Q1: Derive a demand curve

By knowing what bundle maximizes an individual's utility under various price levels, we can derive a demand curve for that person. Consider the following setup:

Situation 1: Income = 20, $P_x = 5$, $P_y = 2$ Situation 2: Income = 20, $P_x = 2$, $P_y = 2$

- a) Draw the budget lines for both situations on one graph, labeling them BL_1 and BL_2 .
- b) Suppose we are told something about the consumer's preferences: in situation 1 she buys X=2 and Y=5, and in situation 2 she buys X=4 and Y=6. Mark and label these points on the appropriate budget lines, and sketch the indifference curve that the consumer reaches in each of the two situations.
- c) Set up a new graph, with "Price of X" on the vertical axis and "Quantity of X" on the horizontal axis. For each of the two prices of X that we have considered, plot the price against the quantity demanded at that price (which you can see on the previous graph). Finally, sketch a line through the points and label it "Demand for X." (Assume that the demand curve for X is a straight line.)
- d) For extra practice, try assuming that the price of Y changes instead of the price of X. Suppose the new situation has price levels $P_x = \$5$ and $P_y = \$5$ (this is our "situation 3"). In this case, the individual consumes X=1 and Y=3. Using this information, along with the information provided for situation 1, derive the demand curve for Y. (Assume that the demand curve for Y is a straight line.)

ANSWER:

a and b. The graph is as follows:



To derive a budget line, we need to use the budget line function: PxX+PyY=I. Plug prices and income into this budget line function we can have:

BL1: 5X+2Y=20; BL2: 2X+2Y=20.

A(X=2, Y=5) is the optimal consumption bundle in situation 1 and B(X=4, Y=6) is the optimal consumption bundle in situation 2. IC1 and IC2 are indifference curves that the consumer reaches in situation 1 and situation2, which are tangent to BL1 and BL2 respectively.

c. Demand curve looks like:



When Px=5, the quantity demanded for X is 2 and when Px=2, the quantity demanded for X is 4. We can find these two points (Px=5, Qx=2) and (Px=2, Qx=4) on the above graph and use a straight line to connect them, thus we derive the demand curve for X.

d.



To derive a budget line, we need to use the budget constraint function: PxX+PyY=I. Plug prices and income into this budget line function we can have:

BL1: 5X+2Y=20; BL3: 5X+5Y=20.

A(X=2, Y=5) is the optimal consumption bundle in situation 1 and C(X=1, Y=3) is the optimal consumption bundle in situation 3. IC1 and IC3 are indifference curves that the consumer reaches in situation 1 and situation 3, which are tangent to BL1 and BL3 respectively.



When Py=2, the quantity demanded for Y is 5 and when Py=5, the quantity demanded for Y is 3. We find these two points (Py=2, Qy=5) and (Py=5, Qy=3) on the above graph and use a straight line to connect them, thus we derive the demand curve for Y.

Q2: Budget Lines and Indifference Curves

An Indifference Curve is a line that shows all the consumption bundles that yield the same amount of total utility for an individual. Please use the three types of indifference curves you learned from your lecture and the discussion section to answer following questions.



- a) Suppose Jack has an income of \$12 to buy two goods: sandwiches and sodas. The price of a bottle of soda is \$1, and the price of a sandwich is \$2. Draw Jack's budget line (BL1) given his income is \$12. (Measure sodas on the X-axis and sandwiches on the Y-axis.) Assume Jack's utility function is U(x,y)=xy (x is the consumption amount of sodas and y is the consumption amount of sandwiches). Jack's marginal utility of consuming sodas and sandwiches at consumption bundle (x, y) are denoted by MUx(x, y) and MUy(x, y) respectively. Jack's preferences are depicted by typical ICs (the left graph). The consumption bundle (x, y) which maximizes Jack's utility satisfies: MUx(x, y)/MUy(x, y)=y/x.
 - (1) Please find the numerical values of x and y of the utility maximization point (x, y). Draw a typical indifference curve (IC1) through this utility maximization point.
 - (2) Suppose the price of a bottle of soda increases from \$1 to \$4, draw Jack's new budget lines (BL₂) and find his new utility maximization consumption bundles.
 - (3) Draw an imaginary budget line (BL_3) parallel to the new budget line (BL_2) and make it tangent to the initial indifference curve (IC_1) . Show the income and substitution effect of the decrease in the consumption of soda as the price of soda increases. At the new price level, at least how much income should Jack get to achieve the original utility level? (Hint: find the tangent point of BL₃ and initial indifference curve (IC_1))

ANSWER:



Denote the price of a bottle of sodas as Px and the price of a sandwich as Py. Denote the units of consumption in soda as X and in sandwiches as Y. The budget line function should be: PxX+PyY=I(income). Plug the prices and income into budget line function we get:

BL2: 4X+2Y=12

Hence we can draw BL1 and BL2 in our above graph.

BL1: X+2Y=12

At the original price level, we assume consumption bundle A maximizes Jack's utility. Point A must lie on BL1. Since point A is the tangent point of indifference curve and BL1, the consumption bundle A(Xa, Ya) (Xa is the consumption amount of sodas and Ya is the consumption amount of sandwiches at point A) also must satisfy:

MUx(Xa, Ya)/MUy(Xa, Ya) = Px/Py = P(soda)/P(sandwich) According to question a4) we know:

MUx(x, y)/MUy(x, y) = y/x.

Combine the two above equations we get: Ya/Xa = Px/Py = (1/2)

Rearrange this equation we have: Xa=2Ya, plug it into BL1, we can solve: Xa=6, Ya=3. Thus we find the coordinates of Point A.

We can find consumption bundle B which maximizes Jack's utility at new price levels by using the same method. Point B must lie on BL2 and it is the tangent point of indifference curve and BL2. The consumption bundle B(Xb, Yb) must satisfy:

MUx(Xb, Yb)/MUy(Xb, Yb) = Px/Py = P(soda)/P(sandwich) According to question a4) we know:

MUx(x, y)/MUy(x, y)=y/x.

Combine the two above equations we get: Yb/Xb=Px/Py(=4/2)

Rearrange this equation we have: Yb=2Xb, plug it into BL2, we can solve: Xb=3/2, Yb=3. Thus we find the coordinates of Point B. Draw two convex and smooth indifference curves through A and B and demote them as IC1 and IC2. By using the utility function u(x, y) = xy, we know IC1 represents 6*3 = 18 utils and IC2 represents (3/2)*3 = 9/2utils.

Draw an imaginary budget line (BL₃) parallel to the new budget line (BL₂) and make it tangent to the initial indifference curve (IC₁), we get the tangent point C. Point C (Xc, Yc) has the same utility level as point A, which means Xc*Yc = 18. Also we know point C is Jack's optimal consumption choice given BL3, so we have the following equation: MUx(Xc, Yc)/MUy(Xc, Yc) = Yc/Xc = Px/Py = 4/2. Rearrange this equation we know: Yc = 2Xc. Combine this equation with Xc*Yc = 18 we know Xc = 3, Yc = 6. At the new price level, Jack needs (PxXc+PyYc = 4*3+2*6 = 24) amount of money to achieve the original 18 utils. Jack's current income is only \$12, so he needs (24-12 = 12) dollars to achieve the original utility. From Xa to Xc is the substitution effect (A and C are on the same indifference curve, but they are achieved at different relative price levels); from Xc to Xb is the income effect (B and C are achieved by same price levels but different income levels.)

- b) Lisa loves drinking coffee and tea. Drinking one cup of tea gives Lisa 10 utils, and drinking one cups of coffee gives her the same utility. Suppose Lisa has an income of \$12 to buy coffee and tea. The price of a cup of tea is \$1, and the price of a cup of coffee is \$2. Draw Lisa's budget line (BL₁) given her income is \$12. (Measure tea on the X-axis and coffee on the Y-axis.) Assume that the utility from consumption of an additional unit of either good is constant (this is just a simplifying assumption to make the math easier).
 - (1) Please find the utility maximization point and draw an indifference curve (IC₁) through the utility maximization point. (Hint: in this example, coffee and tea are perfect substitutes.)
 - (2) Suppose the price of a cup of tea increases from \$1 to \$4, draw Lisa's new budget lines (BL₂) and find her new utility maximization consumption bundles.
 - (3) Draw an imaginary budget line (BL₃) parallel to the new budget line (BL₂) and make it cross the initial indifference curve (IC₁) at the lowest income level. Show the income and substitution effect of the decrease in the consumption of tea as the price of tea increases.

ANSWER:



Denote the price of a cup of tea as Px and the price of a cup of coffee as Py. Denote the units of consumption in tea as X and in coffee as Y. The budget line function should be: PxX+PyY=I(income). Plug the prices and income into budget line function we get:

BL1: X+2Y=12 BL2: 4X+2Y=12 Hence we can draw BL1 and BL2 in our above graph. Because coffee and tea are perfect substitutes in this example, the indifference curves of Lisa are linear. On BL1, we find Point A(X=12, Y=0) is the optimal consumption choice of Lisa. Why? We can draw any linear indifference curves cross BL1, IC1 through point A represents the highest utility level given BL1. (IC1 is the green linear indifference curve in our graph). The economic meaning is obvious, since coffee and tea are perfect substitutes for Lisa and the price of tea is cheaper than the price of coffee, she would only consume tea! On BL2, we find Point B(X=0, Y=6) is the optimal consumption choice of Lisa. Why? We can draw any linear indifference curves cross BL2, IC2 through point B represents the highest utility level given BL2. (IC2 is the pink linear indifference curve in our graph). The economic meaning is also obvious, since coffee and tea are perfect substitutes for Lisa and the price of coffee is cheaper than the price of tea, she would only consume coffee now! Draw an imaginary budget line (BL₃) parallel to the new budget line (BL_2) and make it cross with the initial indifference curve (IC_1) at least income level, we get point C. Point C (Xc, Yc)has the same utility level as point A, but at Point C, Lisa only consumes coffee. When Lisa is given an income which allows her to achieve her initial level of utility she chooses to consume a consumption bundle that is contains only the cheaper good. The total effect of the decrease in the consumption of tea is the substitution effect.

- c) Mary is a student in the Math department who has a lot of math homework. In doing the math homework she will use pencils (assume these pencils have no erasers on their ends) to make all her calculations and an eraser to correct her answers. Mary knows that for every 2 pencils, 1 eraser will be needed. Any more pencils will serve no purpose, because she will not be able to erase the calculations. The price of an eraser is \$2, and the price of a pencil is \$1. Draw Mary's budget line (BL₁) given her income is \$12. (Measure pencils on the X-axis and erasers on the Y-axis.)
 - (1) Please find the utility maximization point and draw an indifference curve (IC₁) through the utility maximization point. (Hint: in this example, pencils and erasers are perfectly complements).
 - (2) Suppose the price of a pencil increases from \$1 to \$4, draw Mary's new budget lines (BL₂) and find her new utility maximization consumption bundles.
 - (3) Draw an imaginary budget line (BL₃) parallel to the new budget line (BL₂) and make it cross to the initial indifference curve (IC₁) at the lowest income level. Show the income and substitution effect of the decrease in the consumption of pencils as the price of pencils increase.



ANSWER:

Denote the price of a pencil as Px and the price of an eraser as Py. Denote the units of consumption in pencils as X and in erasers as Y. The budget line function should be: PxX+PyY = I(income). Plug the prices and income into budget line function we get: BL1: X+2Y = 12 BL2: 4X+2Y = 12Hence we can draw BL1 and BL2 in our above graph. Because pencils and erasers ar

Hence we can draw BL1 and BL2 in our above graph. Because pencils and erasers are perfect complements in this example, the indifference curves of Mary have the shape

depicted in the third graph shown at the beginning of this question. On BL1, we find Point A (Xa, Ya) which satisfies (Xa/Ya=2/1) is the optimal consumption choice of Mary. Why? For every 2 pencils, 1 eraser will be needed. Any more pencils will serve no purpose, vice versa. At consumption bundle (x, y), if Mary chooses x/y>2, then (x-2y) amount of pencils are useless but Mary has to pay for them; if Mary chooses x/y<2, then (y-x/2) amount of erasers are useless but Mary has to pay for them. So the optimal choice A (Xa, Ya) given BL1 must satisfy: Xa/Ya=2/1. Combine this equation with BL1: X+2Y=12 we know Xa=6, Ya=3. We can draw a similar shaped indifference curve IC1 through point A. For the same reason, Point B which maximizes Mary's utility given BL2 must satisfy: Xb/Yb=2/1. Combine this equation with BL2: 4X+2Y=12 we have: Xb=2.4, Xa=1.2. We can draw a similar shaped indifference curve IC2 through point B. Draw an imaginary budget line (BL₃) parallel to the new budget line (BL_2) and make it cross with the initial indifference curve (IC_1) at least income level, we get point C. Point C (Xc, Yc) is actually the same point with point A, both are the kink point of the right angled indifference curve. When Mary is given an income that allows her to attain the same level of utility as she had initially, she does not change her consumption choice. The total effect of the decrease in the consumption of pencils is the income effect.

Q3: Production Cost I

The following table gives cost information for a firm. Assume that labor is paid a constant wage and capital is paid a constant price, i.e., our firm is a price-taker both in the labor and capital markets.

Note: L is labor; K is capital; Q is output; VC is variable cost; FC is fixed cost; TC is total cost; AVC is average variable cost; AFC is average fixed cost; ATC is average total cost; MC is marginal cost; and MPL is the marginal product of labor.

L	K	Q	VC	FC	ТС	AVC	AFC	ATC	MC	MPL
0	45	0			90					
1	45									1
2	45									3
3	45									6
4	45									5
5	45									3
6	45		108							1

a) Please explain when output is equal to zero, why does the firm still incur costs in the short run? What is the fixed cost of this firm?

ANSWER: When output is equal to zero in the short run the firm still incurs fixed cost. It cannot get rid of its capital instantaneously so it will continue to have these costs even though it is not producing any output. The first row gives that the total cost is 90; and the

fixed cost is actually equal to the total cost, since the firm hires no labor at this stage. Thus, the fixed cost is 90. In the short run the fixed cost is constant over all production levels.

b) What is the wage rate? What is the price of a unit of capital?

ANSWER: Wage rate is 18 and the price of capital is 2. The input prices are both constant in this example. For the price of labor, note that the variable cost is equal to 108 when the firm hires 6 units of labor: thus, the price of a single unit of labor must equal 108/6 or 18. For capital price, note that the fixed cost is 90, and 45 units of capital are always used.

c) Please complete the above table with specific numbers (compute your answer to one place past the decimal).

L	K	Q	VC	FC	TC	AVC	AFC	ATC	MC	MPL
0	45	0	0	90	90					
1	45	1	18	90	108	18	90	108	18	1
2	45	4	36	90	126	9	22.5	31.5	6	3
3	45	10	54	90	144	5.4	9	14.4	3	6
4	45	15	72	90	162	4.8	6	10.8	3.6	5
5	45	18	90	90	180	5	5	10	6	3
6	45	19	108	90	198	5.7	4.7	10.4	18	1

ANSWER: the results are:

At what level of output and labor usage does marginal cost attain its minimum?
 ANSWER: Marginal cost attains its minimum at output level 10 and labor usage 3.

 e) At what level of output and labor usage is AVC at its minimum? ANSWER: AVC attains its minimum at output level 15 and labor usage 4. In the general case with continuous production, the minimum of AVC always occurs where AVC=MC.

- f) At what level of output and labor usage is ATC at its minimum?
 ANSWER: ATC attains its minimum at output level 18 and labor usage 5. In the general case with continuous production, the minimum of ATC always occurs where ATC=MC.
- g) At what level of labor usage does the law of diminishing returns first occur?
 ANSWER: The law of diminishing returns first occurs at output level 15 and labor usage 4, because when hiring the fourth unit of labor, MPL starts to decrease.
- h) As output increases, why does AVC first decrease, but then eventually increase? Please explain your answer.

ANSWER: AVC eventually increases due to diminishing returns to labor: as additional units of labor are hired the amount of capital per unit of labor decreases until eventually the additional units of labor are less productive and hence, this leads to rising average variable costs of production.

i)	Suppose the price of output is \$18, fill in the following table using the information you
	gathered in the above table.

L	Q	TR	ТС	Profit
0	0		90	
1				
2				
3				
4				
5				
6				

ANSWER: The results are:

L	Q	TR	ТС	Profit=TR-TC
0	0	0	90	-90
1	1	18	108	-90
2	4	72	126	-54
3	10	180	144	36
4	15	270	162	108
5	18	324	180	144
6	19	342	198	144

j) At what production level does the firm get its maximum profit? Please explain your answer.

ANSWER: The firm gets its maximal profit at output level 19 and labor usage 6. That is because, for a perfectly competitive firm, the profit is maximized when its marginal cost equals the price.

<u>Q4: Production Cost II</u>

The figure below shows three curves, MC, ATC and AVC, for the firm in a perfectly competitive market. In this market, the price of the good is \$10. Use the information given in the figure to answer the following questions.



a) (1) In the short run, will the firm shut down the production when price equals 10?
(2) If the firm does not shut down in the short run when price equals 10, will the firm make a positive economic profit or a negative economic profit? What is the value of the firm's economic profit when price equals 10?

(3) If the firm does not shut down in the short run when price equals 10, what will be the firm's production level? Calculate the firm's fixed cost and variable cost at this level of production.

ANSWER: (1) The firm will not shut down its production in the short run, but will suffer an economic loss of 500. Although this price is lower than ATC, it is still higher than AVC. Thus, the firm will continue producing.

(2) At this price, the firm will produce 100 units of output. Since,

$$profit = TR - TC = 10 \times 100 - 15 \times 100 = -500$$

which is negative. Thus, the firm has an economic loss or a negative economic profit.

(3) When the production level for the firm is equal to 100 units, the firm's AVC=6 and ATC= 15. Thus,

total cost = $(ATC) \times ($ production level $) = 15 \times 100 = 1500$,

variable cost = $(AVC) \times ($ production level $) = 6 \times 100 = 600$,

fixed cost = total cost - variable cost = 1500 - 600 = 900.

b) What is the break-even price for the firm? What is the shut-down price for the firm? ANSWER: The break-even price is 12 (the minimum point on the ATC curve) and the shut-down price is 5 (the minimum point on the AVC curve).

Q5: Perfect Competition

The market for study desks is characterized by perfect competition. Firms and consumers are price takers and in the long run there is free entry and exit of firms in this industry. All firms

are identical in terms of their technological capabilities. Thus the cost function as given below for a representative firm can be assumed to be the cost function faced by each firm in the industry. The total cost and marginal cost functions for the representative firm are given by the following equations:

$$TC = 2Q_s^2 + 5Q_s + 50$$
$$MC = 4Q_s + 5$$

Suppose that the market demand in this market is given by:

$$P_d = 1025 - 2Q_d$$

a) What is the equilibrium price in this market? (Hint: since the market supply is unknown at this point, it's better not to think of trying to solve this problem using demand and supply equations. Instead you should think about this problem from the perspective of a representative firm.)

ANSWER: The equilibrium price is 25. Use the fact that, in the long run, every firm always produces at the output level such that the minimum of average total cost is attained. Also, note the fact that the minimum of average total cost occurs when the average total cost is equal to the marginal cost. Given TC, we can get ATC by:

$$ATC = \frac{TC}{Q_s} = 2Q_s + 5 + \frac{50}{Q_s}$$

Set ATC = MC, then get:

$$2Q_s + 5 + \frac{50}{Q_s} = 4Q_s + 5$$

which in turn gives,

$$\frac{50}{Q_s} = 2Q_s$$

Then, $Q_s = 5$, which is the amount every representative firm produces in the long run. Plugging it into MC,

$$MC = 4 \times 5 + 5 = 25.$$

Note, in a perfectly competitive market, the equilibrium price also equals MC. Thus, the equilibrium price is 25.

- b) What is the long-run output of each representative firm in this industry? ANSWER: It is 5, solved in the above question.
- c) When this industry is in long-run equilibrium, how many firms are in the industry? ANSWER: There will be 100 firms. Since we know that the equilibrium price is 25, then plugging it into the market demand function gives the equilibrium quantity,

$$25 = 1025 - 2Q_d$$
.

i.e., $Q_d = 500$. Since we know, in the long run equilibrium, every representative firm

produces 5 and all firms are identical, then the number of firms N is

$$N = \frac{500}{5} = 100.$$

Now suppose that the number of students increases such that the market demand curve for study desks shifts out and is given by,

$$P_d = 1525 - 2Q_d$$

d) In the short-run will a representative firm in this industry earn negative economic profits, positive economic profits, or zero economic profits? (Hint: do no calculation.)

ANSWER: In the short-run, the representative firm will earn positive economic profits. That is because the market demand has shifted out and the equilibrium price is higher now, while new firms do not enter until the long run.

e) In the long-run will a representative firm in this industry earn negative economic profits, positive economic profits, or zero economic profits? (Hint: do no calculation.)

ANSWER: The long-run profits will be zero. Zero-profit in the long run is a property of perfect competition. That is because new firms will enter into the market if there are positive short-run profits, which will drive the long-run profits to zero.

f) What will be the new long-run equilibrium price in this industry?

ANSWER: It will still be 25. The long-run equilibrium price under perfect competition is always given by the point where ATC attains its minimum. Since the ATC curve has not changed, it still attains its minimum at a price of 25.

g) At the new long-run equilibrium, what will be the output of each representative firm in the industry?

ANSWER: It also will be 5. Since cost curves did not change for every firm, the output level where ATC attains its minimum will still be the same. And, in perfect competition, this output level is the long-run equilibrium output of each firm.

h) At the new long-run equilibrium, how many firms will be in the industry?

ANSWER: There will be 150. Since the long-run equilibrium price is still 25, then plugging it into the new demand curve will get total demand:

$$25 = 1525 - 2Q_{c}$$

$$\Rightarrow Q_d = 750$$
.

We also know that every firm produce 5, thus, the number of firms are:

$$N = \frac{750}{5} = 150$$

That is, there will be 50 new firms entering into this market in the long run.

Now, consider another scenario where technology advancement changes the cost functions of each representative firm. The market demand is still the original one (before the increase in the number of students). The new cost functions are:

$$TC = Q_s^2 + 5Q_s + 36$$
$$MC = 2Q_s + 5.$$

i) What will be the new equilibrium price? Is it higher or lower than the original equilibrium price?

ANSWER: The new equilibrium price is 17 and is lower than the original equilibrium price of 25. Intuitively, the technology advancement changes make the equilibrium price lower. By the same argument in question (a), given the new TC, the ATC is

$$ATC = \frac{TC}{Q_s} = Q_s + 5 + \frac{36}{Q_s}.$$

Setting ATC = MC gives,

$$Q_s + 5 + \frac{36}{Q_s} = 2Q_s + 5$$
$$\Rightarrow Q_s = 6.$$

which is the amount every representative firm produces in equilibrium. Plugging it into MC,

$$MC = 2 \times 6 + 5 = 17.$$

Note, in perfect competition, price equals MC. Thus, the equilibrium price is 17.

j) In the long-run given this technological advance, how many firms will there be in the industry?

ANSWER: There will be 84 firms now. Since the new equilibrium price is 17, then plugging it into the market demand function gets the new total demand:

$$17 = 1025 - 2Q_d$$

$$\Rightarrow Q_d = 504$$
.

Note that here every representative firm produces 6. Then,

$$N = \frac{504}{6} = 84$$
,

is the number of firms in the long run, under this new scenario.