Economics 101
Fall 2018
Answers to Homework #1
Due Thursday, September 27, 2018

Directions:

- The homework will be collected in a box labeled with your TA's name **before** the lecture.
- Please place **your name, TA name, and section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Please **staple** your homework: we expect you to take care of this prior to coming to the large lecture. You do not need to turn in the homework questions, but your homework should be neat, orderly, and easy for the TAs to see the answers to each question.
- Late homework will not be accepted so make plans ahead of time.
- Show your work. Good luck!

Part I: Math Review

1. For this question use a linear equation that takes the form of y = mx + b.
   a. Write the equation of the straight line that passes through the points (x; y) = (2; -14) and (-4; 19). What is the value of the slope of your equation? What is the value of the y-intercept?

   Slope = m = Δy /Δx = (19 - (-14))/(- 4 - 2) = 33/(-6) = -5.5
   Use one of the points to get the value of y-intercept, b = y – mx
   Let us use the point (2; -14), b = y – mx = -14 – (-5.5)*2 = -3

   Alternatively, we can substitute both points into y = mx + b and then solve the system of linear equations.
   We get the following system of two equations: -14 = 2m + b and 19 = -4m + b.
   From the first one we get b = -14 – 2m.
   Plug it in the second equation: 19 = -4m –14 – 2m = -6m – 14. Hence, m = (19+14)/(-6) = -5.5.
   Plug m = -5.5 in to the first equation and we get b = -14 – (-5.5)*2 = -3.

   Overall, y = -5.5x – 3.
   The value of the slope is m = -5.5, the value of the y-intercept is b = -3

   b. Write the equation of the straight line, line #1, that fits the following description. If we take this straight line, line #1, and shift this line down by 2 units (that is, the y-intercept for line #2 is 2 units less than the y-intercept for the original line) and make this new line (line #2) four times as steep as line #1, then the new line #2 passes through the point (1; 3) and for every 2 unit increase in the X variable, the Y variable increases by 4 units. What is the value of the slope of the initial equation (line #1)? What is the value of the y-intercept for line #1?

Let us start with the equation for line #2. From the given information, we can find the slope of the line #2. Slope = $m = \Delta y / \Delta x = 4/2 = 2$. Then the line passes through the point (1; 3). Hence, the value of y-intercept is $b = y - mx = 3 - 2*1 = 1$.
The modified line is $y = 2x + 1$.

Since the modified line has the slope that is four times as steep as the slope for line #1, in order to get the slope for the original line we need to divide the slope of line #2 by 4. We get $m = 2/4 = 0.5$. In addition, line #2 has y-intercept that is 2 units less that the y-intercept for the line #1, i.e. $b = 1 + 2 = 3$.

Hence, the equation for line #1 is $y = 0.5x + 3$.
The value of the slope is $m = 0.5$ and the value of the y-intercept is $b = 3$.

c. Using the equations that you got from (a) and (b), solve this system of equations for (x, y). Use the equation for line #1 from (b).

The equation from a: $y = -5.5x - 3$
The equation from b: $y = 0.5x + 3$
Set these two equations equal to each other:
$-5.5x - 3 = 0.5x + 3$, so $-6x = 6$. Hence, $x = -1$.
Plug x into one of the equations to find the value for y: $y = 0.5*(-1) + 3 = 2.5$.
$(x, y) = (-1, 2.5)$

d. Now shift the first equation from (a) to the left by 6 units and then solve the modified system of equations for (x', y'). Use the equation for line #1 from (b) and the new line based on this new information and your equation from (a).

When we shift the first equation to the left by 6 units, we get the modified equation:
$y' = -5.5*(x' + 6) - 3 = -5.5x' - 33 - 3 = -5.5x' - 36$.
Then, set the modified equation equal to the equation from (b):
$-5.5x' - 36 = 0.5x' + 3$, so $-6x' = 39$. Hence, $x' = -6.5$.
Plug $x' = -6.5$ into one of the equations to find y': $y' = 0.5*(-6.5) + 3 = -3.25 + 3 = -0.25$.
$(x', y') = (-6.5, -0.25)$

2. Alexander purchases a new car for $15,000, and pays $5,000 as a down payment. He takes out a loan for the remaining $10,000 at 5%, compounded annually.
   a. How much does Alexander owe to the bank at the end of the first, second and third year if he doesn't make any payments to the bank?

   At the end of the first year Alexander owes 10,000 + 10,000*5% = 10,000*1.05 = $10,500
   At the end of the second year Alexander owes 10,500 + 10,500*5% = 10,500*1.05 = $11,025
   At the end of the third year Alexander owes 11,025 + 11,025*5% = 11,025*1.05 = $11,576.25

   b. If Alexander wants to repay his loan as well as the interest that is due on the loan within a year, what does his monthly payment need to be?

   At the end of the first year Alexander owes 10,000 + 10,000*5% = 10,000*1.05 = $10,500
   Thus, in order to repay 10,500 with 12 months, he has to pay 10,500/12 = $875 per month.

   c. If Alexander pays $300 per month, could he repay his loan in two years? Three years?

   If Alexander pays $300 per month, his annual payment is 300*12 = 3,600.
   At the end of the first year Alexander owes (10,000 – 3,600) + (10,000 – 3,600)*5% = 6,400*1.05 = $6,720.
   At the end of the second year Alexander owes (6,720 – 3,600) + (6,720 – 3,600)*5% = 3,120*1.05 = $3,276. Thus, he could not repay his loan in two years.
   But he could repay his loan in three years if he makes monthly payments of $300, because the sum he owes to the bank at the end of the third year is less than his annual payment: $3,276 < $3,600.

3. Suppose the grading system for Econ 101 consists of the following components: 5 homework assignments that together account for 10% (each of them accounts for 2% of the final grade) of the student's final grade; two midterms each of them counting for 25% of the student's final grade; and a final exam that counts the remaining 40% of the student's final grade. Assume the scores on each assignment, midterm, and exam could range from 0 - 100. The table below presents the corresponding scores for 3 students in the class.

	Charlie	Bella	Erik
HW 1 (2%)	60	100	60
HW 2 (2%)	50	95	30
HW 3 (2%)	70	90	20

HW 4 (2%)	70	100	60
HW 5 (2%)	100	90	60
Midterm 1 (25%)	60	90	45
Midterm 2 (25%)	80	90	40
Final (40%)	80	80	50
Total grade	74	86.5	

a. Compute the total grade for Bella.

100*2% + 95*2% + 90*2% + 100*2% + 90*2% + 90*25% + 90*25% + 80*40% = 2 + 1.9 + 1.8 + 2 + 1.8 + 22.5 + 22.5 + 32 = 86.5

b. What is the average score on Midterm 2?

(80 + 90 + 40)/3 = 210/3 = 70

c. What is Charlie's score on the Final Exam if his total grade at the end of the semester is 74?

Let X denote Charlie's score on the Final Exam.
60*2% + 50*2% + 70*2% + 100*2% + 100*2% + 60*25% + 80*25% + X*40% = 74
1.2 + 1 + 1.4 + 1.4 + 2 + 15 + 20 + X*40% = 74
42 + X*40% = 74
X*40% = 74 – 42 = 32
X = 32/0.4 = 80

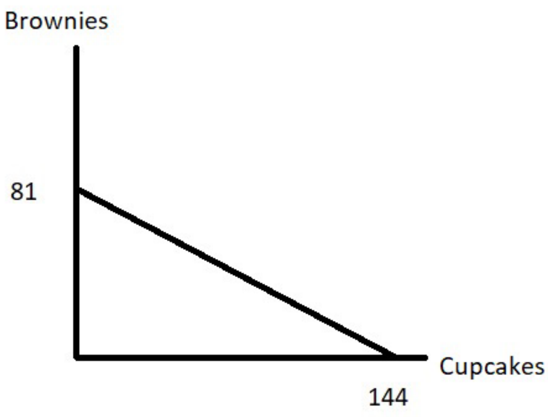
d. What is Erik's score on Midterm 1 if the average score on Midterm 1 is 65?

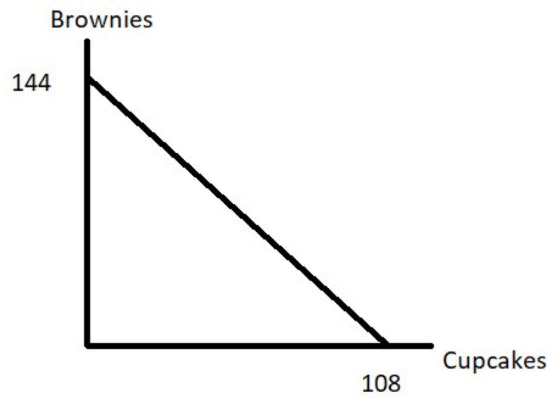
Let Y denote Erik's score on Midterm 1.
(60 + 90 + Y)/3 = 65
60 + 90 + Y = 195
Y = 45

Part II: Production Possibility Frontier, Opportunity Cost, Absolute and Comparative Advantage

4. Annika and Bertrand are in two rival clubs baking brownies (B) and cupcakes (C) for a bake sale. They can each devote 9 hours a day to baking. Annika can bake 9 brownies every hour or 24 cupcakes every 1.5 hours. Bertrand can bake 16 brownies every hour or 12 cupcakes every hour.

a. With brownies on the vertical axis and cupcakes on the horizontal axis, plot the production possibility frontiers for Annika and Bertrand on two separate graphs, and provide an equation for each of these graphs. Make sure your graphs are completely and clearly labeled.





Annika: B = 81 – (9/16)*C

Bertrand: B = 144 – (4/3)*C

b. What is the opportunity cost of 1 cupcake for Annika and Bertrand, in terms of the number of brownies? What about the opportunity cost of 1 brownie in terms of the number of cupcakes?

The opportunity cost of 1 cupcake is 9/16 of a brownie for Annika, and 4/3 brownies for Bertrand. Notice that Annika has to give up less than 1 brownie, whereas Bertrand has to give up more than 1 brownie.

The opportunity cost of 1 brownie is 16/9 cupcakes for Annika, and 3/4 of a cupcake for Bertrand.

c. Who has the absolute advantage in the production of each good? Who has the comparative advantage in the production of each good?

If they each spend all day producing only brownies, Annika can make only 81, whereas Bertrand can make 144, so Bertrand has the absolute advantage in brownie baking. If they each spend all day producing only cupcakes, Annika can make 144, whereas Bertrand can only make 108, so Annika has the absolute advantage.

For comparative advantage, we examine opportunity cost. Notice that the opportunity cost of 1 cupcake is lower for Annika, so Annika has the comparative advantage in cupcake baking.

Similarly, the opportunity cost of 1 brownie is lower for Bertrand, so Bertrand has the comparative advantage in brownie baking.

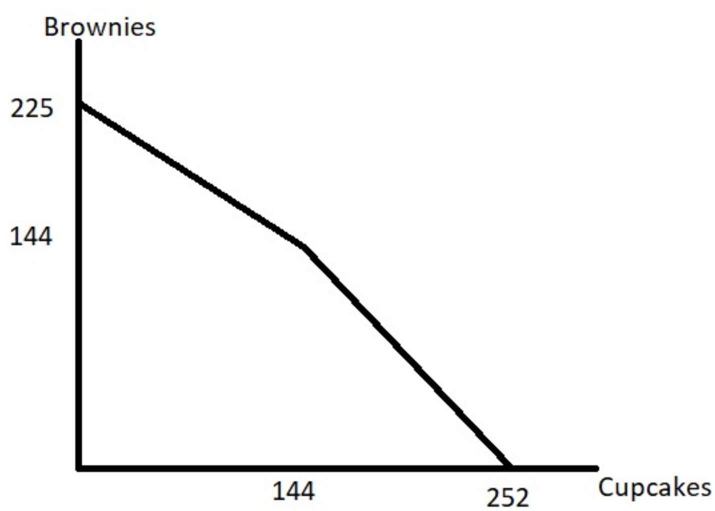
d. Suppose that Annika and Bertrand want to trade brownies and cupcakes before the bake sale. In terms of cupcakes, what is the range of possible trading prices for 1 brownie in terms of cupcakes? In terms of brownies, what is the range of possible trading prices for 1 cupcake?

The trading range of prices should be between Annika and Bertrand's opportunity cost of production. Hence, 1 brownie should cost between 3/4 and 16/9 of a cupcake. If the price is above 3/4, then Bertrand will want to sell brownies to Annika, and if the price is below 16/9, then Annika will want to buy brownies (to make brownies herself, she has to give up 16/9 cupcakes).

Similarly, the trading range of prices for 1 cupcake should be between 9/16 and 4/3 brownies. In this case, Bertrand will buy cupcakes and Annika will sell cupcakes.

e. Plot and give an expression for the joint PPF of Annika and Bertrand.

If both Annika and Bertrand only produce brownies, they can produce a total of 225 brownies. If they both only bake cupcakes, they can bake 252 cupcakes. If they each only produce what they have a comparative advantage in, Annika will bake 144 cupcakes and Bertrand will bake 144 brownies. Then, finding the equation in the usual way, we have



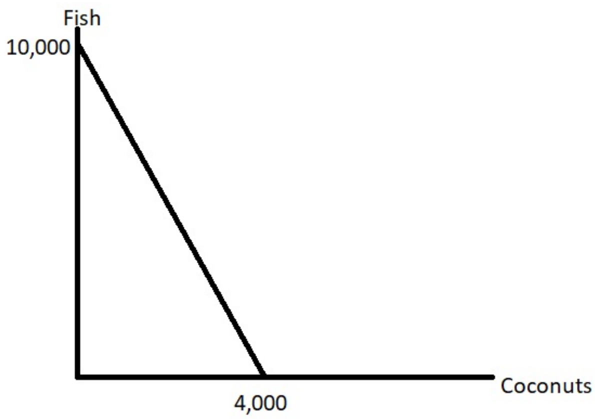
B = 225 – (9/16)*C for C between 0 and 144

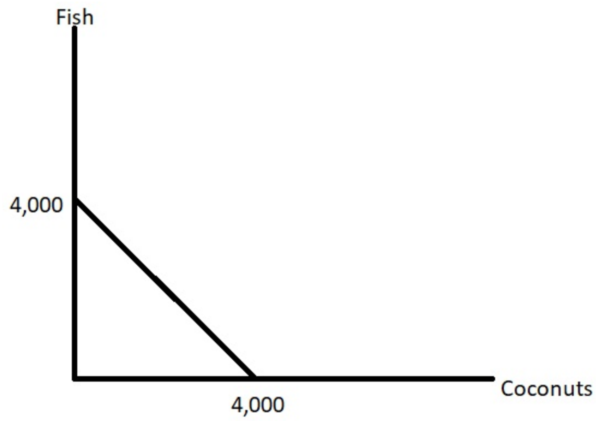
B = 336 – (4/3)*C for C between 144 and 252

5. Consider the small Island countries of Canopia and Dania. Canopia has a population of 200, and Dania has a population of 100. Being close to the Ocean, each country can only

produce two goods – fish and coconuts. Assume that all citizens are employed. In a given month, Canopia can produce 10,000 fish or 4,000 coconuts or any combination of the two goods that lies on the line containing these two points. Meanwhile, Dania can produce 4,000 fish or 4,000 coconuts or any combination of the two goods that lies on the line containing these two points.

a. With fish on the vertical axis and coconuts on the horizontal axis, graph the PPF for these two countries and find the equations for each PPF.





Canopia F = 10,000 – (5/2)*C

Dania F = 4,000 - C

b. What is the opportunity cost of 1 coconut for Canopia and Dania, in terms of the number of fish? What about the opportunity cost of 1 fish in terms of the number of coconuts?

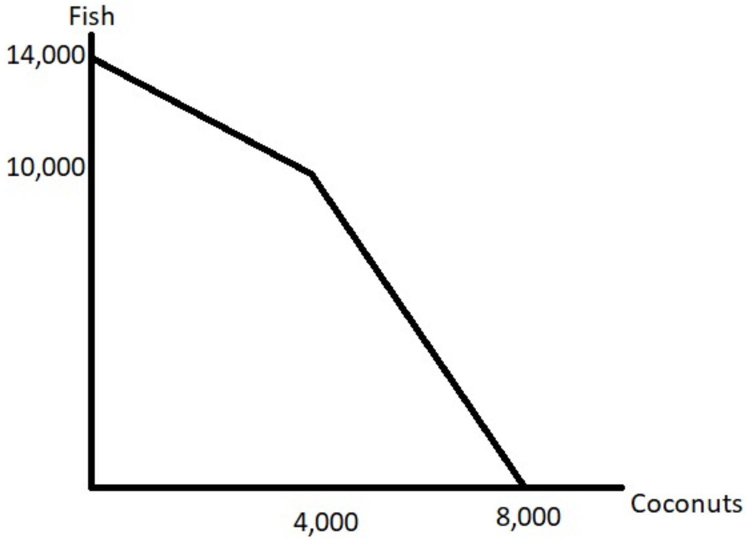
The opportunity cost of 1 coconut is 5/2 fishes for Canopia, and 1 fish for Dania. The opportunity cost of 1 fish is 2/5 of a coconut for Canopia, and 1 coconut for Dania.

c. Who has the absolute advantage in the production of each good? Who has the comparative advantage in the production of each good? Consider the size of the country (that is, calculate the production per person for each good in each country in order to answer this question) in your answer.

Canopia can produce 50 fish and 20 coconuts per person, whereas Dania can produce 40 fish and 40 coconuts per person. Thus, Canopia has an absolute advantage in fish production, and Dania has an absolute advantage in coconut production.

By interpreting our answer to part (b), it is clear that Dania has the comparative advantage in coconut production, and Canopia has the comparative advantage in fish production since comparative advantage asks who has the lower opportunity cost of production for the particular good.

d. Plot and give an expression for the joint PPF of Canopia and Dania.

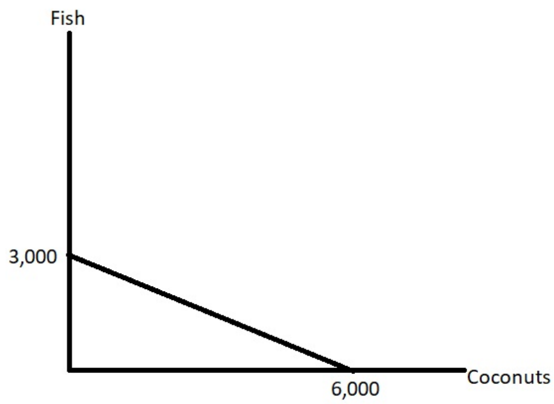


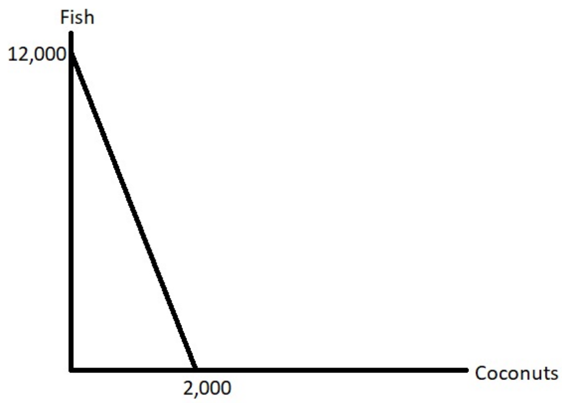
F = 14,000 – C for C between 0 and 4,000

F = 20,000 – (5/2)*C for C between 4,000 and 8,000

e. Suppose that there is a large storm near Dania that brings more fish to the waters, but destroys some of the coconut trees. 50 skilled fishermen from Canopia move to Dania to take advantage of the plentiful fish there, so now each country has a population of 150. Dania can produce 12,000 fish or 2,000 coconuts or any combination of the two goods that lies on the line containing these two points, and Canopia can produce 3,000 fish or 6,000 coconuts or any combination of the two goods that lies on the line containing these two points. With this new information, re-do parts (a) - (c). What has changed in part (c)?

a)





Canopia F = 3,000 – (1/2)*C

Dania F = 12,000 – 6*C

b) The opportunity cost of 1 coconut is 6 fishes for Dania, and 1/2 fish for Canopia. The opportunity cost of 1 fish is 1/6 of a coconut for Dania, and 2 coconuts for Canopia.

c) Dania can produce 80 fish or 13.33 coconuts per person, whereas Canopia can produce 20 fish or 40 coconuts per person. Thus, Dania has an absolute advantage in fish production, and Canopia has an absolute advantage in coconut production.

By interpreting our answer to part (b), it is clear that Canopia has the comparative advantage in coconut production, and Dania has the comparative advantage in fish production since comparative advantage asks who has the lower opportunity cost of production for the particular good.

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