Economics 101
Fall 2016
Answers to Homework #4
Due November 17, 2016

Directions:

- The homework will be collected in a box **before** the lecture.
- Please place **your name**, **TA name** and **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time.
- **Show your work**. Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

Part I (Consumer Theory).
<u>Question 1 (Budget Lines, Income Compensation Lines).</u>

1. Meredith is a freshman student at UW Madison and She works 10 hours a week at the Badger market and her hourly wage is \$8. She spends all of her income to buy coffee or books. Assume that the price of a book is $p_B = \$10$ and the price of a cup of coffee is $p_C = \$2$.

   a. Find the equation for Meredith's budget line (BL1) and then draw a well labeled graph representing this budget line. Measure books (B) on the vertical axis and coffee (C) on the horizontal axis. Assume that her BL in this problem are based on a week's amount of time.

   In Meredith's second year at the university, Meredith's hourly rate rises to \$10 per hour due to negotiations between the student association and the university.

   b. Find the equation for Meredith's new budget line (BL2) in the second year and draw and label BL2 on the same graph that you have from (a).

   In Meredith's third year as a student, the price of a cup of coffee increases to \$4.

   c. Find the equation for Meredith's budget line in the third year (BL3) and then draw and label this BL3 in the same graph as you had in (a) and (b).

a. The equation of budget line is $p_B B + p_c C = I = w \times h$ where w is the wage per hour and h is the number of hours worked. Since the weekly income of Meredith is equal to $8 \times 10 = \$80$, the equation for budget line one, BL1, is:

$$Year\ 1 : 10B + 2C = 80$$

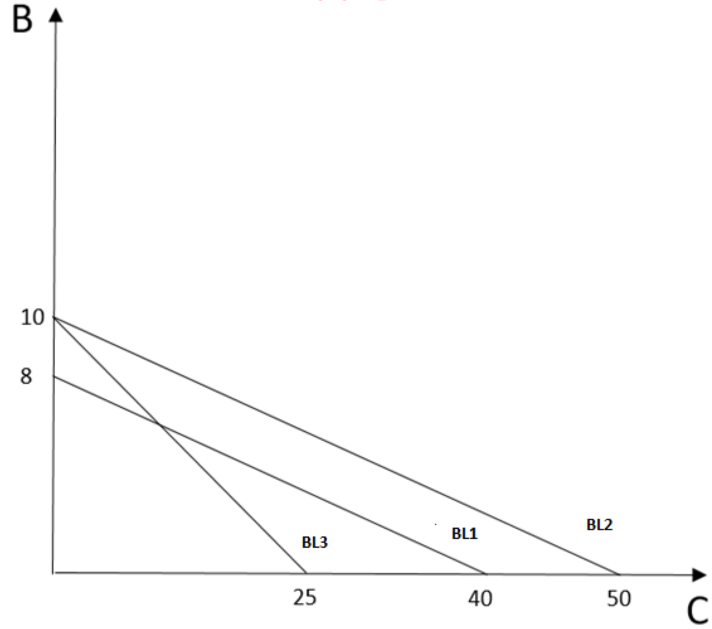
b. For year 2, Meredith's income is $10 \times 10 = \$100$, so the equation of budget line two, BL2, is:

$$Year\ 2 : 10B + 2C = 100$$

c. For year 3, income is the same as in year two, but p_c changes from 2 to 4. Therefore, the equation for the budget line is:

$$Year\ 3 : 10B + 4C = 100$$

These budget lines are drawn in the following graph.



Assume that Meredith's utility function over books and cups of coffee is given by $u(B, C) = \min\{5B, 3C\}$ (this is an example of the perfect complement utility function). Let's examine this information before proceeding. As an example suppose that the bundle of coffee and books we are considering is 4 coffees and 3 books. The utility function (or equation) tells us that the level of utility will be equal to the minimum of 5B or 3C. So, 5B = 5(3) = 15 and 3C = 3(4) = 12. So the utility for this individual will equal 12 since 12 is less than 15. How many books could Meredith consume to have an utility of 12? If B = 12/5, then utility is the minimum of {5b, 3C}: 5 (12/5) = 12 and 3(4) = 12. So, 12/5 books and 4 coffees both yield a utility of 12 for Meredith. This suggests that there is a very particular relationship between the amount of coffee and the amount of books. Ponder this a bit before heading into the rest of this problem.

d. Given this information, determine how many books and how much coffee Meredith will choose to consume each year in order to maximize her utility.

Meredith would always choose $5B = 3C$ (or $B = \frac{3}{5}C$) as her optimal consumption bundle. To see this, assume that $5B > 3C$, so $u(B, C) = \min\{5B, 3C\} = 3C$. This implies that the amount of books in excess of (3/5)C does not provide her with any additional utility. Meredith could have greater utility by buying fewer books and more coffee. So $5B > 3C$ cannot be the optimal allocation for Meredith. You can also check that $5B < 3C$ is not the optimal allocation: in this case Meredith's u(B, C) = min{5B, 3C} = 5B. This implies that the amount of coffee in excess of

3C does not provide her with any additional utility. Meredith could have greater utility by buying less coffee and more books. So, Meredith must maximize her utility when 5B = 3C or when B = (3/5)C.

If we put $B = \frac{3}{5}C$ in the budget line of year 1, then

$$10\left(\frac{3}{5}C\right) + 2C = 80 \Rightarrow 8C = 80 \Rightarrow C = 10 \Rightarrow B = \frac{3}{5}\times 10 = 6.$$

So, she would consume 10 cups of coffee and 6 books in the first year.

If we put $B = \frac{3}{5}C$ in the budget line of year 2, then

$$10\left(\frac{3}{5}C\right) + 2C = 100 \Rightarrow 8C = 100 \Rightarrow C = 12.5 \Rightarrow B = \frac{3}{5}\times 12.5 = 7.5.$$

So, she would consume 12.5 cups of coffee and 7.5 books in the second year.

If we put $B = \frac{3}{5}C$ in the budget line of year 3, then

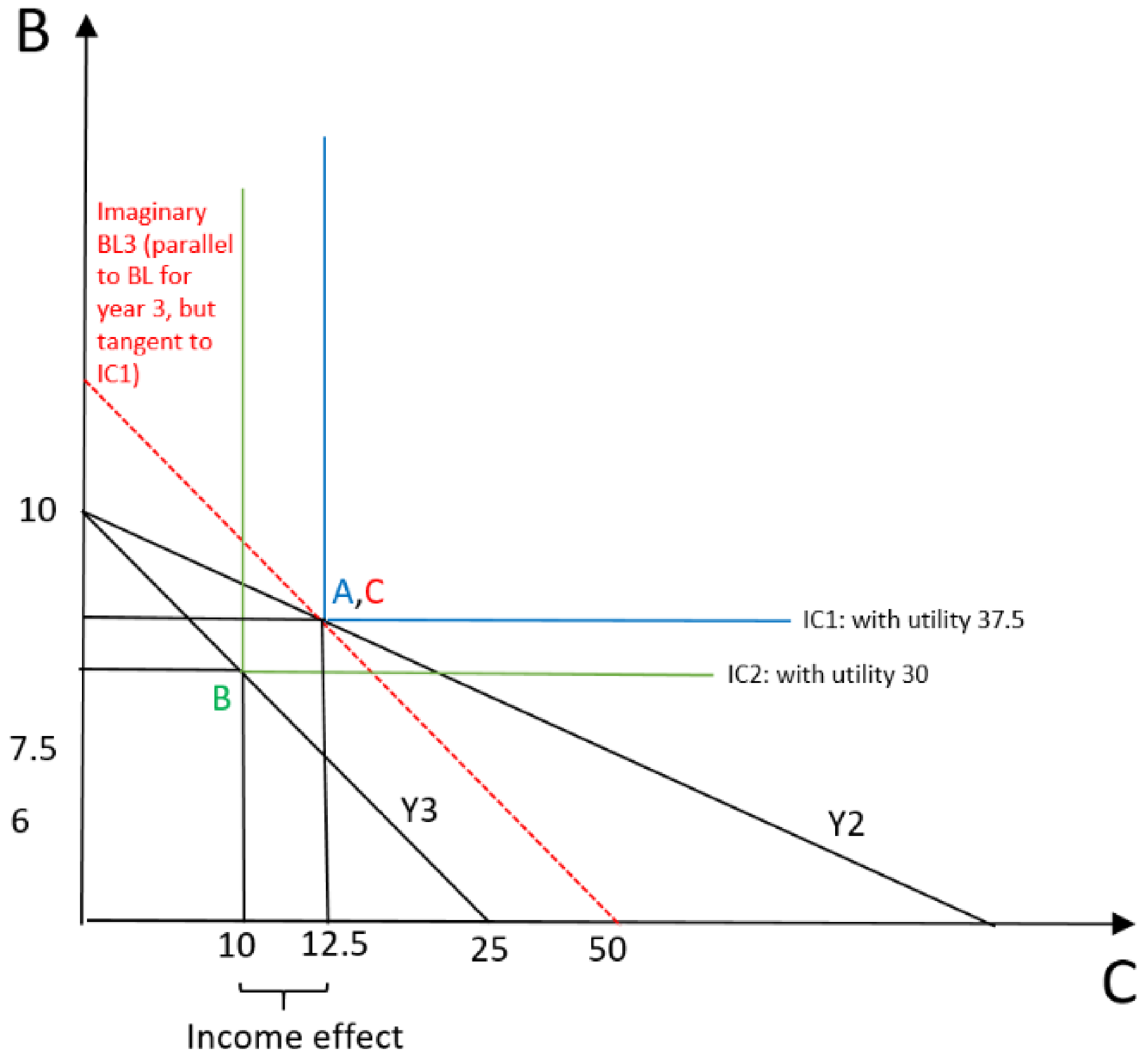
$$10\left(\frac{3}{5}C\right) + 4C = 100 \Rightarrow 10C = 100 \Rightarrow C = 10 \Rightarrow B = \frac{3}{5}\times 10 = 6.$$

So, she would consume 10 cups of coffee and 6 books in the third year.

e. Draw a graph illustrating the Meredith's utility maximization points for year 2 (label this point A) and year 3 (label this point B). Then in your graph determine the substitution and income effects for coffee between these two years. Based on your result with regard to the substitution effect, can you explain why this utility function is called the perfect complement utility function?)

From our work in (d) we know that Meredith consumes (C, B) = (12.5, 7.5) in year 2 and (C, B) = (10, 6) in year 3. See the graph below for these consumer utility optimization points. Then, we should compute the point "C" that lies on an "imaginary" budget line. There are two features of point C: (i) it gives the same utility as the bundle chosen under the original prices, so it lies on the same indifference curve as point A; (ii) the imaginary budget line has the same slope as the second budget line, so the slope of the indifference curve at point C is equal to the slope of budget line for year 3. In this particular case, point A and C are the same. That means for both of the budget line in year 2 and the imaginary budget line, the optimal points are the same. We do not know the income of the imaginary budget line, though we do know its slope. So, the substitution effect (difference between points A and C) is zero. Therefore, there is only an income effect, which is the horizontal difference between points C and B. The income effect is therefore equal to -2.5 units of coffee because that is how many units of coffee are given up due to the change in the real price of coffee between year 2 and year 3.

Because the two goods are perfect complements, then the substitution effect is zero.



Part II: CPI, Real vs. Nominal, and Inflation

<u>Question 2.</u>
As a fieldwork assignment for Econ 101, you investigate life in Madison in the early 2010s: the time period from 2010—2014. You have obtained the following list of **nominal prices** for milk, snow boots and apartments.

year	milk ($/gallon)	snow boots ($/pair)	apartments
2010	$2.5	$30	$530
2011	$3	$24	$500

year	milk ($/gallon)	snow boots ($/pair)	apartments
2012	$3.5	$28	$520
2013	$3	$35	$550
2014	$3.5	$33	$570

Assume that purposes of calculating the CPI that the consumer basket consists of 60 gallons of milk, 4 pairs of snow boots and 1 apartment.

a) Find the cost of market basket for each year.

Year	Cost of Market Basket
2010	
2011	
2012	
2013	
2014	

Since the market basket consists of 60 gallons of milk, 4 pairs of snow boots and 1 apartment,
the cost of market basket of year Y
= (nominal price of milk)*(60 gallons of milk)
+ (nominal price of snow boots)*(4 pairs of snow boots)
+ (nominal price of apartments)*(1 apartment).
Using this formula, you can fill in the blanks of the table.

Year	Cost of Market Basket
2010	($2.5)*60 + ($30)*4 + ($530)*1 = $800
2011	($3)*60 + ($24)*4 + ($500)*1 = $776
2012	($3.5)*60 + ($28)*4 + ($520)*1 = $842
2013	($3)*60 + ($35)*4 + ($550)*1 = $870
2014	($3.5)*60 + ($33)*4 + ($570)*1 = $912

b) Suppose 2010 is the base year. Find the CPI for each year using a 100 point scale.

Year	CPI (with base year 2010)
2010	
2011	
2012	
2013	
2014	

Note that

CPI of year Y

= [(cost of market basket in year Y) / cost of market basket in the base year 2010)] * 100.

Using this formula, you can fill in the blanks of the table. Carry your answers to two places past the decimal.

Year	CPI (with base year 2010)
2010	($800/$800) * 100 = 100
2011	($776/$800) * 100 = 97
2012	($842/$800) * 100 = 105.25
2013	($870/$800) * 100 = 108.75
2014	($912/$800) * 100 = 114

c) The CPI is used to assess the trend of prices. Using the CPI's obtained in (b), find the annual inflation rate for each year. Please round off to two places past the decimal. (Note that you don't have to calculate the annual inflation rate from 2009 to 2010 because there is no data for 2009.)

	Annual Inflation Rate
2010—2011	
2011—2012	
2012—2013	
2013—2014	

Note that the annual inflation rate is the percentage change of the CPI from the last year to this year. Hence, applying the percent change formula to this problem,

Annual inflation rate

= [(CPI of this year - CPI of the last year) / (CPI of the last year)] * 100.

Using this formula, you can fill in the blank of the table.

	Annual Inflation Rate
2010—2011	[(97 - 100)/100] * 100 = -3%
2011—2012	[(105.25 - 97)/97] * 100 = 8.51%
2012—2013	[(108.75 - 105.25)/105.25] * 100 = 3.33%
2013—2014	[(114 - 108.75)/108.75] * 100 = 4.83%

d) Using the CPI's obtained in (b), find the **real prices** of apartments based on dollars in 2010. Please round off to the nearest whole number.

Year	Real Price of Apartments (with base year 2010)
2010	
2011	
2012	
2013	
2014	

We can calculate the real price by removing the inflation from the nominal price by means of CPI. In particular,

real price of year Y
= nominal price of year Y * (CPI of the base year / CPI of year Y).

Recall that CPI of the base year = 100. Using this formula, you can fill in the blanks of the table.

Year	Real Price of Apartments (with base year 2010)
2010	$530 * (100 / 100) = $530 (= the nominal price)
2011	$500 * (100 / 97) = $515
2012	$520 * (100 / 105.25) = $494
2013	$550 * (100 / 108.25) = $506
2014	$570 * (100 / 114) = $500

e) A friend of yours also investigates the cost of living in Madison with the same market basket, but she is interested in the late 2000s: from 2005 to 2009. However, she fails to collect the **nominal price** of milk in 2009. She knows that the **nominal prices** of snow boots and apartments in 2009 are $26 per pair of snowboots and $476 per apartment; furthermore, the **real price** of apartments in 2009 is $544 with the base year 2010. What is the **nominal price** of milk in 2009?

Let's denote by P the unknown nominal price of milk in 2009. (Denoting unknown variables is a very important first step to solve questions.)

Note that

real price of apartments in 2009

= nominal price of apartments in 2009 * (CPI in 2010 / CPI in 2009).

Since the real and nominal prices of apartments in 2009 is $544 and $476, respectively, and since CPI of the base year 2010 = 100,

$544 = $476 * (100 / CPI in 2009),

so CPI in 2009 = 87.5.

Since

CPI in 2009 = (cost of market basket in 2009 / cost of market basket in 2010)*100,

we have

87.5 = (cost of market basket in 2009 / $800) * 100,

and thus,

cost of market basket in 2009 = 87.5 * 8 = 700.

Since

700 = cost of market basket in 2009 = P * 60 + $26 * 4 + $476 * 1,

we have

P = (700 - $26 * 4 - $476 *1) / 60 = $2.

f) If you want to compare the costs of living in Madison and Pennsylvania in 2015 based on dollars in 2010, then how can you do that? Please explain the procedure.

Step 1: Determine the common market basket for Madison and Pennsylvania.

Step 2: Collect data on the nominal prices of the items in the market basket in 2010 and 2015 for both cities.

Step 3: Calculate CPI's for each year in the cities and find the real prices in 2015.

Step 4: Take the ratio

R = real price in Pennsylvania in 2015 / real price in Madison in 2015.

If R>1, then the cost of living in Pennsylvania is higher than that in Madison.

If R<1, then we can say the converse.

If R=1, then the cost of living in the two locations is the same.

Part III: Production and Cost

<u>Question 3.</u>

Luffy and Sanji are mechanical engineering students at UW Madison. They have discovered a new method for producing turbines. They decide to build a factory together that uses their new method. They ask their friend Nico, who studies economics, to help them make plans for their factory. They build their turbines using capital and labor, and they only have 10 units of capital. The table below summarizes the production and cost functions of their method, where q, K and L respectively represent the quantity of turbines, capital and labor. Costs are measured in dollars, average costs are measured in dollars per turbine, marginal cost is measured in dollars per turbine, and the marginal product of labor is measured in turbines per unit of labor. (Before starting this question it would be a good idea to compile all the relevant formulae on a single piece of paper! Also, for some of the calculation on this question you may want to use a calculator! Or a spreadsheet program like EXCEL!)

K	L	q	FC	VC	TC	AFC	AVC	ATC	MC	MPL
10	0	0			20					
10		1							2	0.5
10	3	2								
10		3			26					
10		4					2.5			
10		5								0.2

a. Given the above information, what is the cost of one unit of capital?

Answer: at q = 0 no workers are employed, so the total costs are just the cost of capital. Thus each unit of capital costs $20/10 = $2 per unit of capital.

b. How much labor is required to produce the first unit of output? What is the cost of a unit of labor? (Hint: you can figure this out using marginal cost and the marginal product of labor.)

Answer: at q = 1 the marginal product of labor is 0. 5, so $\frac{\text{Change in q}}{\text{Change in L}} = \frac{1}{\text{Change in L}} = 0.5$ turbine per unit of labor. Thus 2 workers were required to produce one turbine. Since the marginal cost of that increase in output was $2, each unit of labor costs [$2/(2 units of labor)] = $1 per unit of labor.

c. Fill in the blank cells in the table. Check to make sure that your numbers are consistent with each other!

Answer: The table is below.

K	L	q	FC	VC	TC	AFC	AVC	ATC	MC	MPL
10	0	0	20	0	20	___	___	___	___	___
10	2	1	20	2	22	20	2	22	2	0.5

10	3	2	20	3	23	10	1.5	11.5	1	1
10	6	3	20	6	26	6.66	2	8.66	3	0.33
10	10	4	20	10	30	5	2.5	7.5	4	0.25
10	15	5	20	15	35	4	3	7	5	0.2

For most rows you can find total cost by adding together the cost of capital (which is always $20 here) and the cost of labor ($1 times the quantity of labor employed). At q = 3, you need to use the fact that TC = FC + VC, so VC = 26 - 20 = $6. Since the cost of labor is equal to 1, the amount of labor is 6 units of labor. At q = 4, VC = 4*2.5 = $10, so 10 units of labor would be used. At q =5, you can use that change in labor = [(change in q)/MPL], so the amount of labor would be 15 units of labor. The rest is a matter of using the formulae given in lectures.

d. At what quantity does the factory start to exhibit diminishing marginal productivity of labor?

Answer: After q = 2, the marginal product of labor (calculated in the table above) falls as quantity increases.

a. Is their factory operating in the short run or the long run? How do you know?

Answer: In the short run, since capital is being held fixed.

f. If the market price of a turbine is $3, what quantity of turbines would the factory produce (assume that their decisions do not affect the market price)? How large would their profit or loss be given the market price and their production decision? Would they produce at this price?

Answer: Since the factory is a price taker, they choose to produce that quantity where the market price equals marginal cost. Thus they will produce a quantity of 3 units. When they produce three units, their total revenue is $TR = 3\times3 = \$9$. Since their total costs at q = 3 is $26, they makes a loss of $17 (9-26). Note, however, since their total revenue covers their variable costs, they will still operate in the short run.

g. Assume that they see that the price of turbines decreases to $1. Would they continue to produce at this market price?

Answer: recall that the shutdown price (where they stop producing in the short run) is where total revenue equals total variable costs, or where AVC = P = MC. At a price of $1 per turbine, the firm has total revenue of $2 and total costs of $23. But, they have variable costs of $3, and since total revenue is less than this variable cost (the firm is not covering its variable cost) the firm will shut down in the short run.

Part IV: Perfect Competition in Short-run and Long-run

Question 4.

Consider the milk market in the U.S. There are a number of companies selling milk, so that the market is perfectly competitive. Let's look at Kemps, a firm that produces milk. Assume that Kemp's total and marginal costs are given by the following equations:

$$TC = q^2 + 8q + 5$$
$$MC = 2q + 8$$

where q denotes units of milk. The market price is $50 per unit of milk.

a) Given the above information, how many gallons of milk will Kemps produce in the short run?

In the perfectly competitive market, Kemps is a price taker. Hence, P = MR (marginal revenue). By the maximization of its profit, the firm chooses q* such that

$$MC = MR\ (= P)$$

Hence,

$$2q^* + 8 = 50.$$

So

$$q^* = (50 - 8)/2 = 21.$$

Therefore, Kemps produces 21 gallons of milk in the short-run.

b) Given your answer in (a), find the short run profit for Kemps. Show your work.

Since Kemps sells q* = 21 at the market price $50, the total revenue (TR) is

$$TR = (q^*) * P = 21*50 = \$1050.$$

On the other hand, the total cost (TC) is

$$TC = (q^*)^2 + 8*(q^*) + 5 = 21^2 + 8*21 + 5 = \$614.$$

Therefore, the profit is

$$\text{profit} = TR - TC = 1050 - 614 = \$436.$$

c) What will happen in this market in the long-run? Do new companies enter the market or existing companies exit the market? If there is entry or exit, when will the entry/exit stop?

In the perfectly competitive market, new companies are free to enter the market in the long run. Since there is a positive profit in the short-run, the profit exceeds the opportunity cost, which means that there will be entries of new companies. These new companies will mimic Kemps' production procedure, and thus the entries will continue until the profit becomes zero. Then MC = ATC (average total cost TC/Q) in the long-run and the profit for each firm in the industry will be equal to a normal economic profit of $0. Total revenue for each firm will equal the total cost for each firm: revenue will just cover the opportunity costs of production.

Question 5.

Consider the Chinese restaurant industry in Madison. The city has so many Chinese restaurants that that market can be seen as perfectly competitive. Each restaurant can serve at most 30 bowls of fried rice. Assume for mathematical simplicity that the marginal cost of serving a bowl of fried rice is constant and it is $8. The market demand for fried rice in a day is given by

$$Q = 700 - 5P$$

d) How much is a bowl of fried rice in the perfectly competitive market?

At the perfectly competitive equilibrium, P = MC. Hence, by MC = $8, the price must be P = $8.

e) How many bowls of fried rice will be sold in Madison every day?

Since P = $8 by (a), plugging it into the demand function,

$$Q = 700 - 5 * 8 = 660.$$

f) How many Chinese restaurants will be in Madison?

Since each restaurants can serve at most 30 bowls in a day, the number of Chinese restaurants needed is:

$$Q/30 = 660/30 = 22 \text{ restaurants.}$$

Part V: Monopoly

<u>Question 6.</u>

MG&E (Madison Gas and Electronic) is a monopolist in selling electricity in Madison. The market demand curve for electricity faced by this monopolist is given as $p = 6 - \frac{1}{8}Q^D$. MG&E's total cost is given by $p = \frac{1}{16}Q^2 + 5$, and the marginal cost is given by $MC = \frac{1}{8}Q$. Use this information to answer the following questions.

a. What is the equation for the marginal revenue (MR) curve for MG&E?

Since the demand curve faced by the monopolist is linear, the marginal revenue (MR) curve for the monopolist will have the same P-intercept as the demand curve and will have twice the slope of the demand curve. Thus, $MR = 6 - \frac{1}{4}Q^D$.

b. What are the equations for the average total cost (ATC), average variable cost (AVC), and average fixed cost (AFC) for MG&E?

The average total cost for the monopoly is total cost divided by quantity: $ATC = \frac{TC}{Q} = \frac{Q}{16} + \frac{5}{Q}$. Where the first term on the right hand side is the AVC: so AVC = $\frac{Q}{16}$. The second term on the right hand side is the AFC: so AFC = $\frac{5}{Q}$. Remember that fixed cost is $5, and therefore AFC = FC/Q = 5/Q.

c. What is the profit maximizing production quantity, $Q_{M,}$ for MG&E if the monopolist only charges one price and the monopolist is free to set whatever price it chooses (that is, the monopoly is not regulated)? What price, P_M , will it charge? Calculate the value of this monopolist's profits.

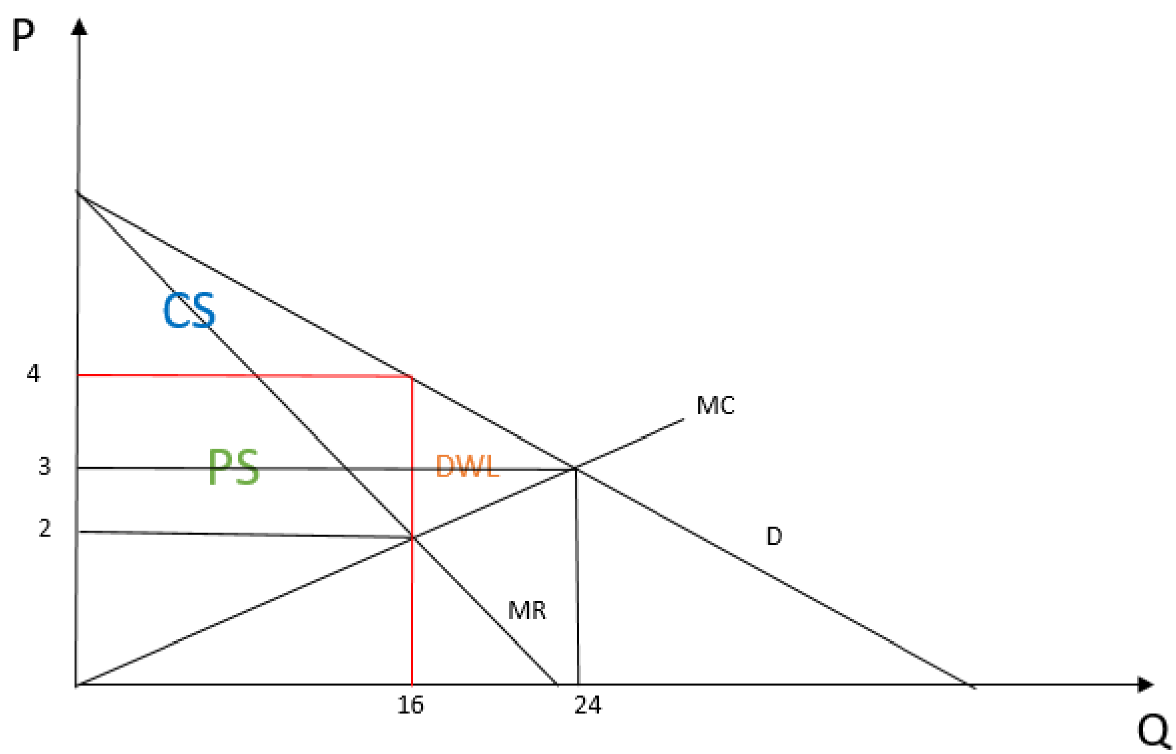
To find the profit maximizing production quantity and price for the single price monopolist we need to set MC = MR.

$$MC = MR \Rightarrow \frac{1}{8}Q = 6 - \frac{1}{4}Q \quad \Rightarrow \frac{3}{8}Q = 6 \Rightarrow Q = 16$$

To find the equilibrium price for the single price monopolist, substitute $Q_M = 16$ into the demand equation: $P_M = 6 - (1/8)(16) = \$4$. The total revenue for the single price monopolist is TR = $P_M * Q_M = 16*4 = \$64$ and the total cost for the single price monopolist is TC = $(1/16)(16)^2 + 5 = 16 + 5 = \$21$. Hence, the profit for single price monopolist = TR – TC = \$64 - \$21 = \$43.

d. Compute the consumer surplus and producer surplus for the monopolist. What is the deadweight loss in this market because of the presence of a monopoly?

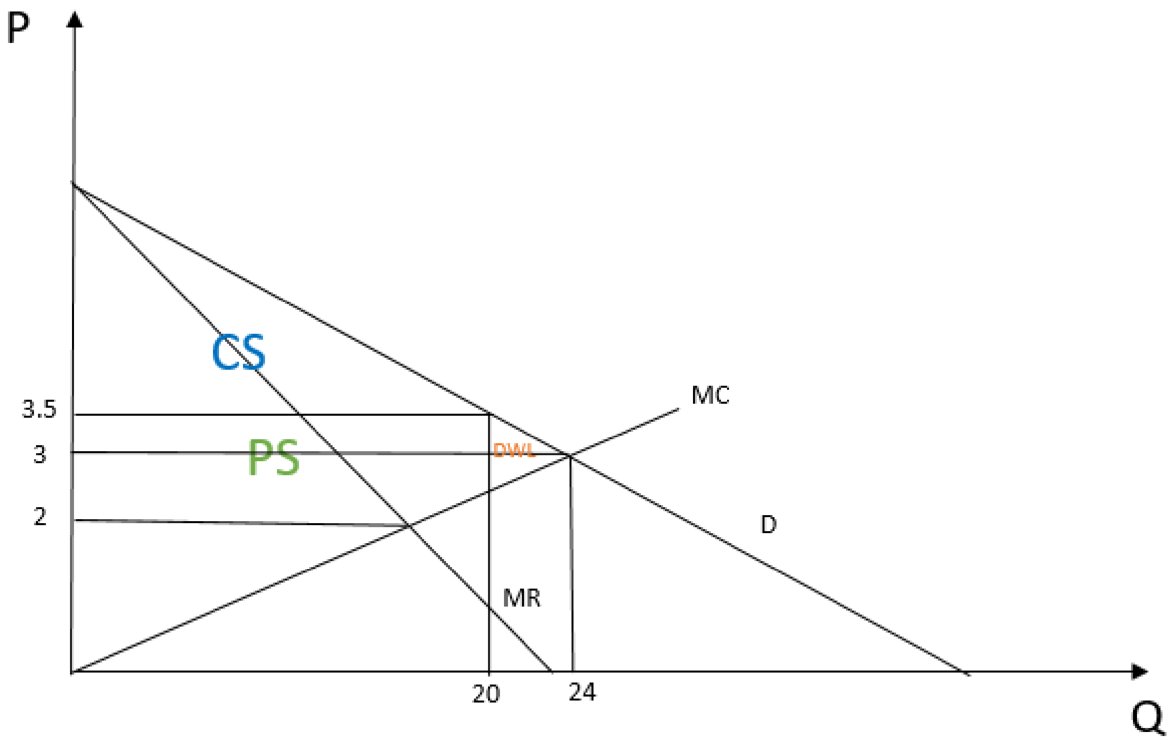
Using the following graph, we have:



$$\begin{cases} CS = \frac{1}{2}\times(6-4)\times 16 = \$16 \\ PS = \frac{1}{2}\times(4+2)\times 16 = \$48 \\ DWL = \frac{1}{2}\times 2\times(24-16) = \$8 \end{cases}$$

e. Now suppose that because of the deadweight loss in the economy, the government decides to set a price ceiling of $3.50 per unit of the good. Is this price ceiling good or bad for consumers and producers?

Since $3.50 < $4.00 the pric ceiling is effective. We can calculate the quantity demanded at this price from the demand function (Q = 20). To calculate the areas we can use the following graph:



$$\begin{cases} CS = \frac{1}{2}\times(6-3.5)\times 20 = \$25 \\ PS = \frac{1}{2}\times\big(3.5+(3.5-2.5)\big)\times 20 = \$45 \\ DWL = \frac{1}{2}\times 1\times(24-20) = \$2 \end{cases}$$

By comparing the results from (d) and (e) we can see that the consumers are better off and the producers are worse off. Also, the deadweight loss is smaller.