

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. Consider an aggregate production function

$$Q = 2K^{1/2}L^{1/2}$$

where Q is the number of widgets, K is the number of units of capital, and L is the number of units of labor. For this question assume K is initially fixed at 100 units. You also know that total cost, TC, is given as

$$TC = P_K K + P_L L$$

where P_K is the price of capital and P_L is the price of labor. Assume that the price of labor and the price of capital are both constant.

a) Fill in the missing cells of the table below based on the above information. (Hint: you might find it fun to do this with Excel: practice your spreadsheet skills and generate the numbers fast!).

L	K	Q	VC	FC	TC	AVC	AFC	ATC	MC
0				\$200		---	---	---	---
1									
4			\$40						
9									
16									
25									
36									
49									

b) What is the price of capital? Explain how you got this answer.

c) What is the price of labor? Explain how you got this answer.

d) Given the above information and your work in (a), fill in the following table. Round your answers to two places past the decimal. (Hint: if you used Excel earlier, you can continue to use Excel in this part of the exercise-just a great way to keep building your spreadsheet skills!)

L	Q	MPL
0		---
1		
4		
9		
16		

25		
36		
49		

e) Given your work, does the production of this good show diminishing marginal returns to labor? Explain your answer.

f) Suppose that K doubles and L doubles. Without using numeric values, can you prove this production function has constant returns to scale? That is, can you show that if K and L both double that output, Q, will also double?

Answers:

a)

L	K	Q	VC	FC	TC	AVC	AFC	ATC	MC
0	100	0	0.00	200.00	200.00	---	---	---	---
1	100	20	10.00	200.00	210.00	0.5	10.00	10.50	0.5
4	100	40	40.00	200.00	240.00	1	5.00	6.00	1.5
9	100	60	90.00	200.00	290.00	1.5	3.33	4.83	2.5
16	100	80	160.00	200.00	360.00	2	2.50	4.50	3.5
25	100	100	250.00	200.00	450.00	2.5	2.00	4.50	4.5
36	100	120	360.00	200.00	560.00	3	1.67	4.67	5.5
49	100	140	490.00	200.00	690.00	3.5	1.43	4.93	6.5

b) The price of capital can be found by recognizing that $FC = P_K K$ and from the table we see that $FC = \$200$, we are told that $K = 100$ and therefore $200 = P_K(100)$ or the $P_K = \$2$ per unit of capital.

c) The price of labor can be found by using the provided information in the table: when $L = 4$ we see that $VC = \$40$. We know that $VC = P_L L$ and so $40 = P_L(4)$ or $P_L = \$10$ per unit of labor.

d)

L	Q	MPL
0	0	---
1	20	20 units of output per unit of labor
4	40	6.67
9	60	4
16	80	2.86
25	100	2.22
36	120	1.82
49	140	1.54

e) Yes. To see this calculate the MPL and see what happens to this measure as you increase your hiring of the variable input (in this case, labor) while holding constant your fixed input. From (d) we can see that the value of the MPL decreases as L increases: this indicates that the change in total product from hiring another unit of labor falls: this is diminishing marginal returns to labor.

$$f) Q = 2K^{1/2}L^{1/2}$$

Then K increases to 2K and L increases to 2L. So,

$$Q' = 2(2K)^{1/2} (2L)^{1/2}$$

$$Q' = 2(2)K^{1/2}L^{1/2}$$

$$Q' = 2Q$$

This production function exhibits constant returns to scale.

2. Consider a perfectly competitive industry composed of ten identical firms. Suppose you are told that the representative firm has the following cost curves:

$$\text{Total Cost: } TC = 9 + 6q + q^2$$

$$\text{Marginal Cost: } MC = 6 + 2q$$

Suppose you also know that the market demand curve is given by the following equation:

$$\text{Market Demand: } P = 20 - (1/2)Q$$

Q represents market quantity and q represents firm quantity.

a) Given the above information write an equation for the market supply curve. Explain how you found this equation.

b) Given the market supply curve you found in (a), calculate the short run market equilibrium quantity and price in this market. How many units of output will the representative firm produce in the short run? Calculate the short-run profits for the representative firm. Explain your work.

c) Given your calculations in (b), will the representative firm produce in the short-run? Explain your answer.

d) Given your answer in (b), what do you predict will happen in the long-run in this industry?

e) Given no changes in the firm's cost curves or the market demand curve, calculate the following and explain how you found your answers:

Long-run equilibrium market price = _____

Long-run equilibrium market quantity = _____

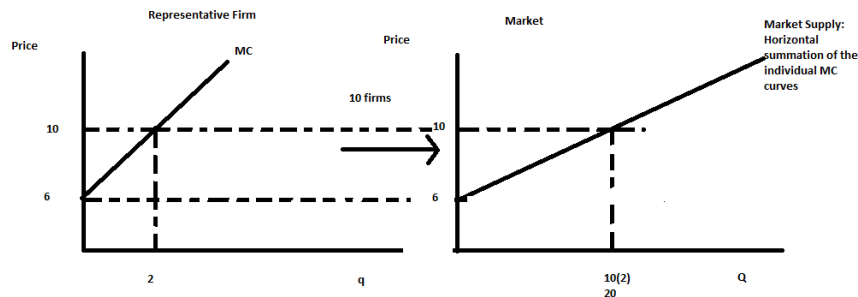
Level of production by the representative firm = _____

Approximate number of firms in industry in the long-run (this will not be a whole number) = _____

Answer:

We know that the firm's MC curve is its supply curve: technically speaking the firm's MC above its AVC curve is its supply curve, but for this exercise we can just take the MC curve as the firm's supply curve (this is because AVC and MC only intersect when $q=0$, so the shutdown point is also where $P = MC$ gives zero quantity anyway). We also know that there are ten firms

in the industry. So, let's put this together graphically to illustrate the connection between these two ideas:



Graphing the representative firm's MC curve we can see that the y-intercept is 6 and then choosing some cost per unit (in the graph I chose \$10 per unit) we can see that the representative firm is willing to supply 2 units of output. Since there are ten identical firms we can deduce that the total amount produced in the market when the cost for the additional unit is \$10 per unit will be 20 units. The market supply curve can therefore be written as $P = 6 + (1/5)Q$.

b) We know that the market demand and market supply curves are as follows:

$$\text{Market Demand: } P = 20 - 1/2Q$$

$$\text{Market Supply: } P = 6 + (1/5)Q$$

Set these two equations equal to one another:

$$20 - (1/2)Q = 6 + (1/5)Q$$

$$14 = (1/2)Q + (1/5)Q$$

$$140 = 5Q + 2Q$$

$$140 = 7Q$$

$$Q = 20$$

$$P = 20 - (1/2)Q = 20 - (1/2)(20) = \$10$$

$$\text{Or, } P = 6 + (1/5)Q = 6 + (1/5)(20) = \$10$$

The representative firm is a price-taking firm, so it will charge \$10 per unit and it will view this market price as equivalent to its MR curve. Thus, $MR = 10$. The representative firm will equate MR to MC to decide its profit maximizing output. The firm does this because it wants to equate the additional to total cost from producing the last unit of the good (the MC) to the addition to total revenue from selling the last unit of the good (the MR): when the firm produces that quantity where $MR = MC$ the firm knows that it is profit maximizing. Thus,

$$10 = 6 + 2q$$

$$4 = 2q$$

$$q = 2$$

In the short run the firm will produce 2 units of output and charge \$10 per unit.

Short-run profit for the firm can be computed as $TR - TC$. For the representative firm:

$$TR = P \cdot q = (\$10 \text{ per unit})(2 \text{ units}) = \$20$$

$$TC = 9 + 6q + q^2 = 9 + 6(2) + (2)(2) = \$25$$

$$\text{Profit in the Short-run for the firm} = \$20 - \$25 = -\$5$$

c) The firm will produce in the short-run provided its total revenue is sufficient to cover its VC of production. From the total cost curve we can compute FC as the amount of cost the firm

incurs when $q = 0$: thus, $FC = 9$. From this we know that $VC = 6q + q^2$ and if the firm produces $q = 2$, then $VC = \$16$. Since $TR > VC$ the firm will produce in the short-run.

d) Since short-run profits are negative, you can predict that firms will exit the industry in the long-run. This will cause the market supply curve to shift to the left and result in an increase in the market price, a decrease in the market quantity, and an increase in the level of production by firms that remain in the industry.

e) In the long-run $ATC = MC$ for the representative firm since the representative firm earns zero economic profit in the long-run. Thus,

$$(9/q) + 6 + q = 6 + 2q$$

$$9/q = q$$

$$q = 3 \text{ units of output}$$

We also know that in the long-run, the firm continues to profit maximize by producing where $MR = MC$ and we can use this idea to find the long-run market price. Thus,

$$MR = MC$$

$$MR = 6 + 2q$$

$$MR = 6 + 2(3) = 12 = \text{Long-run market price}$$

We can use this market price (the price that will result in all firms left in the industry earning zero economic profit) and the market demand curve to calculate the long-run market equilibrium quantity. Thus,

$$P = 20 - (1/2)Q$$

$$12 = 20 - (1/2)Q$$

$$Q = 16 \text{ units of output}$$

To find the number of firms in the industry in the long-run we can divide the market quantity, Q , by the representative firm's production, q : thus,

$$Q/q = (16 \text{ units of output}) / (3 \text{ units of output per firm}) = 5.3 \text{ firms in the industry}$$

To sum up:

Long-run equilibrium market price = \$12

Long-run equilibrium market quantity = 16 units

Level of production by representative firm = 3 units

Approximate number of firms in industry in the long-run (this will not be a whole number) = 5.3 firms

3. Consider a monopoly. Suppose you are told that the monopoly has the following cost curves:

$$\text{Total Cost: } TC = 9 + 6Q + Q^2$$

$$\text{Marginal Cost: } MC = 6 + 2Q$$

Suppose you also know that the market demand curve is given by the following equation:

$$\text{Market Demand: } P = 18 - (1/2)Q$$

a) Given the above information, what is this monopolist's equation for MR?

b) Determine the profit maximizing level of production for this monopolist as well as the price that will be charged for each unit of the good. Assume that this is a single price monopolist, i.e. the monopolist cannot engage in price discrimination. Explain how you found your answer.

c) Given the above information and your answer in (b) calculate the level of profit in the short-run for this monopolist. Explain how you found your answer.

d) Given your answer in (c), what do you predict will happen to this monopolist in the long-run?

e) Calculate the deadweight loss that results from this market being served by a monopolist. Show how you found your answer. Provide a graph that is well labeled to illustrate your answer.

Answers:

a) The monopolist's MR curve has the same y-intercept as the firm's demand curve and for a linear demand curve, has a slope that is twice the slope of the demand curve. The monopolist is the only firm in the market so the market demand curve is the monopolist's demand curve. Thus, the monopolist's MR curve can be written as $MR = 18 - Q$.

b) The profit maximizing amount of output for the monopolist is that level of output where $MR = MC$. Thus,

$$18 - Q = 6 + 2Q$$

$$12 = 3Q$$

$$Q = 4 \text{ units of output}$$

The price the monopolist will charge can be found by plugging in the profit maximizing quantity into the demand curve. Thus,

$$P = 18 - (1/2)(4) = \$16$$

c) To find the monopolist's profit we need to calculate the monopolist's total revenue and its total cost:

$$TR = P \cdot Q = (\$16 \text{ per unit of output})(4 \text{ units of output}) = \$64$$

$$TC = 9 + 6Q + Q^2 = 9 + 6(4) + (4)(4) = \$49$$

$$\text{Profit for the monopolist} = TR - TC = \$15$$

d) This monopolist will continue to earn positive economic profits in the long-run if there are effective barriers to entry that result in the monopoly continuing to operate as a monopoly and, therefore, be safe from competition.

e) To find the deadweight loss we need to first figure out the socially optimal amount of the good: this would be the amount of output where the MC equals the demand curve since for the last unit of output we have the addition to total cost from producing this last unit is equal to the value the consumer places on consuming the last unit (the price they would be willing to pay). So, setting MC equal to the demand curve we have:

$$6 + 2Q = 18 - (1/2)Q$$

$$12 = (5/2)Q$$

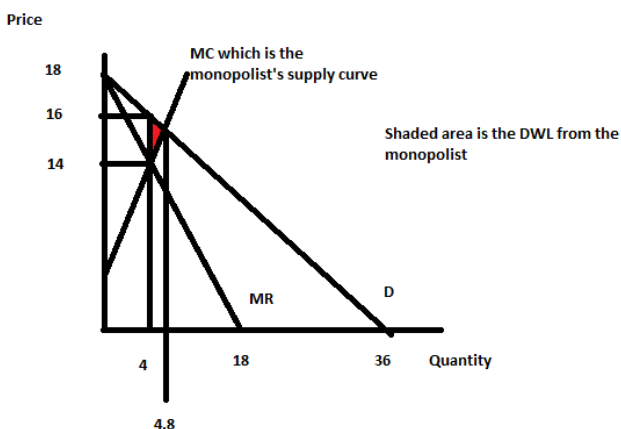
$$Q \text{ socially optimal} = 4.8 \text{ units of the good.}$$

We will also need to find the value of MC when $Q = 4$: so,

$$MC = 6 + 2Q = 6 + 2(4) = \$14$$

$$\text{Deadweight Loss from the monopoly} = (1/2)(\$16 \text{ per unit} - \$14 \text{ per unit})(4.8 \text{ units} - 4 \text{ units})$$

Deadweight Loss from the monopoly = \$0.80



7. Joe has \$100 in income that he can spend on either good X or good Y. Good X costs \$2 per unit while good Y costs \$4 per unit.

a) Given the above information, draw a graph of Joe's budget line (call it BL1) and write an equation in slope-intercept form for Joe's budget line measuring good Y as the good on the vertical axis.

b) Given Joe's income and the prices of these two goods and given Joe's preferences he finds that he maximizes his satisfaction when he chooses to consume bundle A which consists of 30 units of good X and 10 units of good Y. Can Joe afford this bundle given his income and the prices of the two goods? Prove this mathematically. Does consumption of bundle A exhaust Joe's available income?

c) Suppose that the price of good X decreases to \$1. Joe's income and the price of good Y stay constant. Joe now finds that he maximizes his satisfaction when he consumes consumption bundle B which consists of 56 units of good X. Draw a graph that represents Joe's BL1, his new budget line (BL2) and bundle A. Calculate how many units of good Y Joe consumes when he consumes consumption bundle B (make sure you show how you found this answer). Mark bundle B in your graph.

d) Suppose that Joe was constrained to stay on his first indifference curve-the one that bundle A sits on-while paying the new price for good X. We can construct this budget line 3 where Joe's income has been compensated (in this case lowered) so that he can reach the indifference curve that bundle A is on, but he cannot reach a higher level of satisfaction. On budget line 3 Joe finds that he maximizes his satisfaction by consuming bundle C which consists of 36 units of good x and 8 units of good Y. Draw a graph that illustrates BL1, BL2, BL3, bundle A, bundle B, and bundle C. Sketch in indifference curve 1 and indifference curve 2 in your graph.

e) How much would Joe's income have to be decreased by in order for his to have the same utility as he had initially but now face the lower price of good X? You have all the necessary information at hand to calculate this decrease in income. Show how you found your answer.

f) What is the amount of the substitution effect for good X given the above information? What is the amount of the income effect for good X given the above information? Explain your answer.

Answer:

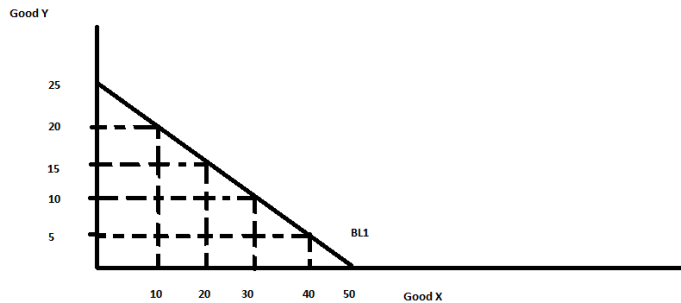
a) The equation for Joe's budget line 1 can be written as $I = P_x X + P_y Y$ where I is Joe's income, P_x is the price of good X and P_y is the price of good Y. X and Y refer to the quantities of good X and good Y, respectively. Thus, Joe's budget line is $100 = 2X + 4Y$. To put this in slope-intercept form we need to solve the equation for Y : thus,

$$4Y = 100 - 2X$$

$$\text{Or, } Y = 25 - (1/2)X$$

Note: that the y-intercept is equal to 25 and this is what you get when you divide Joe's income by the price of good Y. That is, $100/4 = 25$: if Joe spends all of his income on good Y he can afford 25 units of good Y. The slope of the budget line is $(-1/2)$ which is the negative of the ratio of the price of good X to the price of good Y or $-2/4$.

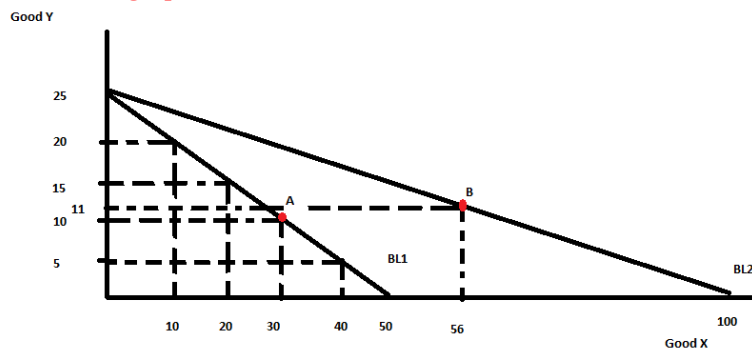
Here's a graph of Joe's budget line BL1.



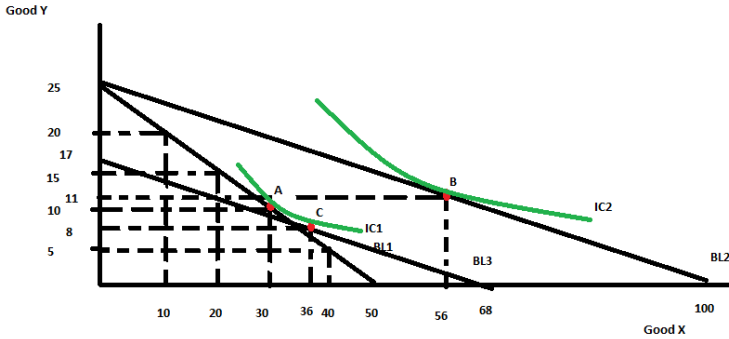
b) Joe can afford bundle A and the consumption of bundle A consumes all of Joe's income. We can see this by calculating the cost of bundle A: $P_x X + P_y Y = \text{Cost of bundle A} =$ which needs to equal Joe's available income. So, $2(30) + 10(4) = \$100$ which is Joe's income.

c) We know that Joe's income is still \$100, the price of good Y is still \$4, and the new price of good X is $P_x' = \$1$. So, the equation for Joe's budget line 2 can be written as $100 = X' + 4Y'$ where X' and Y' refer to the amount of good X and good Y Joe consumes when he is constrained by budget line 2 but still maximizes his satisfaction. We are also told that X' is equal to 56 units of good X: so using this information we can solve for Y' . thus, $100 = 56 + 4Y'$ or $Y' = 11$. So, bundle B consists of 56 units of good X and 11 units of good Y. It is no surprise that Joe consumes more of good X at bundle B than at bundle A since the price of good X has fallen: Joe has more real purchasing power from his income and he will also want to substitute toward the relatively cheaper good X and away from the relatively more expensive good Y.

Here's the graph:



d) Here's the graph:



e) We know that budget line 3 can be written as $I'' = P_x'X'' + P_yY''$ where P_x' is the new price of good X, \$1. X'' and Y'' refer to the amounts of good X and good Y Joe would choose to consume to maximize his satisfaction if he was constrained by budget line 3: we have been given these values as $(X'', Y'') = (36, 8)$. So, $I'' = (36) + 4(8) = \$68$. Joe would need \$68 in income to be on BL3 and with the price of good X \$1 and the price of good Y \$4. This is a \$32 decrease in income from his \$100 in income.

f) The substitution effect is measured as the change in the consumption of good X as Joe moves from bundle A to bundle C (he is keeping his utility constant, but facing the new prices rather than the initial prices). We can see from our work that Joe's consumption of good X increases from 30 units to 36 units: his substitution effect for good X is positive (his consumption of good X went up) and the substitution effect can be measured as 6 units.

The income effect is measured as the change in the consumption of good X as Joe moves from bundle C to bundle B (parallel shift of the budget line from BL3 to BL2: both budget lines have Joe constrained by the new prices but with different levels of income). We can see from our work that Joe's consumption of good X increases from 36 units to 56 units: his income effect for good X is positive (his consumption of good X went up) and the income effect can be measured as 20 units.