

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. a. Suppose you are given the following two points that sit on a straight line:

$$(x, y) = (10, 2) \text{ and } (5, 4)$$

What is the equation for this line in slope-intercept form? Show all your work in finding this equation.

b. Suppose you are asked to find the equation in slope-intercept form of a straight line that has slope = 4 and contains the point (100, 50). Find this equation, showing all the work you did.

c. You are given the two equations:

$$y = 4x + 20$$

$$y = 100 - x$$

Find the (x, y) coordinate values where these two lines intersect each other. Show your work.

d. You are given the equation:

$$y = 10 - 2x \text{ for } y \leq 10$$

Draw a sketch of this line for all values of x and y that are ≥ 0 . Label this line 1. Now, suppose that for every y value, the x value doubles. Draw this new line in your graph showing just the values of x and y that are ≥ 0 . Label this line 2. Write an equation for line 2 showing your work.

e. You are given the equation:

$$y = 10 - 2x \text{ for } y \leq 10$$

and told that for every x value in this original equation, the y value has increased by 10 units. Draw a graph that illustrates the original line as well as this new line. Label these two lines Line 1 and Line 2, respectively. Then, write the equation for Line 2 and provide a verbal explanation for how you found this new equation.

Answer:

a. We can start by finding the slope of the line:

$$\text{slope} = \Delta Y / \Delta X = (4 - 2) / (5 - 10) = (-2/5)$$

Then, general slope-intercept form is $y = mx + b$. Thus, $y = b - (2/5)x$ and using one of the given points we can find the y-intercept, or b, value. Thus,

$$2 = b - (2/5)(10)$$

$$2 = b - 4$$

$$b = 6$$

The equation for the line is $y = 6 - (2/5)x$.

b. Slope-intercept form is $y = mx + b$ and we know the slope, or m , is given as 4. Thus, $y = 4x + b$. Substitute the given point $(x, y) = (100, 50)$ into this equation to find the value of b , the y-intercept.

$$50 = 4(100) + b$$

$$b = -350$$

Thus, the equation is $y = 4x - 350$.

c. To find where these two lines intersect set the two equations equal to one another. Thus,

$$4x + 20 = 100 - x$$

$$5x = 80$$

$$x = 16$$

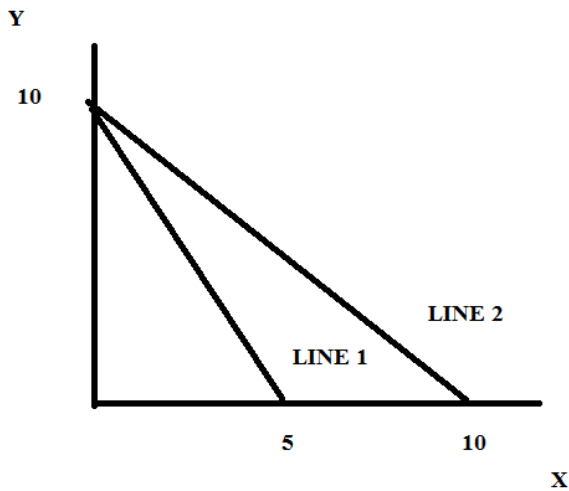
To find the y value, substitute $x = 16$ into either equation. Thus,

$$y = 4(16) + 20 = 64 + 20 = 84 \text{ or}$$

$$y = 100 - 16 = 84$$

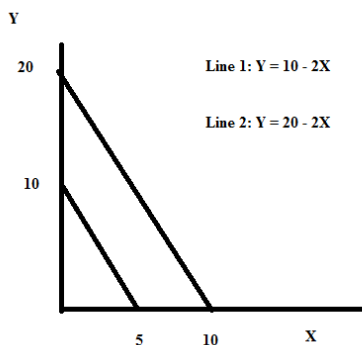
The coordinates (x, y) for where these two lines intersect are $(x, y) = (16, 84)$.

d.



When $y = 10$, $x = 0$ initially. This means that when x doubles for every y value, the point $(x, y) = (0, 10)$ sits on line 2 (as well as line 1). When $y = 0$, $x = 5$ initially. This means that when x doubles for every y value, the point $(10, 0)$ sits on line 2. By looking at our drawn graph we can see that line 2 has slope of -1 and a y-intercept of 10: thus, the equation for line 2 can be written as $y = 10 - x$.

e.



Line 2 is parallel to Line 1, which implies that the slope of both lines is the same and equal to (-2). Since the y-value increases by 10 at every x value this tells us that if the initial point (x, y) = (0, 10) then the new point for this x value will be (0, 20). That is, the y-intercept increases from 10 to 20.

2. More math review:

a. Joe is taking math this semester and on the first day of class his instructor informed the class that grades would be determined by their scores on three midterms and a final. Each midterm would be worth 20% of their final numeric grade while the final would be worth 40% of their final numeric grade. Joe is trying to figure out what score he needs on his final in order to accumulate a final numeric score of 90 on a 100 point scale. Complicating Joe's analysis is that the three midterms have not been on a 100 point scale nor will the final exam be on a 100 point scale, yet the assignment of the final numeric grade will be on a 100 point scale. This is the information that Joe has:

	Total Number of Points Available on the Midterm or Final	Joe's Score on the Midterm or Final
First Midterm	80	60
Second Midterm	40	35
Third Midterm	50	48
Final Exam	25	Not yet taken

- Recalculate Joe's scores for each midterm based on a 100 point scale for each exam (i.e. express Joe's scores as a score out of 100). Show your work.
- Calculate what score Joe must make on the final exam, measured on a 100 point scale, in order to have a final numeric grade of 90 for the class. Show your work.
- Recall that the final exam is actually a 25 point exam, so convert Joe's score on a 100 point final exam to the equivalent score on a 25 point final exam. Show your work. Round to the nearest whole number.

Answer:

i. For the first midterm: $60/80 = x/100$ or, $x = 75$. A score of 60 on an 80 point exam is equivalent to a score of 75 on a 100 point exam.

For the second midterm: $35/40 = y/100$ or, $y = 87.5$. A score of 35 on a 40 point exam is equivalent to a score of 87.5 on a 100 point exam.

For the third midterm: $48/50 = z/100$ or, $z = 96$. A score of 48 on a 50 point exam is equivalent to a score of 96 on a 100 point exam.

ii. To calculate Joe's grade we know that the general formula for the final grade in the class is:

$$M1(.20) + M2(.20) + M3(.20) + F(.40) = \text{final numeric grade}$$

where M1 is the first midterm, M2 is the second midterm, M3 is the third midterm, and F is the final exam. This formula also assumes that all four exams are measured on a 100 point scale. Using the values you calculated in (i) and the final numeric grade of 90 that Joe wants, we get:

$$(75)(.2) + (87.5)(.2) + (96)(.2) + F(.4) = 90$$

$$15 + 17.5 + 19.2 + F(.4) = 90$$

$$.4F = 38.3$$

$$F = 95.75$$

iii. Using the same method as in (i) to convert this score, we have: $95.75/100 = w/25$ or $w = 23.9375$ which we can round to $w = 24$.

3. Susie and Joe produce two goods: brownies (B) and pies (P). Susie and Joe have the same amount of resources available to them. If Susie uses all of her resources to produce brownies she can produce 20 brownies and if she uses all of her resources to produce pies she can produce 10 pies; and if Joe uses all of his resources to produce brownies he can produce 40 brownies and if he uses of his resources to produce pies he can produce 30 pies. Assume that both Susie and Joe have linear production possibility frontiers.

a. Draw a graph that represents Susie's production possibility frontier given the above information. Measure brownies on the vertical axis and pies on the horizontal axis.

b. Write an equation for Susie's production possibility frontier given the above information. Write this equation in slope intercept form. Then fill in the following table using this equation.

Pies produced by Susie	Brownies produced by Susie
2	
	4
8	
	7
9	

c. Draw a graph that represents Joe's production possibility frontier given the above information. Measure brownies on the vertical axis and pies on the horizontal axis.

d. Write an equation for Joe's production possibility frontier given the above information. Determine for each of the following combinations of (brownies, pies) whether the combination lies on Joe's PPF, lies inside Joe's PPF, or lies beyond Joe's PPF.

Combination of (Brownies, Pies)	Location of combination relative to Joe's PPF
(36, 3)	
(33, 6)	
(22, 12)	
(24, 15)	
(2, 27)	

e. Given the above information, who has the absolute advantage in the production of brownies? Explain your answer.

f. Given the above information, who has the comparative advantage in the production of brownies? Explain your answer.

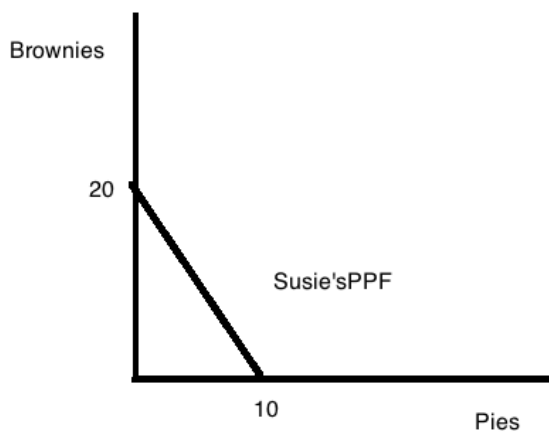
g. Draw Susie and Joe's joint PPF. Measure brownies on the vertical axis and pies on the horizontal axis. Identify any intercepts as well as the coordinates for any "kink" points. Write an equation in slope intercept form for every segment of this joint PPF and provide the relevant range for each segment.

h. What is the acceptable range of trading prices in terms of pies for 10 brownies? Illustrate this using the number line approach presented in class. Make sure to include the arrows and labels that indicate Joe's perspective and Susie's perspective.

i. What is the acceptable range of trading prices in terms of brownies for 5 pies? Illustrate this using the number line approach presented in class. Make sure to include the arrows and labels that indicate Joe's perspective and Susie's perspective.

Answer:

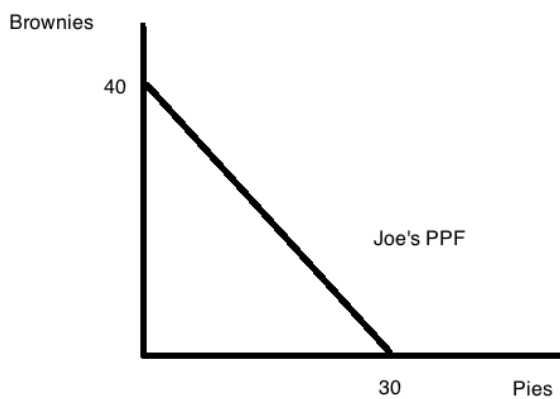
a.



b. $B = 20 - 2P$

Pies produced by Susie	Brownies produced by Susie
2	16
8	4
8	4
6.5	7
9	2

c.



d. $B = 40 - (4/3)P$

Combination of (Brownies, Pies)*	Location of combination relative to Joe's PPF
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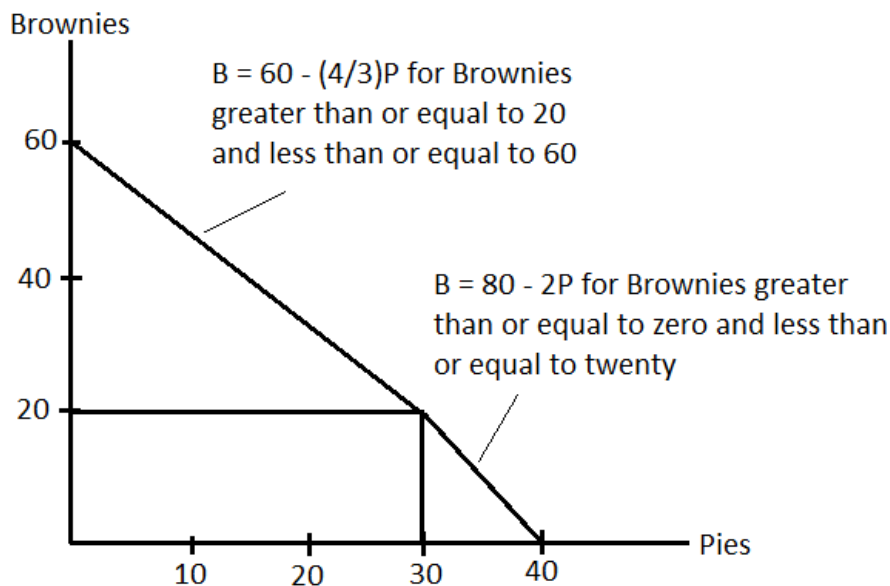
(36, 3)	Lies on the PPF
(33, 6)	Lies beyond the PPF: when Pies = 6, then Brownies = 32 using the equation for the PPF
(22, 12)	Lies inside the PPF: when Pies = 12, then Brownies = 24 using the equation for the PPF
(24, 15)	Lies beyond the PPF: when Pies = 15, then Brownies = 20 using the equation for the PPF
(2, 27)	Lies inside the PPF: when Pies = 27, then Brownies = 4 using the equation for the PPF

*Note: the table purposefully alter the order of the coordinate points from (Pies, Brownies) which is what you graphed in (a) and (c) and instead listed the coordinate points as (Brownies, Pies) to make sure you knew to enter the values in the “right” places in the equation you found in (d).

e. Joe has the absolute advantage in the production of brownies since he can absolutely produce more brownies than Susie (40 brownies versus 20 brownies).

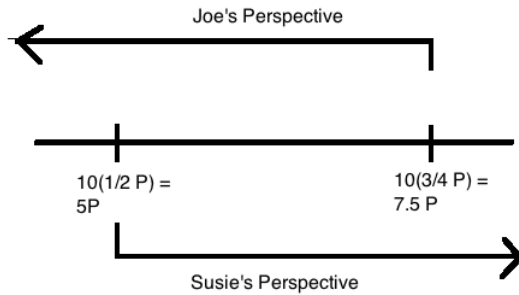
f. Susie has the comparative advantage in the production of brownies since she has a lower opportunity cost for producing brownies than does Joe. Susie’s opportunity cost of producing a brownie is equal to $\frac{1}{2}$ pie, while Joe’s opportunity cost of producing a brownie is equal to $\frac{3}{4}$ pie.

g. Start by finding the intercepts. Clearly if Susie and Joe produced no pies they could make $40 + 20 = 60$ brownies together. Similarly they could make 40 pies if they produced no brownies. To determine the rest of the graph, pretend Susie and Joe were initially thinking about producing only brownies but then decided to make one pie. Who should make this pie? We know that Susie has the comparative advantage in brownies, which means Joe must have the comparative advantage in pies. Hence Joe should make the first pie, and every pie after that until he can make no pies, i.e. until we reach the kink point where Joe makes 30 pies and Susie makes 20 brownies. With the kink point and the intercepts you can find the equations of the two line segments using methods as in 1 (a).



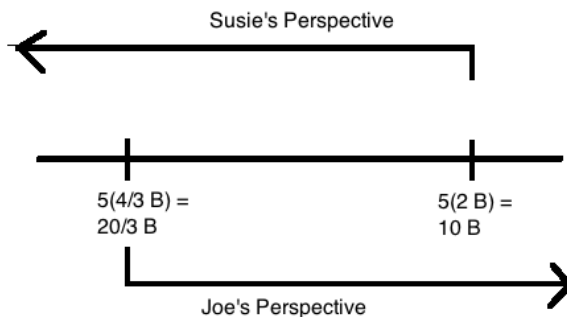
h.

10 Brownies



i.

5 Pies



4. Jacob and Marley produce windows and doors using only their labor. The following table provides you with information about the amount of labor that Jacob and Marley must use in order to produce these doors and windows. Assume that Jacob and Marley both have the same amount of labor and that their PPFs are linear.

	Amount of Labor Needed to Produce One Window	Amount of Labor Needed to Produce One Door
Jacob	2 hours of labor	4 hours of labor
Marley	6 hours of labor	3 hours of labor

- Given the above information, who has the absolute advantage in the production of windows? Explain your answer.
- Given the above information, who has the absolute advantage in the production of doors? Explain your answer.
- Given the above information, what is Jacob's opportunity cost of producing one window? Explain your answer.
- Given the above information, what is Marley's opportunity cost of producing three doors? Explain your answer.

e. If Jacob and Marley each have 120 hours of labor, what combination of (doors, windows) represents each of these individuals completely specializing in their comparative advantage? Explain your answer.

Answer:

a. If Jacob and Marley have the same amount of labor resources, then Jacob has the absolute advantage in the production of windows since it takes Jacob less labor time to produce a window than it does Marley. For example, if both of these individuals each have 60 hours of labor time, then Jacob can produce 30 windows from this amount of labor while Marley can only produce 10 windows from an equivalent amount of labor.

b. If Jacob and Marley have the same amount of labor resources, then Marley has the absolute advantage in the production of doors since it takes Marley less labor time to produce a door than it does Jacob. For example, if both of these individuals each have 60 hours of labor time, then Marley can produce 20 doors from this amount of labor while Jacob can only produce 15 doors from an equivalent amount of labor.

c. Jacob's opportunity cost of producing one window is $\frac{1}{2}$ door. One way to see this is to construct Jacob's PPF based upon his having 60 hours of labor: with 60 hours of labor he can either produce 15 doors or 30 windows: this implies that his opportunity cost of producing one window is $\frac{1}{2}$ door. Alternatively, you could note that it takes twice as much labor for Jacob to produce a door as a window. So, if he has $[(2 \text{ hours of labor})/(\text{window})]/[(4 \text{ hours of labor})/(\text{door})]$ this is equivalent to $[(2 \text{ hours of labor})/(\text{window})][(\text{door})/(4 \text{ hours of labor})]$ or $\frac{1}{2}$ door per window.

d. Marley's opportunity cost of producing 3 doors is $3(1/2)$ window or 1.5 windows. One way to see this is to construct Marley's PPF based upon his having 60 hours of labor: with 60 hours of labor he can either produce 20 doors or 10 windows: this implies that his opportunity cost of producing one door is $\frac{1}{2}$ window and thus, if he produces 3 doors his opportunity cost must be 1.5 windows. Alternatively, you could note that it takes twice as much labor for Marley to produce a window as a door. So, if one door costs $\frac{1}{2}$ window, then 3 doors must cost 1.5 windows.

e. If Jacob and Marley both have 120 hours of labor to use in producing windows and doors and they both specialize according to their comparative advantage, then Jacob will be able to produce 60 windows with his 120 hours of labor while Marley will be able to produce 40 doors from his 120 hours of labor.