Economics 101
Summer 2016
Answers to Homework #1
Due 5/26/16

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. a. Suppose you know that the two points (X, Y) = (12, 6) and (15, 3) sit on the same line. From this information write an equation for this line in slope-intercept form.

b. Suppose that you know that the slope of the line is 5 and that this line also contains the point (20, 25). What is the y-intercept for this line? Show your work.

c. You are given the following two equations:

$$Y = 10X + 100$$
$$Y = 76 – 2X$$

Find the solution (X, Y) for where these two equations intersect. Show your work.

d. Suppose that you know that the relationship between X and Y, where X is the variable measured on the horizontal axis, can be described by the following equation:

$X = 40 – 2Y$ for all values of $X \geq 0$

You are then told that for every Y value the X value has now increased by 10 units. Write the equation in slope-intercept form for this new line. Show your work. Hint: you might find it helpful to draw a "sketch" illustrating these two lines before you start doing your calculations.

e. Suppose that you know that the relationship between X and Y, where X is the variable measured on the horizontal axis, can be described by the following equation:

$Y = 10 + 4X$ for all values of $X \geq 0$

You are then told that for every X value the Y value has now decreased by 20 units. Write the equation in slope-intercept form for this new line. Show your work. Hint: you might find it helpful to draw a "sketch" illustrating these two lines before you start doing your calculations.

Answers:

a. To write the equation start by finding the slope of the line: slope = (change in Y)/(change in X) = (6 – 3)/(12 – 15) = -1. Then, use the general form of the slope-intercept equation and this slope value to find the y-intercept:

$Y = mX + b$ where m = slope and b = y-intercept

$Y = -X + b$

Substitute one of the given points into this equation to find the value of the y-intercept, b:

$6 = -12 + b$

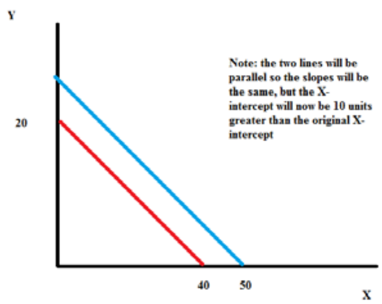
$b = 18$

Equation for the line containing these two points: $Y = 18 - X$

b. Since we know the slope of the line, we can write the equation for this line as:
Y = 5X + b
We are asked to find the value of the y-intercept, or b, in this equation. To do this, we can substitute into the equation the given point and solve for b:
25 = 5(20) + b
b = -75
The y-intercept for this line described by the provided information is -75 and the coordinates for this y-intercept are (0, -75).

c. To find the solution set the two equations equal to one another:
10X + 100 = 76 – 2X
12X = -24
X = -2
Then, use this X value in either equation to solve for Y:
Y = 10(-2) + 100 = 80
Or, Y = 76 – 2(-2) = 80
(X, Y) = (-2, 80)

d. I find it helpful to visualize what is happening to this line by drawing a sketch. Here is my sketch with the red line the original equation and the blue line indicating the new equation.



With my sketch it is easy to see that the X-intercept has increased by 10 units from 40 to 50: so, if the equation is in X-intercept form I simply need to change the X-intercept:
X = 50 – 2Y
But, I am asked to write the equation in Y-intercept form so I will rearrange the equation:
2Y = 50 – X
Y = 25 – (1/2)X

e. I find it helpful to visualize what is happening to this line by drawing a sketch. Here is my sketch with the red line the original equation and the blue line indicating the new equation.

With my sketch it is easy to see that the Y-intercept has decreased by 20 units from 10 to =10: so, if the equation is in Y-intercept form I simply need to change the Y-intercept:
Y = -10 + 4X
Or, I might write it as
Y = 4X – 10 so that the negative sign before the 10 does not get visually lost!

2. More math review:
Consider three individuals who are enrolled in Professor Kelly's Econ 101 class: Howard, Sue, and Mike. In class Professor Kelly has emphasized the importance of turning in the homework and despite this advice these three individuals have taken decidedly different approaches. Professor Kelly anticipates that students will need to earn a total of 90 points on a 100 point scale from the various assignments for the semester in order to earn an A in the class. The assignments are as follows:

- Five homework assignments that are each worth 2 points: if a student turns in all five homeworks and shows good effort they will earn 10 points on that 100 point scale from this effort. If they turn in four homeworks that show good effort they will earn 8 points on that 100 point scale from this effort. And, so on. If the student turns in none of the homeworks, they will earn 0 points on that 100 point scale.
- Two midterms that contribute 25% each to their weighted score total. So, for instance if a student scores a 50 on each of these two midterms they will get 12.5 points on a 100 point scale from each of these exams (a total of 25 points on that 100 point scale from these two exams).
- A final that contributes 40% to their weighted score total. (Note that we have now accounted for 100% of that weighted score total.) If a student scores a 70 on the final, the student will get 28 points on a 100 point scale from this final.

Here is the data you have for Howard, Sue, and Mike. You are asked to calculate the score each of these individuals needs on the final exam (assume it is a 100 point final exam) in order to get an A in the class. Show your work for each calculation. For some individuals you may find that it is impossible to earn that "A" in the class.

	Howard	**Sue**	**Mike**
Number of Homeworks turned in	0	3	5
Score per Homework Submitted	2	2	2
Score on First Midterm out of a Possible 100 Points	84	84	84
Score on Second Midterm out of a Possible 100 Points	88	88	88
Score Needed on Final out of a Possible 100 Points to Earn an "A" in the class			

Answers:

In general we can write this calculation as:
Final Score Needed to Earn an "A" = (Number of Homeworks Submitted)(Score on Submitted Homeworks) + .25(Score on First Midterm) + .25(Score on Second Midterm) + .4(Score on Final)

We also know that the final score needed to earn an "A" in the class is 90. Thus,
90 = (Number of Homeworks Submitted)(Score on Submitted Homeworks) + .25(Score on First Midterm) + .25(Score on Second Midterm) + .4(Score on Final)

For Howard:
90 = (Number of Homeworks Submitted)(Score on Submitted Homeworks) + .25(Score on First Midterm) + .25(Score on Second Midterm) + .4(Score on Final)
90 = (0)(2) + .25(84) + .25(88) + .4(Score for Howard)
90 = 21 + 22 + .4(Score for Howard)
.4(Score for Howard) = 47
Score for Howard = 117
If the final exam has only 100 points, then it is impossible for Howard to earn an "A" in the class. Howard should have done his homework!

For Sue:
90 = (Number of Homeworks Submitted)(Score on Submitted Homeworks) + .25(Score on First Midterm) + .25(Score on Second Midterm) + .4(Score on Final)
90 = (3)(2) + .25(84) + .25(88) + .4(Score for Sue)
90 = 6 + 21 + 22 + .4(Score for Sue)
.4(Score for Sue) = 41
Score for Sue = 102.5
If the final exam has only 100 points, then it is impossible for Sue to earn an "A; in the class. Sue should have done her homework!

For Mike:
90 = (Number of Homeworks Submitted)(Score on Submitted Homeworks) + .25(Score on First Midterm) + .25(Score on Second Midterm) + .4(Score on Final)
90 = (5)(2) + .25(84) + .25(88) + .4(Score for Mike)
90 = 10 + 21 + 22 + .4(Score for Mike)
.4(Score for Mike) = 37
Score for Mike = 92.5
If Mike scores a 92.5 on the final exam, Mike will earn the "A" in the class: Mike was smart to do that homework!

3. Suppose that there are three countries that produce popcorn (P) and juice (J): Merryland, Happyland, and Sedateland. The maximum amount of popcorn and juice each country can produce if they only produce that one good is given in the table below. Use this information to answer this set of questions. Assume that each of the three countries have constant opportunity costs with respect to the production of popcorn and juice: that is, each country has a linear production possibility frontier.

Country	**Maximum Amount of Popcorn Production Possible**	**Maximum Amount of Juice Production Possible**
Merryland	10 units of popcorn	20 units of juice
Happyland	10 units of popcorn	10 units of juice
Sedateland	5 units of popcorn	20 units of juice

a. Given the above information, what is Sedateland's opportunity cost of producing one more unit of popcorn?

b. Given the above information, what is Happyland's opportunity cost of producing one more unit of juice?

c. Given the above information, rank these three countries in order of their comparative advantage in the production of popcorn. List the order from the country with the greatest comparative advantage to the country with the least comparative advantage.

d. Given the above information, rank these three countries in order of their comparative advantage in the production of juice. List the order from the country with the greatest comparative advantage to the country with the least comparative advantage.

e. Construct the joint PPF for these three countries if they specialize according to comparative advantage. For this joint PPF measure popcorn on the vertical axis and juice on the horizontal axis. After constructing this joint PPF, provide the coordinates of any intercept or "kink point" in your diagram. Then write the equation for each segment of the joint PPF and provide a range or domain for each segment.

f. Consider each of the production combinations given in the table below and decide whether this production combination is possible if these three countries specialize according to comparative advantage and then trade with one another. Enter your answer as a "Yes, this combination lies on the joint PPF", "Yes, this combination lies inside the joint PPF" or "No, this combination lies outside the joint PPF" in the provided column.

Combination	Amount of Units of Popcorn in Combination	Amount of Units of Juice in Combination	Is this Combination a Possible Production Combination for these Three Countries?
A	22	12	Yes, this combination lies on the joint PPF
B	22	8	Yes, this combination lies inside the joint PPF
C	20	22	No, this combination lies outside the joint PPF
D	15	30	Yes, this combination lies on the joint PPF
E	3	48	No, this combination lies outside the joint PPF

Answers:

a. Sedateland's opportunity cost of producing one more unit of popcorn is 4 units of juice.

b. Happyland's opportunity cost of producing one more unit of juice is one unit of popcorn.

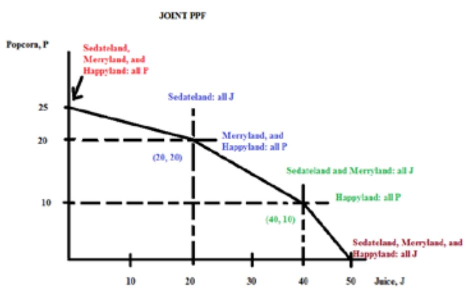
c. Happyland, Merryland, and Sedateland
To see this ordering, first write the opportunity cost of producing one unit of popcorn for each of these countries: you may find it helpful to draw a sketch of each country's PPF and then use the slope measure to guide these opportunity cost measures. For example, if you measure popcorn on the vertical axis and juice on the horizontal axis, then the PPF for Merryland has a slope of -1/2: this tells us that the opportunity cost of one more unit of the good on the X axis (juice) is 1/2 unit of the good on the Y axis (popcorn). We can use the reciprocal of the slope (-2) to find the opportunity cost of one more unit of the good measured on the Y axis: in this case, this means that the opportunity cost of one more unit of popcorn is 2 units of juice.

Using this method: the opportunity cost of producing one more unit of popcorn for Merryland is 2 juices, for Happyland is 1 juice, and for Sedateland is 4 juices. Happyland has the lowest opportunity cost and therefore the comparative advantage in the production of popcorn.

d. Sedateland, Merryland, and Happyland
To see this ordering, first write the opportunity cost of producing one unit of juice for each of these countries: you may find it helpful to draw a sketch of each country's PPF and then use the slope measure to guide these opportunity cost measures. For example, if you measure popcorn on the vertical axis and juice on the horizontal axis, then the PPF for Merryland has a slope of -1/2: this tells us that the opportunity cost of one more unit of the good on the X axis (juice) is 1/2 unit of the good on the Y axis (popcorn). We can use the reciprocal of the slope (-2) to find the opportunity cost of one more unit of the good measured on the Y axis: in this case, this means that the opportunity cost of one more unit of popcorn is 2 units of juice.

Using this method: the opportunity cost of producing one more unit of juice for Merryland is 1/2 unit of popcorn, for Happyland is 1 popcorn, and for Sedateland is 1/4 unit of popcorn. Sedateland has the lowest opportunity cost and therefore the comparative advantage in the production of juice.

e. Here is the joint PPF.



And, now for the equations:
For $0 \leq J \leq 20$, $P = 25 - (1/4)J$

For $20 \leq J \leq 40$, $P = 30 - (1/2)J$
To see this start with your basic y-intercept form for the equation: $Y = mX + b$
Then, replace Y and X with the relevant variables:
$P = mJ + b$
Then, calculate the slope of this segment: $m = \text{slope} = -10/20 = -1/2$ and plug this slope value into the equation:
$P = b - (1/2)J$
Then use one of the points that you know is on the joint PPF in this segment to solve for b: (J, P) = (20, 20) or (40, 10) are known points. Thus,
$20 = b - (1/2)(20)$
$30 = b$
The equation for this segment is thus, $P = 30 - (1/2)J$

For $40 \leq J \leq 50$, $P = 50 - J$
To see this start with your basic y-intercept form for the equation: $Y = mX + b$
Then, replace Y and X with the relevant variables:
$P = mJ + b$
Then, calculate the slope of this segment: $m = \text{slope} = -10/10 = -1$ and plug this slope value into the equation:

P = b – J
Then use one of the points that you know is on the joint PPF in this segment to solve for b: (J, P) = (50, 0) or (40, 10) are known points. Thus,
50 = b – (0)
50 = b
The equation for this segment is thus, P = 50 – J

f. For this question you will find it helpful to use the equations you found in (e). For example, for (J, P) = (12, 22) you will need to use the first equation: P = 25 – (1/4)J since 12 units of juice lies in the domain of $0 \leq J \leq 20$. In this equation, if J = 12 units, then the point on the joint PPF associated with 12 units of juice would provide 22 units of popcorn. Thus, the point (J, P) = (12, 22) is a possible production combination since it lies on the joint PPF.

Repeat this process for each combination using the appropriate equation for the segment of the PPF you are considering.

Here are the final answers:

Combination	Amount of Units of Popcorn in Combination	Amount of Units of Juice in Combination	Is this Combination a Possible Production Combination for these Three Countries?
A	22	12	Yes, this combination lies on the joint PPF
B	22	8	Yes, this combination lies inside the joint PPF
C	20	22	No, this combination lies outside the joint PPF
D	15	30	Yes, this combination lies on the joint PPF
E	3	48	No, this combination lies outside the joint PPF

4. Helen and Charlie produce windows (W) and doors (D). The table below provides information about how many hours of labor they need individually to produce a window or a door. Assume that they only need labor to produce these two goods and assume that both Helen and Charlie have linear PPFs.

	Number of Hours of Labor Needed to Produce One Window	Number of Hours of Labor Needed to Produce One Door
Helen	2 hours of labor	1 hour of labor
Charlie	4 hours of labor	5 hours of labor

a. Suppose that Helen and Charlie each have 40 hours a week that they can devote to producing windows and doors. In two separate graphs draw Helen's and Charlie's production possibility frontiers: label each graph clearly and completely. In your graphs, measure doors on the vertical axis and windows on the horizontal axis.

b. Given the above information, who has the comparative advantage in the production of doors? Explain your answer.

c. Given the above information, who has the comparative advantage in the production of windows? Explain your answer.

d. Given the above information, fill in the following table:

	Opportunity Cost of Producing One More Window	Opportunity Cost of Producing One More Door
Helen		
Charlie		

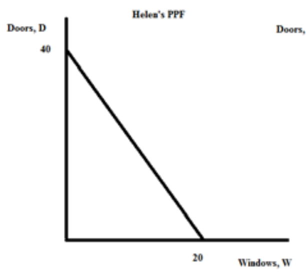
e. Based upon Helen and Charlie each having forty hours of labor available per week, construct the joint PPF for these two individuals if they decide to specialize and trade with one another. In your graph measure doors on the vertical axis and windows on the horizontal axis. Make sure that the coordinates of all kink points are identified.

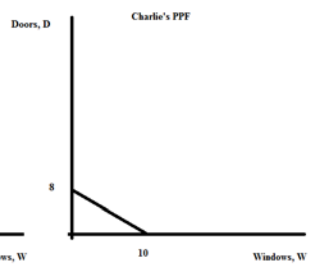
f. Given the joint PPF you constructed in (c), write the equation(s) for each segment of this joint PPF. Make sure you identify either the relevant range or domain for any equation you provide.

g. Using the number line approach discussed in class show the range of acceptable trading prices for 5 windows if Helen and Charlie specialize according to comparative advantage and then trade with one another.

Answers:

a.





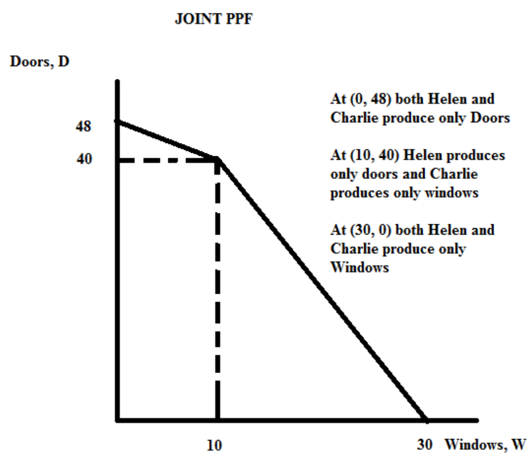
b. Helen has the comparative advantage in the production of doors: Helen's opportunity cost of producing an additional door is 1/2 window while Charlie's opportunity cost of producing an additional door is 5/4 window.

c. Charlie has the comparative advantage in the production of windows. Charlie's opportunity cost of producing an additional window is 4/5 door while Helen's opportunity cost of producing an additional window is 2 doors.

d.

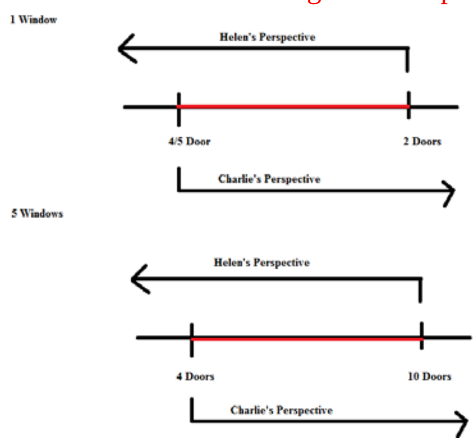
	Opportunity Cost of Producing One More Window	Opportunity Cost of Producing One More Door
Helen	2 doors	1/2 window
Charlie	8/10 door or 4/5 door	5/4 window or 1.25 windows

e.

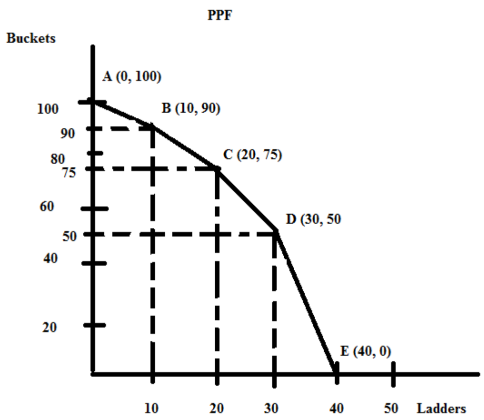


f. The top segment of the joint PPF can be written as:
D = 48 – (4/5)W for the 0 ≤ W ≤ 10
The bottom segment of the joint PPF takes a bit more work:
y = mx + b
D = (-2)W + b
We know that the points (W, D) = (10, 40) and (30, 0) sit on this part of the PPF. So, use one of these points to solve for the value of b, the y-intercept for the equation.
40 = (-2)(10) + b
b = 60
The equation for this lower segment of the joint PPF can be written as:
D = 60 – 2W for 10 ≤ W ≤ 30

g.
Let's start by drawing this sketch showing the acceptable range of trading prices in terms of doors for 1 window and then we can "gross this up" for five windows.



5. The graph below depicts the production possibility frontier for a small economy that produces only buckets (B) and ladders (L). This PPF is linear between any two adjacent points on the PPF: e.g., the PPF is linear between points A and B, between points B and C, and between points C and D.…



a. Suppose this economy is currently producing at point B. What is the opportunity cost of producing one additional bucket given this information? Explain your answer. Make sure your answer provides the units of measurement.

b. Suppose that this economy is currently producing at point B. What is the opportunity cost of producing one additional ladder given this information? Explain your answer. Make sure your answer provides the units of measurement.

c. Suppose this economy is currently producing at point C. What is the opportunity cost of producing one additional bucket given this information? Explain your answer. Make sure your answer provides the units of measurement.

d. Suppose this economy is currently producing at point C. What is the opportunity cost of producing one additional ladder given this information? Explain your answer. Make sure your answer provides the units of measurement.

e. Suppose this economy is currently producing at point D. What is the opportunity cost of producing one additional bucket given this information? Explain your answer. Make sure your answer provides the units of measurement.

f. Suppose this economy is currently producing at point D. What is the opportunity cost of producing one additional ladder given this information? Explain your answer. Make sure your answer provides the units of measurement.

g. Given the above PPF, write the equation(s) for each segment of the PPF. Identify the relevant range or domain for each equation. Show your work and how you found these equations.

Answers:

a. If this economy is initially at point B and wants to produce an additional bucket, then the economy is moving along the PPF from point B toward point A. To find the opportunity cost of an additional bucket we would need to find the reciprocal of the slope of the segment of the PPF between points A and B. The slope of this segment is -1, so the reciprocal of the slope of this segment is also -1. The opportunity cost of an additional bucket if this economy is at point B is therefore 1 ladder.

Alternatively, you could write the equation for the PPF between points A and B:
B = 100 – L and then plug in B' = 91. When you do this, you find that L' = 9. Instead of having 10 ladders you now only have nine ladders: the opportunity cost of producing that additional bucket (going from 90 buckets to 91 buckets) is measured by what you gave up...in this case, this economy gives up 1 ladder.

b. If this economy is initially at point B and wants to produce an additional ladder, then the economy is moving along the PPF from point B toward point C. To find the opportunity cost of an additional ladder we would need to find the slope of this segment of the PPF between points B and C. The slope of this segment is -3/2: the opportunity cost of an additional ladder if this economy is at point B is therefore 3/2 buckets.

Alternatively, you could write the equation for the PPF between points B and C: this takes a bit more work. So, here are the steps:
y = mx + b is the general form of an equation for a straight line
B = (-3/2)L + b
Then, use one of the known points that lies on this segment to find the value of the y-intercept, b: we know that (L, B) = (10, 90) and (20, 75) both are on this segment.
90 = (-3/2)(10) + b
b = 105
B = 105 – (3/2)L and then plug in L' = 11. When you do this, you find that B' = 88.5. Instead of having 90 buckets you now only have 88.5 buckets: the opportunity cost of producing that additional ladder (going from 10 ladders to 11 ladders) is measured by what you gave up...in this case, this economy gives up 1.5 buckets.

c. If this economy is initially at point C and wants to produce an additional bucket, then the economy is moving along the PPF from point C toward point B. To find the opportunity cost of an additional bucket we would need to find the reciprocal of the slope of the segment of the PPF between points B and C. The slope of this segment is -3/2, so the reciprocal of the slope of this segment is -2/3. The opportunity cost of an additional bucket if this economy is at point C is therefore 2/3 ladder.

Alternatively, you could write the equation for the PPF between points B and C: this takes a bit more work. So, here are the steps:
y = mx + b is the general form of an equation for a straight line
B = (-3/2)L + b
Then, use one of the known points that lies on this segment to find the value of the y-intercept, b: we know that (L, B) = (10, 90) and (20, 75) both are on this segment.
90 = (-3/2)(10) + b
b = 105
B = 105 – (3/2)L and then plug in B' = 76. When you do this, you find that L' = 19.3. Instead of having 20 ladders you now only have 19.3 ladders: the opportunity cost of producing that additional bucket (going from 75 buckets to 76 buckets) is measured by what you gave up...in this case, this economy gives up 2/3 ladder.

d. If this economy is initially at point C and wants to produce an additional ladder, then the economy is moving along the PPF from point C toward point D. To find the opportunity cost of an additional ladder we would need to find the slope of this segment of the PPF between points C and D. The slope of this segment is -5/2: the opportunity cost of an additional ladder if this economy is at point C is therefore 5/2 buckets.

Alternatively, you could write the equation for the PPF between points C and D: this takes a bit more work. So, here are the steps:

y = mx + b is the general form of an equation for a straight line
B = (-5/2)L + b
Then, use one of the known points that lies on this segment to find the value of the y-intercept, b: we know that (L, B) = (30, 50) and (20, 75) both are on this segment.
50 = (-5/2)(30) + b
b = 125
B = 125 – (5/2)L and then plug in L' = 21. Instead of having 75 buckets you now only have 72.5 buckets: the opportunity cost of producing that additional ladder (going from 20 ladders to 21 ladders) is measured by what you gave up...in this case, this economy gives up 2.5 buckets.

e. If this economy is initially at point D and wants to produce an additional bucket, then the economy is moving along the PPF from point D toward point C. To find the opportunity cost of an additional bucket we would need to find the reciprocal of the slope of the segment of the PPF between points C and D. The slope of this segment is -5/2, so the reciprocal of the slope of this segment is -2/5. The opportunity cost of an additional bucket if this economy is at point D is therefore 2/5 ladder.

Alternatively, you could write the equation for the PPF between points C and D: this takes a bit more work. So, here are the steps:
y = mx + b is the general form of an equation for a straight line
B = (-5/2)L + b
Then, use one of the known points that lies on this segment to find the value of the y-intercept, b: we know that (L, B) = (30, 50) and (20, 75) both are on this segment.
50 = (-5/2)(30) + b
b = 125
B = 125 – (5/2)L and then plug in B' = 51. When you do this, you find that L' = 29.6. Instead of having 30 ladders you now only have 29.6 ladders: the opportunity cost of producing that additional bucket (going from 75 buckets to 76 buckets) is measured by what you gave up...in this case, this economy gives up .4 or 2/5 ladder.

f. If this economy is initially at point D and wants to produce an additional ladder, then the economy is moving along the PPF from point D toward point E. To find the opportunity cost of an additional ladder we would need to find the slope of this segment of the PPF between points D and E. The slope of this segment is -5: the opportunity cost of an additional ladder if this economy is at point D is therefore 5 buckets.

Alternatively, you could write the equation for the PPF between points D and E: this takes a bit more work. So, here are the steps:
y = mx + b is the general form of an equation for a straight line
B = (-5)L + b
Then, use on of the known points that lies on this segment to find the value of the y-intercept, b: we know that (L, B) = (30, 50) and (40, 0) both are on this segment.
0 = (-5)(40) + b
b = 200
B = 200 – 5L and then plug is L' = 31. When you do this, you find that B' = 45. Instead of having 50 buckets you now only have 45 buckets: the opportunity cost of producing that additional ladder (going from 30 ladders to 31 ladders) is measured by what you gave up…in this case, this economy gives up 5 buckets.

g.
For the segment between points A and B: B = 100 – L
This equation holds from $90 \leq B \leq 100$ or for $0 \leq L \leq 10$.

For the segment between points B and C: $B = 105 – (3/2)L$
This equation holds from $75 \leq B \leq 90$ or for $10 \leq L \leq 20$.

For the segment between points C and D: $B = 125 – (5/2)L$
This equation holds from $50 \leq B \leq 75$ or for $20 \leq L \leq 30$.

For the segment between points D and E: $B = 200 – 5L$
This equation holds from $0 \leq B \leq 50$ or for $30 \leq L \leq 40$.