

Economics 101
Spring 2019
Answers to Homework #1
Due Thursday, February 7th, 2019

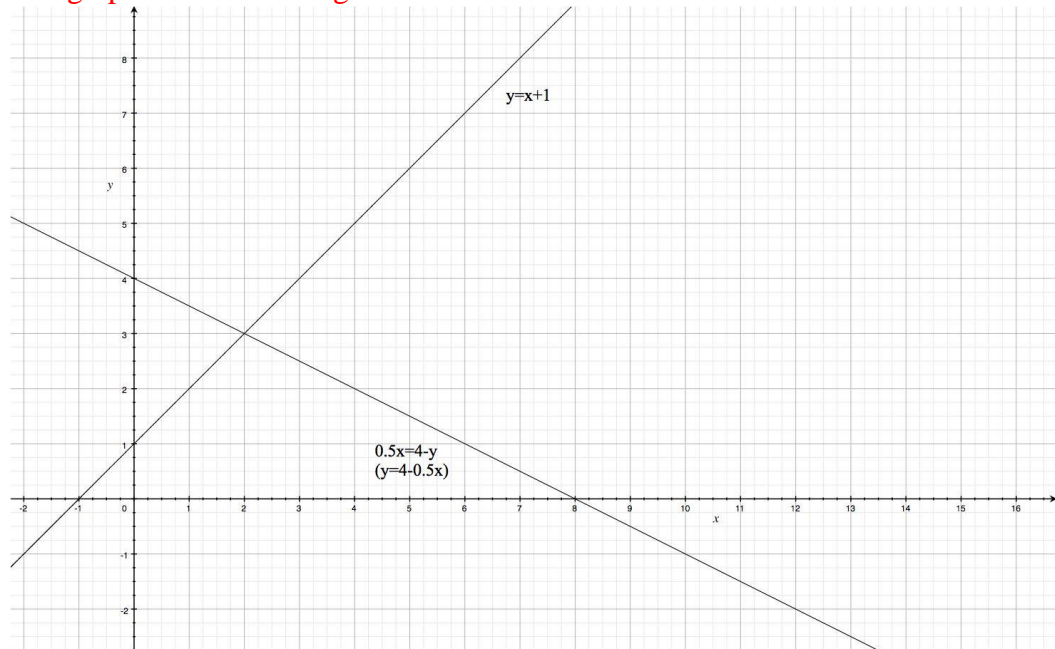
Directions:

- The homework will be collected in a box labeled with your TA's name **before** the lecture.
- Please place **your name, TA name, and section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Please **staple** your homework: we expect you to take care of this prior to coming to the large lecture. You do not need to turn in the homework questions, but your homework should be neat, orderly, and easy for the TAs to see the answers to each question.
- Late homework will not be accepted so make plans ahead of time.
- Show your work. Good luck!

Part I: Math Review

1. Suppose you are given the following two equations where y is measured on the vertical axis and x is measured on the horizontal axis:
(1) $y = x + 1$
(2) $0.5x = 4 - y$
 - a. Draw the two lines in a graph. Make sure you identify any intercepts in your graphs.

The graph is the following:



- b. Find the intersection of the two lines. That is, find (x, y) where these two lines intersect one another.

There are many ways to solve this problem: here is one method. First, rewrite equation (2) so that it is in y-intercept form:

$$0.5x = 4 - y$$

$$y = 4 - 0.5x$$

$$\text{LHS (1)} = \text{LHS (2)} \Leftrightarrow x + 1 = 4 - 0.5x \Leftrightarrow 1.5x = 3 \Leftrightarrow x = 2$$

$$\text{Plug } x = 2 \text{ into (1) or (2): } y = 2 + 1 \text{ or } y = 4 - 0.5 \cdot 2$$

$$\text{Find } y = 3$$

The intersection of these two lines is $(x, y) = (2, 3)$.

- c. Suppose the y-intercept of equation (2) changes to 7 while the slope of the line is unchanged. In a new graph draw the new lines. Find the new intersection point where line 1 intersects with line 2': $(x', y') = \underline{\hspace{2cm}}$.

$$\text{Equation (2) become (2)': } y = 7 - 0.5x$$

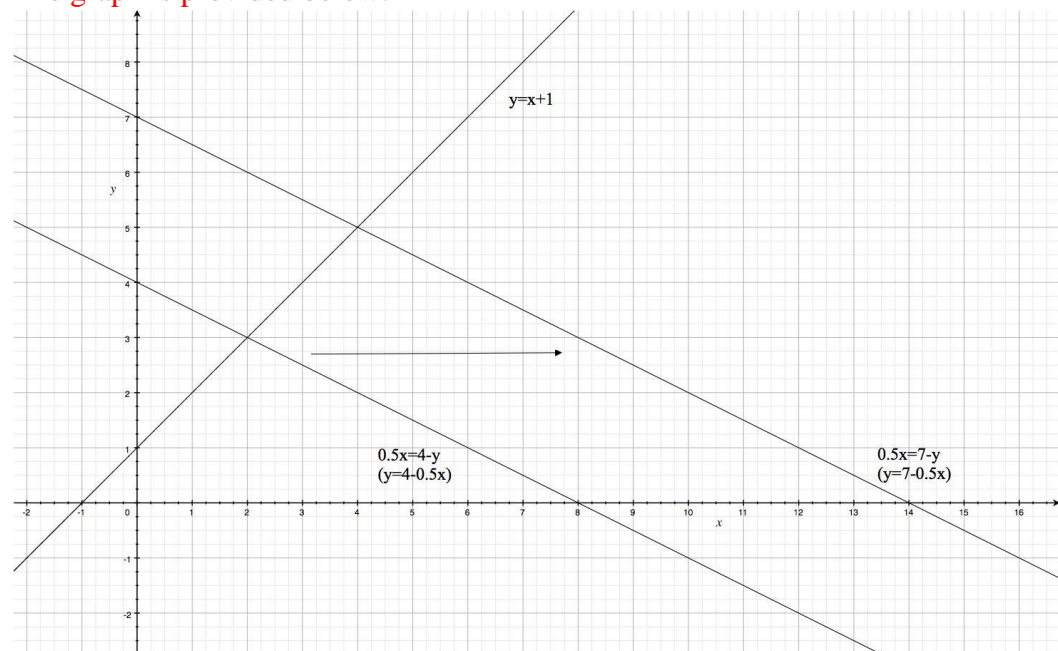
$$x + 1 = 7 - 0.5x \Leftrightarrow 1.5x = 6 \Leftrightarrow x = 4$$

$$\text{Plug } x = 4 \text{ into (1) or (2)': } y = 4 + 1 \text{ or } y = 7 - 0.5 \cdot 4$$

$$\text{Find } y = 5$$

The new intersection is $(x', y') = (4, 5)$

The graph is provided below:



2. This is a question where you just need to do a few calculations (and, it's okay to use a calculator here but show the underlying equations driving your calculator use). Don't make

it more difficult than it is! Here's the general scenario: someone borrows money and at the end of the year the person owes the money they borrowed plus the interest payment that is due on the loan. So, for instance if I borrow \$10,000 for a year at 2% interest, then at the end of the year I will owe \$10,000 plus $(10,000)(.02)$ or \$10,200. And, if I don't pay off any of the loan that first year then at the start of the second year I will owe \$10,200 and at the end of the second year I will owe $10,200 + (10,200)(.02)$ or \$10,404. Use this example to guide your work for this problem.

Bob purchased a new car for \$30,000, and paid \$10,000 in cash and then borrowed the rest of the money from a bank to make the purchase. He takes out a loan for the remaining \$20,000 at 1% compounded annually.

- a. How much does Bob owe to the bank at the end of the first, second and third year if he doesn't make any payments to the bank? Carry your calculations out to two places past the decimal.

At the end of the first year Bob owes $20,000 + 20,000 \cdot 0.01 = \$20,200$.

At the end of the second year Bob owes $20,200 + 20,200 \cdot 0.01 = \$20,402$

At the end of the third year Bob owes $20,402 + 20,402 \cdot 0.01 = \$20,606.02$

- b. If Bob wants to repay his loan as well as the interest that is due on the loan within a year, what does his monthly payment need to be?

At the end of the first year Bob owes $20,000 + 20,000 \cdot 0.01 = 20,000 \cdot 1.01 = \$20,200$. Thus, in order to repay 20,200 with 12 months, he has to pay $20,200/12 = \$1,683.333\dots$ per month.

- c. If Bob pays \$600 per month, could he repay his loan in two years? Three years?

If Bob pays \$600 per month, his annual payment is $600 \cdot 12 = \$7,200$.

At the end of the first year Bob owes $(20,000 - 7,200) + (20,000 - 7,200) \cdot 0.01 = 12,800 \cdot 1.01 = \$12,928$.

At the end of the second year Bob owes $(12,928 - 7,200) + (12,928 - 7,200) \cdot 0.01 = 5,728 \cdot 1.01 = \$5,785.28$.

Thus, he could not repay his loan in two years.

But he could repay his loan in three years if he makes monthly payments of \$600, because the sum he owes to the bank at the end of the third year is less than his annual payment: $\$5,785.28 < \$7,200$.

3. Suppose the grading system for Econ 101 consists of the following components: 5 homework assignments that together account for 10% (each of them accounts for 2% of the final grade) of the student's final grade; two midterms each of them counting for 25% of the student's final grade; and a final exam that counts the remaining 40% of the student's final grade. Assume the scores on each assignment, midterm, and exam could range from 0 - 100. The table below presents the corresponding scores for 3 students in the class. Assume the class only has three students.

	Adam	Bob	Chris
HW 1 (2%)	60	80	60
HW 2 (2%)	50	80	30
HW 3 (2%)	70	80	20
HW 4 (2%)	70	80	60
HW 5 (2%)	100	80	60
Midterm 1 (25%)	60	100	35
Midterm 2 (25%)	60	100	80
Final (40%)	70	80	50
Total weighted grade	65	90	

- a. Compute the total grade for Adam.

$$60*0.02 + 50*0.02 + 70*0.02 + 70*0.02 + 100*0.02 + 60*0.25 + 60*0.25 + 70*0.4 = 1.2 + 1.0 + 1.4 + 1.4 + 2.0 + 15 + 15 + 28 = 65$$

- b. What is the average score on Midterm 2?

$$(60 + 100 + 80)/3 = 240/3 = 80$$

- c. What is Bob's score on the Final Exam if his total weighted grade at the end of the semester is 90?

Let X denote Bob's score on the Final Exam. We can get X by solving the following equation:

$$\begin{aligned} (80*0.02)*5 + (100*0.25)*2 + X*0.4 &= 90 \\ \Leftrightarrow 8 + 50 + X*0.44 &= 90 \\ \Leftrightarrow 58 + X*0.44 &= 90 \\ \Leftrightarrow X*0.4 &= 90 - 58 = 32 \\ \Leftrightarrow X &= 32/0.4 = 80 \end{aligned}$$

- d. What is Chris's score on Midterm 1 if the average score on Midterm 1 is 65?

Let Y denote Chris's score on Midterm 1. We can get Y by solving the following:

$$(60 + 100 + Y)/3 = 65$$

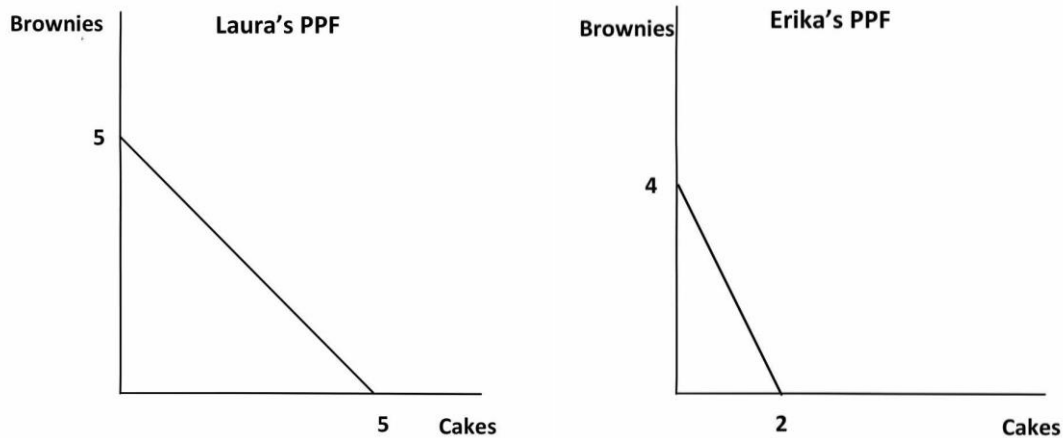
$$\Leftrightarrow 60 + 100 + Y = 195$$

$$\Leftrightarrow Y = 35$$

Part II: Production Possibility Frontier, Opportunity Cost, Absolute and Comparative Advantage

4. Erika and Laura both have bakeries. Laura can bake 1 cake or 1 brownie in 1 hour and Laura only works 5 hours every day. Erika can bake 4 brownies in one day or two cakes in one day and like Laura, Erika only works 5 hours per day.

a. In two separate graphs draw an individual PPF for both Laura and Erika measuring brownies per day on the vertical axis and cakes per day on the horizontal axis. Identify all intercepts in the two graphs and make sure your graphs are completely and clearly labeled.



- b. What is the opportunity cost of 1 cake for Laura and Erika, in terms of the number of brownies? What about the opportunity cost of 1 brownie in terms of the number of cakes for Laura and Erika?

Laura's opportunity cost for 1 cake is 1 brownie.
Erika's opportunity cost for 1 brownie is 0.5 cakes.
Laura's opportunity cost for 1 brownie is 1 cake.
Erika's opportunity cost for 1 cake is 2 brownies.

- c. Who has a comparative advantage in the production of brownies and why? Who has a comparative advantage in the production of cakes and why?

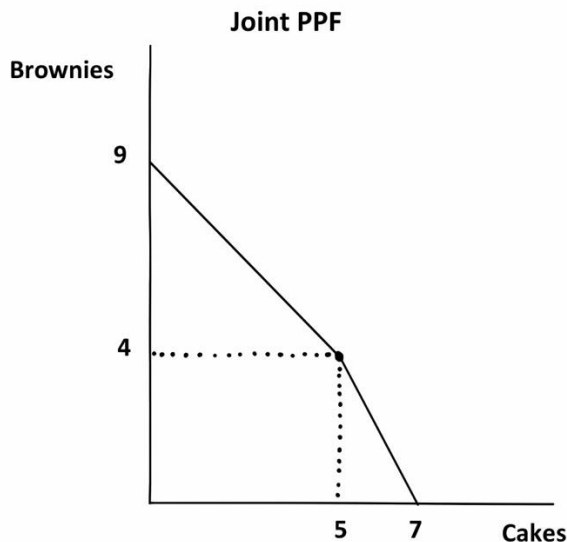
Erika has a comparative advantage in the production of brownies because she has a lower opportunity cost in terms of cakes when producing brownies.
Laura has a comparative advantage in the production of cakes because she has a lower opportunity cost in terms of brownies when producing cakes.

d. Who has the absolute advantage in producing brownies? Who has the absolute advantage in producing cakes?

Laura has both the absolute advantage in producing cakes and brownies. This is because she can bake 5 cakes and Erika can only produce 4 cakes and Laura can produce 5 brownies and Erika can only produce 2 brownies in one day.

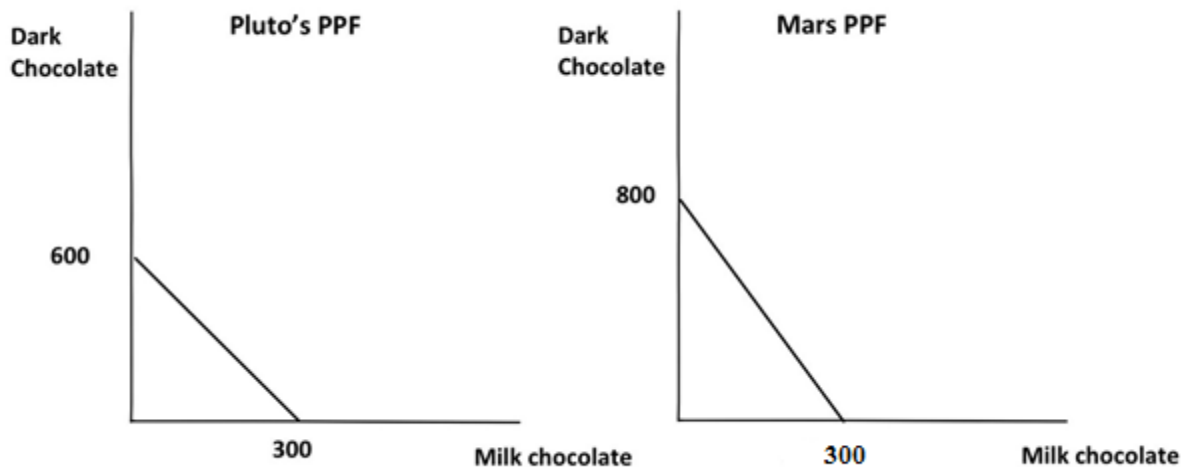
e. Draw a graph of the joint PPF of Erika and Laura. In this graph, measure brownies per day on the vertical axis and cakes per day on the horizontal axis. Identify all intercepts and the coordinates of any kink points that you find in your graph.

If both Erika and Laura only produce brownies, they can produce $4 + 5 = 9$ brownies in one day. This will be the y-intercept on the joint PPF graph. If both Erika and Laura only produce cakes, then they can produce $2 + 5 = 7$ cakes in one day. This will be the x-intercept on the joint PPF graph. If both Erika and Laura completely specialize in what they have their comparative advantage in, then Laura would produce 5 cakes and Erika would produce 4 brownies. This will be the kinked point on the joint PPF graph.



5. Consider two countries Mars and Pluto. Pluto can produce 600kg of dark chocolate in one hour and 300 kg of milk chocolate in one hour. Mars can produce 800kg of dark chocolate in one hour and 300kg of milk chocolate in one hour.

a. Draw two separate graphs of the PPFs for Pluto and Mars. In your graphs measure milk chocolate on the x-axis.



b. Which country has a comparative advantage in the production of dark chocolate and why? Which country has a comparative advantage in the production of milk chocolate and why?

First we need to calculate the opportunity cost for each country for producing these two kinds of chocolate.

Pluto's opportunity cost of producing dark chocolate is 0.5 kg of milk chocolate.

Mars' opportunity cost of producing dark chocolate is $\frac{3}{8}$ kg of milk chocolate.

Pluto's opportunity cost of producing milk chocolate is 2kg of dark chocolate.

Mars's opportunity cost of producing milk chocolate is $\frac{8}{3}$ kg dark chocolate.

Mars opportunity cost of producing dark chocolate is lower than Pluto's opportunity cost of producing dark chocolate. Therefore, Mars has the comparative advantage in producing dark chocolate.

Pluto's opportunity cost of producing milk chocolate is lower than Mars' opportunity cost of producing milk chocolate. Therefore, Pluto has the comparative advantage in producing milk chocolate.

c. Which of the following combinations of milk chocolate and dark chocolate can Pluto produce in one hour? Of the ones that are feasible, which combinations are efficient?

i) 700kg of dark chocolate and 20kg of milk chocolate

First we need to find the functional form for the PPF graph for Pluto. We have the points (0,600) and (300,0).

The slope is the following, $\frac{0-600}{300-0} = \frac{-600}{300} = -2$.

Using $\text{dark} = m \cdot \text{milk} + b$, where m is the slope, we can substitute -2 in for m and 600 for the y-intercept.

$\text{dark} = -2 * \text{milk} + 600$

Let's check the given combination by plugging in 20kg for milk chocolate (what we are calling "milk" in our equation):

Dark = $-2 \cdot 20 + 600 = -40 + 600 = 560\text{kg}$. $560\text{kg} < 700\text{kg}$ and therefore, the given point lies above the PPF and is not feasible. That is, our equation tells us that when Pluto produces 20 kg of milk chocolate, then the maximum amount of dark chocolate Pluto can produce given this milk chocolate production is 560 kg of dark chocolate.

ii) 180kg of dark chocolate and 200kg of milk chocolate

Let's check this combination by plugging in 200kg for milk chocolate:

Dark = $-2 \cdot 200 + 600 = -400 + 600 = 200\text{kg}$. Since $200\text{kg} > 180\text{kg}$, this point lies underneath the PPF and is feasible. It is not efficient because this combination does not lie on the PPF line.

iii) 550kg of dark chocolate and 10kg of milk chocolate

Let's check this combination by plugging in 500kg for milk chocolate:

Dark = $-2 \cdot 10 + 600 = -20 + 600 = 580\text{kg}$. Since $580\text{kg} > 550\text{kg}$, this point lies underneath the PPF and is feasible. It is not efficient because this combination does not lie on the PPF line.

iv) 300kg of dark chocolate and 150kg of milk chocolate

Let's check this combination by plugging in 150kg for milk chocolate:

Dark = $-2 \cdot 150 + 600 = -300 + 600 = 300\text{kg}$. $300\text{kg} = 300\text{kg}$ and therefore, this point lies on the PPF and is feasible and efficient.

v) 150kg of dark chocolate and 250kg of milk chocolate

Let's check this combination by plugging in 350kg for milk chocolate:

Dark = $-2 \cdot 250 + 600 = -500 + 600 = 100\text{kg}$. $100\text{kg} < 150\text{kg}$ and therefore, this point lies above the PPF and is not feasible.

d. Find the range of prices that Mars and Pluto are going to be willing to trade for. Provide the range of trading prices in terms of dark chocolate that one kg of milk chocolate will trade for and then provide the range of trading prices in terms of milk chocolate that one kg of dark chocolate will trade for.

From part b, we know that Pluto has the comparative advantage in producing milk chocolate and therefore, Pluto will sell milk chocolate. We also know that Mars has the comparative advantage in dark chocolate and therefore, Mars will sell dark chocolate.

Mars is willing to sell one kg of dark chocolate for any price that is equal to or greater than their opportunity cost of producing the one kg of dark chocolate: Mars will therefore be willing to sell one kg of dark chocolate for $\frac{3}{8}$ kg of milk chocolate. Pluto will be willing to pay any price that is equal to or less than their opportunity cost of producing one kg of dark chocolate. Pluto will therefore be willing to pay $\frac{1}{2}$ kg of milk chocolate or less for one kg of dark chocolate. So, the two countries will agree to a price of between $\frac{3}{8}$ kg of milk chocolate and $\frac{1}{2}$ kg of milk chocolate for 1 kg of dark chocolate.

Pluto is willing to sell one kg of milk chocolate for any price that is equal to or greater than 2 kg of dark chocolate. Mars is willing to pay any price that is equal to or less than their opportunity cost of producing one kg of milk chocolate. Mars will therefore be willing to pay $\frac{8}{3}$ kg of dark chocolate or less for one kg of milk chocolate. So, the two countries will agree to a price of between 2 kg of dark chocolate and $\frac{8}{3}$ kg of dark chocolate for 1 kg of milk chocolate.