Economics 101
Fall 2013
Homework 4
Due Tuesday, November 5, 2013

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!
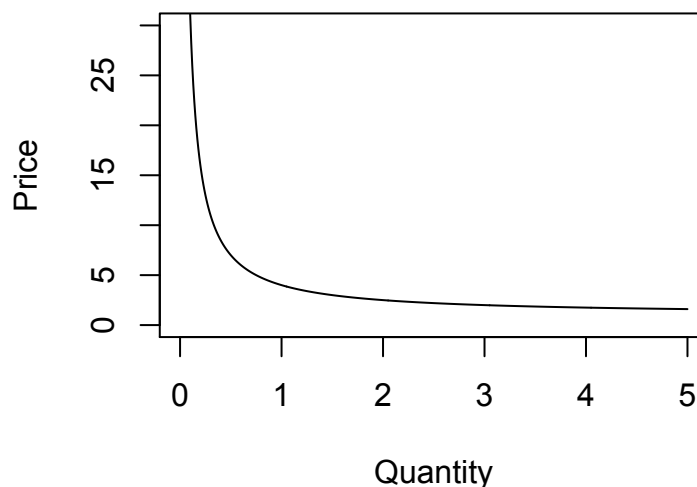
Elasticity and Total Revenue

1.) Consider the demand equation $P = 3/Q^D + 1$, where P is the price in dollars and Q^D is the quantity demanded.

a.) Plot the demand curve qualitatively (you may plot a few points and then fill in the general shape from there, because the line is nonlinear). Does this demand curve obey the law of demand?

The demand curve bows in towards the origin as shown below. It does obey the law of demand as lowering the price always increases the quantity demanded.
b.) For what region on this curve is demand elastic? Where is demand inelastic? Note, some of you may solve this question using the brute force of calculus, but it is actually much easier without calculus. Instead, use a total revenue argument. You may find it helpful to make a graph relating quantity to total revenue measuring quantity on the horizontal axis and total revenue on the vertical axis.
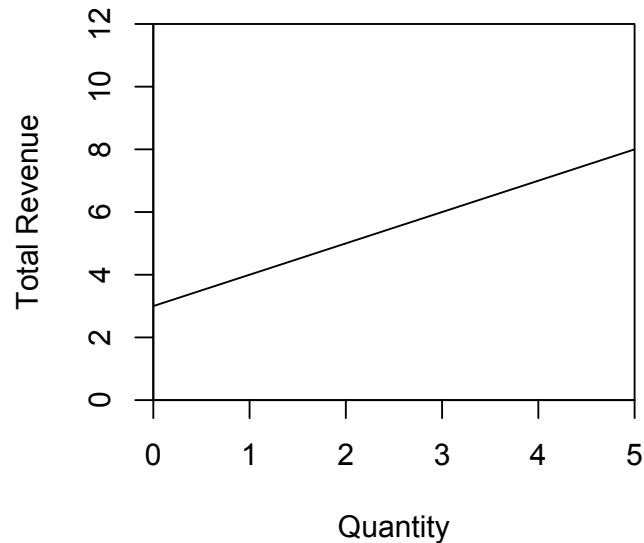

This demand curve is everywhere elastic (for all prices above $1).

Note Total Revenue (TR) = P*Q
Substitute in for P, TR = P*Q = 3 + Q
So total revenue is increasing in quantity. Equivalently, total revenue increases as the price falls to P = 1 (the demand curve is actually not defined for prices below P = 1). This implies that on this demand curve

2.) Bledi and Nisha are the only consumers in the market for pet turtles. Their demands are described below, where P is the price in dollars and Q is the number of turtles.
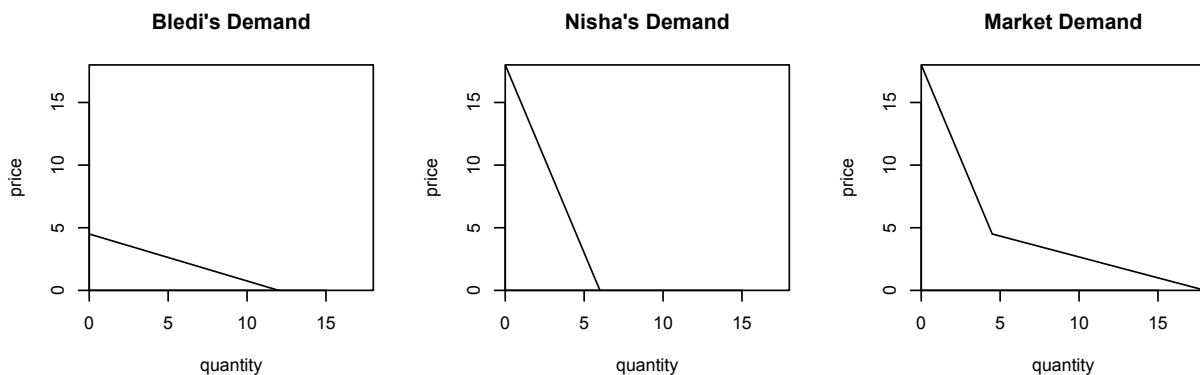
Bledi: $Q^D = 12 - (8/3)P$

Nisha: $Q^D = 6 - (1/3)P$

a.) Plot the individual demand curves and the market demand curve in three separate graphs (you will find it helpful to align these three graphs horizontally as we did in class). Label each graph carefully and completely.

The market demand has equation:
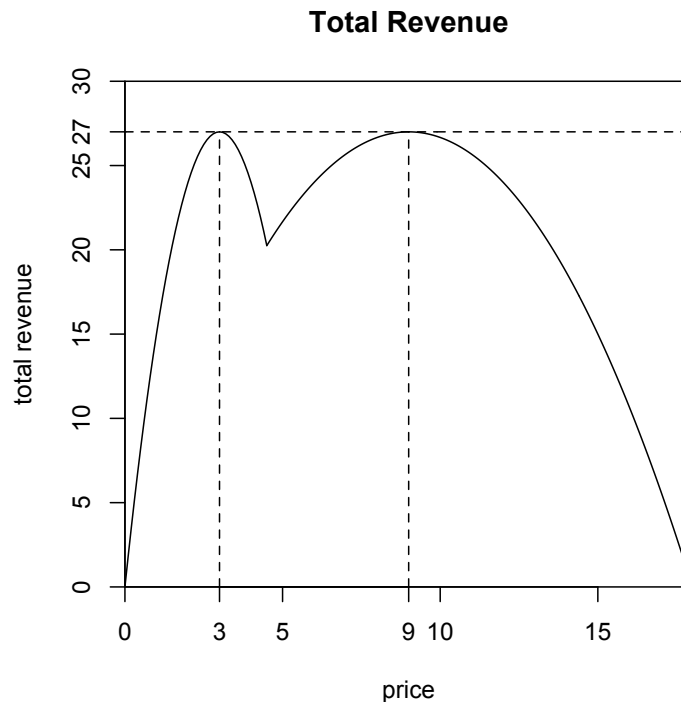
$Q = 6 - (1/3)P$ for $P > (36/8)$

$Q = 18 - 3P$ for $P < (36/8)$

b.) Plot, qualitatively, the total revenue corresponding to the market demand as a function of price. That is, draw a graph measuring total revenue on the vertical axis and price on the horizontal axis. Completing the table below and then plotting the points (price, total revenue) will help you construct this graph. From these points you found in the table, use your knowledge of elasticity to help fill in the general shape of the line that represents the relationship between price and total revenue (hint: don't expect this line to be linear!).

Price (ascending)	Quantity (descending)	Total Revenue
0	18	0
1	15	15
3	9	27
4.5	4.5	81/4 = 20.25
9	3	27
18+	0	0

Remembering that total revenue will peak at the price halfway between the y-intercept and zero (for linear demand), we can hypothesize a double-peaked shape to our total revenue plot, as revenue will attain a local maximum on each segment of the kinked demand curve.

Total Revenue

c.) Where is total revenue maximized? Label this on your graph from part (b).

P = \$3, Q = 9 *and* at P = \$9, Q = 3

This piece-wise market demand will have two prices for which elasticity of demand is equal to one: that is, on each segment of the market demand you should be able to find a price that maximizes total revenue for that segment of the market demand and the price elasticity of demand (using the point elasticity formula) will be equal to one. Computing elasticity for a point is done according to the usual formulas, according to the equation for the relevant piece of the demand curve.

For low prices, revenue peaks at a price of $3. For high prices (when only Nisha demands positive quantity), revenue peaks at a price of $9. These prices give a quantity demanded of 9 and 3 units respectively. So revenue is maximized at $27 by charging $3 and selling 9 units, or by charging $9 and selling 3 units. This is shown on the graph from part (b). Note, solving for a total revenue function is not necessary. But if you did solve for total revenue, note further the shape. The total revenue function is a combination of the total revenue corresponding to Nisha's demand curve (for high prices) and the total revenue corresponding to the combined demand (for prices low enough that Bledi will demand).

For higher prices, the total revenue curve is a bit more stout--the slope changes more slowly. This reflects the fact that Nisha's demand is more inelastic at any price than is Bledi's. A change in price has a smaller impact on the total revenue acquired from Nisha than from Bledi.

CPI/Inflation/Nominal/Real

3.) Serena Van der Woodsen spends her summer vacation in Venice every year. Although her budget is generally unlimited thanks to her trust funds and thus she doesn't have to worry about the prices of the goods she consumes, she is quite interested in how the cost of living has changed over time. Serena decides to make a table that provides the cost of her "Venice market basket" over time: here is the data she collects.

	Cost of "Venice Market Basket" (euros)
1998	256
1999	300
2000	368
2001	412
2002	450
2003	468

a.) Using 1998 as the base year, compute the CPI based on the "Venice market basket" and a 100 point scale. In your answer provide the general formula for the CPI in a specific year and then place your calculations in a table like the one below. Round the CPI figures you calculate to the nearest whole number.

	CPI with base year 1998
1998	
1999	
2000	
2001	
2002	

2003	

	CPI with base year 1998
1998	(256/256)(100) = 100
1999	(300/256)(100) = 117
2000	(368/256)(100) = 144
2001	(412/256)(100) = 161
2002	(450/256)(100) = 176
2003	(468/256)(100) = 183

b.) With base year 1998, compute the rate of inflation from 2000 to 2001 using the CPI with base year 1998 as the inflation index for this computation.

The CPI's for 2000 and 2001 are 144 and 161 respectively, and so the inflation rate is equal to [(161 − 144)/144](100%) = 12%

c.) If the real price for high heels sold in Venice in 1999 was 55 euros per pair, what is the nominal price of high heels in Venice based on the CPI calculated using the "Venice market basket?" Carry your answer out to two places past the decimal.

Since the CPI in 1999 is 117, we can solve for the nominal price by using the formula: Real price = [(Nominal Price)/(Inflation Index)](Scale Factor) where the inflation index is the CPI. To find the nominal price we will need to rearrange this formula and solve for the nominal price. Thus:

$$\frac{Nominal\ price}{CPI} \times 100 = real\ price$$

Thus, 55 * 117/100 = 64.45 Euros

d.) Describe a way to figure out if the cost of living in Venice is higher than that in Paris.

Start by defining a market basket that will be priced in both Paris and Venice. Let's call this our "Venice Market Basket". Then collect the data to find out the cost of this market basket in Paris and in Venice during the same time period. Then, we could calculate the ratio of the cost of the market basket in Paris in year n to the cost of the same market basket in Venice in year n times the scale factor. For example, for the year 1998 we would have $\frac{Cost\ of\ basket\ for\ Paris\ in\ 1998}{Cost\ of\ basket\ for\ Venice\ in\ 1998} \times 100$

If this calculation has a value greater than 100, then living in Paris in 1998 was more costly than living in Venice in 1998; and if the calculation has a value lower than 100, then living in Paris in 1998 is less costly than living in Venice in 1998.

4.) Blair Waldorf lives in Manhattan along with her friend Dorota. Blair has asked Dorota to help her analyze the cost of living on the upper east side of Manhattan. Dorota obliges and collects the following

data about the nominal prices of typical products upper east side residents consume. The following table summarizes the data she has collected.

	Nominal Price of Hairbands	Nominal Price of Dresses	Nominal Price of Handbags
2000	$10	$20	$50
2001	$12	$34	$65
2002	$15	$36	$75
2003	$14	$45	$90

a.) Assuming that the upper east siders typically consume 5 hairbands, 6 dresses and 8 handbags, compute the cost of the basket. Consolidate your calculations in a table like the following one.

	Nominal Price of Hairbands	Nominal Price of Dresses	Nominal Price of Handbags	Cost of basket
2000	$10	$20	$50	
2001	$12	$34	$65	
2002	$15	$36	$75	
2003	$14	$45	$90	

	Nominal Price of Hairbands	Nominal Price of Dresses	Nominal Price of Handbags	Cost of basket
2000	$10	$20	$50	$570
2001	$12	$34	$65	$784
2002	$15	$36	$75	$891
2003	$14	$45	$90	$1,060

b.) Suppose Blair, while reviewing the data, realizes that there is an error for the nominal price of hairbands in 2001. She does not know the nominal price of hair bands in 2001 but she does know that the correct inflation rate from 2000 to 2001is 50% using 2000 as the base year. From this information calculate the correct nominal price for hairbands in 2001.

Let x to be the cost of basket in 2001. Then the CPI for 2001 using 2000 as the base year is $\frac{x}{570} \times 100$
Since we know that the inflation rate is 50%, we can deduce x by setting up an equation as follows:
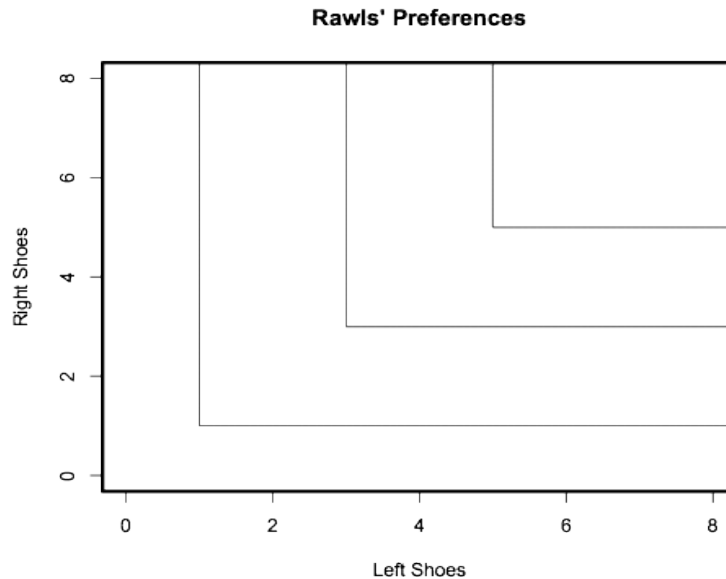$$\frac{\frac{100x}{570} - 100}{100} \times 100 = 50$$

Solving for x, we get, x = 855
Then let p to be the nominal price for hairbands, and 5p+204+520 = 855
Thus, price of hairband = $26.20

5.) Rawls considers left and right shoes to be perfect complements (that is, Rawls always wears both a left shoe and a right shoe). For some reason, his local shoe store sells left and right shoes separately.

a.) Plot Rawls' indifference curves. Put left shoes on the *x*-axis and right shoes on the *y*-axis.

Rawls' Preferences

b.) Suppose Rawls owns 3 right shoes and 3 left shoes. He then inherits 10 left shoes from his Uncle Wilson (Wilson had two left feet). What is the marginal utility of each of these 10 left shoes?
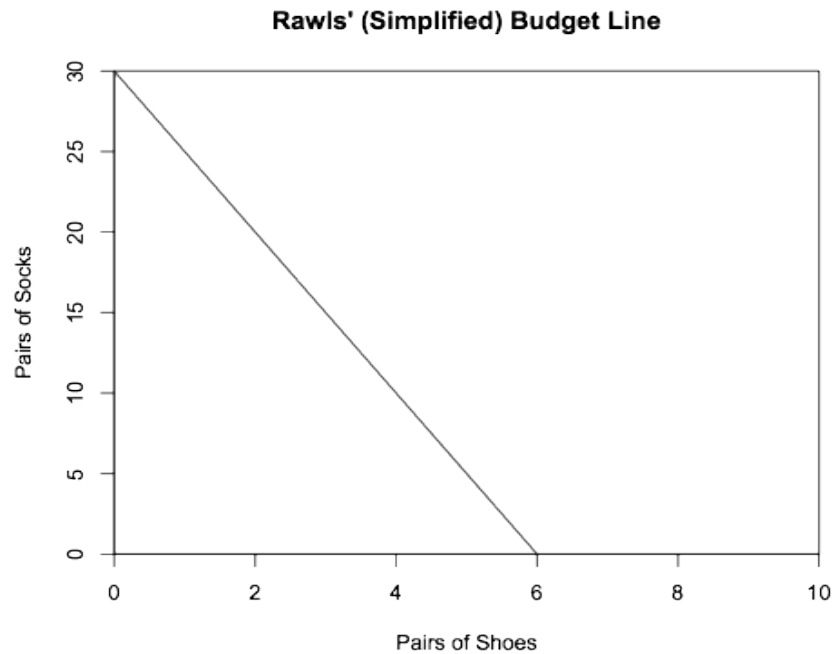
From point (3,3) to point (10,3), the slope of the indifference curve is always zero. The slope of the indifference curve is $-MU_x/MU_y$. Therefore, we can conclude that $MU_x=0$ for each of these shoes.

Instead, now suppose Rawls has $30 to spend on any combination of left shoes, right shoes, and pairs of socks. Left shoes cost $2 each, right shoes cost $3 each, and socks cost $1 for each pair.

c.) Rawls has preferences over all three of these goods. This means we have a utility function of three variables, U(x,y,z). Argue that we can simplify this into a function of just two variables by reinterpreting two of these goods to be just one good. Then draw Rawls' budget line for these two goods. Put the combined two-in-one good on the *x*-axis, give it a name, and put the other good on the *y*-axis.

To optimally buy left and right shoes, Rawls will purchase equal quantities of both. So if we know that Rawls buys X quantity of left shoes, he will also buy Y=X quantity of right shoes and vice versa. In essence, one left shoe and one right shoe are together a single good (a pair of shoes) since they are never purchased separately. A pair of shoes will have a price of $5 (combined cost of a left and right shoe).

A bit more technically: we started with an optimization across three variables--maximize the value of the utility function U(left shoes, right shoes, socks) subject to the budget constraint. We already know exactly how Rawls will optimize across two of these variables, irrespective of any positive prices for any of these three goods, so we can figure out the optimal quantity of left shoes as a function of right shoes. This gives us a simpler maximization of U(left shoes as a function of right shoes, right shoes, socks). This is a function of only two variables now.

Rawls' (Simplified) Budget Line

d.) What is the slope of the budget line? What is the meaning of this number?

The slope is -5. This is the opportunity cost of purchasing one of the good on the *x*-axis, a pair of shoes. Buying one pair of shoes means you can't buy five pairs of socks. Equivalently, the slope is price of shoes over the price of socks (the relative price of shoes).
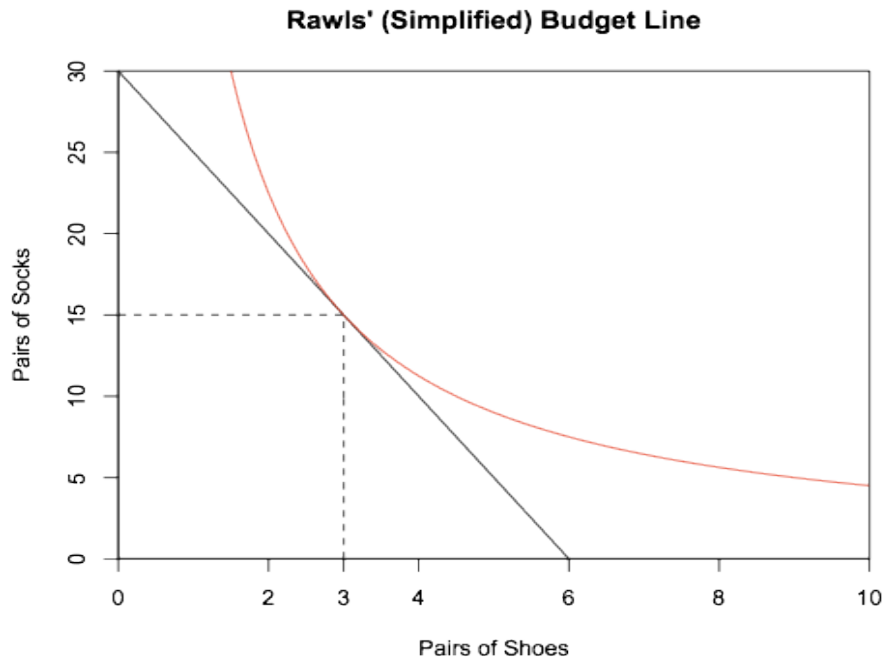
e.) Suppose Rawls has a utility function of $U(x,y) = xy$, where x is the quantity of the combined good from part (c) and y is the quantity of the remaining good (hint: we are being vague with the names of these goods because we don't want to give you the answer to (c)-but, you should name them more clearly in your answer!). The marginal utility of good x is $MU_x=y$. The marginal utility of good y is $MU_y=x$. In what ratio will Rawls purchase these goods? Given his income of $30, how many will he buy of each if we assume that Rawls seeks to maximize his utility from consuming these goods given his tastes and preferences and his income?

We equate the marginal utility per dollar of each of these goods (we can do this because we have a diminishing MRS). X is the number of pairs of shoes and Y is the number of pairs of socks.

$MU_y/P_y = x/1 = MU_x/P_x = y/5$
$y = 5x$

Substitute this into the budget constraint, $30 = 5x + 1y$
$10x = 30$
x = 3 pairs of shoes
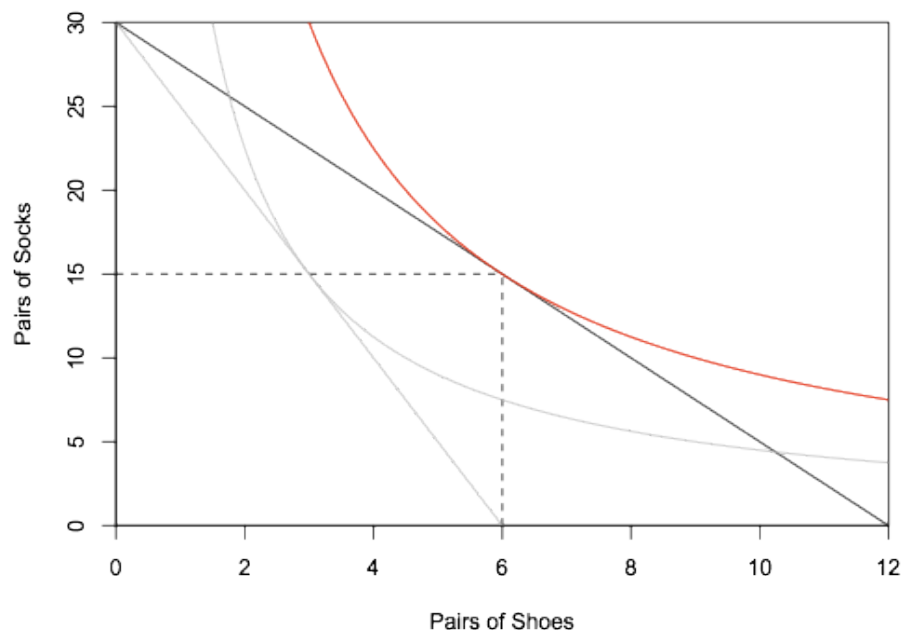y = 15 pairs of socks

Rawls' (Simplified) Budget Line

f.) Now, suppose the price for the pair of shoes changes from $5 to $2.5. Holding everything else constant, what is Rawls' new consumption point that will maximize his satisfaction? Draw your new budget line and identify the new utility maximizing point. Assume goods to be divisible.

Again, we equate the marginal utility per dollar of each of these goods (we can do this because we have a diminishing MRS). X is the number of pairs of shoes and Y is the number of pairs of socks.

$MU_y/P_y = x/1 = MU_x/P_x = y/2.5$
$y = 2.5x$

Substitute this into the budget constraint, $30 = 2.5x + 1y$
$5x = 30$
$x = 6$ pairs of shoes
$y = 15$ pairs of socks

Pairs of Shoes

g.) Now, in your graph, identify the substitution effect and the income effect with respect to Rawls' consumption of pair of shoes. How do each of these effects affect the change in the quantity of pairs of shoes consumed by Rawls when the price of a pair of shoes changes? In your answer, numerically analyze each effect and give economic interpretations for these two effects. Don't be surprised if you end up with some "challenging" numbers in this exercise!

Rawls optimal consumption bundle before the price change was $(x, y) = (3, 15)$ and his new optimal consumption bundle after the price change is $(x', y') = (6, 15)$.

Now, consider an imaginary budget line that reflects this price change but, allows Rawls to enjoy the same utility level he used to enjoy with his original consumption bundle. Thus we are looking for a point (x'', y'') that satisfies the following equations:
$U(x, y) = 3*15 = 45 = x''*y'' = U(x'', y'')$ and $2.5x'' + 1y'' = I$ where I is income.
Since $MU_y/P_y = x/1 = MU_x/P_x = y/2.5$
$y = 2.5x$
$x''*2.5x'' = 45$ (here we are just substituting $(2.5x'')$ for (y'')).

$x'' = \sqrt{18} = 3\sqrt{2}$ and $y'' = \frac{15}{\sqrt{2}} = \frac{15\sqrt{2}}{2} = 7.5*(2)^{1/2}$
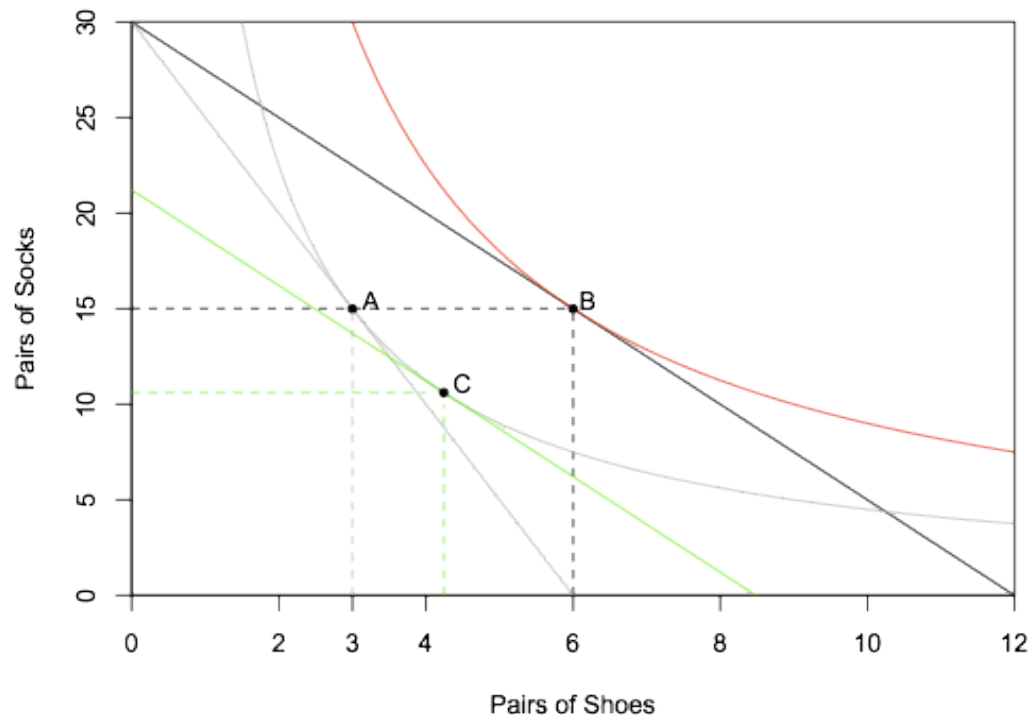Thus, we can separate the movement from one optimal consumption point to another optimal consumption point, that is from (x, y) to (x', y'), into the following:

$(x, y) = (3, 15) \rightarrow (x'', y'') = (3\sqrt{2}, \frac{15\sqrt{2}}{2}) \rightarrow (x', y') = (6, 15)$
The first part corresponds to the substitution effect and the second part, to the income effect.
Due to a price decrease in good x, Rawls would like to consume more of good x, which has become cheaper than before. (Substitution effect) Also, consuming more of the good x is reinforced due to a rise in the actual purchasing power of Rawls. (Income effect)

Refer to the following for a graphical illustration of this analysis: the horizontal change in the number of pairs of shoes from point A to point C is the substitution effect and the horizontal change in the number of pairs of shoes from point C to point B is the income effect.
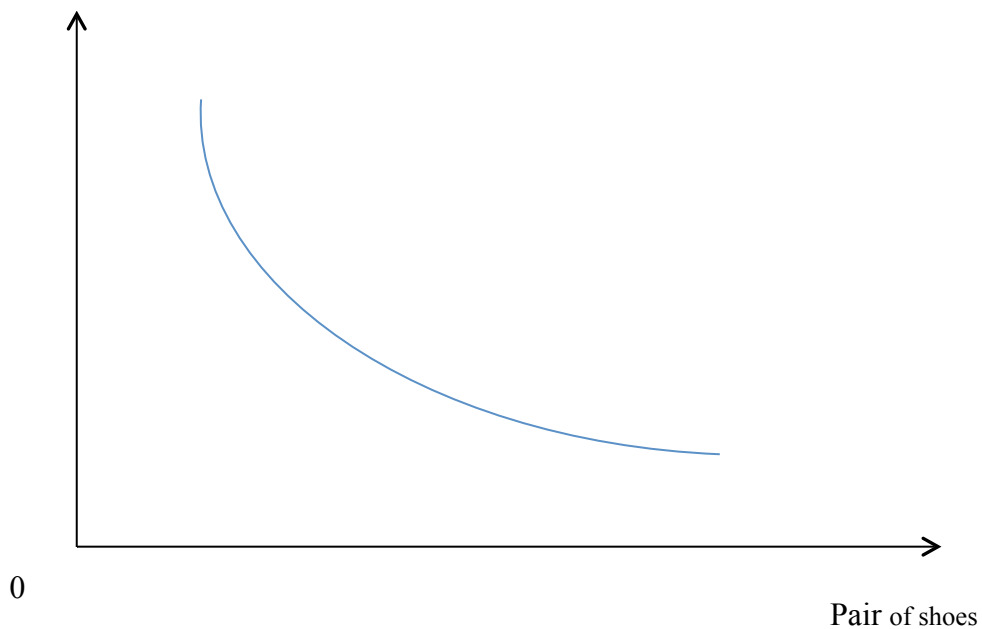

h.) Does Rawls' choice of consumption for the pair of shoes obey the law of demand? (Hint: draw a graph of Rawls' demand curve for the various price changes you have been given.)

Yes, Rawls' choice of consumption for the pair of shoes obeys the law of demand.

Connecting points A and B from the figure in part (g) will give two points on the price-consumption line. We see that as the price of shoes decreases, Rawls will demand more shoes.
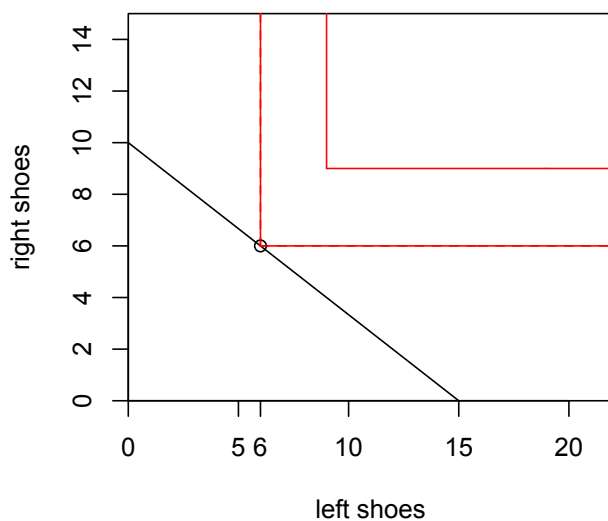
Alternatively, you could take the two consumer optimal points (points A and B) that you found and graph them in (quantity of good X, price of good X space) to get two points on the demand curve for good X. You would need to think about what the price of good X is when constructing BL1 and what the price of good X is when constructing BL2. If you plotted these two points you would find that the relationship between the price of good X and the quantity demanded of good X is an inverse relationship. The graph below provides a schematic to illustrate this idea. Of a downward sloping demand curve like the one below. Each tangency is mapped into a point on a demand curve using the number of shoes bought according to the price.

Price of the pair of shoes

Pair of shoes

6.) Let's return to just the market for left shoes (x) and right shoes (y) for Rawls. Rawls considers left and right shoes to be perfect complements (that is, Rawls always wears both a left shoe and a right shoe, or mathematically speaking $x = y$). For some reason, his local shoe store sells left and right shoes separately. Suppose Rawls has $30 to spend on any combination of left shoes and right shoes. Left shoes cost $2 each and right shoes cost $3 each.

a.) Draw Rawls' indifference curves together with the budget line and graphically find the optimal consumption point for Rawls given the above information.

b.) Numerically solve for Rawls' optimal consumption point given the above information.

Since the utility function is such that Rawls consumes the same amount of x with the same amount of y, i.e. x = y, we can solve for the optimal consumption bundle by plugging this into the budget constraint 2x + 3y = 30
5x = 30
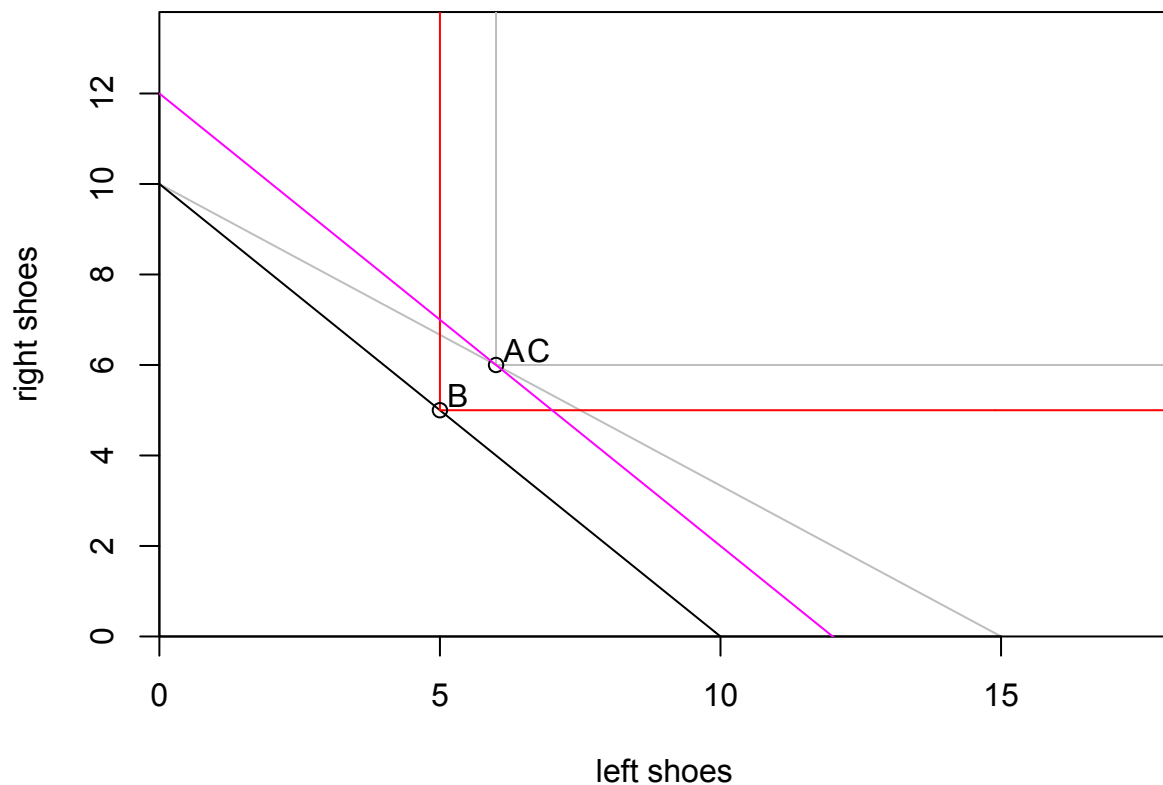Thus, x = 6, and y = 6 will be the new optimal consumption bundle.

c.) Suppose the price of a left shoes increases from $2 to $3. Given this information, what is the new optimal consumption bundle? Indicate the changes in your graph in (a) and interpret your results in terms of the income effect and the substitution effect.

With the change in the price of a left shoe, we have a new budget constraint of 3x+3y=30
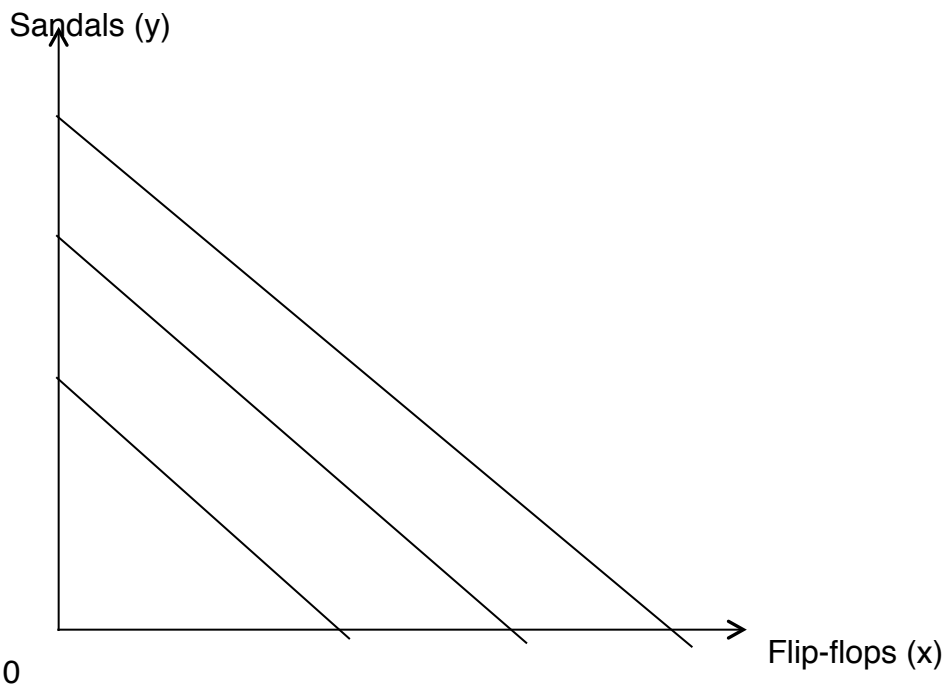Since x=y, we have 6x = 30 and thus, x=5, y=5 (denoted as B on the graph below)
Because in this example the goods are perfect complements, we do not have any substitution effect here but just an income effect. Note points A and C are the same.

7.) Now let's consider Rawls' brother John and his preferences with regard to flip-flops (measured on the x-axis) and sandals (measured on the y-axis). John considers flip-flops and sandals to be perfect substitutes for one another: that is, from John's perspective he is perfectly indifferent between a pair of flip-flops and a pair of sandals. In other words, John considers 1 pair of flip-flops as providing exactly the same satisfaction as 1 pair of sandals.

a.) Given the above information draw a graph that illustrates at least three different indifference curves for John with flip-flops measured on the x-axis and sandals on the y-axis. Draw John's indifference curves over flip-flops and sandals, with putting flip-flops on the x-axis.

These lines have a slope of -1.

b.) Suppose that John has $30 to spend on sandals and flip-flops and that flip-flops cost $5 a pair while sandals cost $6 a pair. Given this information determine John's budget line in equation form and then solve to find John's optimal consumption bundle for sandals and flip-flops.

John's budget constraint can be written as $5x+6y=30$ or $y = 5 – (5/6)x$. In a more verbal representation we could write this equation as Sandals = 5 – (5/6)(Flip-flops). Since they are perfect substitutes, John will always go for the cheaper kind of footwear. Thus, John will choose to consume flip flops only. i.e. y=0, and $5x = 30$. So, x=6 and y=0. John will consume 6 pairs of flip-flops.

c.) Suppose John got an extra source of income and now has $50 instead of $30 to spend on sandals and flip-flops. Holding everything else constant how does this change John's optimal consumption bundle?

Now, John's new budget constraint is $5x+6y=50$. John will continue to only purchase flip-flops however since flip-flops and sandals are perfect substitutes to him and the prices of the two goods have remained constant. So John will have his optimal consumption bundle be y=0 and x =10. John will consume 10 pairs of flip-flops.

d.) Go back to John's original income, which is $30, and consider a price change for flip-flops: suppose that flip-flops now cost $6 per pair instead of the original $5 per pair. Holding everything else constant how does this price change alter the optimal consumption bundle for John?
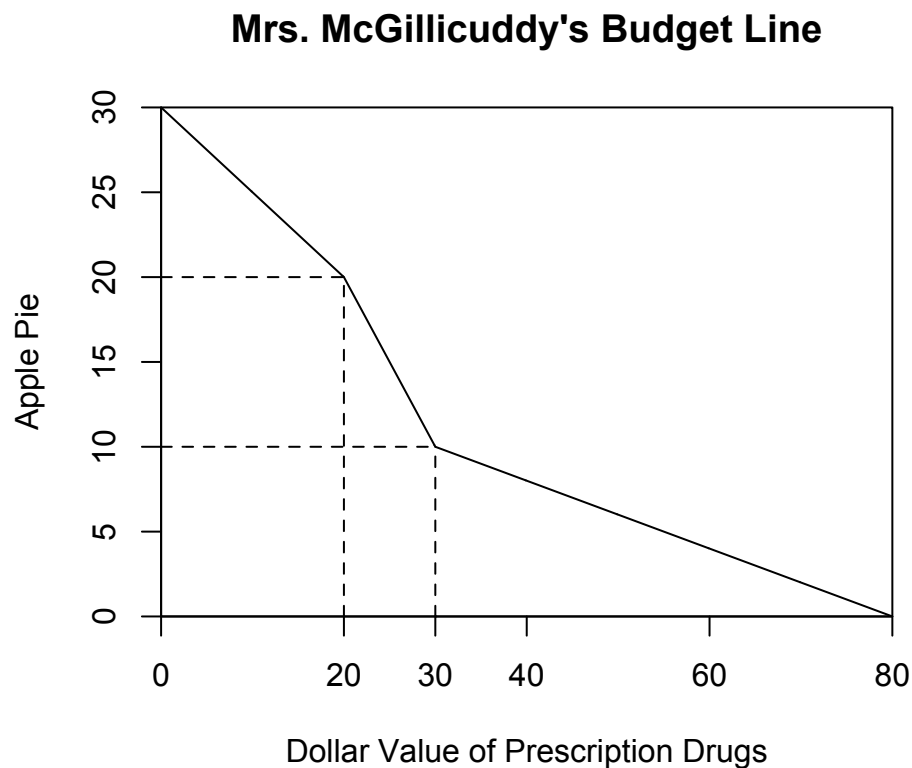
The modified budget constraint is $5x+5y=30$. Now, since the prices are the same, John will be entirely indifferent between the two. Thus, any combination of x and y will be possible as long as x+y = 6 is satisfied.

8.) Mrs. McGillicuddy spends her money on only two things, apple pie and prescription drugs. On Medicare, her prescription drug costs are subsidized according to the following schedule. Medicare covers 50% of initial drug costs until Mrs. McGillicuddy incurs an out-of-pocket expense of $10. Then Medicare covers 0% of the drug costs for her next $10 in prescription drug expenses. For any remaining expenses, Medicare will cover 80% of the cost. Suppose Mrs. McGillicuddy has $30 to spend and that apple pies cost $1.

a.) Suppose Mrs. McGillicuddy spends $10 on prescription drugs, and the rest of her income on apple pie. How many apple pies will she have? What will be the dollar value of her prescription drug purchase?

She will have 20 apple pies and $20 worth of drugs. Her first $10 expenditure on drugs is subsidized at a 50% rate, effectively making the price 50 cents on the dollar. Here, we've scaled our units to be dollars' worth of drugs, so the price is $0.50 per unit. Therefore, she will be able to purchase $20 worth of drugs for $10.

b.) Plot Mrs. McGillicuddy's budget line measuring apple pies on the vertical axis and the **dollar value of prescription drugs** on the horizontal axis. Your budget line should have two kinks (this is a more complicated problem then you saw in class lecture-so be prepared to have to think hard on this one). Label the kinks and the slopes. What do the kinks signify?

Mrs. McGillicuddy's Budget Line

Start at the point (0, 30), so that Mrs. McGillicuddy is spending all of her money on apple pie. At this point, Medicare will cover 50% of prescription drug expenses. So $20 worth of drugs will only cost Mrs. McGillicuddy $10. Effectively, the price of $1 worth of drugs is $0.50, so we get a slope of 1/2. After Mrs. McGillicuddy has spent $10 (at coordinate point (20, 20)), this discount goes away and both goods cost $1 (gives a budget line slope of 1). After Mrs. McGillicuddy has spent another $10 on drugs, or $20 in total, the 80% coverage will kick in (starting at point (30,10)). At this point, Mrs. McGillicuddy has $10 left to spend, with which she can purchase $50 of drugs given the 80% discount. This is calculated by

taking the money left ($10) and dividing through by the price of drugs with the discount (she pays 20% so the price is effectively $0.20), giving 10/(1/5)=10*5= $50 of drugs. This gives us the x-intercept of 80 and slope of 1/5 for this final segment of the budget line as Mrs. McGillicuddy is responsible for paying only 1/5 of the price. The kinks signify the changes in insurance coverage leading to these different prices.

How strange looking. For thematic justification and a wonky real life application, google "Medicare donut hole." Medicare Part D does indeed have a coverage gap like the one shown.
There is recognition of this issue and the Affordable Care Act ("Obamacare") includes some important changes that address this issue: by 2020 the Medicare donut hole will have been phased out.