Economics 101
Spring 2018
Answers to Homework #1
Due Thursday, February 8

Directions:

- The home will be collected in a box **before** the lecture.
- Please place **your name, TA name, and section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Please **staple** your homework: we expect you to take care of this prior to coming to the large lecture. You do not need to turn in the homework questions, but your homework should be neat, orderly, and easy for the TAs to see the answers to each question.
- Late homework will **not** be accepted so make plans ahead of time.
- **Show your work**. Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional? For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you submit any work for someone else.

Part I: Math Review

1) Consider the following two equations:

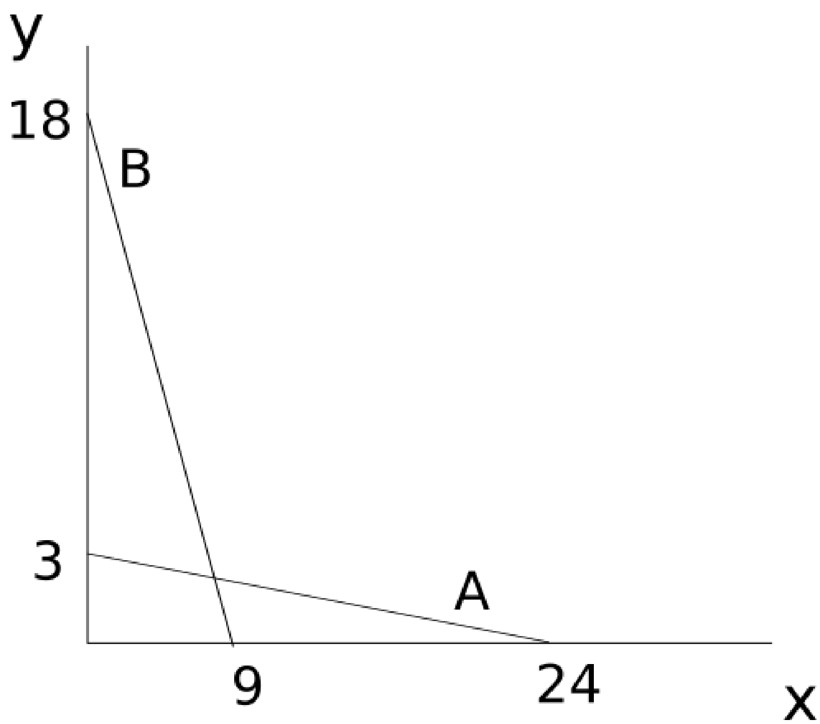
Equation A: $x = 24 – 8y$ and Equation B: $y = -2x + 18$

Take careful note of the algebraic form of each equation. (I.e. is each in x-intercept or y-intercept form?)

a) Plot both equations on the same graph with x and y on the horizontal and vertical axes respectively. (Restrict your attention to the first quadrant; that is, consider only values equal to or greater than zero for both x and y.)

Solution:

We proceed by computing the intercept points:
Equation A has an x-intercept of 24 and a y-intercept of 3. Equation B has a y-intercept of 18 and an x-intercept of 9. Using this information, we have the following plot:



b) Find the intersection point of these two lines.

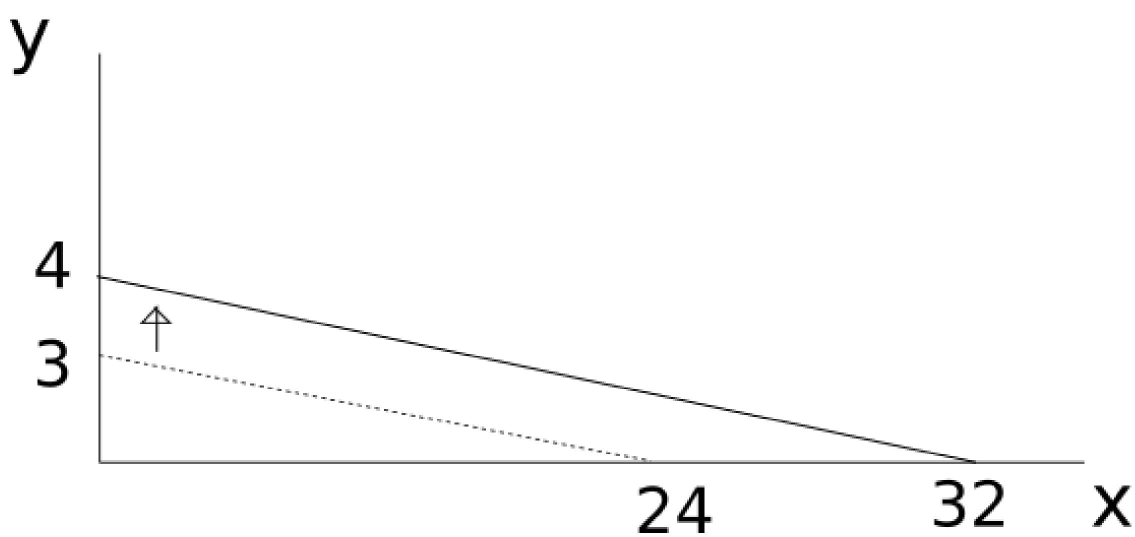
Solution: We will want to set the two equations equal to each other, but we must first take care to ensure that they are in the same form. Rearranging A into a y-intercept form, we have y = 3 – (1/8)x. Now setting this equal to B, we have

$3 - (1/8)x = 18 - 2x$
$(15/8)x = 15$
$x = 8.$

Plugging this back into either equation, we have y = 2. Thus the intersection is at (8,2).

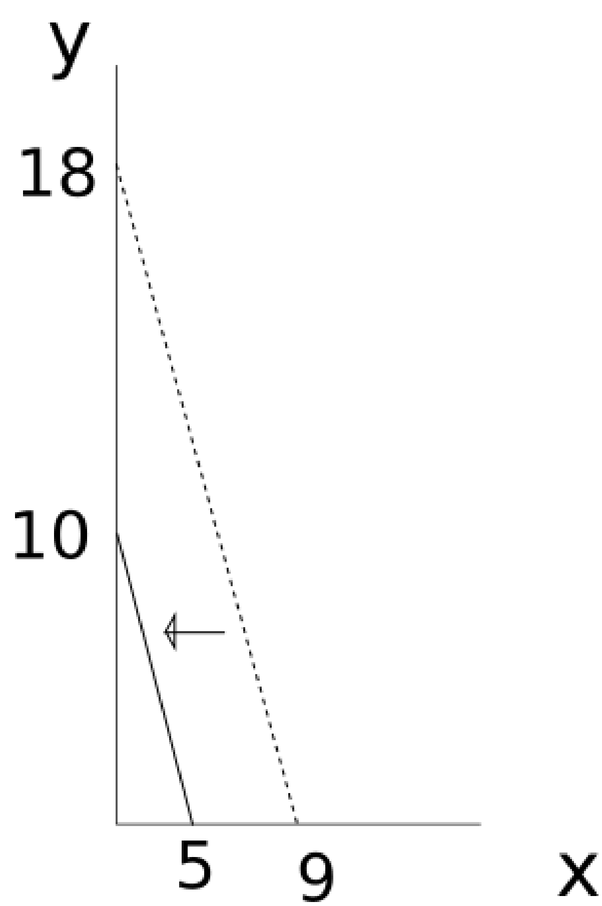
c) Suppose the line represented by equation A shifts *vertically* up by 1 unit. Plot and find the new equation (labelled A') for this line. Give the expression in both x-intercept and y-intercept form.

Solution: Shifting A up by 1 unit increases its y-intercept to 4. This leads to a new y-intercept of 32. Thus the new equation is y = 4 – (1/8)x or equivalently x = 32 – 8y.



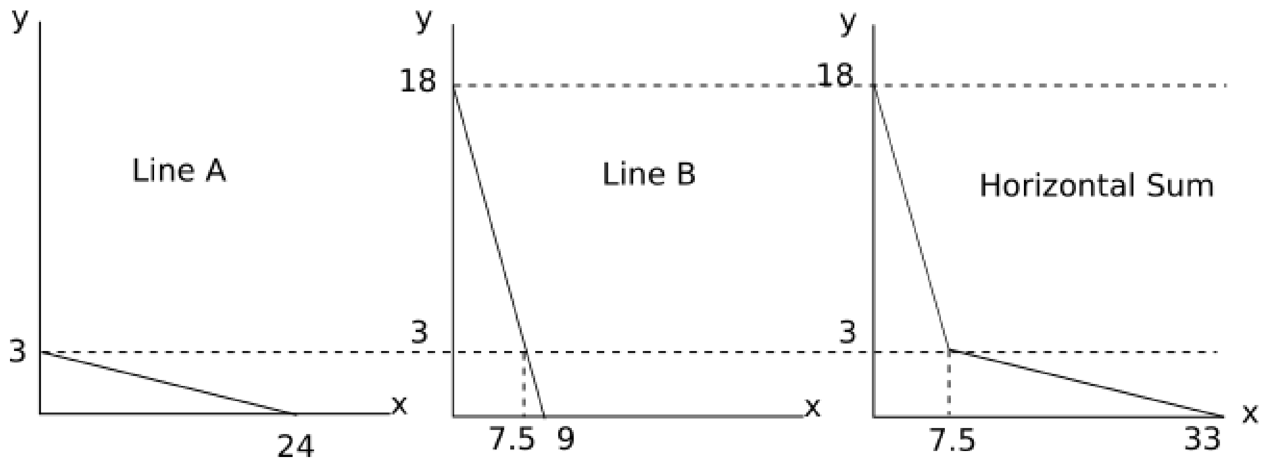
d) Suppose the line represented by equation B shifts *horizontally* left by 4 units. Plot and find the new equation (labelled B') for this line. Give the expression in both x-intercept and y-intercept form.

Solution: Shifting equation B horizontally left by 4 units, reduces its x-intercept to 5 and thus its y-intercept to 10. This gives a new equation of y = 10 – 2x or equivalently x = 5 – (1/2)y.



e) Using only the values from the first quadrant, plot and find the equation for the horizontal sum of equations A and B. Give the equation in both x-intercept and y-intercept form.

Solution: We proceed in the usual way, by finding the intercepts and the kink point. The y-intercept of the horizontal sum is simply the higher of the two y-intercepts, so y = 18. The kink point occurs on equation B at the y-value where equation A begins to have a positive x-value, so at y = 3, x = 7.5. Finally, the x-intercept occurs at the sum of the two x-intercepts, so x = 33. Plotting this we have



From the plot, we can then find the equation in the usual way
$y = 18 – 2x$ for $0 \leq x \leq 7.5$ and
$y = (66/17) – (2/17)x$ for $7.5 \leq x \leq 33$

Converting to x-intercept form, we have
$x = 33 – (17/2)y$ for $0 \leq y \leq 3$ and
$x = 9 – (1/2)y$ for $3 \leq y \leq 18$

2) Suppose you are running a shoe-making firm. This firm has two factories: Factory R can **only** make **right** shoes, and Factory L can **only** make **left** shoes.

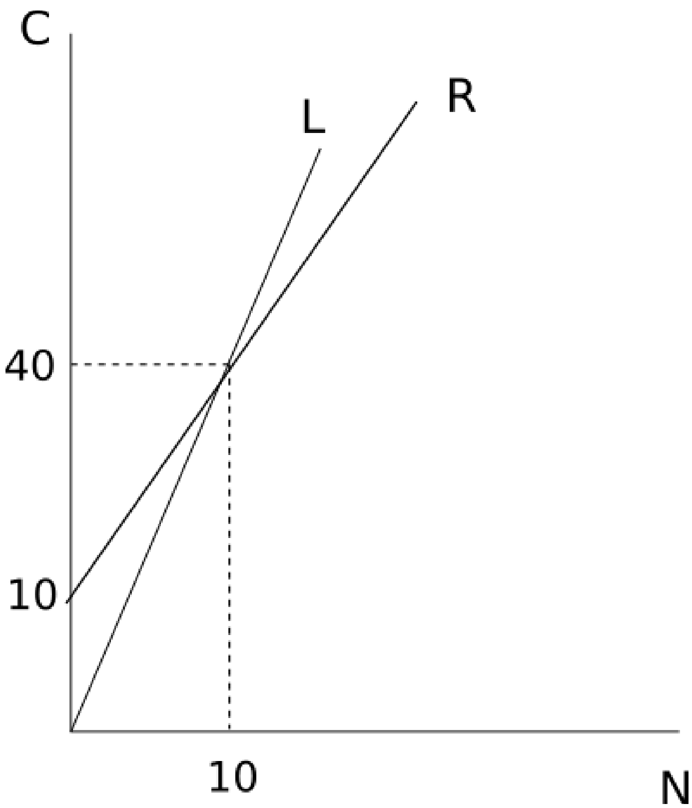
Factory R produces right shoes for a cost of \$3 per shoe, and has a \$10 maintenance cost that must be paid regardless of the number of shoes produced.
Factory L produces left shoes for a cost of \$4 per shoe, and has no maintenance costs.

a) Given the above information, write an equation for the total cost of producing shoes at each factory. (There should be two different equations and each equation should be of the form C = aN + b, where C is the total cost, and N is the number of shoes produced.)

Solution: Given the above information, we can express the total cost of producing right shoes as CR = 3N + 10 and the cost of producing left shoes as CL = 4N.

b) Plot the equations from the previous part with total cost (C), on the vertical axis, and number of shoes (N) on the horizontal axis.

Solution: Using the information from the previous part we have the following plot:

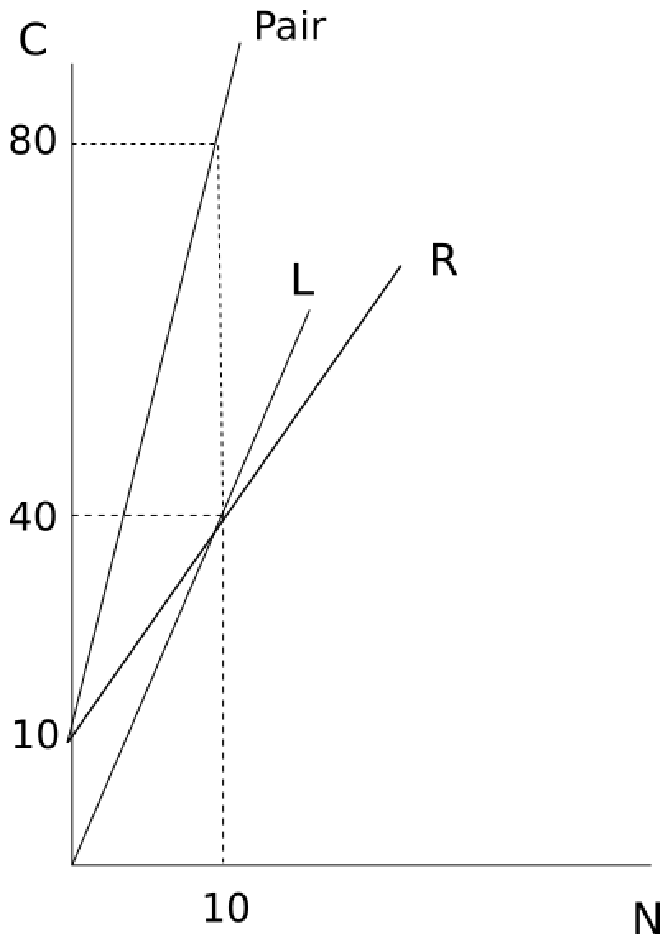


c) As the manager of these factories, you only care about how many **pairs** of shoes you can produce. Given the above information, plot and give an equation for the total cost of producing N **pairs** of shoes.

Solution: To get the total cost of producing N pairs of shoes, we must add the cost of producing N right shoes to the cost of producing N left shoes (since cost is on the vertical axis in this problem, this corresponds to the process of vertical summation). Since cost is positive for all numbers of shoes, we don't need to worry about kink points, so we can simply add the two expressions from part A together:

$C = CR + CL$
$C = (3N + 10) + 4N$
$C = 7N + 10$

Plotting this, we have



Part II: Percentages and Interest Rates

3) Suppose Dave earned $50,000 last year, and saved 25% of his income.

a) How much money did Dave save in total last year?

Solution: 0.25*50,000 = (1/4)*50,000 = 15,000/4 = $12,500.

b) Suppose this year, Dave earns $50,000 and saves $10,000. What is his savings rate this year?

Solution: r * 50,000 = 10,000, Solving for r we have r = 1/5 = 20%

4) The "rule of 70" is a handy rule of thumb for estimating the amount of time it takes for an initial investment that grows with interest rate r, to double. To estimate this doubling time, simply divide 70 by the interest rate. For example, if an investment grows at 5% per year, it would take approximately 70/5 = 14 years to double in value.

a) Suppose you invest $1000 this year with a 7% annual return. Using the rule of 70, give an approximation for how much this investment will be worth in 15 years. Then, use a calculator (but still show your work!) to find the exact value after 15 years. Hint: after the first year the value of this investment would be equal to (1000)(1 + .07); after the second year, the value of this investment would be (1000)(1 + .07)(1 + .07), etc. Compare this to your approximation.

Solution: Applying the rule, we have that an investment should double in value approximately every 10 years. Thus in 15 years, the investment should double approximately 1.5 times. You could use a calculator to compute 2^1.5 = 2.82, giving a total amount in the account of about $2820. Alternatively, you could use roughly approximate this by doubling the amount once (2*1000 = 2000) and adding half of another doubling (0.5*2(2000) = 1000) resulting in $3000 total. (The *true* amount is 1000*1.07^15 = $2760.)

b) Suppose 20 years ago you invested $1000. You now find that you have $4000 in the account. Approximately, using the Rule of 70, what was the annual rate of return on this account? Use a calculator (but still show your work!) to find the exact interest rate. Compare this to your approximation.

Solution: Since the initial investment doubled in value twice in 20 years, it should double approximately every 10 years. Thus the approximate interest rate was 7%. (The true value solves (1+r)^20 = 4. This gives r = 7.2%)

Suppose your employer offers you 2 pension plans. Plan A initially invests $100,000 which then grows 2% per year until you retire. Plan B initially invests $50,000 which then grows at 4% per year until you retire.

c) If you intend to retire in 20 years, which plan should you choose? (Hint: how long would it take for you to be indifferent between the two plans?)

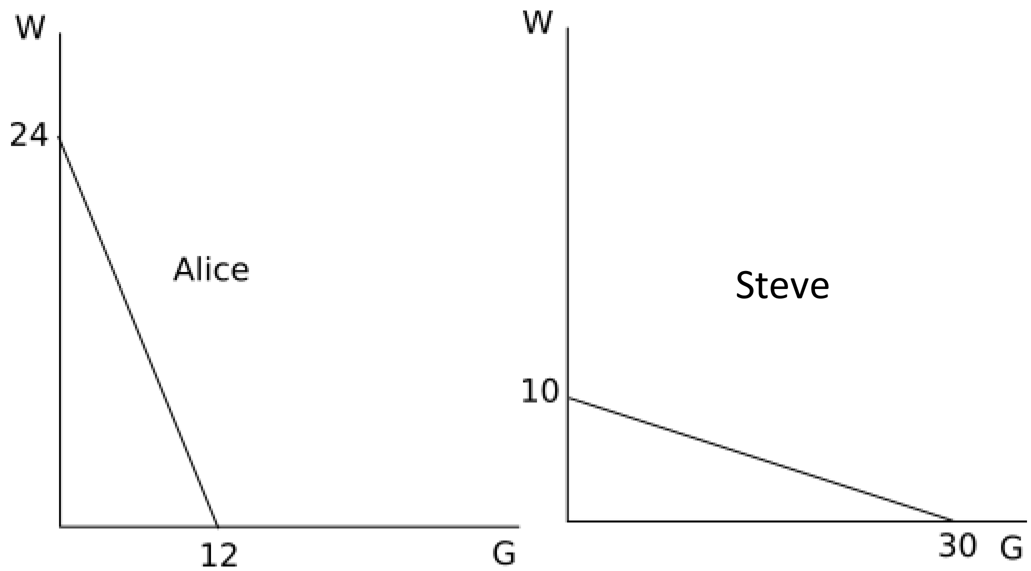
Solution: Plan A doubles in value roughly every 35 years, and plan B doubles roughly every 12.5 years. Since plan B doubles in value in half the time, after 35 years, plan A will have doubled once, and plan B will have doubled twice. Since plan B started with exactly half as much as A, after 35 years, then, they will have the same value. For any time less than 35 years, plan A will have a higher value, and for any time more than 35 years, plan B will have a higher value. Thus, if the time until retirement is 20 years, plan A is the better option. (To solve, for the exact time until indifference, you must solve (1.04)^t = 2*(1.02)^t. Using logarithms, you can find t = 35.70. If the math here is hard for you to follow, don't worry we will not be doing this particular type of calculation in the class. We just want to provide you with a very precise answer here.)

Part III: Opportunity Costs, Absolute vs. Comparative Advantage, and Production Possibility Frontiers

5) Alice and Steve can each produce widgets (W) and gadgets (G). Both have 12 hours each day that they can devote to the production of these goods. Alice can produce 6 widgets every 3 hours, or 1 gadget per hour. Steve can produce 5 widgets in 6 hours or 5 gadgets in 2 hours.

a) With widgets on the vertical axis, and gadgets on the horizontal axis, plot the production possibility frontiers for Alice and Steve on two separate graphs, and provide an equation for each of these graphs.

Solution:



Finding the equations we have:

Alice: W = 24 – 2G
Steve: W = 10 – (1/3) G

b) In terms of the number of widgets, what is the opportunity cost of 1 gadget for Alice and Steve respectively? What is the opportunity cost of 1 widget in terms of the number of gadgets for these two individuals?

Solution: Using the graph from the previous part, it becomes easy to see that the opportunity cost of 1 gadget for Alice is 2 widgets. Alternatively, 1 gadget takes 1 hour for Alice, during which time she can produce (1/3)*6 = 2 widgets. By similar reasoning, Steve's opportunity cost of 1 gadget is 1/3 of a widget. By taking reciprocals, we have that Alice's OC for 1 widget is 1/2 a gadget, and Steve's OC for 1 widget is 3 gadgets.

c) Who has the absolute advantage in the production of each good? Who has the comparative advantage in the production of each good?

Solution: Alice can produce more widgets in total and thus has the absolute advantage in widgets. Similarly, Steve can produce the most gadgets and thus has the absolute advantage in gadgets. To find comparative advantage, we must compare opportunity costs. Since Alice has the lower opportunity cost for producing 1 widget, she has the comparative advantage in the production of widgets. Similarly, Steve has the lower OC for 1 gadget and thus has the comparative advantage in the production of gadgets.

d) Suppose Alice and Steve wish to trade gadgets and widgets. In terms of the number of widgets, what is the range of possible trading prices for 1 gadget? In terms of the number of gadgets, what is the range of possible trading prices for 1 widget?

Solution: The trading range of prices for 1 gadget must be between the opportunity costs of Alice and Steve, otherwise at least one of them would do better to produce the good themselves rather than trading for it. Thus the trading range of prices for 1 gadget must be no lower than 1/3 of a widget (otherwise Steve would never sell gadgets) and no higher than 2 widgets (otherwise Alice would never buy gadgets). Similarly, the trading range of prices for 1 widget must be no lower than 1/2 a gadget and no higher than 3 gadgets.

e) (Challenge) Suppose the trading price for 1 gadget is the one most favorable to Steve (is this the highest or lowest possible price?). Plot the possible bundles of widgets and gadgets that Steve can consume if he makes efficient use of his own productive capacity and the possibility of trade with Alice.

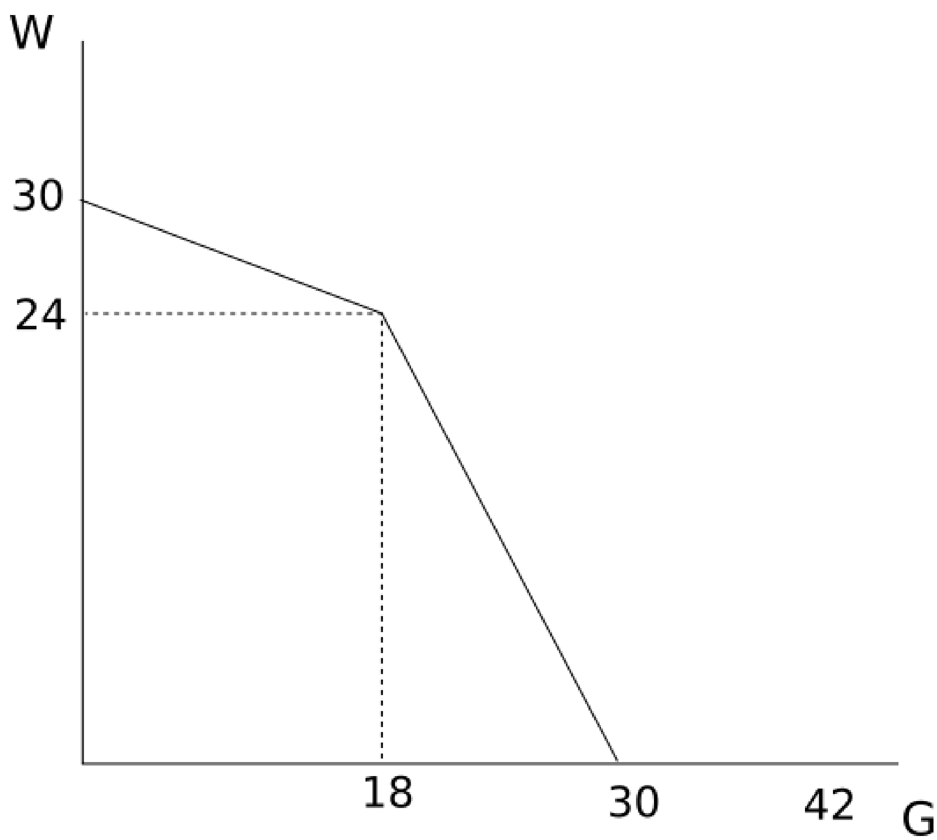
Solution: First, notice that the price of 1 gadget most favorable to Steve is the highest price (he will only ever sell gadgets since he has comparative advantage). Thus the most favorable price for Steve is 2 widgets per gadget.

To plot the consumption frontier, first consider how many gadgets Steve can consume if he only consumes gadgets. Since Steve has the comparative advantage in the production of gadgets, he will never buy gadgets; he will only make them himself. Thus if he only consumes gadgets, he can consume at most 30 gadgets.

Now, if wants to consume some widgets, he will first buy from Alice since he can buy them for 2 widgets per gadget (producing them himself would only net him 1/3 of a widget per gadget. However, he can only purchase up to 24 widgets, since Alice can't produce more than 24, so we must have a kink point. At this price, 24 widgets would cost Steve 12 gadgets, so if Steve consumes 24 widgets, he would have 18 gadgets left.

Finally, if Steve wants more than 24 widgets, he must make them himself, at an opportunity cost of 1/3 of a widget per gadget. Thus, he can consume at most 6 more gadgets (if he gave up production of all 18 of his remaining gadgets. Thus if he only consumes gadgets, he can consume 30.

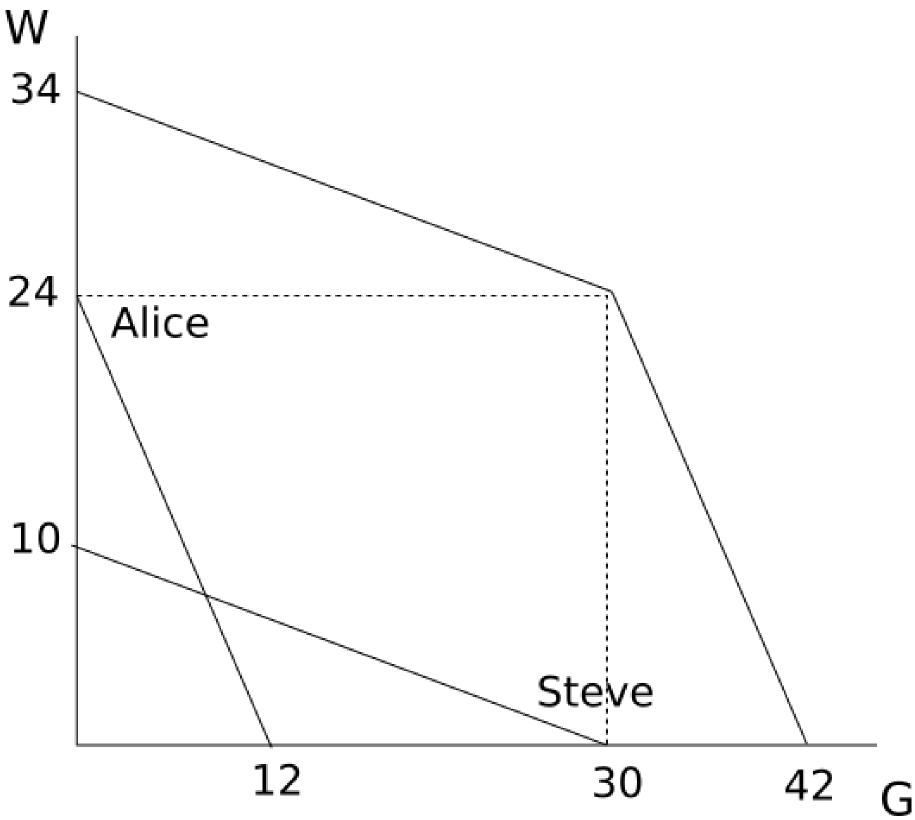
Plotting all of this results in the following:



Putting it all together to get an algebraic expression, we have
$W = 30 – (1/3)G$ for $0 \leq G \leq 18$ and $W = 60 – 2G$ for $18 \leq G \leq 30$

f) Plot and give an expression for the joint PPF of Alice and Steve.

Solution: If both only produce widgets, they can produce at most 34 widgets. From there, Steve will produce the first gadgets since he has the comparative advantage in the production of gadgets. Only Steve will produce gadgets until Steve uses all his resources on gadget production. At this point, (24,30), both are producing only the good for which they have comparative advantage. To produce more gadgets, Alice will have to start producing gadgets. If they both only produce gadgets, they can produce 42 gadgets. Plotting this information, we have



Finding the equation in the usual way, we have
W = 34 – 1/3 G for $0 \le G \le 30$ and W = 84 – 2G for $30 \le G \le 42$

6) Sally lives alone on her own little separate island. She has 10 hours a day to either catch fish or gather coconuts. If she spends her time catching fish, she can catch 1 fish per hour. If she instead spends her time gathering coconuts, she can gather 4 coconuts per hour until she has gathered 8 coconuts. After that, only coconuts high in the trees remain, and she can only gather 1 coconut every 4 hours.

a) Consider the following production bundles for Sally:

i) 5 coconuts and 9 fish
ii) 8 coconuts and 6 fish
iii) 9 coconuts and 4 fish

Label each as either feasible or infeasible, and, if feasible, efficient or inefficient. Justify your solution by showing how much time it would take for Sally to produce each bundle.

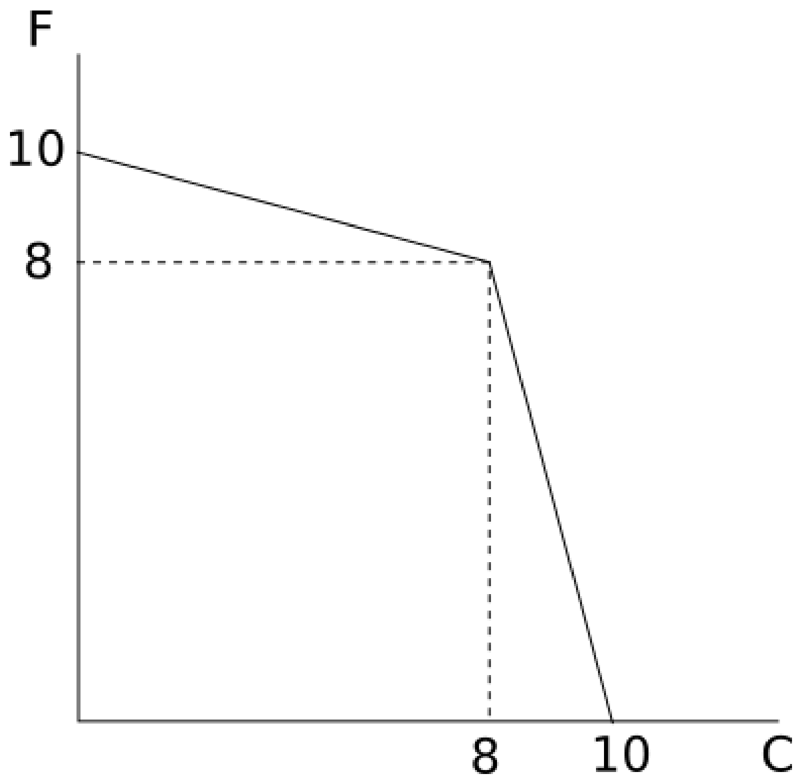
Solution:

Bundle (i) requires 1.25 hours of coconut gathering and 9 hours of catching fish. Thus, this bundle would take 10.25 hours and is infeasible.
Bundle (ii) requires 2 hours of coconut gathering and 6 hours of catching fish. Thus, this is feasible, but inefficient since it takes strictly less than 10 hours.

Bundle (iii) requires 6 hours of coconut gathering (2 hours for the first 8 plus four hours for the last one), and 4 hours of gathering fish. Thus, this is feasible and efficient since it takes precisely 10 hours total.

b) With coconuts (C) on the horizontal axis and fish (F) on the vertical axis, plot Sally's production possibility frontier and then give an algebraic expression for Sally's PPF.

Solution:



Finding the expression in the usual way we have

$F = 10 - (1/4)\,C$ for $0 \leq C \leq 8$ and $F = 40 - 4\,C$ for $8 \leq C \leq 10$

c) Suppose if Sally goes to the mainland, she can buy or sell as many coconuts as she wants at a price of 2 fish per coconut. Given this trade possibility, how many coconuts and fish will she produce?

Solution: Sally must be producing 8 of each. To see why this must be the case, Suppose Sally were producing strictly more than 8 coconuts. Suppose she were to instead produce 1 fewer coconut. If she did so, she could produce 4 more fish, which she could sell for 2 coconuts. Thus, if she wanted more than 8 coconuts, she would do better to buy them rather than produce them herself. Similar reasoning shows that she also will not produce more than 8 fish.