Economics 101
Spring 2017
Answers to Homework #1
Due Thursday, Feburary 9, 2017

Directions:

- The homework will be collected in a box **before** the large lecture.
- Please place <u>your name</u>, <u>TA name</u> and <u>section number</u> on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

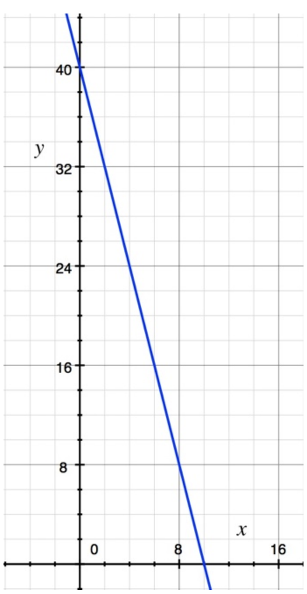
Part I: Math Review Questions
Remember to show all your work. Also remember that calculators are not permitted during exams, so you should try these questions by hand.

1. Equations for lines
   a. Consider a line that passes through the points (x, y) = (10, 0) and (1, 36). Give the equation for this line in y-intercept form and then graph this line.

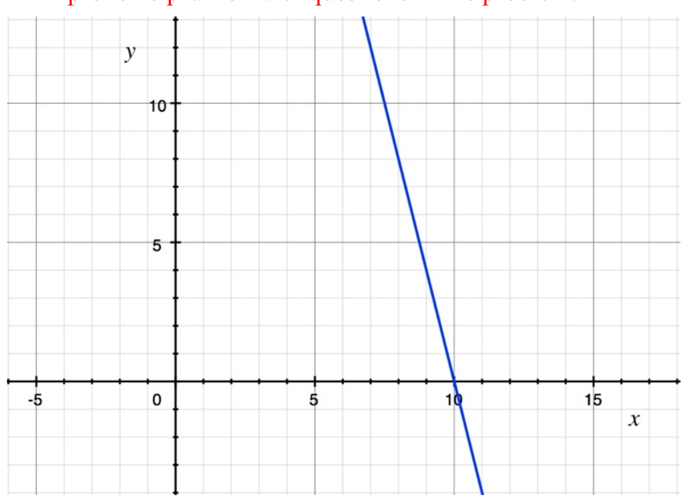
The slope (rise/run) of the line is -36/9 = -4. Solve for the y-intercept using the slope and either of the points.

$$0 = -4(10) + b$$
$$b = 40$$

So we have the equation: $y = 40 - 4x$.



This second graph below is drawn to a scale that will prove helpful for later questions in the problem.



b. Consider a line with slope equal to 1/2. This line passes through the point (x, y) = (2, 5). Give the equation for this line and then graph this line in the same graph you used in (a).

The slope is given as 1/2. The slope-intercept form of the equation is:

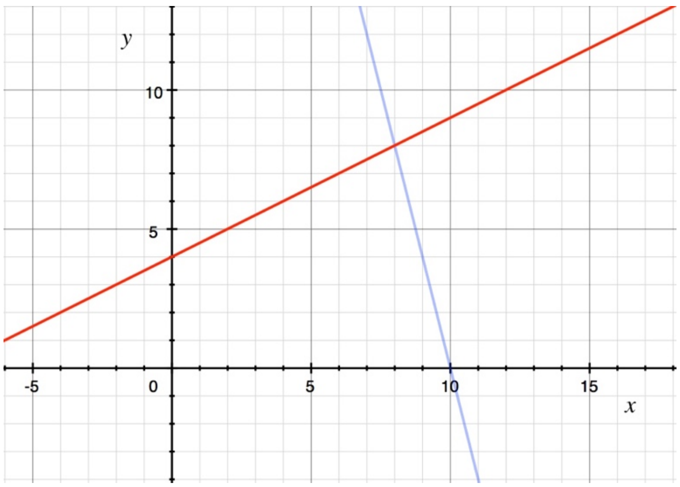
$y = (1/2)\,x + b.$

Using the point (2, 5),

$5 = (1/2)(2) + b$

$b = 4$

So we have equation $y = (1/2)\,x + 4$.



c. Find the point, (x, y), where the two lines you found in (a) and (b) intersect.

Equate $y = 40 - 4x$ and $y = (1/2)\,x + 4$, and then solve for the value of x.

$40 – 4x = (1/2)x + 4$

$(1/2)x + 4x = 40 – 4$

$(9/2)x = 36$

$x = 8$

Use $x = 8$ to find the value of y using one of the equations:

$y = 40 – (4)(8) = 8$

The point where these two lines intersect is given as (x, y) = (8, 8).

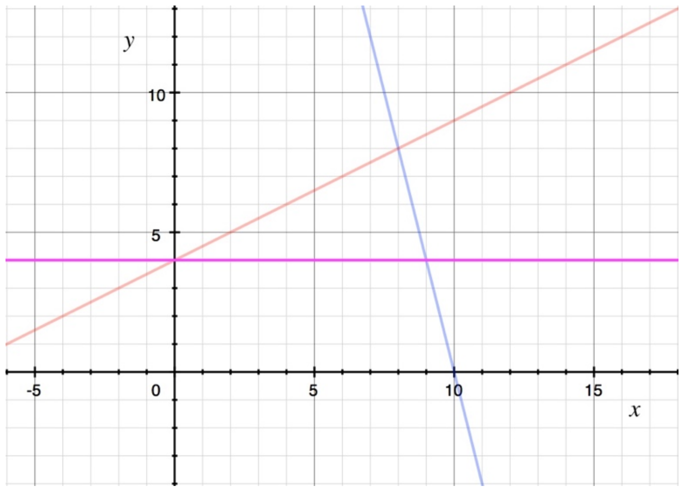
d. Consider a third line where the y value does not change as the x value changes. You also know that the y-intercept of this third line is 4. Write the equation for this third line in slope-intercept form and then graph this line in your graph.

The y value does not change as the x value changes, so this implies that the slope of this third line is 0.

$y = 0x + b = b$

We also know that the y-intercept is 4. We can write the equation for this third line as:

$y = 4$



2. Consider the line described by the equation y = 30 + 4x.
   a. What are the x-intercept and y-intercept of the line? What is the slope of this line?

   The x-intercept is found by plugging in 0 for y and solving for x.

   y = 0 = 30 + 4x

   -4x = 30

   x = -7.5

   The y-intercept is recognized as the constant on the right hand side of the original equation, 30. Verify by plugging in x = 0. The slope is recognized as the number in place of m in y = mx + b format (slope-intercept form). So the slope is 4.

   b. Does the line in (a) intersect with the line described by the equation 12x = 90 - 3y? If these two lines intersect, provide the coordinates (x, y) for where they intersect.

   Converting the second equation to slope-intercept form, we have y = 30 - 4x. We can add the two equations,

   y = 30 + 4x

   <u>+ y = 30 - 4x</u>

   2y = 60 then y = 30 and x = 0. So the two lines do intersect at the point (x, y) = (0, 30).

   Alternatively, you can set the two equations equal to one another and then solve for y:

   30 – 4x = 30 + 4x

   8x = 0

   x = 0

   And, if x = 0 then using either equation we can solve for the y value:

   y = 30 – 4xy

   y = 30 – 4(0) = 30

   Thus, the point (x, y) = (0, 30) is where these two lines intersect with one another.

   c. Shift the original line (y = 30 + 4x) downward by 5 units. What are the x-intercept and y-intercept of this new line? What is the slope of this new line? Does this line intersect the line with equation 12x = 90 - 3y? Where does the new line intersect with the equation 12x = 90 – 3y?

The equation for the shifted line is $y = (30 - 5) + 4x = 25 + 4x$. Since the line is shifting down we are moving the line vertically and therefore we want to use the y-intercept form of the original equation to alter the value of the y-intercept. The new y-intercept will be 5 units less than the initial equation's y-intercept.

The x-intercept is found by plugging in 0 for y and solving for x.

$y = 0 = 25 + 4x$

$-4x = 25$

$x = -6.25$

The y-intercept is recognized as the constant on the right hand side of the original equation, 25. Verify by plugging in $x = 0$. The slope does not change by the shift, so it is 4. So the equation for the new line is:

$y = 25 + 4x$.

This line intersects with the line with equation 12x = 90 - 3y.

Converting the second equation to slope-intercept form, we have $y = 30 - 4x$. We can add the two equations,

$$\begin{array}{r} y = 25 + 4x \\ \underline{+\ y = 30 - 4x} \\ 2y = 55 \end{array}$$

then $y = 55/2 = 27.5$ and $x = 5/8 = 0.625$. So the two lines intersect at the point $(x, y) = (5/8, 55/2)$.

Alternatively, you can set the two equations equal to one another and then solve for y:

$30 – 4x = 25 + 4x$

$8x = 5$

$x = 5/8$

And, if $x = 5/8$ then using either equation we can solve for the y value:

$y = 30 – 4x$

$y = 30 – 4(5/8) = 27.5 = 55/2$

Thus, the point $(x, y) = (5/8, 55/2)$ is where these two lines intersect with one another.

d. Shift the original line ($y = 30 + 4x$) toward the right by 7.5 units. What are the x-intercept and y-intercept of this new line? What is the slope of this new line? Provide the equation for this new line in slope-intercept form. Does this new line intersect the line with equation $12x = 90 - 3y$? Provide the coordinates (x, y) of this point of intersection.

Since the line is shifting to the right we are moving the line horizontally and therefore we will need to use the x-intercept form of the original equation to alter the value of the x-intercpept. The new x-intercept will be 7.5 units greater than the initial equation's x-intercept. Converting the original equation to x-intercept form, we have $x = (1/4)y - (15/2)$, or $x = (0.25)y - 7.5$, and the equation for the shifted line is $x = (0.25)y - 7.5 + 7.5 = (0.25)y$, or $y = 4x$.

The new line passes through the origin, so both the x-intercept and y-intercept are 0. The slope does not change with the shift, so the slope is still 4.

This new line intersects with the line with the equation $12x = 90 - 3y$.

Converting the second equation to slope-intercept form, we have $y = 30 - 4x$. We can add the two equations,

$$\begin{array}{r} y = 4x \\ \underline{+\ y = 30 - 4x} \\ 2y = 30 \end{array}$$

then $y = 15$ and $x = 15/4 = 3.75$. So they have an intersection at $(x, y) = (3.75, 15)$.

Alternatively, you can set the two equations equal to one another and then solve for y:

$30 - 4x = 4x$

$8x = 30$

$x = 3.75$

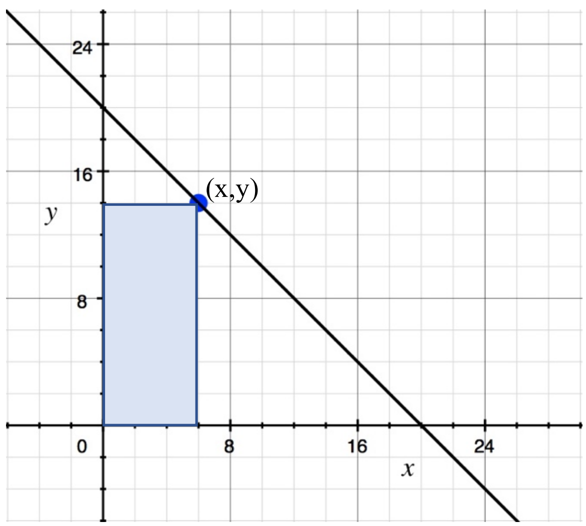
And, if $x = 3.75$ then using either equation we can solve for the y value:

$y = 30 - 4x$

$y = 30 - 4(3.75) = 15$

Thus, the point (x, y) = (3.75, 15) is where these two lines intersect with one another.

3. Consider a downward sloping line with the equation $y = 20 - x$. Consider only points in the first quadrant: that is consider only positive values of x and y. Find the point that sits on this line and provides the maximum area for a specific point located on this line. What is the maximum area of the rectangle bounded by the x-axis and the y-axis, (x, y) where both x and y are greater than 0?



The area of the rectangle bounded by x-axis, y-axis, x, and y is (x*y), and we can rewrite this as x*(20-x) since y = 20 - x.

Area = $x*y = x*(20-x) = 20x - x^2$

But what point (x, y) that lies in the first quadrant and sits on the line gives the maximum area? We could do this with calculus (and if you have studied calculus do it this way but then come up with a non-calculus method to find the solution), but let's instead try to reason our way to the solution. Start with the point (0, 20) which is the y-intercept for the line: the area under this point is 0 (0*20 = 0). Similarly, the x-intercept provides an area equal to 0. So it appears that we need to move down the line from the y-intercept and up the line from the x-intercept. So, let's try two points that sit on the line: for example, (x, y) = (2, 18) and (18, 2).

Area under the line at the point (2, 18) = 36

Area under the line at the point (18, 2) = 36

So, okay this is looking good moving down the line from the y-intercept increased the area as did moving up the line from the x-intercept. So let's try it again: now for the two points (x, y) = (5, 15) and (15, 5).

Area under the line at the point (5, 15) = 75

Area under the line at the point (15, 5) = 75

So, this is still working….but, can you see the logic of where this should lead us to for the single point that maximizes the area under this line in the first quadrant? I am hoping that you are thinking about the line's midpoint between its y-intercept and its x-intercept. This would be the point (x, y) = (10, 10). The area under this point is equal to 100. And, that is where the area under this line and in the first quadrant is maximized.

4. The total number of point on Midterm 1 is 110 points, and Kevin scored an 80 out of 110 points on the first midterm.
    a. What is Kevin's Midterm 1 score as a percent? Show your work and round your answer to the nearest hundredth.

       Midterm 1 score in % = (80/110) * 100 = 72.73 (%)

    b. What is the equivalent score for him on Midterm 2 if the full score on Midterm 2 is 137 points? Give his Midterm two score based on 137 points. Show your work and round your answer to the nearest hundredth.

       We can set an equation:

       $80/110 = x/137$

       $x = 99.64$

       A Midterm 2 score of 99.64 out of 137 points is equivalent to a Midterm 1 score of 80 out of 110 points.

    c. Assume that the full score on the Final Exam is 137 points. Kevin studied harder for his Final Exam and his score on the Final increased by 10% from his score on Midterm 2. What is his Final Exam score on a 137 point scale?

       His Final Exam score was 10% greater than his Midterm 2 score. Let y be his Final Exam score, then we can set use the simple percentage change equation to do our calculation:

       Percentage Change = [(new value – initial value)/(initial value)]*(100%)

       10% = [(Final Exam score – Midterm 2 Score)/(Midterm 2 Score)]*(100%)

       $10\% = (y – 99.64)/99.64 * 100\%$

       $1/10 = (y – 99.64)/99.64$

       $99.64 = (y – 99.64)(10)$

       $9.964 + 99.64 = y$

       $y = 109.61$

       Kevin's Final Exam score is 109.61 points out of 137 possible points.

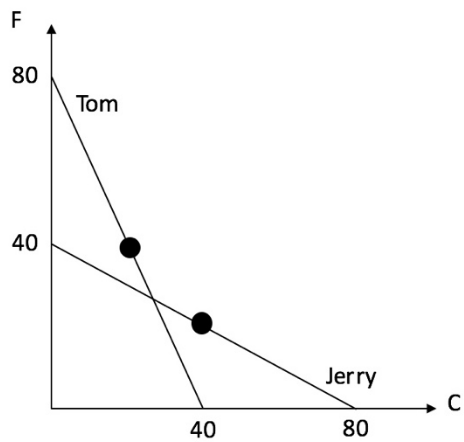
Part II: Opportunity Cost, Absolute Advantage, Comparative Advantage, Production Possibility Frontier

5. Tom and Jerry produce fish and cheese. Tom can produce 10 fish or 5 pound of cheese in an hour, and Jerry can produce 5 fish or 10 pounds of cheese in an hour. Suppose that both Tom and Jerry have 8 hours available to work.
    a. Who has the absolute advantage in producing fish? Who has the absolute advantage in producing cheese?

    Tom has the absolute advantage in producing fish, and Jerry has the absolute advantage in producing cheese. We can see this by comparing the maximum amount of fish the two can produce if they spend all of their time producing fish: Jerry if he works for 8 hours producing fish produces 40 fish while Tom if he works for 8 hours producing fish produces 80 fish. Tom can absolutely produce more fish than Jerry if they have the same resources. If they spend all of their time producing cheese Jerry and produce 80 pounds of cheese while Tom can produce 40 pounds of cheese: Jerry can absolutely produce more cheese than Tom if they have the same resources.

b. Both Tom and Jerry spend half of their available work time producing each good. How many fish and how many pounds of cheese does Tom produce? How many fish and how many pounds of cheese does Jerry produce? Graph their production possibility frontier (PPF) curves with F (fish) and C (cheese) on the vertical and horizontal axes, respectively. Mark their current levels of production on your graph.

Tom produces 40 fish and 20 pounds of cheese, and Jerry produces 20 fish and 40 pounds of cheese.



c. What is Tom's opportunity cost of producing one more fish (in terms of pounds of cheese)? What is Tom's opportunity cost of producing one more pound of cheese (in terms of fish)? What is Jerry's opportunity cost of producing one more fish (in terms of pounds of cheese)? What is Jerry's opportunity cost of producing one more pound of cheese (in terms of fish)? Fill out the following table.

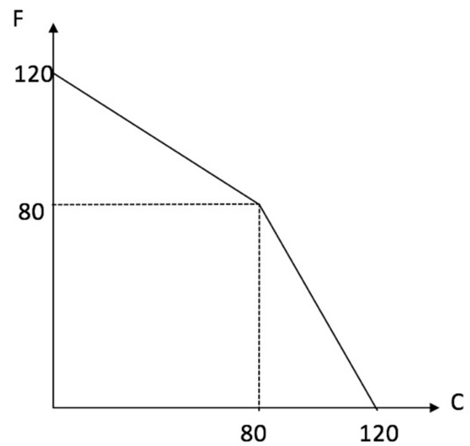
	Maximum Fish Production per Hour	Maximum Cheese Production per Hour	Opportunity Cost of Producing One Fish (in Terms of Pounds of Cheese)	Opportunity Cost of Producing One Pound of Cheese (in Terms of Fish)
Tom	10 (fish)	5 (pounds of cheese)	(pounds of cheese)	(fish)
Jerry	5 (fish)	10 (pounds of cheese)	(pounds of cheese)	(fish)

	Maximum Fish Production per Hour	Maximum Cheese Production per Hour	Opportunity Cost of Producing One Fish (in Terms of Pounds of Cheese)	Opportunity Cost of Producing One Pound of Cheese (in Terms of Fish)
Tom	10 (fish)	5 (pounds of cheese)	1/2 (pounds of cheese)	2 (fish)
Jerry	5 (fish)	10 (pounds of cheese)	2 (pounds of cheese)	1/2 (fish)

d. Who has the comparative advantage in producing fish? Who has the comparative advantage in producing cheese?

Tom has the comparative advantage in producing fish, and Jerry has the comparative advantage in producing cheese. We can see this since the opportunity cost of producing one pound of cheese for Tom is 2 fish and for Jerry the opportunity cost of producing one pound of cheese is 1/2 fish: Jerry can produce cheese more cheaply than can Tom. The opportunity cost of producing one fish is 1/2 pound of cheese for Tom and the opportunity cost of producing one fish is 2 pounds of cheese for Jerry: Tom can produce fish more cheaply than can Jerry.

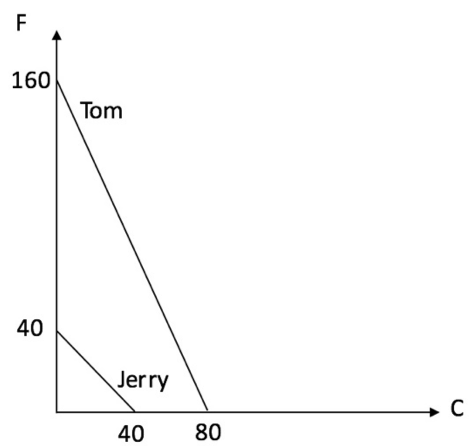
e. Graph the joint PPF for Tom and Jerry. Measure Fish on the vertical axis and Cheese on the horizontal axis. Make sure you identify any "kink points" in your graph.



6. Now assume that Tom can produce 20 fish or 10 pounds of cheese in an hour, and Jerry can produce 5 fish or 5 pounds of cheese in an hour. Suppose that both Tom and Jerry have 8 hours to work.

a. Who has the absolute advantage in producing fish? Who has the absolute advantage in producing cheese? Graph Tom and Jerry's PPFs with F (fish) and C (cheese) on the vertical and horizontal axes, respectively.

Tom has the absolute advantage in producing fish and cheese since with the same amount of resources he can absolutely produce more fish and more cheese than can Jerry.



b. What is Tom's opportunity cost of producing one more fish (in terms of pounds of cheese)? What is Tom's opportunity cost of producing one more pound of cheese (in terms of fish)? What is Jerry's opportunity cost of producing one more fish (in terms of pounds of cheese)? What is Jerry's opportunity cost of producing one more pound of cheese (in terms of fish)? Fill out the following table.

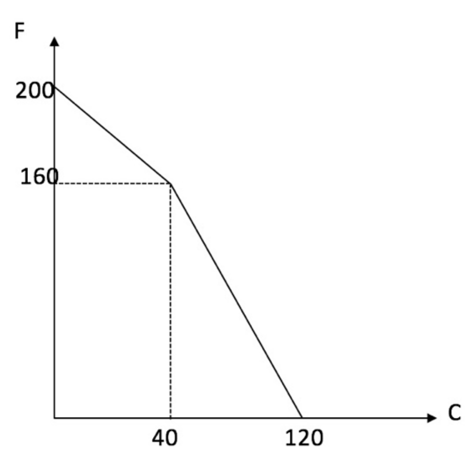
	Maximum Fish Production per Hour	Maximum Cheese Production per Hour	Opportunity Cost of Producing Fish (in Terms of Pounds of Cheese)	Opportunity Cost of Producing A Pound of Cheese (in Terms of Fish)
Tom	20 (fish)	10 (pounds of cheese)	(pounds of cheese)	(fish)
Jerry	5 (fish)	5 (pounds of cheese)	(pounds of cheese)	(fish)

	Maximum Fish Production per Hour	Maximum Cheese Production per Hour	Opportunity Cost of Producing Fish (in Terms of Pounds of Cheese)	Opportunity Cost of Producing A Pound of Cheese (in Terms of Fish)
Tom	20 (fish)	10 (pounds of cheese)	1/2 (pounds of cheese)	2 (fish)
Jerry	5 (fish)	5 (pounds of cheese)	1 (pounds of cheese)	1 (fish)

c. Who has the comparative advantage in producing fish? Who has the comparative advantage in producing cheese?

Tom has the comparative advantage in producing fish, and Jerry has the comparative advantage in producing cheese. Tom's opportunity cost of producing 1 fish is 1/2 pound of cheese while Jerry's opportunity cost of producing 1 fish is one pound of cheese: Tom has the comparative advantage in producing fish. Jerry's opportunity cost of producing 1 pound of cheese is 1 fish while Tom's opportunity cost of producing 1 pound of cheese is 2 fish: Jerry has the comparative advantage in producing cheese.

d. Graph the joint PPF. Label all intercepts as well as the coordinates for any "kink point".



e. What is the minimum price for one fish (in terms of pounds of cheese) in which trade would take place?

1/2 pound of cheese
If the price of the one fish is less than 1/2 pound of cheese, then Tom would not sell the one fish to Jerry.

f. What is the maximum price for one fish (in terms of pounds of cheese) in which trade would take place?

1 pound of cheese
If the price of the one fish is greater than 1 pound of cheese, then Jerry would not buy the one fish from Tom.

g. What is the range of trading prices for one fish?

1/2 pound of cheese $\leq$ Trading Price for one Fish $\leq$ 1 pounds of cheese
Trading price of one fish has to be greater than or equal to 1/2 pound of cheese, and it has to be less than or equal to 1 pound of cheese.