Economics 101
Fall 2012
Homework #3
Due 11/06/2012
Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered,** because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. **Please show your work; otherwise you will not receive full credit.** Good luck!

1. Elasticity

In "Jormungand", Koko Hekmatyar is an arms dealer. Suppose she faces the market for Javelin ATGM missiles, and the demand is given by Px = 10,000 - 2Qx.

1) Using the **midpoint method**, calculate the price elasticity of demand when price changes from $ 8,000 to $ 6,000.

Ans: {(2000 -1000)/(1000+2000) }/ {(8,000-6,000)/(8,000+6,000)}= 7/3.

To answer the following questions, please use the **point elasticity formula**.
Suppose you're hired by Koko as her chief economist.

2) Calculate the price elasticity of demand when price equals $ 7500. Should Koko raise or lower the price of ATGM to gain more revenue if the current price is $7500? Why?

Ans: (1/2)*(7,500/1,250) =3, Koko should lower the price because at a price of $7500, the demand is elastic.

3) Calculate the price elasticity of demand when Qx equals 3500. Should Koko raise or lower the price of ATGM to gain more revenue when Qx equals 3500? Why?

Ans: (1/2)*(3,000/3,500) =3/7, Koko should raise the price because at a quantity of 3500 unit, the demand is inelastic.

4) What is the optimal price of ATGM you should suggest to Koko for total revenue maximization? At the optimal price, what is the point elasticity of demand? How much is the maximized total revenue?

Ans: from b. and c. intuitively we should select the price where elasticity equals 1. So, (1/2)*(P/Q) =1, substitute the relation back to demand. We get P= 5,000; Q=2,500. This is where total revenue is maximized given the demand curve. Total revenue is equal to $12,500,000.

Suppose that there exists another company HCLI, who sells FIM-92 Stingers. The price of FIM-92 Stingers can be represented as Py.

5) When the price of FIM-92, Py, decreases from $3000 to $1500, the demand for ATGM, Qx, also decreases from 6000 to 5000. What is the cross-price elasticity of demand equal to given this information? Are ATGM and FIM-92 substitutes or complements? Use the standard formula for percentage changes to compute this value.

Ans: cross-price elasticity = {(5,000-6,000)/6,000}/{(1,500-3,000)/3,000} = 1/3. They are substitutes because the cross-price elasticity of demand is positive.

2. **Excise Tax**

Consider the market for cigarettes in New York City and Los Angeles. Suppose the daily demand for cigarettes in NYC is given as Q_d=1000-100P, and the demand in LA is Q_d=900-200P. The market supply for the two markets is the same: Q_s=100+200P. Assume these two markets are totally separated.

1) Find the equilibrium price and quantity in the cigarette market for both NYC and LA.

Ans: NYC: Q_E=700, P_E=3; LA: Q_E=500, P_E=2.

2) Calculate the point price elasticities of demand for both NYC and LA at equilibrium.

Ans:

NYC: $\varepsilon_d = \left|\frac{1}{slope} \cdot \frac{P}{Q}\right| = \left|\frac{1}{-0.01} \cdot \frac{3}{700}\right| = \frac{3}{7}$

LA: $\varepsilon_d = \left|\frac{1}{slope} \cdot \frac{P}{Q}\right| = \left|\frac{1}{-0.005} \cdot \frac{2}{500}\right| = \frac{4}{5}$

3) Calculate the point price elasticities of supply for both NYC and LA at equilibrium.

Ans:

NYC: $\varepsilon_s = \frac{1}{slope} \cdot \frac{P}{Q} = \frac{1}{0.005} \cdot \frac{3}{700} = \frac{6}{7}$

LA: $\varepsilon_s = \frac{1}{slope} \cdot \frac{P}{Q} = \frac{1}{0.005} \cdot \frac{2}{500} = \frac{4}{5}$

Suppose that the government plans to impose an excise tax on sellers of cigarettes in order to reduce the market quantity of cigarettes by 100 units in both the NYC market as well as the LA market (that is, a total reduction of 200 units).

4) Find the corresponding excise taxes for NYC and LA.

Ans: The excise tax in NYC should be $1.5/unit, and the tax in LA should be $1/unit.
NYC: Let Q=700-100=600, then P_d=4 by plugging Q_d=600 into the NYC demand equation, and P_s=2.5 by plugging Q_s=600 into the supply equation. Therefore, excise tax should be equal to the difference between P_d and P_s: P_d-P_s=4-2.5=1.5.
LA: Let Q=500-100=400, then P_d=2.5 by plugging Q_d=400 into the LA demand equation, and P_s=1.5 by plugging Q_s=400 into the supply equation. Therefore, excise tax should be equal to the difference between P_d and P_s: P_d-P_s=2.5-1.5=1.

5) Calculate the government revenues, consumer tax incidence, producer tax incidence, and dead weight losses for both NYC and LA due to the imposition of this excise tax.

Ans: NYC: Tax revenue=tax per unit * market quantity=1.5*600=$900. CTI=($4/unit-$3/unit)(600 units)=$600. PTI=($3/unit-$2.5/unit)(600 units)=$300. DWL=0.5*1.5*100=$75.
LA: Tax revenue=1*400=$400. CTI=($2.5/unit-$2/unit)(400 units) = $200. PTI=($2/unit-$1.5/unit)(400 units)=$200. DWL=0.5*1*100=$50.

6) For both NYC and LA compare the fraction of the tax paid by consumers to the fraction of the tax paid by producers. Explain your results from the point of elasticity.

Ans: NYC: fraction of CTI > fraction of PTI since consumers pay $600/$900 (2/3 of the tax) while producers pay $300/$900 (1/3 of the tax). At the equilibrium point, the price elasticity of demand (3/7) is less than the price elasticity of supply (6/7). Because the demand is more inelastic than the supply according to the change of price, consumers should bear more of the excise tax than producers in NYC. LA: fraction of CTI = fraction of PTI since consumers pay $200/$400 (1/2 of the tax) while producers pay $200/$400 (1/2 of the tax). At the equilibrium point, the price elasticity of demand (0.5) is the same as the price elasticity of supply (0.5). Because the demand and the supply are inelastic at the same level, consumers and producers should share the excise tax equally in LA.

3. **Elasticity and Total Revenue**

The table below describes the demand of undergraduate students for chocolate muffins at a café on the campus. Suppose the students have a linear demand curve for muffins.

Quantity demanded (Muffins)	**Price ($/muffin)**	**Total revenue for the café ($)**	**Point price elasticity of demand**
200	a	0	d
b	2.4	192	e
f	g	h	3
Demand equation of undergraduate students for muffins: c			

1) Complete the table above.

Ans:

a. Price=revenue/quantity=0/200=0.

b. 80. Quantity=revenue/price=192/2.4=80.

c. Q_d=200-50P. The demand curve goes through (Q=200, P=0) and (Q=80, P=2.4). Then according to these two points, the equation for demand curve can be written as P=4-0.02Q_d.

d. The slope for the demand curve is -0.02. The price elasticity of demand at this point is $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.02}\cdot\frac{0}{200}\right| = 0.$

e. The price elasticity of demand at this point is $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.02}\cdot\frac{2.4}{80}\right| = 1.5.$

f. 50. The price elasticity of demand at this point is $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.02}\cdot\frac{4-0.02Q}{Q}\right| = 3.$ Solve this equation in terms of Q and get Q=50.

g. P=4-0.02Q=4-0.02*50=3.

h. 150. Revenue=price*quantity=3*50= $150.

2) Find the price with which the café can maximize its revenue from undergraduate students. What's the point price elasticity of demand at this maximum point for revenue?

Ans: P=2. The point elasticity of demand is 1.
The total revenue is maximized at the point with unit price elasticity for this linear demand curve. Let the point price elasticity of demand be $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.02}\cdot\frac{4-0.02Q}{Q}\right| = 1.$ Solve this equation and get Q=100. P=4-0.02*100=2.
Another way to solve maximal point is based on the fact that the mid-point of the demand curve has unit elasticity. Since the X intercept point is (200, 0) and the Y intercept point is (0, 4), the mid-point is (100, 2).

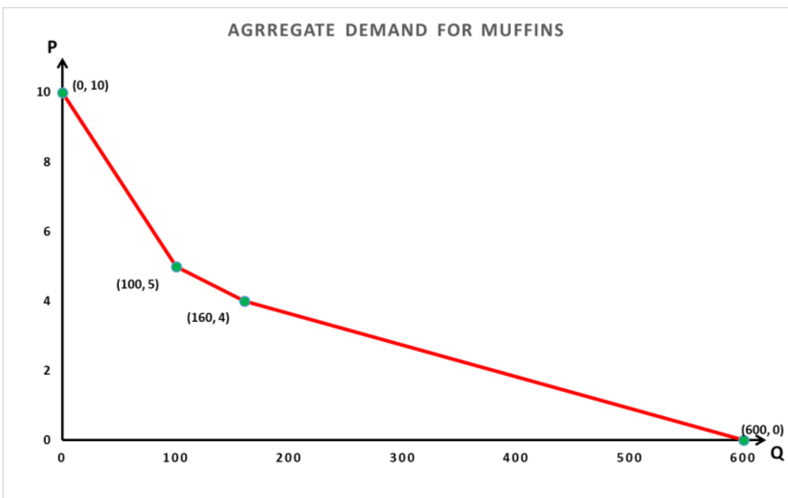
Now the demand equation for graduate students for chocolate muffins is Q_d=200-40P, and the demand equation for faculty for chocolate muffins is P=10-0.05Q_d.

3) Suppose that this café can charge different prices to different groups of customers. What are the maximum revenues from graduate students and faculty if the café charges different prices to the two groups?

Ans: Maximum revenue from graduate students is $250, and maximum revenue from faculty is $500. The revenue for each group is maximized at the point with unit price elasticity of demand. That is the mid-point of the linear demand curves for each group.
Graduate students: X intercept point is (Q=200, P=0), Y intercept point is (0, 5), mid-point is (100, 2.5), and maximum revenue is 2.5*100=$250.
Faculty: X intercept point is (Q=200, P=0), Y intercept point is (0, 10), mid-point is (100, 5), and maximum revenue is 5*100=$500.

4) Suppose that the café cannot discriminate among those three groups of customers and that the market is only composed of these three groups of customers. In order to maximize the total revenue from all the customers, which price should this café charge? What's the maximum revenue if there is only one price for muffins? Calculate the point price elasticities of demand, respectively, for each group of customers at this single price? Hint: these numbers will get a bit messy, but persevere!

Ans:
We need to figure out the market demand for muffins at first.



$$Q_d = \begin{cases} Q_d^F = 200 - 20P, & P \geq 5 \\ Q_d^F + Q_d^G = (200 - 20P) + (200 - 40P) = 400 - 60P, & 4 \leq P < 5 \\ Q_d^F + Q_d^G + Q_d^U = (200 - 20P) + (200 - 40P) + (200 - 50P) = 600 - 110P, & P < 4 \end{cases}$$

$$Revenue = P \cdot Q_d = \begin{cases} 200P - 20P^2, & P \geq 5 \\ 400P - 60P^2, & 4 \leq P < 5 \\ 600P - 110P^2, & P < 4 \end{cases}$$

Then solve this maximization problem by case,

$$Max\ Revenue = \begin{cases} 200 \times 5 - 20 \times 5^2 = 500, & P \geq 5 \\ 400 \times 4 - 60 \times 4^2 = 640, & 4 \leq P < 5 \\ 600 \times \frac{30}{11} - 110 \times \left(\frac{30}{11}\right)^2 = \frac{9000}{11}, & P < 4 \end{cases}$$

Apparently, the maximum revenue (9000/11) is attained when P=30/11 and Q_d=300. For undergraduate students, Q_d^U=700/11, and price elasticity of demand is $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.02}\cdot\frac{\frac{30}{11}}{\frac{700}{11}}\right| = \frac{15}{7} > 1$. For graduate students, Q_d^G=1000/11, and price elasticity of demand is $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.025}\cdot\frac{\frac{30}{11}}{\frac{1000}{11}}\right| = 1.2 > 1$. For faculty, Q_d^F=1600/11, and price elasticity of demand is $\left|\frac{1}{slope}\cdot\frac{P}{Q}\right| = \left|\frac{1}{-0.05}\cdot\frac{\frac{30}{11}}{\frac{1600}{11}}\right| = 0.375 < 1$.

4. Real and Nominal Price

Suppose John Titor is a time traveler from a future year. He hopes to travel back in time to get one IBM 5100 which is unavailable in the future year that he lives in. He seeks to travel back in time to get this computer so that he can use it to fix a bug that threatens the computer system in the future. After several time travel trips back in time he has the following data about the IBM 5100 computer.

Years	CPI	Nominal Price of IBM 5100	Real Price of IBM 5100
1975	100		$10,000
1982		$24,000	$20,000
1996	180	$54,000	
2000	200		$40,000
2010	250	$125,000	$50,000

In addition, suppose there is a time travel rule that states that purchases at times other than the future time can only be made using real prices.

1) Examining the above table, what year is being used as the base year? That is, which year is the current dollar being used to measure the real price of the IBN 5100?

Ans:
Year 1975 because in 1975 the value of the CPI is 100 and we know that in the base year the CPI is equal to 1 or 100 (depending upon the scale factor of the index) since the CPI value for a particular year is given as [(The cost of the market basket in that year)/(The cost of the market basket in the base year)]*(the scale factor). If the CPI value is 100 this indicates that the cost of the market basket in that year is equal to the cost of the market basket in the base year and that the scale factor is 100.

2) Fill in the missing values in the table.

Ans:

Years	CPI	Nominal Price of IBM 5100	Real Price of IBM 5100
1975	100	$10,000	$10,000
1982	120	$24,000	$20,000
1996	180	$54,000	$30,000
2000	200	$80,000	$40,000
2010	250	$125,000	$50,000

3) Suppose John travels from 1975 to 1996 and when he travels he brings gold with him. This gold has a value of $20,000 in 1996 dollars. John knows that he does not have enough gold to buy the IBM 5100 in 1996 and he is trying to figure out how much additional nominal dollars (1996 dollars) he would need in order to have enough dollars to purchase one IBM 5100 in 1996. Can you help him figure out the additional nominal dollars he would need?

Ans:
We know that the real price of the IBM 5100 in 1996 is $30,000 and that John has $20,000 of that amount in gold. That means that John needs an additional $10,000 in real dollars to make the purchase. But how much is $10,000 real dollars in nominal dollars? To find this remember that Real Value = [(Nominal Value)/(Inflation Index)](Scale Factor) or 10,000 = [(Nominal Value)/180]*100. Solving

this for Nominal Value we have Nominal Value = 10,000(180)/100 = $18,000 nominal dollars (these dollars are measured as 1996 dollars).

4) Suppose John has nominal income of $79,000 in 2010. Given this nominal income, which years could John travel back in time and be able to afford one IBM 5100?

Ans:
To answer this question we first need to calculate the real income John has in 2010. To find John's real income remember that real income = [(Nominal Income)/(Inflation Index)]*(Scale Factor) or Real Income = ($79,000/250)*100 . To say some time we might just realize that 79,000/250 > 75,000/250 = 30,000. This implies that John's nominal income in 2010 is sufficient to buy one IBM 5100 computer in 1975, 1982, or 1996 since the real price of a computer in each of these years is $30,000 or less.

Suppose now we change the base year from 1975 to 2000. Assume that the **nominal price** of an IBM 5100 in each year in the table does not change.

5) What is the CPI now in 1975?

Ans: 50.
To find the CPI in 1975 when the base year is changed to 2000 you need to first recognize that in order for 2000 to be the base year the CPI in 2000 must equal 100 (the scale factor). So, (CPI in 2000 with base year 1975)/(CPI in 2000 with base year 1975)*(Scale factor) = 100. But, if you are going to divide the CPI number for 2000 by 200, then you must divide all the other CPI numbers by 200: this will change the base year from 1975 to 2000. Thus, (CPI in 1975 with base year 1975)/(CPI in 2000 with base year 1975)*(Scale factor) = (100/200)*100 = 50.

6) If the base year is 2000 for the CPI, then what is the real price of an IBM 5100 in 1975?

Ans:
Recall that the real price = [(nominal price)/CPI] * (scale factor). Thus, the real price = [($10,000)/50]*100 = $20,000.

John is also interested in how the CPI is calculated for different years. He has the following information.

Year	CPI	Cost of Market Basket	Nominal income	Real income
1975	100	$500	$10,000	$10,000
1980		$550	$11,000	
1985		$750	$15,000	

7) Please help John fill out the above table.

Ans:

Year	CPI	Cost of Market Basket	Nominal income	Real income
1975	100	$500	$10,000	$10,000
1980	110	$550	$11,000	$10,000

1985	150	$750	$15,000	$10,000

8) What is the inflation rate between 1975 and 1985?

Ans:
Use the formula: (CPI_1985 – CPI_1975) / CPI_1975. We get (150 -100)/100 = 50%. Inflation rate between these two years is 50%.

9) Given the table you computed in part (7), what is happening to consumer's real income over this period of time? Does consumer welfare as measured by real income improve, stay the same, or decline between 1975 and 1985?

Ans: The real incomes are the same between these years. There is no change in the welfare of the consumer as measured by their real income.

5. **Budget Constraint**

Consider an economy composed of two goods, X and Y. The prices producers receive for a unit of these goods are, respectively, $2 for X and $3 for Y. The consumer has income of $60 to spend on the two goods. (Hint: The following questions are independent from each other.)

1) Write the consumer's budget constraint. Graph the consumer's budget constraint with X on the horizontal axis and Y on the vertical axis.

Ans: 2X+3Y=60

2) Suppose there is a tax of $1/unit imposed on good X. Write the new budget constraint, and graph it.

Ans:(2+1)X+3Y=60 => 3X+3Y=60

3) Suppose there is a subsidy of $1/unit bestowed on good Y. Write the new budget constraint, and graph it.

Ans:2X+(3-1)Y=60 => 2X+2Y=60

4) Suppose there is a tax on both goods that raises the price of each good by 50%. Write the new budget constraint, and graph it.

Ans: 2(1+50%)X+3(1+50%)Y=60 => 3X+4.5Y=60

5) Suppose there is a lump-sum tax of $10 levied on the consumer. That is, suppose the individual must pay the government $10 due to this new tax. Write the new budget constraint, and graph it.

Ans: 2X+3Y+10=60 => 2X+3Y=50