24X = 600 and X = 600/24 = 300/12 = 150/6= 75/3 = 25.

Y = 8(25) = 200. Plugging this bundle into the utility function, we get U = 1,000,000, so the student does strictly better under this compensation scheme.



Part III – Producer Theory

4) You have just graduated, and your first job is as a manager at Widget Co., a company that makes widgets. Your engineer gives you the following table of costs, but, for some reason, he has forgotten to fill in most of it. The per unit cost of labor (the wage) is $10, and the per unit cost of capital is $50.

Q	**K**	**L**	**MPL**	**APL**	**FC**	**VC**	**TC**	**AFC**	**AVC**	**ATC**	**MC**
0	10	0	-	-				-	-	-	-
	10	1									$1
	10			12.5				$20			
35	10	3									
	10					$40				$13.50	
	10						$550				$10

a) Using your knowledge of producer theory, fill in the missing information from the table. (You may need a calculator for some of the entries.) Remember that:

MPL is the [(change in total product)/(change in total labor)]
APL is the [(total product)/(total labor)]
AFC = FC/Q
AVC = VC/Q
ATC = TC/Q
MC = [(change in total cost)/(change in output)]

Start by filling in the easy entries: Cost of capital is $50 per unit, so fixed costs are $500 all the way down. Now we systematically move down the table one row at a time. In the first row, no labor is used so VC = 0, and TC = FC = $500.

Moving down one row, we are now using 1 unit of labor, so VC = $10 and TC = $510. We know that MC = $1 on this line, and that tells us that the (change in total cost)/(change in output) = 1. The change in total cost is 510 – 500 or $10. We can then solve this equation to find the change in output. The change in output must be 10 units. Or, since MC = $1, and we increased total spending by $10, we can infer that Q = 10. Since labor increased by 1 and quantity increased by 10, MPL = 10. Using this, we can derive the various average costs, by dividing by quantity.

Moving down again, in the third row we are given AFC = 20. Since AFC = FC/Q, we can solve for Q, finding Q = 25. Since APL = Q/L, we have L = 2. Since labor increased by 1 and quantity increased by 15, we can see MPL = 15. Since we increased spending by another $10 and increased production by 15, we have MC = 10/15 = $0.67. We can then fill in average costs as usual.

In the fourth row, we're given Q = 35 and L = 3. MPL thus is 10 since quantity increased by 10 in response to increasing labor by 1 unit. APL is simply 35/3 = 11.67. Spending increased by $10, and the quantity increased by 10, so MC = $1. The remaining costs can be filled in directly using the standard formulae.

In the fifth row, we are given VC = $40, so using the cost of labor, we have L = 4. With these, we can compute all the relevant total costs. Since we now know TC = $530, and ATC = $13.50, we must have Q = 40. From there we can compute the remaining average costs and marginal cost in the usual way.

Finally in the last row, total cost is $550, so L = 5. Since MC = $10, and the cost of the last unit of labor was $10, Q must have only increased by 1, so Q = 41. With that the remaining columns can be filled from the usual formulae.

Q	K	L	MPL	APL	FC	VC	TC	AFC	AVC	ATC	MC
0	10	0	-	-	$500	$0	$500	-	-	-	-
10	10	1	10	10	$500	$10	$510	$50	$1	$51	$1
25	10	2	15	12.5	$500	$20	$520	$20	$0.80	$20.80	$0.67
35	10	3	10	11.67	$500	$30	$530	$14.29	$0.86	$15.14	$1

40	10	4	5	10	\$500	\$40	\$540	\$12.50	\$1	\$13.50	\$2
41	10	5	1	8.20	\$500	\$50	\$550	\$12.19	\$1.21	\$13.41	\$10

5) Suppose the market for cheese curds is perfectly competitive. The demand curve for cheese curds is given by

$$Q = 600 - 50P$$

Suppliers of cheese curds are all identical and have the following cost structure:

Marginal cost for the representative firm: $MC = 2q$
Total cost for the representative firm: $TC = q^2 + 9$

a) Find the equations for the fixed costs, variable costs, average variable costs, and average total costs for a representative firm. Plot marginal costs, average variable costs, and average total costs for a representative firm on the same graph.

From the total cost equation, we can see fixed costs are simply FC = \$9 and variable costs are $VC = q^2$. Using the regular formulae, we have $AVC = q$ and $ATC = q + 9/q$. Plotting these we have