Economics 101
Spring 2015
Answers to Homework #1
Due Thursday, February 5, 2015

Directions: The homework will be collected in a box **before** the lecture. Please place your name on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. For each of the following pairs of coordinates (X, Y) write the equation for the straight line that contains both points. Show your work. Show how you calculated the slope of the line.
a. (X, Y) = (10, 2) and (X, Y) = (8, 4)
b. (X, Y) = (8, 5) and (X, Y) = (5, 5)
c. (X, Y) = (8, 5) and (X, Y) = (10, 10)
d. (X, Y) = (1/2, ¼) and (X, Y) = (1/5, 1/10) [Note: keep this as fractions and do not use a calculator: this problem is designed to help you review some basic "rules of fractions" that you will need over the course of this semester.] Show all your work on this one: no calculator shortcuts-your TA wants to see how you did the work!
e. (X, Y) = (0.2, 3.5) and (X, Y) = (2.7, 4.2) [Note: keep this in decimals and do not use a calculator: this problem is designed to help you review some basic "rules of decimals" that you will need over the course of this semester.] Show all your work on this one: no calculator shortcuts-your TA wants to see how you did the work! You will need to carry your work to at least three places past the decimal.
f. (X, Y) = (1/4, ¾) and (X, Y) = (0.5, 1.5) [Note: you will need to either convert all numbers to fractions or all numbers to decimals to answer this problem.] Show all your work on this one: no calculator shortcuts-your TA wants to see how you did the work!

Answer:
a. Slope = rise/run = (2 – 4)/(10 – 8) = -1
Y = b – X
2 = b – 10
12 = b
Equation for line containing these two points: Y = 12 – X

b. Slope = rise/run = (5 – 5)/(8 – 5) = 0/3 = 0

$Y = b - (0)X$

$Y = b$

$5 = b$

Equation for line containing these two points: $Y = 5$

c. Slope = rise/run = $(5 - 10)/(8 - 10) = (-5)/(-2) = 5/2$

$Y = b + (5/2)X$

$10 = b + (5/2)(10)$

$-15 = b$

Equation for line containing these two points: $Y = (5/2)Q - 15$

d. Slope = rise/run = $[(1/4) - (1/10)]/[(1/2) - (1/5)] = (3/20)/(3/10) = (3/20)(10/3) = (1/2)$

$Y = b + (1/2)X$

$1/4 = b + (1/2)(1/2)$

$b = 0$

Equation for line containing these two points: $Y = (1/2)X$

e. Slope = rise/run = $[(3.5 - 4.2)/(0.2 - 2.7)] = (-0.7)/(-2.5) = .28$

$Y = b + (.28)X$

$3.5 = b + (.28)(.2)$

$3.5 = b + 0.056$

$b = 3.444$

Equation for line containing these two points: $Y = (.28)X + 3.444$

f. Method One: doing the problem in decimals:

The two points are $(X, Y) = (0.25, 0.75)$ and $(0.5, 1.5)$

Slope = rise/run = $[(0.75 - 1.5)/(0.25 - 0.5) = (-.75)/(-.25) = 3$

$Y = 3X + b$

$0.75 = 3(0.25) + b$

$b = 0$

Equation for line containing these two points: $Y = 3X$

Method Two: doing the problem in fractions:

The two points are $(X, Y) = (1/4, 3/4)$ and $(1/2, 3/2)$

Slope = rise/run = $[(3/4) - (3/2)]/[(1/4) - (1/2)] = (-3/4)/(-1/4) = (3/4)*(4/1) = 3$

$Y = 3X + b$

$3/2 = 3(1/2) + b$

$b = 0$

Equation for line containing these two points: $Y = 3X$

2. Let's do a bit more math practice:

a. Suppose you are told that there is a straight line with slope of -4. This line includes the point (.05, 10.2). Write the equation for this line in x-intercept form. Note: do not use a calculator on this problem-we want you to practice with some decimals for a bit!

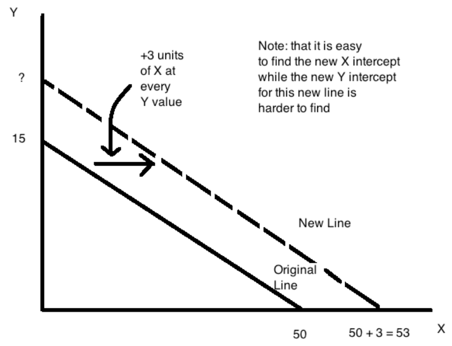
b. Suppose you are given an equation for a line: $Y = 15 - 0.3X$ and then are told that at every Y value the X value has increased by 3 units. First draw a graph of the original equation (showing just the first quadrant values) and then sketch the new line in this same graph. Measure the Y value on the vertical axis and the X value on the horizontal axis. (Here the intent is to get you to "visualize" the two lines and their relationship to one another. You might find it fun to do this sketch using a computer drawing program like "Paint" or "Paintbrush" particularly if you have never used this kind of program.) What is the equation for this new line given this information?

c. Suppose you are given an equation for a line: $X = 50 – (10/3)Y$ and then you are told that at every Y value the X value has increased by 3 units. What is the equation for this new line written in X-intercept form? Again, you might find it helpful to draw a sketch of the situation before you work on writing the new equation.

d. Suppose you are given an equation for a line: $Y = 100 – 0.5X$ and then you are told that at every X value the Y value has decreased by 10 units. What is the equation for the new line written in Y-intercept form? What is the equation for the new line written in X-intercept form?

Answer:
a. Let's start by writing this equation in Y-intercept form:
$Y = b + mX$
$Y = b + (-4)X$
Then, find the value of b by substituting into this equation the coordinate values you were given for the point that you know is on the line.
$10.2 = b + (-4)(.05)$
$10.2 = b – 0.2$
$10.4 = b$
Equation for the line in Y-intercept form is: $Y = 10.4 – 4X$
Now, let's rewrite the equation in X-intercept form:
$4X = 10.4 – Y$
$X = (10.4/4) – (1/4)Y$
$X = 2.6 – (0.25)Y$

b. Here's the sketch:



Now, for the equation for the new line: first, we can note that the lines are parallel to one another and that tells us that the two lines have the same slope. So, we can start by writing our new equation as:

Y = b - 0.3X

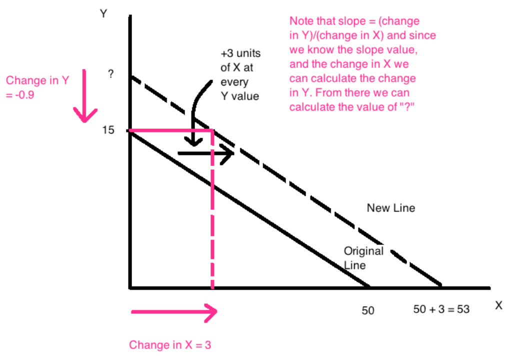
Now, we need to find the value of b: to do this we need to find one point that we know "sits" on the new line. We know from our sketch that the point (X, Y) = (53, 0) is on the new line. Thus,

0 = b - (0.3)(53)

b = 15.9

Equation for the new line: Y = 15.9 – 0.3X

Note: the Y-intercept did not increase by 3 units: since the X-intercept increased by 3 units and the slope is -0.3 for this line this tells us that the Y value must have increased by 0.9 to maintain that relationship….here's an illustration to help you see this-



c. This is just the equation we had from the last problem (but's let just "prove" to you):

X = 50 – (10/3)Y can be rearranged as follows:

(10/3)Y = 50 – X

Y = 50(3/10) – (3/10)X

Y = 15 – 0.3X

So, the new equation should be the same as the equation we found in (c), except we need to write the equation in X-intercept form:

New equation: X = 53 – (10/3)Y

Note: that if the line shifts horizontally and we have the original equation in X-intercept form, then we need to only change the constant in the equation (the X-intercept) to reflect the shift and we have the new equation.

d. The new equation in Y-intercept form is easy to write since the new line is parallel to the initial line and simply intersects the Y-axis at a value that is 10 units smaller than the original Y-intercept. We can write this equation as: $Y = 90 - 0.5X$. To write the new equation in X-intercept form we need to rearrange the equation so that we are solving for X (so that X is alone on the left hand side of the equation):

$0.5X = 90 – Y$

$X = 180 – 2Y$

3. In this set of exercises we will combine working with equations and graphs.

a. Suppose you are given the following two equations:

$Y = 10 – (1/4)X$

$Y = 1 + (1/8)X$

Draw a graph (first quadrant only) illustrating these two lines. In your graph identify the values of any Y or X intercepts. Then, solve for the coordinate values (X, Y) where these two lines intersect. Show your work and keep all fractions as fractions as you work through the problem. Do not use a calculator or a graphing calculator: we are working on some very important skills that you will need as the course progresses!

b. Suppose you are given the following two equations:

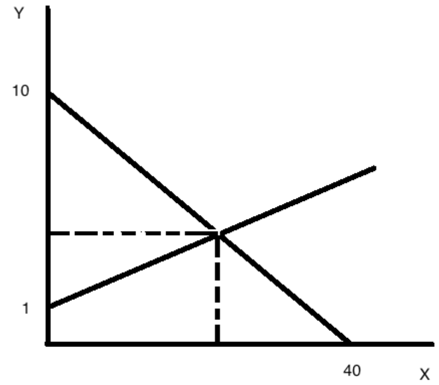
Line 1: $Y = 10 – (1/4)X$

Line 2: $Y = 1 + (1/8)X$

But, you are also told that Line 2 has shifted to the left so that at every Y value the X value is now 16 units smaller. Write the equation for this new Line 3 and then solve for the coordinate values (X, Y) where Line 1 intersects Line 3. Show your work! Keep your answer as an improper fraction.

Answer:

a.



To find where these two lines intersect we need to set the two equations equal to one another:

$10 - (1/4)X = 1 + (1/8)X$

$9 = (1/4)X + (1/8)X$

$9 = (2/8)X + (1/8)X$ [Note: here we are getting a common denominator for the fractions so that we can add them together!]

$9 = (3/8)X$

$(8/3)(9) = X$

$X = 24$ [Note: this value seems reasonable since our graph shows that the intersection of the line occurs somewhere in the domain of X values from 0 to 40.]

To find the Y value use either equation:

First equation: $Y = 10 - (1/4)X = 10 - (1/4)(24) = 10 - 6 = 4$

Second equation: $Y = 1 + (1/8)X = 1 + (1/8)(24) = 1 + 3 = 4$

Coordinate values for where the two lines intersect: $(X, Y) = (24, 4)$

b. Let's start by writing the equation for Line 3. I am going to give you two methods and you should think about which is easier for you. Here's one based on using Line 2 in slope-intercept form and a new point that you know "sits" on Line 3. Since $(X, Y) = (24, 4)$ was on Line 2 we can find a new point that is on Line 3 by subtracting 16 units from the X value: thus, the point $(X, Y) = (8, 4)$ is on Line 3. So, using the equation for Line 2 we have:

$Y = 1 + (1/8)X$

We know that Line 2 and Line 3 are parallel: so same slope. We also know that the y-intercepts for the two lines are different. So,

Equation for Line 3: $Y = b + (1/8)X$ and to find the value of b use our new point (8, 4). Thus, $4 = b + (1/8)(8)$ or $b = 3$. The equation for Line 3 is $Y = 3 + (1/8)X$.

Alternatively, we can write Line 2 in X-intercept form: $X = 8Y - 8$ and then realize that Line 3 is just shifting horizontally along the X-axis by 16 units to the left of the Line 2. Thus, the new X-intercept must be 16 units smaller than the X-intercept of Line 2. This implies that the equation of Line 3 in X-intercept form is $X = 8Y - 24$. Rearranging this into Y-intercept form we have $Y = (1/8)X + 3$. Note that either method provides you with the same equation: this is a sign that you are on the right path.

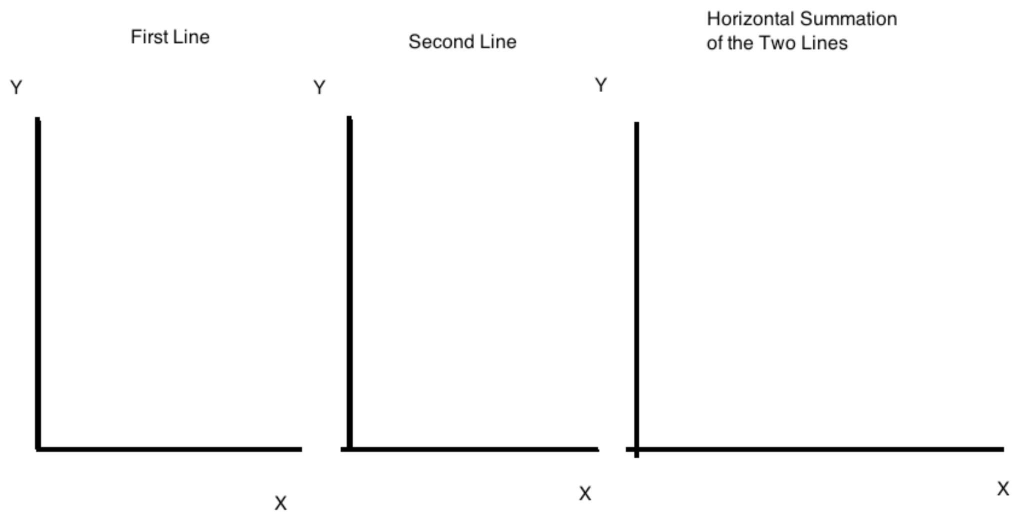
Now, to find the new point of intersection between Line 1 and Line 3 we need to set these two equations equal to one another:
$10 – (1/4)X = 3 + (1/8)X$
$7 = (3/8)X$
$56/3 = X$
$Y = 10 – (1/4)(56/3) = 10 – (56/12) = 10 – (14/3) = (30/3) – (14/3) = 16/3$
Or, $Y = (1/8)(56/3) + 3 = (7/3) + (9/3) = 16/3$
The point of intersection for Line 1 and Line 3 is $(X, Y) = (56/3, 16/3)$

4. Suppose you are given the following two equations:

$Y = 10 – X$ for all values of X that are greater than or equal to 0 and all values of Y that are greater than or equal to 0 (hence, just the first quadrant values)

$Y = 8 – (2/5)X$ for all values of X that are greater than or equal to 0 and all values of Y that are greater than or equal to 0 (hence, just the first quadrant values)

a. Draw three graphs of the first quadrant so that all three graphs are lined up in a row: here's the image we are looking for:

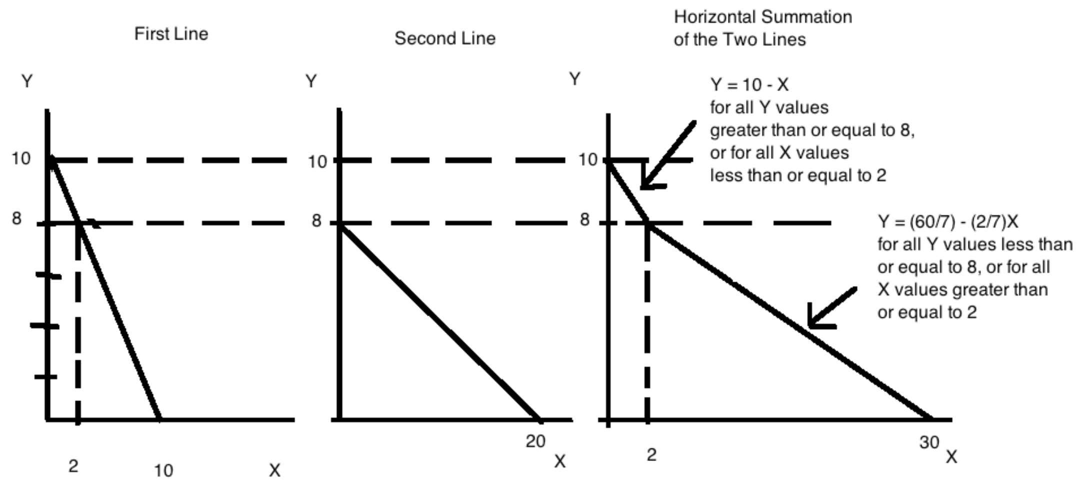


In this problem it will be important that you draw "to scale"! In the first graph (the one to the far left) draw in the first line. In the middle graph draw the second line.

b. Now, we want to draw the image for the third graph. To do this, we are going to add the two lines together horizontally. To add lines together horizontally we will hold the Y value constant and then sum the X values together. So, for instance if the Y value is 2, then you would want to calculate the X value for the first equation (X1) when the Y value is set at 2 and the X value for the second equation (X2) when the Y value is set at 2. Then add these two X values together to get the new coordinate point in the third graph where (X, Y) will be (X1 + X2, 2). Be careful here: you should get two linear segments in the third graph and if you don’t get this-then you are not getting the right answer!

c. Write the two equations for the lines drawn in the third graph. For each equation provide the range of Y values for that equation and the domain of X values for that equation. Show your work.

Answer:
a. and b.



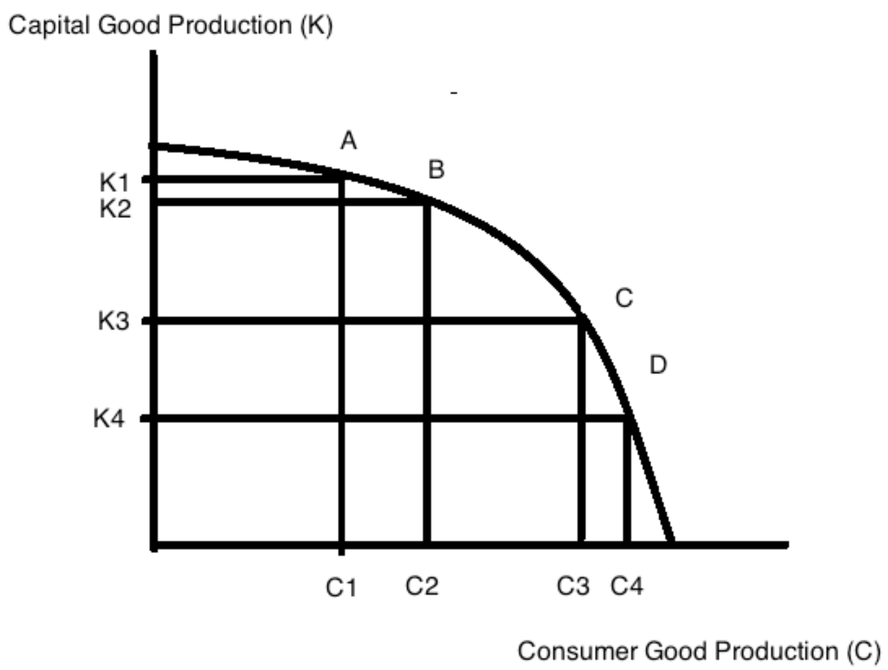
c. The equation for the upper segment of the horizontally added lines in the third graph is easy to write: it is just the equation for line 2. Essentially what is happening in this segment is that when the Y value exceeds 8 then we are only considering the first line. So, the equation for the upper segment is Y = 10 – X for all Y values greater than or equal to 8 or for all X values greater than or equal to zero and less than or equal to 2.

The equation for the lower segment in the third graph is a bit more challenging to write: we can start with our familiar slope intercept form equation:
Y = mX + b and then proceed.
Slope of this segment = rise/run = -8/28 = -2/7
We know two points on this lower segment: (X, Y) = (2, 8) and (30, 0). So, use one of these points to find the b value:
Y = b + (-2/7)X
8 = b = (-2/7)(2)
8 + (4/7) = b
56/7 + 4/7 = b
Equation for lower segment: Y = 60/7 – (2/7)X for all Y values between zero and 8 and all X values between 2 and 30.

5. For each of the following statements determine whether the statement is a positive or a normative statement. Explain your answer.
a. Joe is working hard and by the end of the year his savings from his work should be sufficient to pay for a year of college tuition.
b. Susie is working hard and she ought to get more sleep than she is getting in order to function better in her day-to-day activities.
c. Mark is working hard and he ought to get straight A's given his work effort.
d. Paul is working hard and this work effort seems to be paying off: he has been on the Dean's List the last four semesters.

Answer:
a. Positive: at the end of the year you can verify or not that Joe has saved enough income to pay for a year of college tuition.
b. Normative: this is your opinion and this opinion cannot be verified.
c. Normative: this statement seems initially positive because at the end of the semester you can verify whether he got those straight A's, but there is also a normative element to the statement. I am going with normative since it seems to me that the statement strongly suggests that work effort is all it takes to get those A's and that is a value-ladened statement. Not everyone would agree that work effort is all that matters: for instance, would you agree to have brain surgery from a hard-working surgeon that has only a 10% success rate for the surgery you need?
d. Positive: the work effort can be verified by the student's being on the Dean's List.

6. Suppose you are given the following production possibility frontier (PPF) for the economy of Grantsville. Grantsville produces consumer goods (C) and capital goods (K) from its available resources.



a. Given the above graph, what is the opportunity cost of moving from Point A to Point B? Make sure you identify the type of units that opportunity cost is measured in when you provide your answer.

b. Given the above graph, what is the opportunity cost of moving from Point C to Point A? Make sure you identify the type of units that opportunity cost is measured in when you provide your answer.

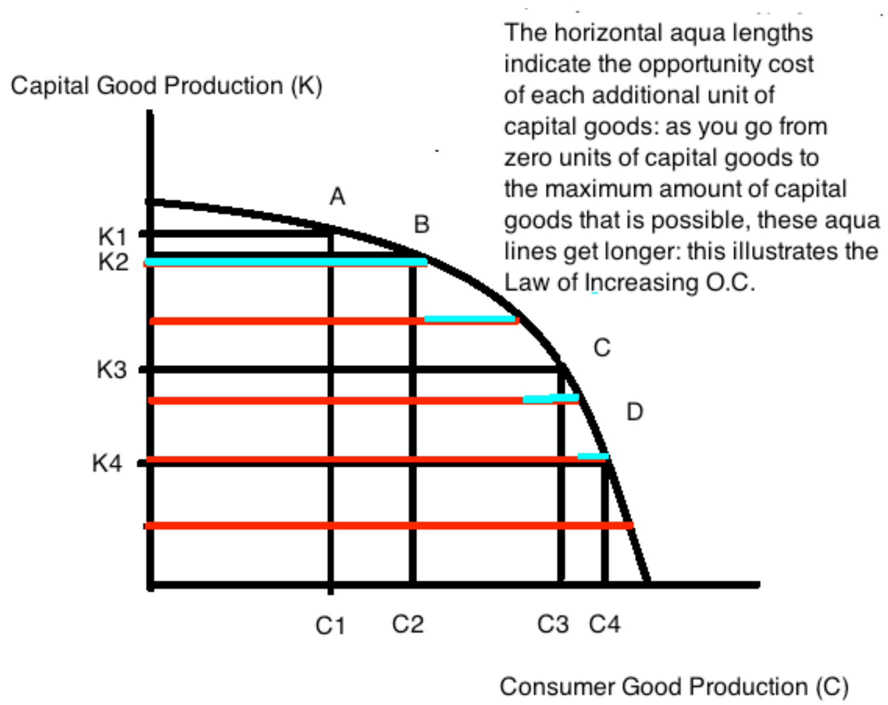
c. Given the above graph, does the Law of Increasing Opportunity Cost hold for capital goods? The Law of Increasing Opportunity Cost states that the opportunity cost of getting each additional unit of the good increases as you get more and more of that good. Explain your answer and provide a graph to illustrate this idea.

Answers:

a. The opportunity cost of going from Point A to Point B is measured by the number of capital goods you are giving up as you acquire more consumer goods: in this case you are giving up (K1 – K2) units of capital goods.

b. The opportunity cost of going from Point C to Point A is measured by the number of consumer goods you are giving up as you acquire more capital goods: in this case you are giving up (C3 – C1) units of consumer goods.

c. Yes, the Law of Increasing Opportunity Cost does hold for capital goods. Let's amend the graph to show what the opportunity cost of an additional unit of capital looks like:



7. Janine is taking Econ 101 and Psych 202 this semester and has figured out that she has ten hours available each week to devote to the study of economics or psychology. The table below provides information about the amount of study time she allots to each subject and the anticipated final grade she will get in each subject given this allocation of study time.

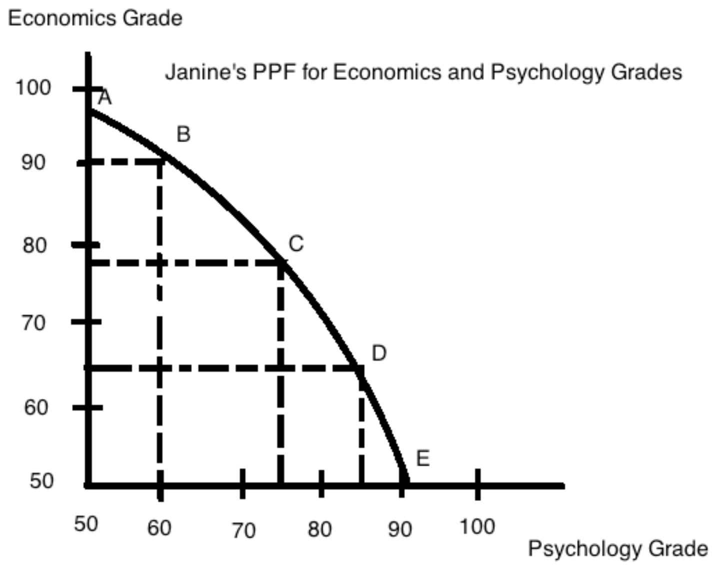
Possible Study Combination	Number of Hours of Study for Economics	Number of Hours of Study for Psychology	Anticipated Final Grade in Economics	Anticipated Final Grade in Psychology
A	10	0	98	50
B	8	2	90	60
C	6	4	78	75
D	2	8	65	85
E	0	10	50	90

a. In a graph plot out Janine's production possibility frontier given the above information. Measure her Economics grade on the vertical axis and her Psychology grade on the horizontal axis. Instead of using (0, 0) as the origin, use (50, 50) as the point of intersection for the two axes: do this since even if she studies zero hours for Economics or Psychology she will get a grade of 50 in the classes: not a bad assumption for many classes where the professor expects a student to be able to get approximately 50% of the material by just coming to class and hearing the lectures. Mark combinations A, B, C, D, and E on your graph and then connect these five combinations to make a PPF for Janine's production of these two goods (the grades in Economics and the Psychology). To make this a more interesting assignment: figure out how to draw the graph using some kind of computer program: for example, Excel (here you will need to enter the data and then convert to a graph), Paint, or Paintbrush (I tend to use Paint or Paintbrush if I am making "sketches" and Excel if I want a "too-scale" drawing). (By doing this with a computer program you will build your skill sets and can add "comfortable with graphic design programs to your resume.)

b. What is Janine's opportunity cost of changing her study habits from 8 hours of Economics study a week to 2 hours of Economics study a week? Be sure to include some kind of unit of measurement for the answer you provide.

c. Given the above information, is there an optimal allocation of study time to these two subject areas? Explain your answer.

Answers:
a.



b. Opportunity cost is measured in terms of what you are giving up: in this case Janine by devoting fewer study hours to economics reduces her grade in economics from a 90 in the class to a 65 in the class: the opportunity cost of choosing this combination of study hours is a decrease of 25 points in her anticipated final grade in economics.

c. No there is no optimal choice of study time. The above combinations provide us with a set of efficient points: each combination tells us the anticipated maximum score in both subject areas that Janine can expect to get if she allocates her study time as given for that combination in the table. But, the table provides no guidance as to the optimal combination: Janine will need to think about her short-term as well as long-term goals in order to determine how best to allocate her scarce resource-study time-between these two subject areas. Clearly, if Janine anticipates pursuing a career direction more closely related to economics it is likely she will choose to devote more hours of study to economics. And, if she anticipates pursuing a career direction more closely related to psychology she will choose to devote more hours of study to psychology.

8. Consider two countries: Country X and Country Y. Both countries produce autos and shoes and the following table provides you with information about the maximum amount of autos and shoes that each country can produce if they only produce one of these goods. Both countries have linear production possibility frontiers in these two goods.

	Maximum Amount of Autos that can be Produced in the given time period	Maximum Amount of Pairs of Shoes that can be Produced in the given time period
Country X	20,000	50,000
Country Y	10,000	20,000

a. Given the above table, which country has the absolute advantage in the production of autos?

b. Given the above table, which country has the absolute advantage in the production of shoes?

c. Draw two graphs: in the first graph represent Country X's PPF measuring autos on the vertical axis and pairs of shoes on the horizontal axis; in the second graph represent Country Y's PPF measuring autos on the vertical axis and pairs of shoes on the horizontal axis. Again, you might do this on a computer graphic program (see the last question for names of some of the readily available programs).

d. What is the opportunity cost of the following (in your answer provide units of measurement):

  i. One pair of shoes for Country X?
  ii. One pair of shoes for Country Y?
  iii. One auto for Country X?
  iv. One auto for Country Y?

e. Which country has the comparative advantage in the production of autos? Explain your answer. Which country has the comparative advantage in the production of shoes? Explain your answer.

f. Draw the joint PPF for these two countries in a single graph, measuring autos on the vertical axis and pairs of shoes on the horizontal axis. Label clearly the coordinates for any "kink points".

g. What is the acceptable range of trading prices for 10,000 pairs of shoes? Provide a number line like the one presented in class to illustrate your answer. Include the directional arrows and the Country Perspective in your number line illustration.

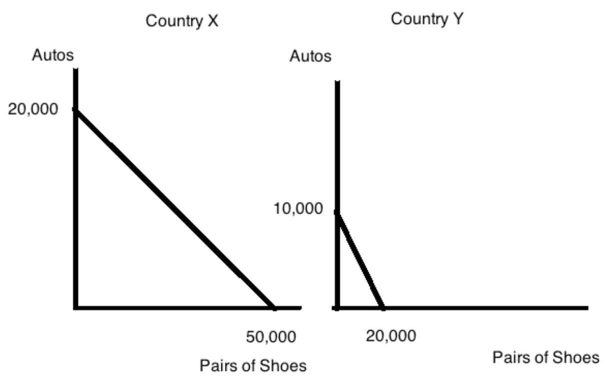
h. What is the acceptable range of trading prices for 5,000 autos? Provide a number line like the one presented in class to illustrate your answer. Include the directional arrows and the Country Perspective in your number line illustration.

Answers:

a. Country X has the absolute advantage in the production of autos since Country X can absolutely produce more autos than can Country Y (20,000 autos versus 10,000 autos.

b. Country X has the absolute advantage in the production of shoes since Country X can absolutely produce more shoes than can Country Y (50,000 pairs of shoes versus 20,000 pairs of shoes).

c.



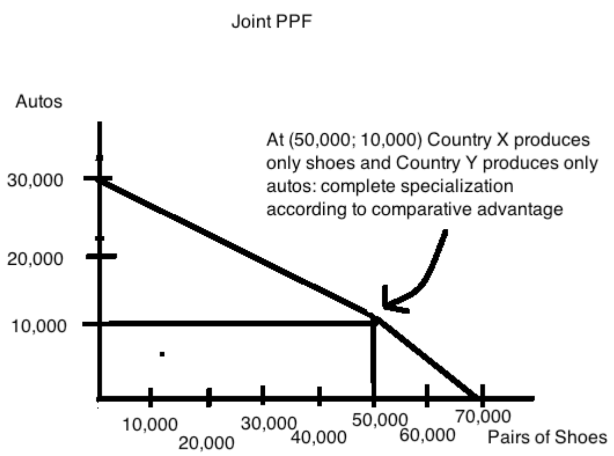
d.

i. The opportunity cost of one pair of shoes is 2/5 auto for Country X (note that this is the absolute value of the slope of the PPF for Country X)
ii. The opportunity cost of one pair of shoes is ½ auto for Country Y (note that this is the absolute value of the slope of the PPF for Country Y)

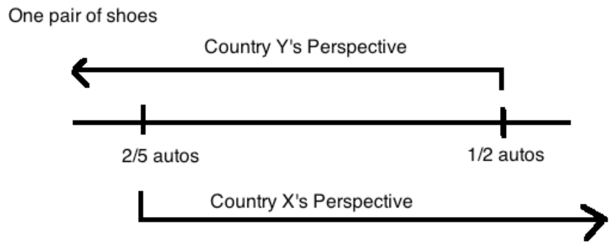
iii. The opportunity cost of one auto is 5/2 pairs of shoes for Country X (note that this is the absolute value of the reciprocal of the slope of the PPF for Country X)
iv. The opportunity cost of one auto is 2 pairs of shoes for Country Y (note that this is the absolute value of the reciprocal of the slope of the PPF for Country Y)

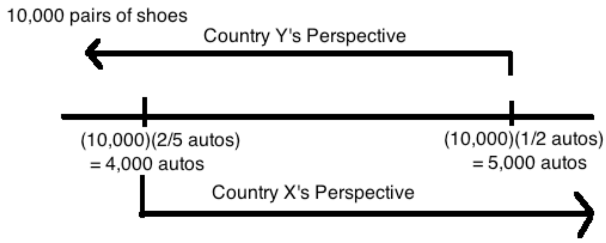
e. Country Y has the comparative advantage in the production of autos since Country Y's opportunity cost of producing an auto is lower than Country X's opportunity cost of producing an auto (2 pairs of shoes versus 2.5 pairs of shoes). Country X has the comparative advantage in the production of shoes since Country X's opportunity cost of producing a pair of shoes is lower than Country Y's opportunity cost of producing a pair of shoes (2/5 auto versus ½ auto).

f.



g. Here is the acceptable range of trading prices for 10,000 pairs of shoes: note that I started by making the number line for the trading prices for 1 pair of shoes and then just grossed this up by multiplying all the numbers by 10,000.





h. Here is the acceptable range of trading prices for 5,000 autos: note that I started by making the number line for the trading prices for 1 auto and then just grossed this up by multiplying all the numbers by 5000.