Economics 101
Spring 2015
Homework #3
Due March 19, 2015

Directions: The homework will be collected in a box **before** the lecture. Please place your name on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

Part I – Excise Taxes

1. (You are not *required* to plot anything for this part, but, as usual, plotting provides valuable insight into both the calculations and the intuition.)
Suppose the market for hot chocolate in January in Madison can be described by the following supply and demand curves:

Supply: Qs = 5P – 5 for $P \geq 1$
Demand: Qd = 100 – 10P for $0 \leq P \leq 10$

where Qs is the quantity supplied, Qd is the quantity demanded, and P is the price.

a) Find the equilibrium quantity and supply in this market.

We must find a price such that quantity supplied equals quantity demanded. Set the 5P – 5 = 100 – 10P, and solve for P. By solving we will get an equilibrium price, P = 7. By plugging this result back into the equation, we find equilibrium quantity, Q = 30.

b) What is consumer and producer surplus in this market?

Consumer surplus is the area of the triangle above the price, and below the demand curve. The height of the triangle is the P-intercept of demand minus the equilibrium price (10 – 7 = 3). The base is the quantity purchased (Q = 30). Thus, CS = (1/2)*3*30 = $45.

Producer surplus is the area of the triangle below the price and above the supply curve. The height of the triangle is the price minus the P-intercept of supply (7 – 1 = 6). The base is as before. So PS = (1/2)*6*30 = $90.

c) Using the point elasticity formula, calculate the price elasticities of demand and supply at the equilibrium point.

Point elasticity of demand is | (1/m)*(P/Q) |, where m is the slope of the demand curve. Point elasticity of supply is defined similarly but using the slope of the supply curve (and the absolute value is unnecessary). Note the supply and demand equations are both presented in an 'x-intercept' form, so the coefficient on P is (1/m).

Using these formulae, we get:

Price Elasticity of Demand = | (-10)*(7/30) | = 7/3 [Demand is elastic at this point since the elasticity value is greater than one]

Price Elasticity of Supply = 5*(7/30) = 7/6 [Supply is elastic at this point since the elasticity value is greater than one]

d) Suppose February is more bitterly cold than usual, increasing demand for hot chocolate by 60 units at any given price. Supply remains as before. Given this information, what is the new equilibrium price and quantity?

Given the change, demand can now be expressed by Q = 160 – 10P. So using the same practice as before, we set the supply equal to demand and solve for P. Thus we get equilibrium price, P = 11, and quantity Q = 50.

e) What is the new consumer and producer surplus, CS' and PS'?

The new P intercept for demand is 16. So the height of the CS' triangle is 16 – 11 = 5. Quantity is now 50 so:

CS' = (1/2)*(5)*(50) = $125

The supply curve did not change, so all we have to do is take into account the new quantity and price. The height of the PS' triangle is 11 – 1 = 10, and the base is 50. Thus,

PS' = (1/2)*(10)*(50) = $250

f) What are the new price elasticities of supply and demand using the point elasticity formula?

Price Elasticity of Demand = | (-10)*(11/50) | = 11/5 = 2.2
Price Elasticity of Supply = 5*(11/50) = 11/10 = 1.1

g) Suppose the city of Madison decides to tax hot chocolate with a $3 per unit excise tax on suppliers. Calculate the equilibrium quantity (Qe^t), price (Pe^t), consumer surplus (CS^t), producer surplus (PS^t), government revenue (Govt. Tax Rev.), tax incidence on consumers (CTI), tax incidence on producers (PTI), and deadweight loss (DWL) from this tax for January (before the change in demand).

(A graph of the solution follows the explanation)
The tax has the effect of adding a $3 per unit cost to suppliers, shifting the supply curve leftward. The original supply curve had a P-intercept of $1, so the new will have a P-intercept of $4. The slope is unchanged, so using this information we can determine that the new supply curve will be Q = 5P -20. As before solve for the equilibrium price and quantity. You should find $Pe^t = 8$ and $Qe^t = 20$.
From the plot, we can see CS^t is

CS^t = (1/2)*2*20 = $20

Although the price is $8, producers only receive $5 after paying the tax (this is P_{net} as discussed in lecture), thus Producer surplus is the triangle below $5 and above the original supply curve.

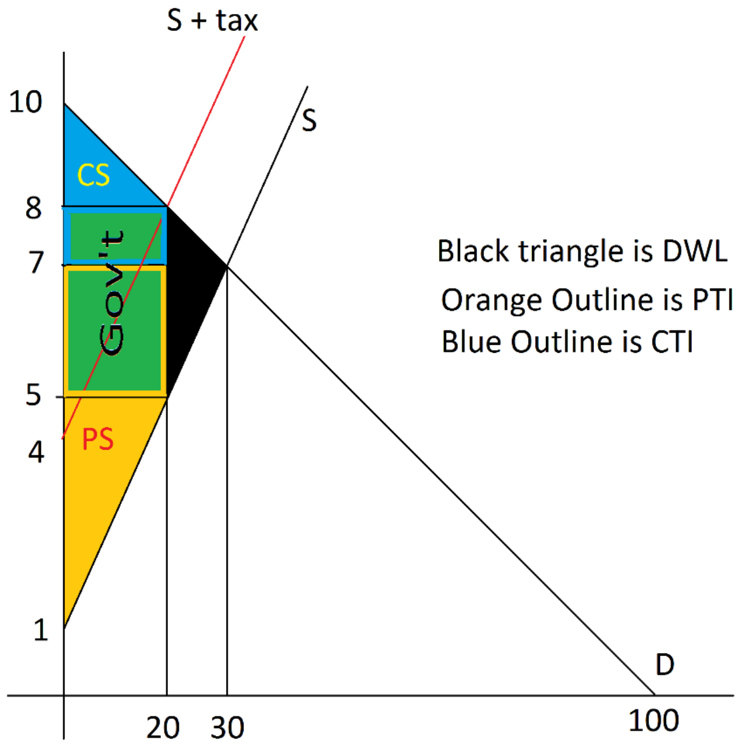
$PS^t = (1/2)*20*4 = \$40$

The tax is \$3 per unit, and 20 units were bought, thus Government Revenue is
Govt. Tax Rev. $= 3*20 = \$60$

Deadweight loss can be observed in the plot as the dark triangle in the middle.
$DWL = (1/2)*3*10 = \$15$

We can see that the DWL is not government revenue, producer surplus or consumer surplus, so it is potential surplus that is lost due to the imposition of the excise tax.

Again, from the plot, we can see that of the \$3 tax, consumers effectively pay \$1 per unit of the good consumed and producers pay \$2 per unit of the good produced and sold (the upper and lower vertical heights of the government revenue rectangle respectively). So total consumer tax incidence (CTI) is \$20, and producer tax incidence (PTI) is \$40.



h) What is the ratio of the absolute value of the slope of the demand curve to the slope of the supply curve? What is the ratio of consumer tax incidence to producer tax incidence? Is this a coincidence? Make a conjecture about what the relationship between the two ratios. Briefly discuss an intuition for this result.

The slope of the demand curve is -1/10 and the slope of the supply curve is 1/5. Thus, the absolute value of the ratio of their slopes is = ½. CTI is equal to \$20 and PTI is equal to \$40. Thus, the ratio of CTI/PTI = 1/2.

The absolute value of the ratio of slopes will always equal the ratio of tax incidences. The intuition for this is as follows:

If the magnitude of the slope of the demand curve is higher than that of the supply curve, we can think of the demand as being more inelastic than supply, thus if there is a tax, producers have a better ability to pass the tax to consumers without losing too much business. Conversely, if demand is less steep than supply, then demand is more elastic than supply, and if producers try to pass too much of the tax to consumers, they will see a sharp drop in demand.

i) [Note: this is a bit of a CHALLENGE QUESTION: don't despair, but do think!] **Without fully solving for it**, what would be the deadweight loss if we instead used the demand curve from February (i.e. after the demand shift)? (Draw a quick sketch to see what will happen.) What does this tell you about the relationship between deadweight loss and parallel shifts of one of the curves?

Deadweight loss is unchanged. If we draw a quick sketch, we would see that by shifting the demand curve out parallel to the original, we simply slide the deadweight loss triangle along the supply curve until it fits between the curves at the new equilibrium point. From this, we can conclude that parallel shifts don't affect the magnitude of the DWL.

Part II – International Trade

2. Suppose Narnia is a small country with a domestic supply and demand for Turkish Delight given by:

Domestic demand: P = 10 – (1/500)Qd
Domestic supply: P = 2 + (1/500)Qs

where P is the price, Qd is the quantity demanded domestically, and Qs is the quantity supplied domestically.

a) Suppose Narnia is currently in autarky (i.e. it does not participate in the international market for Turkish Delight-Narnia's economy is a closed economy when it comes to Turkish Delight), what is the market clearing price and quantity in the domestic market? What is Consumer, Producer, and Total Surplus?

To find the equilibrium point, follow the usual method: set supply equal to demand.
10 – (1/500)Q = 2 + (1/500)Q
Solving, we find Q = 2000. Plugging this back into either supply or demand, we find P = 6.

Consumer surplus is the triangle below the demand curve but above the equilibrium price. The P-intercept of demand is 10 and the equilibrium price is 6, so the height of the triangle is 4. The base length is simply the equilibrium quantity. Thus,
CS = (1/2)*4*2000 = $4000

To find producer surplus, we can follow a similar method to find the area of the triangle below equilibrium price but above the supply curve. Notice, in this specific instance where the absolute value of the slopes of the two curves are the same, and the distance from the equilibrium price to the P-intercept is the same, we can appeal to symmetry to see PS is the same as CS. Thus,
PS = (1/2)*4*2000 = $4000

Total surplus is merely the sum of the two so, TS = 4000 + 4000 = $8000

b) Narnia now decides to enter the international market for Turkish Delight. Once the market clears, we find that Narnia is importing 2000 units of Turkish Delight. Given this fact, what must the world price? What is the new Consumer, Producer, and Total Surplus? Illustrate your answers graphically.

We know imports are 2000. This implies that the domestic quantity demanded, Qd, is 2000 more than the domestic quantity supplied, Qs, i.e.,

Qd = Qs +2000

Further by rearranging the equations for domestic supply and demand, we find

Qd = 5000 – 500P

Qs = 500P -1000

So by substituting these two equations into the one above, we get

(5000 – 500P) = (500P – 1000) + 2000

Simplifying we get

1000P = 4000

Thus, the world price must be P = 4.

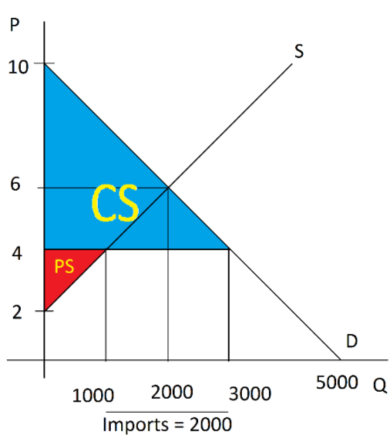
To find Consumer and Producer surpluses, we proceed as before, but now CS is a much larger triangle and PS a smaller one (see plot below).

CS = (1/2)*6*3000 = $9000

PS = (1/2)*2*1000 = $1000

TS = 9000 + 1000 = $10,000

Notice, that although PS is much smaller, the increase in CS is much greater than the loss, so TS increases.



c) Suppose Aslan, ruler of Narnia, fearing that the domestic Turkish Delight industry is unduly suffering from the influx of cheap foreign Turkish Delight, decides to implement a $1 per unit tariff on imports. What is the new price for Turkish Delight in the domestic market, the quantity consumed, the quantity imported, Consumer Surplus, Producer Surplus, Government Revenue, Total Surplus (recalling the convention that we include Government Revenue in Total Surplus), and Deadweight Loss? Illustrate your answers graphically.

Since the tariff is $1, the domestic price will be $1 more than the world price, thus P = 5. Plugging this into the demand curve, we find quantity consumed is 2500. By plugging P = 5 into the supply curve, we find the 1500 units are provided by domestic producers, thus 1000 units are imported.

We can calculate Consumer Surplus and Producer Surplus in the usual manner, finding

CS = (1/2)*5*2500 = $6250

PS = (1/2)*3*1500 = $2250

Since the tariff is $1 per unit imported, and 1000 units are imported we know Government Revenue must be

Govt. Tax Rev. = 1*1000 = $1000

Total Surplus is CS + PS + GR so

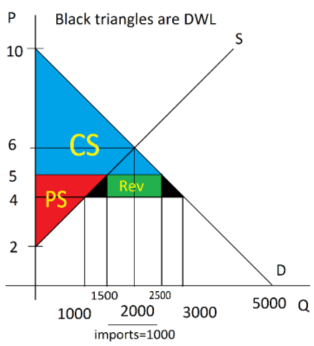
TS = 6250 + 2250 + 1000 = 9500

Deadweight Loss is the difference between the TS without the tariff and with the tariff, thus

DWL = 10,000 – 9500 = $500

Alternatively, DWL is the area of the two black triangles in the plot, each of which has a base of 500, and height of 1, so

DWL = 2*((1/2)*(500*1) = $500



d) Provide an intuitive explanation for the sources of deadweight loss under this tariff.

From the plot, we can see there are two deadweight loss triangles. The one on the right (under the demand curve) can be thought of as deadweight loss from some consumption being lost due to the tariff. The one on the left (under the supply curve) can be thought of as loss from the reallocation of productive resources. In the absence of the tariff, the only domestic production was by firms who could produce more efficiently than the firms on the international market. The remaining firms would have then put the unused resources towards production of a good for which they had a comparative advantage against the rest of the world. With the implementation of the tariff, some domestic resources that could have been better used elsewhere are now required for the production of Turkish Delight. To illustrate the point further, imagine Wisconsin put some limit on oranges imported from

Florida. Wisconsin could make up for some of the lost imports by growing oranges domestically in heated greenhouses, but that would come at great costs.

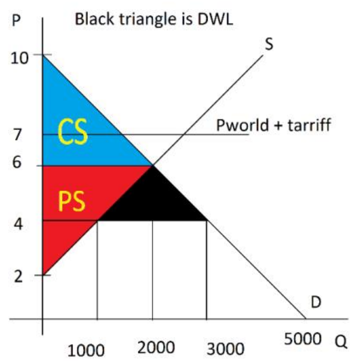
e) Suppose, Aslan, seeing the success of this tariff at generating revenue for his government decides to increase the tariff to $3 per unit in the hope of raising even more revenue. What is the new quantity consumed domestically, quantity imported, Consumer Surplus, Producer Surplus, Government Revenue, Total Surplus, and Deadweight Loss? Illustrate your answers graphically.

If the tariff is $3, then the world price plus the tariff is $7, but recall, that in autarky, the domestic price was $6. So, given this, no units will be imported since the domestic price would have to be at least $7 for it to be worth importing. Thus all answers are the same as from part (b).

Since no units are imported, government revenue is zero.

To find DWL, we can use the fact that TS without any tariff was $10,000 and TS surplus with the tariff is $8000 (the same as TS in autarky), so DWL is $2000.

Alternatively we can find the area of the DWL triangle in the plot, which will give the same result.



f) Suppose Aslan decides to try an alternative policy. Instead of a tariff, he decides to set an import quota. If Aslan wants Producers to receive as much surplus under this new quota as they do with the $1 tariff, what should he set the quota at? Suppose he decides to auction the import rights to a single firm. At most how much would a firm be willing to pay for the import license?

To maintain producer surplus at the level of the $1 tariff, the price and quantities must remain unchanged. Since 1000 units were imported with the $1 tariff, that must be where the quota is set. With a quota of 1000 imported units, we know that the domestic price will be $5, since it must be the same as the price with the $1 tariff.

The license-holder gets to purchase Turkish Delight on the international market for $4 per unit and sell it in the domestic market for $5 per unit, netting $1 per unit. Thus revenue to the license holder will be 1000*1 = $1000 (the same as government revenue with the tariff). A license-holder is not going to purchase the license if it costs more than the $1000 they would earn from it. Thus $1000 is the upper limit to the cost of the license.

g) [Note: this is a CHALLENGE QUESTION! You can get there through reasoning or through a mathematical analysis-I would recommend using your comparative advantage when selecting the type

of analysis you choose to use.] Aslan quickly realized that increasing the tariff to $3 was a bad idea if he was concerned about revenue; however, he is still interested in finding the tariff that maximizes revenue. Find the revenue maximizing tariff. Can he do better than the $1 tariff? Provide some intuition for the relationship between revenue and tariff.
Hint: Let T denote the tariff, then use the following facts:
Imports = Qd – Qs
Price = world price + T
$0 \leq T \leq 2$
Think of Imports like a demand curve, but with T instead of P. At what price is revenue maximized on a linear demand curve? Think in terms of elasticity.)

First we need to find as expression for imports. We know that Imports = Qd – Qs. Recall,

Qd = 5000 – 500P
Qs = 500P – 1000

Further P = 4 + T so,

Qd = 5000 – 500(4+T) = 3000 – 500T
Qs = 500(4+T) – 1000 = 1000 + 500T

Which implies:

Imports = 3000 – 500T – (1000 + 500T) = 3000 – 500T – 1000 – 500T
= 2000 – 1000T

So now if we think of this like a linear demand curve, we know revenue will be maximized when the price elasticity of demand (or really a "tariff elasticity of imports" in this case) is equal to 1.

So recall our formula for price elasticity of demand

e = |1/m|*(P/Q)

So in our application, this converts to

e = |1/m|*(T/Imports)

Imports = 2000 – 1000T is a x-intercept form, so 1000 = |1/m|, thus

e = 1000 * (T/(2000 – 1000T))

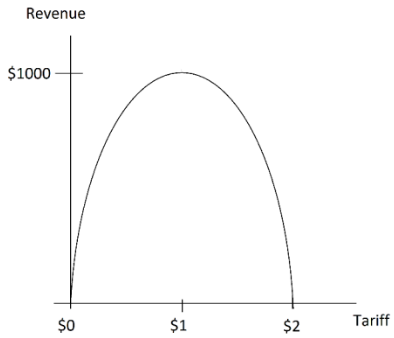
Now, we want e = 1 so

1 = 1000 * (T/(2000 – 1000T))
2000 – 1000T = 1000T
2000 = 2000T
T = $1

(Alternatively, we could have just used the fact that the unit elastic point is the half-way point, and just jumped straight to the answer.)

If we plot Revenue = T*Imports, we should get a parabola that hits the horizontal axis at T = 0 and T = 2, and is above the axis between those two points. These two points make sense because, if T = 0, clearly the government will raise no revenue regardless of how much is imported, and if T = 2, no units are imported, so revenue will be zero there as well.

For intuition, as you increase the tariff from zero, imports are relatively inelastic, so a 1% change in tariffs causes a less than 1% fall in imports, so the price effect dominates the quantity effect, and revenues increase. Once you pass $1, the demand for imports becomes more elastic, so a 1% change in tariffs causes a more than 1% fall in imports, so the quantity effect dominates, revenue falls.



3. Somelandia is a small nation with domestic supply and demand for widgets given by

Domestic Demand: Qd = 300 – 6P for $0 \leq P \leq 50$
Domestic Supply: Qs = 2P – 20 for $P \geq 10$

where Q is the quantity demanded domestically, Qs is the quantity supplied domestically, and P is the price.

Suppose the World Price for the good is $20.

a) What is the equilibrium price and quantity in autarky (remember "autarky" is the term used to describe a closed market)?

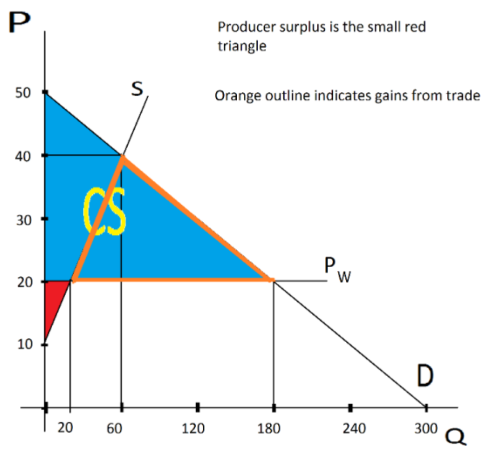
Follow the usual procedure. Set supply equal to demand:
300 – 6P = 2P – 20
So P = 40. Plugging this back into supply or demand, we find Q = 60.

b) What is the equilibrium price, quantity, imports, consumer and producer surplus if this market is closed to trade? How much is the gain in surplus from international trade if this market is opened to trade? Illustrate your answers graphically. Specifically mark which region represents the surplus gains from trade.

From part (a) we know the market price without trade is $40 which is above the world price, thus the price with trade will be the world price of $20.
Plugging this into the supply and demand curves we find
Qd = 180
Qs = 20
Since the domestic quantity demanded is 180, and the domestic quantity supplied is only 20, the difference must be made up by imports. Thus
Imports = 160
Consumer Surplus is the triangle above the world price, below the demand curve, so
CS = (1/2)*30*180 = $2700
Producer Surplus is the area below world price, above the supply curve, so
PS = (1/2)*10*20 = $100

Notice, without international trade, total surplus would be the triangle to the left of the domestic equilibrium, above $10 and below $50. Thus, from the plot, we can see that the gains from trade can be represented by the triangle below the domestic equilibrium point and above the world price, between the supply and demand curves. So

Gains from Trade = (1/2)*20*160 = $1600



c) Now suppose the government decides to set an import quota of 200 widgets; i.e. only 200 widgets may be imported. What is the new equilibrium price, quantity, revenue to import license-holders, and deadweight loss?

From part (b) we know only 160 units would be imported in the absence of regulation. Thus, if the government limits imports to 200 units, the policy has no effect. So price, quantity, imports, and surpluses are all as they were in (b) when the market was open to trade.

Since the policy has no effect, there is no revenue to those who hold the import license (license holders import the good at the world price and sell it at the world price for a net gain of zero).

Similarly, there is no deadweight loss since the outcome is the same as the unregulated one.

d) Now suppose the government of Somelandia decides to try a quota of 80 units. What will be the new equilibrium price and quantity, Consumer and Producer Surplus, Revenue to license-holders, and Deadweight Loss? Illustrate your answers graphically.

First we find the equilibrium price with the quota noting the fact that, at the equilibrium price

Qd = Qs + Quota
300 – 6P = (2P – 20) + 80
300 – 6P = 2P + 60

Solving for P we find P = 30.
By plugging 30 into the demand curve, we find the quantity consumed domestically is 120 units. 80 are imported (up to quota), so domestic supply is 40 units.

Consumer Surplus is the usual triangle (see plot) so

CS = (1/2)*120*20 = $1200

Similarly for Producer Surplus

PS = (1/2)*40*20 = $400

License holder revenue is the number of imported units (the quota) times the difference between the domestic price and the world price (the importer buys widgets for the world price and sells for the domestic price, netting the difference).

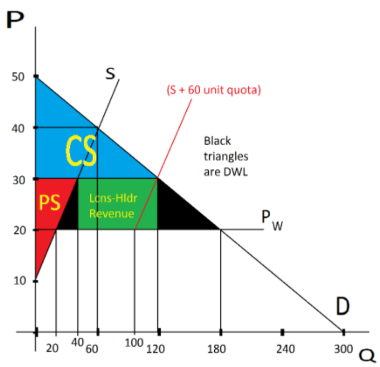
License Holder Rev = 80*10 = $800

Total surplus is the sum of CS, PS, and License-Holder Revenue so

TS = $2400

Recalling that TS before the quota was $2800, we see that DWL must be $400. Alternatively, we could calculate the area of the DWL triangles from the plot.

DWL = (1/2)*20*10 + (1/2)*60*10 = 100 + 300 = $400



Part III – Elasticities

4. Suppose the demand for gadgets is given by the following equation where P is the price and Q is the quantity demanded:

$$P = 50 - Q$$

a) Suppose the price is originally $10 and increases to $20. Using the regular or standard percentage formula, find the price elasticity of demand for gadgets.

Regular percentage elasticity of demand formula is

e = | (% change Q / % change P) | = | ((new Q – old Q) / old Q) / ((new P – old P) / old P) |

= | ((30 – 40) / 40) / (20 – 10) / 10) | = 1/4

b) Suppose the price is originally $20 and decreases to $10. Using the regular percentage formula, find the price elasticity of demand for gadgets.

Using the formula again we find

e = | ((40 – 30) / 30) / (10 – 20) / 20) | = (1/3) / (1/2) = 2/3

c) Now using the arc formula, calculate the price elasticity of demand using the prices from the previous 2 parts. What is the advantage of using the arc formula over the regular percentage formulas?

The arc formula is

$$e = \left| \frac{(Q_1 - Q_2)/(Q_1 + Q_2)}{(P_1 - P_2)/(P_1 + P_2)} \right|$$

Thus we find

e = (10/70) / (10/30) = 3/7

We can see that the arc elasticity is between the regular elasticities from (a) and (b), and the arc elasticity measure does not depend on the ordering of the price change.

d) Using the point formula, calculate the price elasticity of demand at a price of $15.

The point elasticity formula is
e = | (1/m)(P/Q) |

Notice the equation is given in a y-intercept form, so the slope is simply the coefficient on Q, thus m = -1. So
e = | -1 * (15/35) | = 3/7

e) Alice is the sole producer of gadgets, and thus gets to set the price. If Alice's sole concern is maximizing revenue, what price should she set?

Revenue is maximized when the demand is unit elastic. We can use the fact that a linear demand curve is unit elastic at its midpoint, implying that revenue maximizing price is $25.

Alternatively, we could solve for the point algebraically
1 = | -1 * (P/(50-P))|
1 = P/(50-P)
50 – P = P
50 = 2P
P = 25

5. Suppose the demand for frozen chicken nuggets and fresh tuna are given by the following equations where Pn is the price of nuggets (in dollars), Pt is the price of tuna (in dollars), I is median income (in tens of thousands of dollars), and Q is the quantity demanded:
Demand for Frozen Nuggets: Q = 20 – 2Pn – Pt – I
Demand for Fresh Tuna: Q = 40 – 4Pt - Pn + 2I

a) Suppose the price for frozen nuggets is fixed at $2 and the price of tuna is fixed at $4. What is the quantity of frozen nuggets demanded when median income is $10,000? How about when income is $20,000? Given these quantities and using the arc formula, what is the income elasticity of demand for frozen nuggets between these two income levels?

First by plugging in the values of the fixed prices, we find Q = 12 – I. By plugging in 1 and 2 for incomes we find quantities are 11 and 10 respectively.

So using the arc formula (note we only use absolute value for price elasticity) from above, we get
e = ((11 – 10) / (11 + 10)) / ((1 – 2) / (1 + 2)) = (1/21) / (- 1/3) = - 1/7

b) Suppose the prices for nuggets and tuna are still fixed as they were in part (a). What is the quantity of fresh tuna demanded when median income is $10,000? How about $20,000? Given these quantities and using the arc formula, what is the income elasticity of demand for fresh tuna?

By plugging in the givens as before we find Q = 22 + 2I. Plugging in 1 and 2 for incomes, we find quantities 4 and 6 respectively.

So using the arc formula we get

$e = ((26 – 24) / (26 + 24)) / ((2 – 1) / (1 + 2)) = (2/50) / (1/3) = 3/25$

c) Given your answers to parts (a) and (b), what can you say about Chicken Nuggets and Fresh Tuna? Are they inferior goods, normal goods and/or luxury goods?

Income elasticity for frozen nuggets is negative so nuggets are an inferior good. Income elasticity for tuna is positive, but less than 1 so it is a normal, but not a luxury, good.

d) Now fix the median income at $20,000 and the price of nuggets at $2. If the price of tuna is $4, what is quantity of nuggets demanded? What if the price of tuna is $6? Using this information and the arc formula, calculate the cross-price elasticity of demand for nuggets.

Plugging in the given values, we get Q = 14 – Pt. Plugging in 4 and 6 for price of tuna, we get quantities 10 and 8 respectively.

So using the arc formula to calculate the cross-price elasticity of demand:

$e = ((10 – 8) / (10 + 8)) / ((4 – 6) / (4 + 6)) = (2/18) / (- 2/10) = - 5/9$

e) Given your answer to part (d), what can you say about the relationship of tuna to nuggets? Are the substitutes or complements?

Since cross-price elasticity is negative, tuna and chicken nuggets are complements for this economy.

Part IV – Consumer Theory

6. Use the following information in answering this next set of questions.

Sue likes to eat cheese curds and make long-distance phone calls. Currently, she has no cell phone, and must use a pay phone to make calls, thus she must pay for each call separately at a cost of $5 per call. A dish of Cheese Curds costs $10. Sue's income is $50.

a) Supposing long-distance phone calls and cheese curds are the only goods Sue buys, plot her budget line with cheese curds on the horizontal axis, and long-distance phone calls on the vertical axis. What is the equation of this line? Use P as the symbol for "phone calls" and C as the symbol for "cheese curds". Label this budget line as BL1 or "budget line part (a)".

Sue's budget can be expressed by

$5P + 10C = 50$

So by rearranging, terms (remember, P is the vertical axis)

$P = 10 – 2C$

The graphs are shown in part (c).

b) Suppose Sue's income increases to $100. Plot her new budget line. What is the equation of this new budget line? In your graph, label this budget line as BL2 or "budget line part (b)".

Now Sue's budget line can be expressed by:

$$5P + 10C = 100$$

So rearranging terms:

$$P = 20 – 2C$$

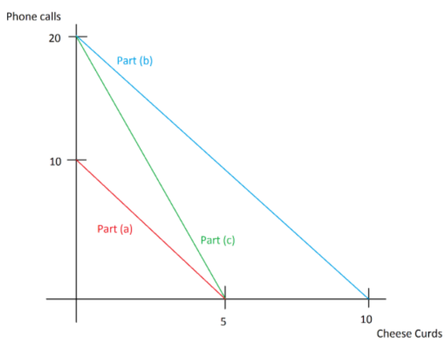
c) Suppose Sue's income is still $100, but now, due to an unfortunate dairy shortage, the price of cheese curds doubles to $20 a dish. Plot this new budget line. In your graph, label this budget line as BL3 or "budget line part (c)". What is the equation of the budget line?

Now Sue's budget line can be expressed by

$$5P + 20C = 100$$

So rearranging terms

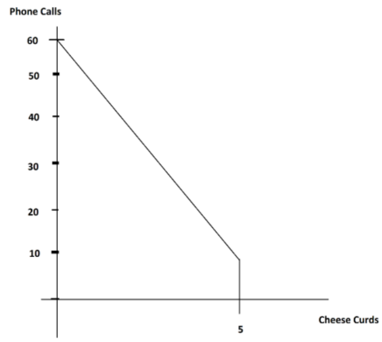
$$P = 20 – 4C$$



d) [Note: (d) and (e) of this problem is a bit of a CHALLENGE and a logic puzzle: do not despair-you can do this, and to get full credit on the homework you CANNOT avoid these challenge questions. So START EARLY AND THINK SERIOUSLY!] Let's suppose the dairy shortage has been resolved, so the price of cheese curds is back to normal (i.e. Sue's budget line is as in part b). A cell phone provider offers Sue a service plan that costs $50. For the $50, Sue gets 10 long-distance calls to start. After the first 10, Sue may pay for additional long-distance calls at a price of $1 per call. Plot the budget line Sue would face if she paid for the service plan. *(Hint: First, subtract $50 from Sue's income to account for the cost of the plan. Be careful; there will be a kink in the budget line.)*

First consider how many dishes of Cheese Curds Sue could buy if she spent all her remaining income on curds; this will be the x-intercept. Sue has $50 left to spend, and curds cost $10 per dish, so the x-intercept is 5.

Now, even if she spends all her remaining income on curds, she gets 10 calls as part of the phone plan, so there will be a vertical line up to P = 10. After the tenth phone call, Sue can pay $1 per call to purchase additional calls. If she spends all of her income on phone calls, she can afford to buy 50 additional phone calls, after the first 10, so the y-intercept is 60.



e) Suppose if Sue does not buy the phone service plan, she would consume 8 dishes of cheese curds and 4 long distance phone calls (call this bundle A). If she purchases the phone plan, she would consume the 5 cheese dishes of cheese curds and use her 10 starter calls, but no more (call this bundle B). Should she purchase the phone plan? Provide some intuition for your solution.

(Hint: Sue can afford Bundle A if she doesn't purchase the service plan since that's what she consumes, and similarly, she can afford Bundle B if she purchases the service plan. But could Sue have afforded Bundle A with the service plan, and could she afford Bundle B without the service plan?)

Note, if Sue doesn't purchase the service plan, she could have afforded the Bundle B anyways, but she chose to consume Bundle A anyways. If she purchases the service plan, she can no longer afford Bundle A.

If Sue purchases the phone plan, it must be because she prefers Bundle B to Bundle A, but she could have afforded B without the phone plan when she decided to consume bundle A instead of bundle B. Since she chose A over B, it must be that she prefers A to B. Thus, she would not want to purchase the phone plan.

7. Use the following information to answer this next set of questions.

Suppose Bob gets Utility from consuming two goods, *x* and *y*, according to the following equation where U is the total amount of utility Bob gets measured in utils, X is the amount of good X, and Y is the amount of good Y:

$$U = X*Y$$

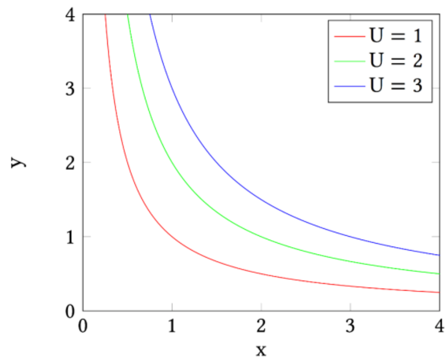
a) Plot indifference curves for Bob for U = 1, U = 2, and U = 3, with X on the x-axis, and Y on the y-axis. Provide equations for each of these curves. What are some properties that make preferences of this sort reasonable in many contexts?

Plug 1 in for U, then solve for Y.

$$1 = X*Y$$
$$Y = 1/X$$

Similarly, for U = 2, Y = 2/X, and for U = 3, Y = 3/X.

A utility function of this sort is useful in economics for a number of reasons. First of all, it is a very simple equation that is easy to work with mathematically. Economically, it satisfies increasing marginal rates of substitution. That is, holding U fixed, X and Y are partially substitutable for each other, but as you consume more X and less Y, more and more units of X are required to make up for each unit of Y lost. This also captures a "preference for variety." We generally like to consume a variety of different goods, rather than spending all our income on a single good.



b) Now suppose Bob's utility takes this form where U is the total amount of utility Bob gets measured in utils, X is the amount of good X, and Y is the amount of good Y:

$$U = X + Y$$

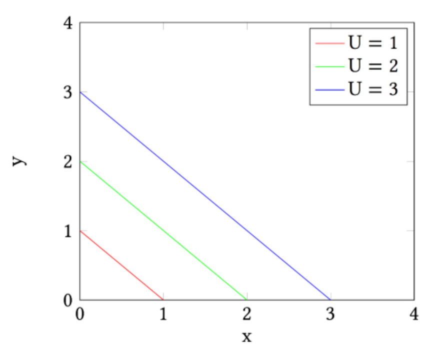
Plot indifference curves for U = 1, U = 2, and U =3, with X on the x-axis, and Y on the y-axis. Provide equations for each of these curves. What is the economic relationship between goods X and Y?

Following a similar procedure to part (a), we get for U = 1

$$1 = X + Y$$
$$Y = 1 - X$$

And similarly, for U = 2, Y = 2 – X, and for U = 3, Y = 3 – X.

With this utility function, X and Y are *perfect substitutes*. If you lose 1 unit of Y, you must gain 1 unit of X to maintain the same level of utility.

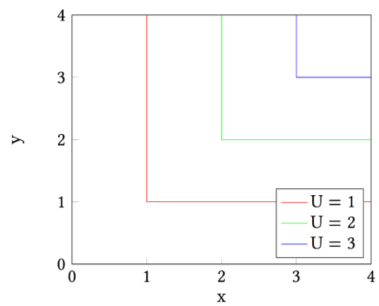


c) Now suppose Bob's utility takes the form where U is the total amount of utility Bob gets measured in utils, X is the amount of good X, and Y is the amount of good Y:

$$U = \min\{X, Y\}$$

i.e. Bob's level of utility is equal to the amount of the good of which he has less (for example, if (X, Y) = (10, 20) then Bob's level of utility is 10). Again, plot indifference curves for U = 1, U = 2, and U =3, with X on the x-axis, and Y on the y-axis. What is the economic relationship between goods X and Y? You do not need to provide equations for these indifference curves.

With preferences like this, X and Y are *perfect complements*. Utility can only increase if the amount consumed of both increases. If Y increases, but X remains fixed, utility does not increase, assuming Y was at least as much as X to begin with.



8. Suppose Alice has preferences for xylophones (x) and yachts (y) with utility given by the following equation where U is Alice's level of utility measured in utils, x is number of xylophones and y is number of yachts:

$$U = x^3y$$

With this utility, Alice has the following marginal utilities:

$$MU_x = 3x^2y$$
$$MU_y = x^3$$

a) Suppose the price of a xylophone is \$1,000, and the price of a yacht is \$5,000. Alice currently has an income of \$100,000. What is Alice's optimal consumption bundle? Remember, the optimal consumption bundle is the point at which Alice equates marginal utilities per dollar across both goods *and* spends all her income. You will have two equations (one equation equating marginal utilities per dollar and the second equation being the budget constraint) and two unknowns (number of xylophones and number of yachts).

For this solution we are going to provide two analyses: read both carefully and work on understanding the logic!

First Analysis: we know that to maximize utility the slope of Alice's indifference curve at the optimal bundle must equal the slope of her budget line and we also know that we can write the absolute value of the slope of her indifference curve as:

$$MU_x / MU_y = P_x / P_y$$

Using the information we have been given we can substitute as follows into this equation:

$$3x^2y/ x^3 = 1000/5000$$

Simplifying we get:

$$3y/x = 1/5$$

$x = 15y$

Now, let's look at that budget line:

$P_x x + P_y y = \text{Income}$

$1000x + 5000y = 100{,}000$

$x + 5y = 100$

Now, use these two equations to solve for x and y:

$15y + 5y = 100$

$20y = 100$

$y = 5$ yachts

$x = 15y = 15(5) = 75$ xylophones

Alice will maximize her utility when she consumes (75 xylophones, 5 yachts). You can verify that she can afford this bundle and that this bundle exhausts her income.

Second Analysis: At the optimal consumption bundle, Alice equates the Marginal Utilities per dollar across both goods, i.e.

$MU_x / P_x = MU_y / P_y$

And she spends her budget

$P_x x + P_y y = 100{,}000$

So plugging things in we get:

$3x^2y / 1000 = x^3 / 5000$

$1000x = 3*5000y$

Notice, this equation implies that Alice always spends three times as much on xylophones as she does on yachts. Let S_x be spending on xylophones and S_y be spending on yachts. From above we have

$S_x = 3S_y$

And

$S_x + S_y = \text{Income}$

So substituting we find

$3S_y + S_y = \text{Income}$

$4S_y = \text{Income}$

$S_y = \text{Income} / 4$

So Alice spends ¼ of her income on yachts. The remaining ¾ must be spent on xylophones.

Thus, for Income = \$100,000, Alice's spending on xylophones is \$75,000, and her spending on yachts is \$25,000. Now we can solve for the final quantities:

$1000x = 75{,}000$

$x = 75$

$5000y = 25{,}000$

$y = 5$

b) Now suppose Alice's income doubles to \$200,000. What is her new optimal bundle?

We know that to maximize utility the slope of Alice's indifference curve at the optimal bundle must equal the slope of her budget line and we also know that we can write the absolute value of the slope of her indifference curve as:

$MU_x / MU_y = P_x / P_y$

Using the information we have been given we can substitute as follows into this equation:

$3x^2y / x^3 = 1000/5000$

Simplifying we get:

$3y/x = 1/5$

$x = 15y$

Now, let's look at that budget line:

$P_x x + P_y y = \text{Income}$

$1000x + 5000y = 200{,}000$

$x + 5y = 200$

Now, use these two equations to solve for x and y:

$15y + 5y = 200$

$20y = 200$

$y = 10$ yachts

$x = 15y = 15(10) = 150$ xylophones

Alice will maximize her utility when she consumes (150 xylophones, 10 yachts). You can verify that she can afford this bundle and that this bundle exhausts her income.

Or, alternatively you can repeat the second analysis but change Alice's income in the budget constraint. You will get the same results.

We can simply use the condition that we found in (a) that Alice spends ¾ of her income on xylophones and ¼ on yachts. Since her income doubled, but the proportion of her income that she spends on each good doesn't change, she will simply double her purchases of both goods.

$1000x = 150{,}000$

$x = 150$

$5000y = 50{,}000$

$y = 10$

c) Now suppose Alice's income is still \$200,000, but the price of yachts doubles to \$10,000. Now what is Alice's optimal bundle?

We know that to maximize utility the slope of Alice's indifference curve at the optimal bundle must equal the slope of her budget line (but, remember that the price of yachts has changed!) and we also know that we can write the absolute value of the slope of her indifference curve as:

$MU_x / MU_y = P_x / P_y$

Using the information we have been given we can substitute as follows into this equation:

$3x^2y / x^3 = 1000/10{,}000$

Simplifying we get:

$3y/x = 1/10$

$x = 30y$

Now, let's look at that budget line:

$P_x x + P_y y = \text{Income}$

$1000x + 10{,}000y = 200{,}000$

$x + 10y = 200$

Now, use these two equations to solve for x and y:

$30y + 10y = 200$

$40y = 200$

$y = 5$ yachts

$x = 30y = 30(5) = 150$ xylophones

Alice will maximize her utility when she consumes (150 xylophones, 5 yachts). You can verify that she can afford this bundle and that this bundle exhausts her income.

Or, here is an alternative analysis:

Alice will still spend ¾ of her income on xylophones and ¼ on yachts. Thus she will still consume 150 xylophones, but now:

$10000y = 50000$

$y = 5$

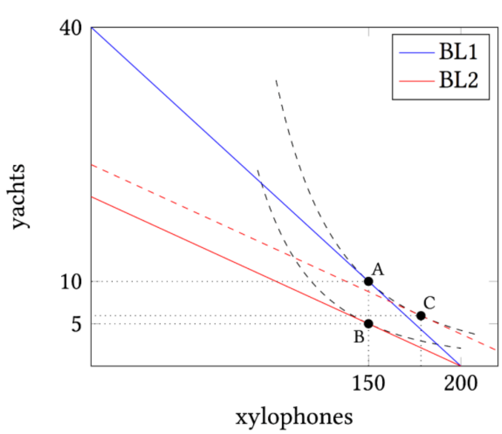
d) Draw a well labeled graph that illustrates the income and substitution effects of this price change described in (c). [Note that you will not have the numeric values for (x, y) for point C: just indicate where "C" should be given your analysis.]

Plot Alice's budget line when she faces the original prices (yacht price is $5000). Call this BL1. Draw in an indifference curve tangent to BL1 at the point (150, 10) (i.e. her original consumption bundle). Call this point A.

Now repeat this process with her budget line for the higher yacht price of $10,000 (BL2), and the new consumption bundle of 150 xylophones and 5 yachts (point B).

Now shift out BL2 parallel until it just touches the original indifference curve at a tangent point. In the plot below, this imaginary budget line is the red dashed line. The tangent point is point C.

The substitution effect is represented by the change from point A to point C (the outward shift of the budget line eliminates any income effects). Thus the remaining effect (the income effect) is represented by the change from point C to point B.



e) Notice this increase in price has reduced Alice's total consumption, so her welfare is negatively affected by the change. How much extra income would you have to give Alice to compensate her for this change? That is, how much additional income would she need so that her optimal bundle with the new prices would lie on her previous indifference curve? (**You will need a calculator for this part**.)

Let I* be the total income she would need to reach her previous indifference curve. Recall, before the price change, her optimal bundle was (150, 10), giving her utility

$U = (150^3)(10) = 33{,}750{,}000$

So we wish to find x, y, and I* such that

$x^3y = 33{,}750{,}000$ (must be on original indifference curve)

$1000x = 3*10000y$ (follows from optimality condition that marginal utilities per dollar are equal)

$1000x + 10000y = I^*$ (budget constraint)

From the second equation, we have

$x = 30y$

Plugging this into the first equation, we get

$27000y^4 = 33{,}750{,}000$

$y^4 = 1250$

$y \approx 5.95$

Plugging this into $x = 30y$ we find

$x \approx 178.38$

(Note, this bundle is Point C from the previous part.)

Now all that remains is to find I* required to afford this bundle

$1000*178.38 + 10000*5.95 = 237{,}880$

So Alice would need about $237,880 to be able to afford to consume on her original indifference curve. Thus she would need about $37,880 to be fully compensated for the price change.

Part V – Real vs Nominal

(You may need a calculator for certain parts of the next 2 questions.)

9. Suppose prices for goods in some fictional Midwestern city over time are given by the following table:

Year	Price of Cheese Curds	Price of a Sweater	Price of a Gold Bar
2010	$5	$25	$500
2011	$6	$25	$800
2012	$6	$30	$900

a) Suppose the average market basket in this city is composed of 300 dishes of Cheese Curds, 50 Sweaters, and 1 Gold Bar. Calculate the cost of the market basket in each year.

Year	Cost of Market Basket
2010	
2011	
2012	

Cost of Basket = (300 * price of cheese curds) + (50 * price of a sweater) + (1 * price of a gold bar)

Year	Cost of Market Basket

2010	$3250
2011	$3850
2012	$4200

b) Now, using 2010 as the base year, calculate the CPI for each year using a 100 point scale. Then, for 2011 and 2012, calculate the annual inflation rate. Show your answers to two decimal places.

Year	**CPI**	**Inflation rate**
2010		-
2011		
2012		

CPI = ((price of basket this year) / (price of basket in base year)) * 100

Inflation rate = ((CPI this year – CPI last year) / (CPI last year)) * 100

Year	**CPI**	**Inflation rate**
2010	100	-
2011	118.46	18.46%
2012	129.23	9.09%

c) Now, still using 2010 as base year, calculate the real price of cheese curds in each year. Again, show your answers to two decimal places.

Year	**Real price of cheese curds**
2010	
2011	
2012	

Real price = (Nominal Price / CPI) * 100

Year	**Real price of cheese curds**
2010	$5
2011	$5.07
2012	$4.64

10. Now, using everything you've learned so far about real and nominal prices, fill in the blanks of the following table. The market basket for this economy is composed of 10 loaves of bread and 5 gallons of milk. Compute real prices using a base year of 2005 and a CPI computed on a 100 point scale. Express your answers to 2 decimal places when necessary.

Year	**Nom. Price of Bread**	**Nom. Price of Milk**	**Nom. Cost of Market Basket**	**Real Price of Bread**	**Real Price of Milk**	**Real Cost of Basket**	**CPI**	**Annual Inflation Rate**
2005	$5	$2						-
2006		$2						5%
2007			$70	$3				
2008		$3			$4			

Year	Nom. Price of Bread	Nom. Price of Milk	Nom. Cost of Market Basket	Real Price of Bread	Real Price of Milk	Real Cost of Basket	CPI	Annual Inflation Rate
2005	$5	$2	$60	$5	$2	$60	100	-
2006	$5.30	$2	$63	$5.05	$1.90	$60	105	5%
2007	$3.50	$7	$70	$3	$6	$60	116.67	11.11%
2008	$3	$3	$45	$4	$4	$60	75	-35.71%

First, we can fill the entire first row. We are given the nominal price of both goods, so using the fact that a market basket is 10 loaves of bread and 5 gallons of milk, we get that the nominal basket cost is $60. Furthermore, since the base year is 2005, the nominal prices for that year are all also the real prices. Lastly, again because it is the base year, CPI in 2005 is 100.

Notice now, we can get the real basket cost in every year for free now that we have the real cost in the base year. In every year, the real cost of the basket will be the same (i.e. $60)

To see this, note

CPI = ((Nominal price of basket this year) / (price of basket in base year)) * 100
and
Real basket = (Nominal Basket / CPI) * 100

Rearranging the second equation, we get
Nominal Basket this year = (Real Basket this year * CPI) / 100

Plugging this into the first equation, many terms cancel, leaving

1 = Real Basket this year / Price of Basket in Base Year

Thus
Real Basket this year = Price of Basket in Base Year

Now, find CPI for 2006. Using the equation for inflation rate given in the previous question, we can find that CPI in 2006 was 105.

Using the equation for CPI given in part (b) we can back out that the nominal cost of the market basket is $63. Since we are given the nominal price of milk, we can use this with the basket cost to find that the nominal price of bread is $5.30.

Now using the formula for real prices given in part (c) of the previous question, we can fill in the real prices.

Finally, the inflation rate follows from the equation from part (b).

Moving on to 2007, we are given the nominal basket cost, so we can directly compute the CPI. Then, using this CPI with the given real price of bread, we can back out the nominal price of bread. With that, the remaining information fills out immediately.

Finally, for 2008, we can use the real and nominal prices for milk, in conjunction with the formula for real prices, to back out CPI. The rest of the information follows from formulae already discussed.