Economics 101
Fall 2018
Answers to Homework #4[1]
Due Tuesday, November 20, 2018

Directions:

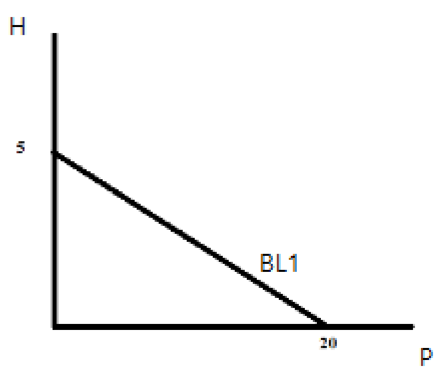
- The homework will be collected in a box labeled with your TA's name **before** the lecture.
- Please place **your name**, **TA name** **and** **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Please **staple** your homework: we expect you to take care of this prior to coming to the large lecture. You do not need to turn in the homework questions, but your homework should be neat, orderly, and easy for the TAs to see the answers to each question.
- Late homework will not be accepted so make plans ahead of time.
- **Please show your work.** Good luck!

Part I: Consumer Theory (Budget lines, Indifference Curves, Demand Curve, Income and Substitution Effects)

1. Jim is a Cubs fan with a fixed income of $100. Jim only purchases two goods: bags of peanuts (P) and Cubs baseball hats (H). Initially each bag of peanuts costs $5 and each baseball hat costs $20.

a. Find the equation for Jim's budget line and then graph this budget line (BL1) in a well-labeled graph. Measure hats on the vertical axis and bags of peanuts on the horizontal axis.

Answer: Let P and H denote the quantity of bags of peanuts and Cubs' hats respectively. 5P + 20H =100
Rearranging the terms, we get H = 5 - 0.25P



[1] This homework is partially adapted from Econ 101, 2018 Spring semester.

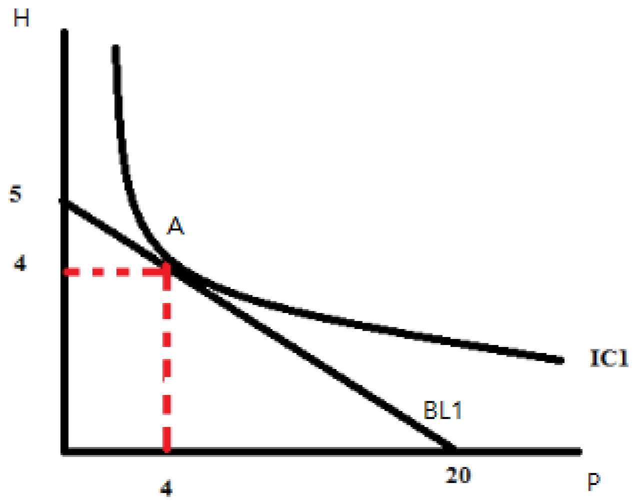
b. Can Jim afford the bundle (P, H) = (10,2)? Can Jim afford the bundle (P, H) = (6,4)? Provide a proof and complete explanation for your answer to these two questions.

Answer: Plugging (10,2) and (6,4) into our budget constraint:

5(10)+20(2)= 90 <100, so it is affordable. Jim can afford this bundle and still have income left over.
6(5) + 4(20) = 110 >100 so it is not affordable. This bundle costs more than Jim's income: he cannot afford it.

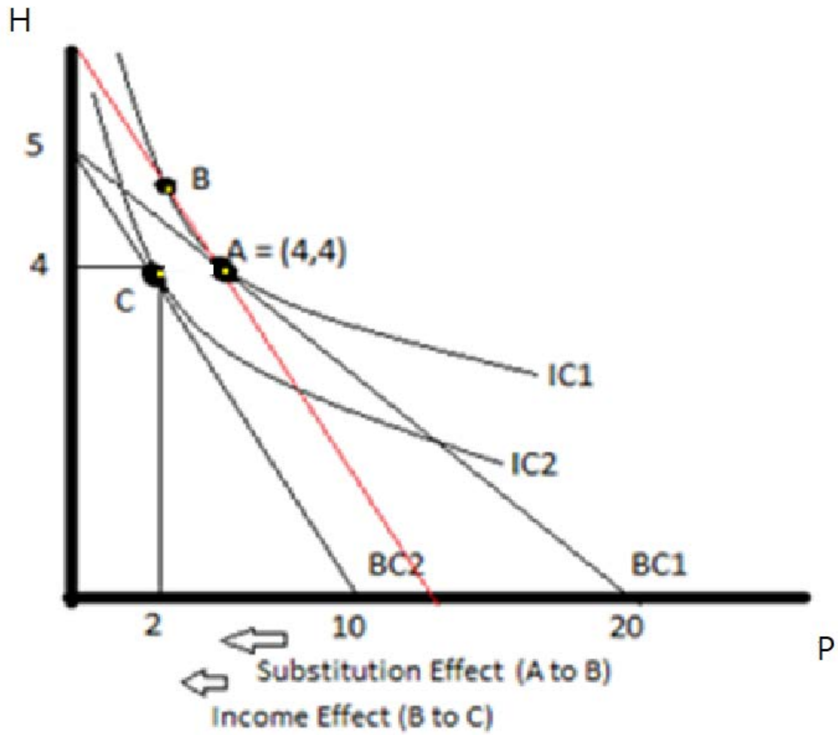
c. Suppose you are told that given the above information, Jim maximizes his utility when he consumes 4 bags of peanuts and 4 baseball hats. Given this information draw a graph that illustrates Jim's budget line, his indifference curve (IC1), and his consumer utility maximizing point (Point A). Make sure your graph is completely and clearly labeled.

Answer:



d. Now suppose that the price of peanuts increases from $5 per bag to $10 per bag. Furthermore you are told that with this price change Jim maximizes his utility when he consumes bundle B consisting of 2 bags of peanuts and 4 baseball hats. Graph Jim's new budget constraint (BL2) and new indifference curve (IC2). Label his new utility maximizing point B in your graph. Once you have drawn all this in your graph construct BL3, the imaginary budget line. BL3 should be parallel to BL2 and just tangent to IC1. Once you have BL3 drawn in your graph, label point C where BL3 is just tangent to IC1. Then complete the graph by indicating the income effect and substitution effect with regard to bags of peanuts on the graph.

Answer: The change in consumption of bags of peanuts from point A to point B captures the substitution effect while the change in consumption of bags of peanuts from point B to point C captures the income effect.



e. Given the information you have been given thus far in this question, derive Jim's demand curve for peanuts, assuming that his demand curve is a straight line. For the demand equation use P as the symbol for the price per bag of peanuts and Q as the quantity of peanuts.

Answer: We know that when the price is $5 per bag Jim chooses to consume 4 bags of peanuts. When the price is $10 per bag Jim chooses to consume 2 bags of peanuts. Assuming the demand curve is a straight line, we can graph these two points and derive the demand curve equation: $P = (-2.5)Q + 15$.

f. Let's return to the original situation where Jim has $100 to spend on peanuts and hats and where the price of a bag of peanuts is $5 and the price of a hat is $20. Now suppose peanuts and baseball hats are perfect substitutes for Jim: Jim's utility function is such that buying 2 bags of peanuts gives him the same satisfaction as buying one baseball hat. Given this information what bundle of (peanuts, hats) will Jim consume to maximize his satisfaction given his income? Explain your answer.

Answer: Jim's indifference curve map consists of parallel straight lines with each line having the slope of -1/2. With perfect substitutes we know that Jim is going to choose one of the endpoints of his budget line (recall that the budget line is $H = 5 – (1/4)P$ where H is hats and P is bags of peanuts). The question is which endpoint will he choose? Jim will choose the endpoint that maximizes his utility. If Jim selects (P, H) = (0, 5) he will be on one indifference curve and if he selects (P, H) = (20, 0) he will on a different indifference curve. The point (P, H) = (20, 0) is on a higher indifference curve and will therefore be the point he selects in order to maximize his utility.

g. Let's return to the original situation where Jim has $100 to spend on peanuts and hats and where the price of a bag of peanuts is $5 and the price of a hat is $20. Now suppose that peanuts and baseball hats are perfect complements for Jim. That is, he always buys exactly 1 baseball hat with each bag of peanuts. Given this information and holding everything else constant, what bundle will he consume? Would Jim consume 10 bags of peanuts and 12 baseball hats at any income level in this scenario? (use initial prices)

Answer: For the perfect complements case we know that P = H. We know that the budget line can be written as 5P + 20H = 100. With these two equations we can solve for the value of P and H. Thus, 5H + 20H = 100 or H = 4 and P = 4. Jim will consume 4 bags of peanuts and 4 baseball hats given this information.

Would Jim consume 10 bags of peanuts and 12 baseball hats at any income level? No, because he will always make P = H no matter what the prices are for the two goods or what his level of income is. If it is possible (that is the prices and/or income is adjusted so that Jim can do this) for Jim to consume 10 bags of peanuts and 12 baseball hats, he would prefer to sell the extra 2 baseball hats and use the money to purchase additional, but equal amounts of bags of peanuts and baseball hats.

2. Pam, a cat, only consumes two goods: fish (F) and candy (C). Through careful negotiations with Michael (her owner), Michael agrees to spend $72 per week to buy Pam these two goods and Pam can choose whatever bundle she prefers provided Michael can afford that bundle given the money he has to spend on the two goods. One can of fish cost $9 and one pack of candy costs $6. The table below shows some of Pam's marginal utilities relating to her consumption of these two goods:

Q_F	1	2	3	4	5	6	7	8	9
MU_F	72	36	24	18	14.4	12	10.28	9	8
Q_C	1	2	3	4	5	6	7	8	9
MU_C	72	36	24	18	14.4	12	10.28	9	8

a. Given the above information and holding everything else constant, find the equation for Pam's budget line and then graph this budget line as BL1 in a graph where candy (C) is measured on the vertical axis and fish (F) is measured on the horizontal axis.

Answer: The budget line is given by the equation 72 = 9F + 6C, and it is shown as BL1 in the graph below. The budget line in y-intercept form is: C = 12 – (3/2)F.

b. What is Pam's utility maximizing bundle at this budget constraint? What is her total utility from consuming this bundle? In your graph illustrate this optimal bundle and label it point A. Draw an indifference curve that goes through bundle A and is tangent to BL1: label this indifference curve IC1.

Answer: To answer this question you will need to examine the table carefully and look for a bundle of (fish, candy) that will exhaust the total income of $72 and that satisfies the utility maximizing rule (MUx/MUy = Px/Py). Recall that this utility maximizing rule states that the consumer's utility will be maximized if they consume that bundle where the slope of the indifference curve is equal to the slope of the budget line at that consumption point.

This gives us the consumption bundle: (F, C) = (4, 6). In the graph below this point has been labeled as point A.

The total utility from consuming fish is 72 + 36 + 24 + 18 = 150 utils.

Total utility from consuming candy is 72 + 36 + 24 + 18 + 14.4 + 12 = 176.4 utils.

Thus the total utility from this bundle is 150 + 176.4 = 326.4 utils.

c. Suppose the price of a can of fish decreases to $4 per can. Given this information and holding everything else constant, find the equation of Pam's new budget line (BL2) and graph it in the same diagram as in part (a). Label this new budget line BL2.

Answer: The new budget line is given by the equation 72 = 4F + 6C and is shown as BL2 in the graph below. The budget line in y-intercept form is: C = 12 – (2/3)F.

d. What is her utility maximizing bundle with the new budget line? Remember you have that initial table to consult to help you figure this out! What is the change in her consumption of good F when Pam moves from point A to point B?

Answer: To answer this question you will need to go again and look for a consumption bundle of fish and candy that both exhausts her total income while simultaneously satisfying the utility maximizing rule (Px/Py = Pf/Pc = Muf/MUc). This will occur when Pam consumes 9 cans of fish and 6 units of candy. This is point B in the graph below.

Pam now consumes 5 more cans of fish than she did at point A.

e. Observe the table carefully. What other consumption bundle from this table gives Pam the same utility level as she got at point A (the amount you calculated in (b))? Once you have found this point label it point C on your graph and draw an indifference curve that contains both point A as well as point C. Label this indifference curve IC1 in your graph.

Answer: From the provided table we can find that the bundle (F, C) = (6, 4) gives the same total utility of 326.4 utils as was found at point A. This new point C is labeled in the graph below and both point A and point C sit on same indifference curve.

f. How much additional money would Pam need from Michael to afford this bundle C? Why will Pam need less money to afford this bundle than she needed to afford bundle A?

Answer: To be able to afford this bundle, Pam would need to ask Michael for 6*4 + 4*6 = $48.

Pam needs less money to buy bundle C than she needed to buy bundle A because the price of fish has fallen relative to its initial price. Her purchasing power from a given level of nominal income is increased when the price of one of the goods falls, holding everything else constant.

g. Verify that the consumption bundle you found in (e) is the utility maximizing bundle for the level of income you determined in (f). This new level of income is based upon the new price of fish.

Answer: First check that this new consumption bundle satisfies the utility maximizing rule: MUx/MUy = 12/18 = 4/6 = Px/Py.

Second check that it exhausts the income of $48. 6 cans of fish at $4 per can will cost $24 and 4 packs of candy at $4 per pack will cost $24: this exhausts the amount of income that was determined in (f).

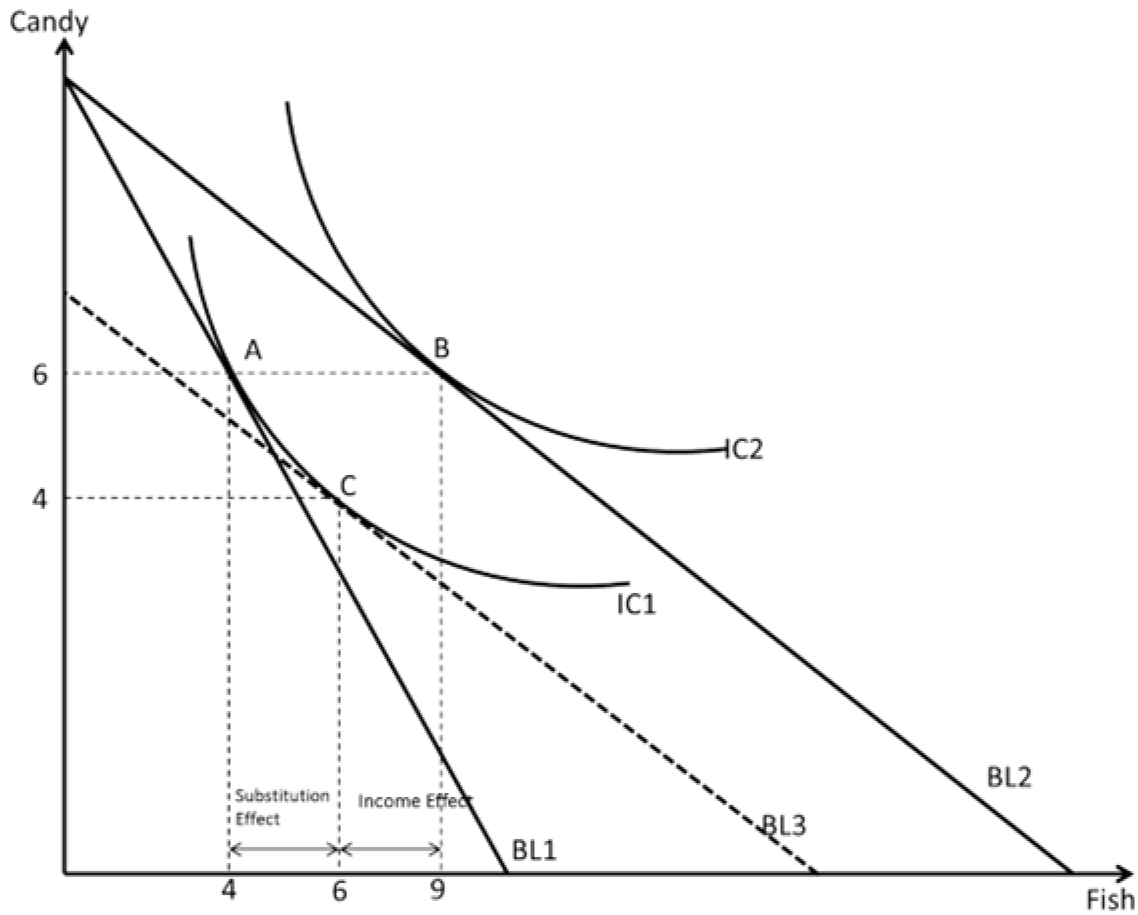
Since both requirements for utility maximization are satisfied, the bundle in (e) is the utility maximizing bundle for Pam when she has her income compensated so that she remains on her initial indifference curve but now faces the new prices.

h. What is the equation for Pam's imaginary budget line, BL3? Graph it in the same diagram and label it BL3.

Answer: His imaginary budget line is given by this equation 48 = 4F + 6C, and it is labeled as BL3 in the graph below. The budget line in y-intercept form would be: C = 8 – (2/3)F.

i. Identify graphically and numerically how much of the change in Pam's consumption of fish is due to the income effect and how much of the change is due to the substitution effect. Explain your answer fully and make sure your graph identifies clearly both the substitution and the income effect.

Answer: In the graph below, the movement along the X-axis from point A to point C measures the substitution effect, which is 2 cans of fish. The movement along the x-axis from point C to point B measures the income effect, which is 3 cans of fish.



3. A poor graduate student survives on two types of food items, pizza (P) and mushroom soup (S). He spends all of his monthly income of $80 on food. One slice of pizza and one cup of soup both cost $1. You are also provided the following information about the graduate student's utility function:

$$U = P*S$$
$$MU_P = S$$
$$MU_S = P$$

a. Graph the budget line (BL1), with pizza on the x-axis and soup on the y-axis. Under current price levels, how many slices of pizza and how many cups of soup does the student consume? Show how you found this optimal bundle and then label this optimal bundle as Bundle A in your graph. What is his total utility at this optimal bundle?

Answer: The tangency condition for utility maximization is:

$$\frac{S}{P} = \frac{MUp}{MUs} = \frac{p_p}{p_s} = \frac{1}{1} = 1$$

So we have S = P for the utility maximizing bundle. The student will also spend all the $80 on the two goods, so $P * p_p + S * p_s = P + S = 80$.

Together, the two conditions give us P = S = 40: that is, the student will consume 40 slices of pizza and 40 cups of soup. His utility level is P*S = 40*40 = 1600 utils.

The graph of this information is in the answer to (c).

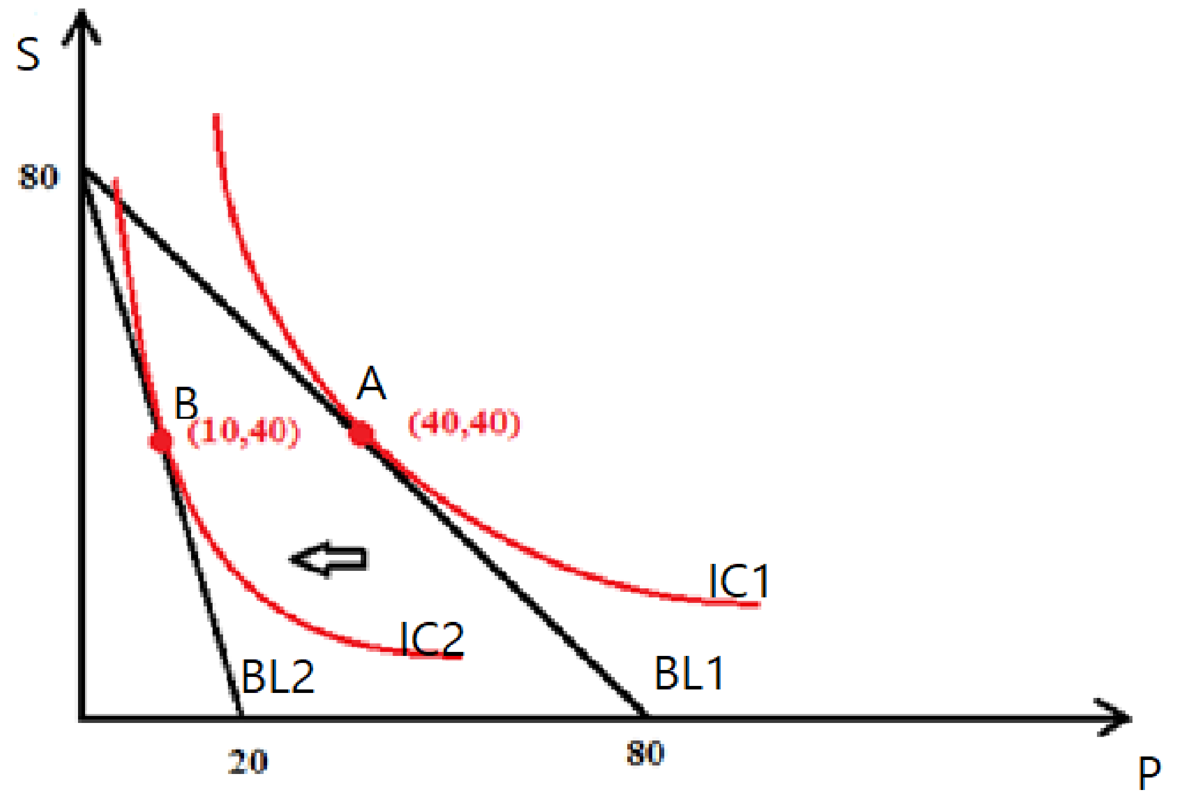
b. To encourage healthy living, the mayor imposes an excise tax of $3 on each slice of pizza. How does this tax affect the graduate student's budget line? Provide an equation for this new budget line, BL2. What is his optimal consumption bundle, Bundle B, now? Show how you found this optimal consumption bundle. When this student maximizes his utility now, how much utility will he have? Show how you found this answer. How much tax revenue is collected from this student? Illustrate in your graph BL2 and Bundle B. Make sure your graph identifies all intercepts as well as the coordinates of any known points. Also in your graphs include the indifference curves that represent the level of utility this student has at Bundle A and at Bundle B. Label these indifference curves IC1 and IC2, respectively.

Answer: By imposing the excise tax on pizza, the mayor essentially quadruples the (post-tax) price of one slice of pizza. Therefore, the student now faces $p_p = \$4$ and $p_s = \$1$. The budget line changes to 4P + S = 80, or equivalently S = 80 – 4P.

The tangency condition changes to

$$\frac{S}{P} = \frac{MUp}{MUs} = \frac{p_p}{s} = \frac{4}{1} = 4$$

So now we have S = 4P, and 4P + S = 80. Therefore, the optimal consumption bundle is now (P', S') = (10, 40). The utility level shrinks by 1200 utils to 400 utils. The new utility is: U' = P'S' = (10)(40) = 400 utils.



The tax revenue collected is given by: Tax Revenue = (excise tax per unit)(number of units consumed) or Tax Revenue = ($3 per slice of pizza) * (10 slices of pizza) = $30.

c. Find the income and substitution effects on pizza consumed due to the pizza tax. This will take some work: remember that you need to find that consumption bundle, Bundle C, that has the same level of utility as Bundle A but which is based upon the new price of a slice of pizza. Show the work you did to find this optimal consumption Bundle C and then draw the imaginary budget line, BL3, and Bundle C in a graph that includes BL1, BL2, IC1, IC2, Bundle A and Bundle B. In this new graph indicate the change in consumption of pizza due to the substitution effect and the change in consumption of pizza due to the income effect.

Answer: To find the income and substitution effects, one needs to find the intermediate point, where the original indifference curve passing through (40, 40) hits a hypothetical line parallel to the new budget line. In other words, we want the old utility level of 1600 utils, but with the new price ratio. This gives the conditions:

P*S = 1600

$\frac{S}{P} = \frac{MUp}{MUs} = \frac{Pp}{Ps} = \frac{4}{1}$ = 4 or 4P = S

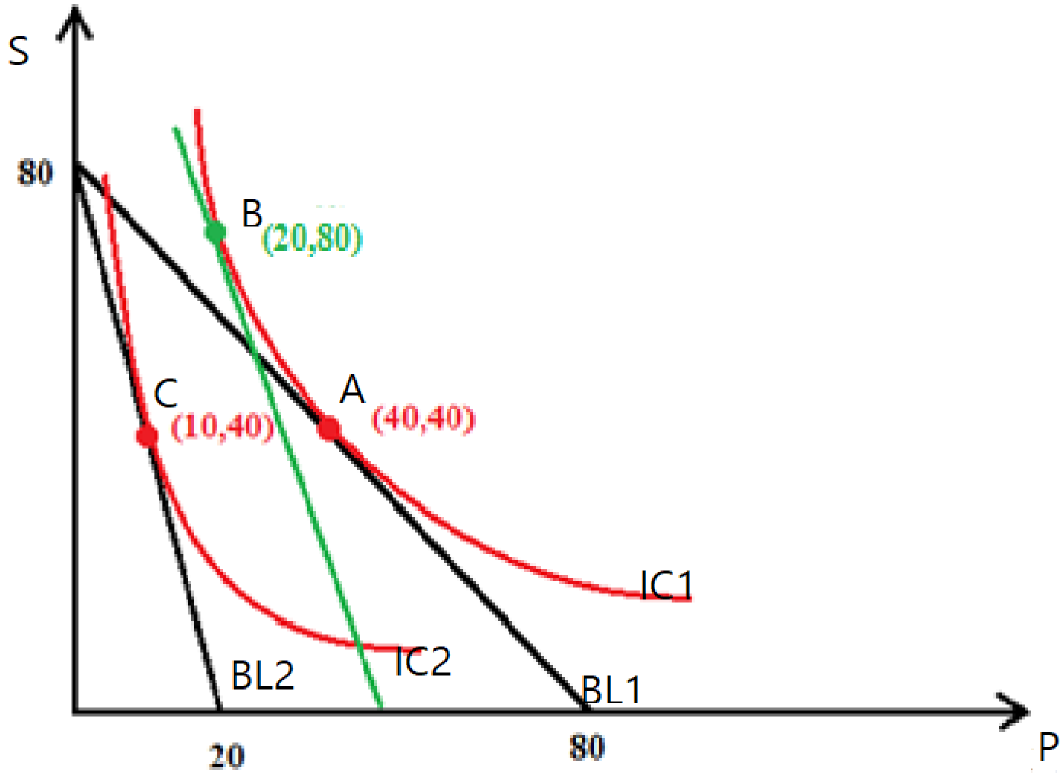
Use these two equations to find (P, S) = (20, 80). Here's the work:

P*S = 1600

P*(4P) = 1600

$4P^2 = 1600$
$P = 20$
And therefore $4P = S = 80$.

Therefore, (20, 80) is the intermediate point we want. It is represented as Bundle C in the graph below.



From A to B, the transition is called the substitution effect and from B to C, the transition is called the income effect.

The substitution effect on pizzas consumed is the distance between the original level of pizza consumption at Bundle A (40 slices) and the level of pizza consumption at Bundle C (20 slices). Hence the substitution effect is 40 – 20 = 20 or a reduction in the consumption of pizza by 20 slices.
The income effect on pizzas consumed is the distance between the level of pizza consumption at Bundle C (20 slices) and the level of pizza consumption at Bundle B (10 slices), so it is 20 – 10 = 10 or a reduction in the consumption of pizza by 10 slices.

d. Suppose the excise tax described in this question has been implemented, but at the same time the University provides the graduate student with a cash grant so that the student can attained his original (pre-tax utility level) level of utility. The University does this because they recognize that the excise tax on pizza causes the real income or purchasing power of the graduate student's nominal income to fall. Given this information and holding everything else constant, what is the amount of this cash grant?

Answer: Receiving a cash grant pushes the budget line outwards, while keeping the slope (and thus the price ratio) fixed. So the tangency condition is still $\frac{Y}{X} = \frac{MUX}{MUY} = \frac{p_X}{p_Y} = \frac{4}{1} = 4$, but the utility level is back to the original level 1600. Obviously, this is just the green hypothetical budget line in the graph for part c, and the optimal bundle the student chooses after receiving the cash grant will be exactly (P, S) = (20, 80), the point labeled as Bundle C in the graph.

To be able to consume (P, S) = (20, 80) under the current price levels, the student needs an income of 20*4 + 80*1 = $160. So he needs a cash grant of $80 in addition to his original income of $80.

Part II: Producer Theory (Production and costs)

4. J.J.'s Diner makes Leslie Knope's favorite waffles using capital (K) and labor (L). The table summarizes the production and cost functions for J.J.'s Diner, where Q, K and L respectively represent the quantity of waffles, capital and labor. Costs are measured in dollars. This firm also happens to be a producer in a perfectly competitive waffle market.

K	L	Q	FC	VC	TC	AFC	AVC	ATC	MC	MPL
2		0			10					
2		1			18					0.25 units of output/unit of labor
2	5	2								
2	6	3								
2		4							4	
2		5			32					
2		6			38					
2	20	7								

a. Given the above information and holding everything else constant, how much does one unit of capital cost to J.J.'s Diner? Explain how you found your answer to this question.

Answer: at Q = 0 there are zero units of labor employed, so the total costs of production are equal to the fixed costs since there are no variable costs. So one unit of capital costs $10/2 = $5 per unit of capital.

b. If J.J.'s produces 10 waffles, what are the average fixed costs for the firm? Show how you calculated this average fixed cost.

Answer: Fixed costs are $10 since there are two units of capital employed and the price per unit of capital is $5. At a quantity of 10, the average fixed cost = (fixed cost)/(quantity of output) or AFC = FC /Q. Thus, AFC = $10/10 units of output or $1 per unit of output.

c. What is the price of a unit of labor? Explain how you found your answer to this question.

Answer: From the second row of this table, we know at Q = 1 the marginal product of labor is 0.25. Using the definition of the marginal product of labor as (the change in output)/(change in labor) and this value we can solve for the change in labor: 4 workers are required to move from first row to second row and produce one additional unit of waffle ((change in q)/(change in L) = ¼. Since total cost increases by $8, and the amount of capital required is not changed, each unit of labor costs $8/4 = $2 per unit of labor.

d. Fill in the blank cells in the above table. Round each answer as required to two places past the decimal.

K	L	Q	FC	VC	TC	AFC	AVC	ATC	MC	MPL
2	0	0	10	0	10	-	-	-	-	-
2	4	1	10	8	18	10	8	18	8	0.25 units of output/unit of labor
2	5	2	10	10	20	5	5	10	2	1
2	6	3	10	12	22	3.33	4	7.33	2	1
2	8	4	10	16	26	2.5	4	6.5	4	0.5
2	11	5	10	22	32	2	4.4	6.4	6	0.33
2	14	6	10	28	38	1.67	4.67	6.33	6	0.33
2	20	7	10	40	50	1.43	5.71	7.14	12	0.17

e. What is the minimum price for a waffle at which J.J.'s is willing to stay in the market in the short run? Explain your answer.

Answer: The price at which J.J.'s stops producing in the short run is where total revenue is equal to total variable costs, or where AVC = P = MC. This occurs at a price of $4 per waffle and that occurs when the quantity of waffles is equal to 4 waffles (Q = 4).

f. What is the minimum price for a waffle at which J.J.'s is willing to stay in the market in the long run? How many waffles will J.J.'s produce at this price? Explain your answer.

Answer: The breakeven price in the long run should satisfy ATC = P = MC. This occurs at an output of somewhere between 6 and 7 waffles, but closest to Q=6, which is associated with a price of $6 per waffle.

g. At what quantity is the average total cost minimized?

Answer: Average total cost is minimized when the quantity of waffles is equal to 6 waffles.

Part III: Perfect competition

5. Ann runs a shoe factory in a perfectly competitive industry. You are told the following information about her shoe factory and the market where q is the level of output produced

by Ann's factory, Q is the total amount produced in the industry, and P is the price per unit:

Total cost function for Ann's factory: $TC = 2q + (1/50)q^2 + 50$
Marginal Cost for Ann's factory: $MC = 2 + (1/25)q$
Market Demand Curve in the industry: $Q = 5020 – 5P$

a. Given the above information, find the equations for FC, VC, ATC, and AVC.

Answer: To find FC, recall that FC = TC when q = 0. So FC = 50. VC is the part of TC that varies with q, so $VC = 2q + (1/50)q^2$. $ATC = TC/q = 2 + (1/50)q + 50/q$. $AVC = VC/q = 2 + (1/50)q$.

b. How many pairs of shoes will Ann's factory produce when the market price is equal to $5? How many will it produce when price is equal to $10? Show how you found your answers to these questions.

Answer: Since the firm is a price taker in a perfectly competitive market, the firm will choose to produce that quantity when P = MC. When the market price is $5 per unit, set $2 + (1/25)q = 5$, and we get q = 75units. When the market price is $10 per unit, set $2 + (1/25)q = 10$, and we get q = 200 units.

c. What is the factory's breakeven point in the long run? What is the price at that quantity?

Answer: The breakeven price is where the minimum point of the ATC curve intersects the MC curve. So to find the quantity associated with this breakeven point, set MC = P = ATC:

$2 + (1/50)q + 50/q = 2 + (1/25)q$

$-(1/50)q + 50/q = 0$

$(1/50)q^2 = 50$

q = 50 units at the breakeven point.

Ann's factory will produce 50 units when it operates at the breakeven point.

We can then use this quantity and either the ATC curve or the MC curve to calculate the price the firm will charge for each unit of output: $P = MC = 2 + (1/25)q = 2 + 2 = \$4$ per unit of output.

d. Calculate the value of profits for Ann's factory in the long run.

Answer: From part (c) we know that the breakeven (or long run) price in this market is $4 per unit of output. At this price Ann's factory will produce 50 units of output and her profit can be calculated as:

profit = TR - TC

profit = $q*p – TC = 4*50 – [2*50 + (1/50)*50^2+50] = \$0$

e. Given the above information and holding everything else constant, how many shoe factories will there be in this market in the long run? When answering this question assume all factories are identical with the same cost functions as Ann's.

Answer: We know that the long run price in this market is P = $4 per unit. Using this price and the market demand curve we can solve the total quantity produced in the market: Q =5000 units. Since each factory in the long run produces 50 units and since total production is equal to 5000 units, this implies that there must be 100 factories in the industry.

f. Derive the market supply curve.

Answer: To find the market supply curve you will need to find two different combinations of quantity and price that are on this market supply curve. When q = 0, MC = $2 per unit of output for the firm. So when P = $2 per unit of output, the quantity produced by a representative factory is 0 units and therefore the market quantity is also 0 units. When q = 25, MC = $3 per unit of output for the firm. So when P = $3 per unit of output the quantity produced by a representative firm is 25 units and therefore the market quantity supplied is 25*100 = 250 units of output. With these two points we can derive the market supply curve as: P = (1/2500)Q + 2.

Suppose Ann adopts a new technology to produce shoes, which causes her marginal cost to decrease by $1 per pair of shoes, and her fixed cost to increase by $5.

g. Given this information and holding everything else constant, what is the TC equation for Ann now? What is her MC equation now?

Answer: Given this information her new MC is given by the equation: MC = 1 + (1/25)q. Her new TC is given by the equation: TC = q + (1/50)q^2 + 55.

PartIV: Elasticity

6. The market demand curve and the market supply curve for Masks-Я-Us, a company that makes masquerade ball masks in Verona, can be described by the following equations where P is the price per unit and Q is the quantity of masks:

   Market Demand Curve: P = 10 - (1/5)Q
   Market Supply Curve: P = 2 + (1/5)Q

a. Using the mid-point method, calculate the price elasticity of demand when price changes from $8 to $6.

Answer: When P = $8, the quantity demanded is 10 units. When P = $6, the quantity demanded is 20 units. Price Elasticity of Demand = |{(20 - 10)/(20 + 10)}/{(6 - 8)/(6 + 8)}|= 7/3.

b. Using the point method, calculate the price elasticity of demand when the price is equal to $8. Then, using the point method, calculate the price elasticity of demand when the price is equal to $6.

Answer: The point elasticity of demand formula is: Elasticity of demand = [1/(-slope of the demand curve)][P/Q]. When P = $8 the point elasticity of demand = [1/ (-1/5)] [8/10] = 4. When P = $6 the point elasticity of demand = [1/ (-1/5)] [6/20] = 3/2.

c. Should Masks-Я-Us lower or raise its price to gain more revenue if its current price is $6 per unit? Explain your answer.

Answer: When P = $6 per unit, the price elasticity of demand is greater than one. This indicates that demand is elastic at this price and if the firm wishes to increase its revenue it should therefore lower the price that it sells the good for.

d. Find the market equilibrium in this market. Calculate the point elasticity of demand and the point elasticity of supply at this market equilibrium.

Answer: equilibrium (p, q) = ($6, 20).
Point elasticity of demand = (-1/slope)(P/Q)= (5)(6/20) = 3/2
Point elasticity of supply = (1/slope)(P/Q) = 3/2
Note that these two elasticities are the same but that is only because the underlying slopes in absolute value terms are the same for the two curves.