Economics 101
Fall 2017
Answers to Homework #1
Due Tuesday, 26 September 2017

Directions:

- The home will be collected in a box **before** the lecture.
- Please place **your name, TA name, and section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will **not** be accepted so make plans ahead of time.
- **Show your work**. Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional? For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you submit any work for someone else.

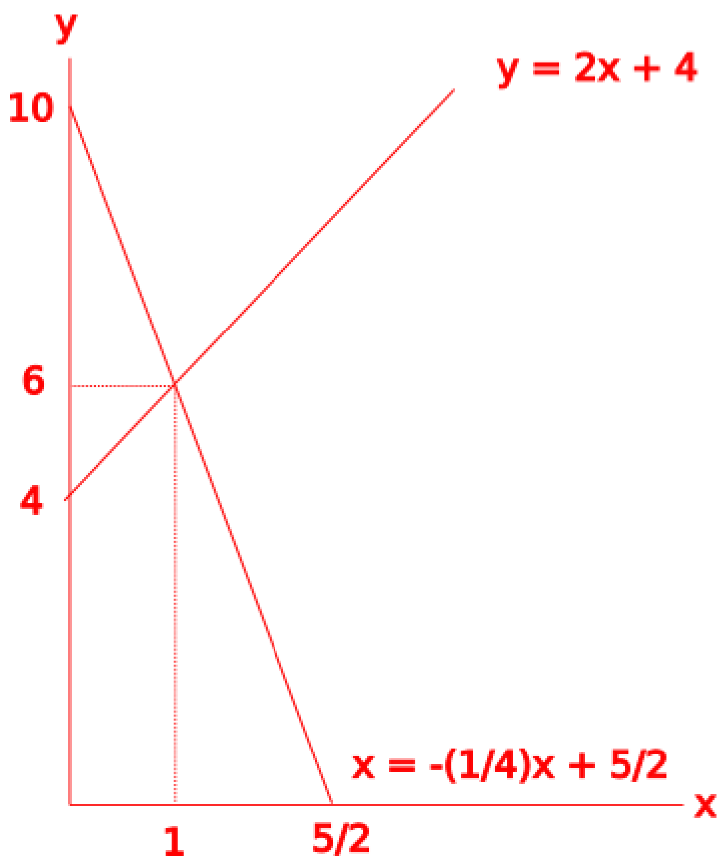
Part I: Math Review

1) Consider the following two equations:

Equation A: $y = 2x + 4$ and Equation B: $x = -(1/4)y + (5/2)$

a. Plot both equations on the same graph with x and y on the horizontal and vertical axes respectively. (Restrict your attention to the first quadrant: that is, consider only values equal to or greater than zero for x and y.)

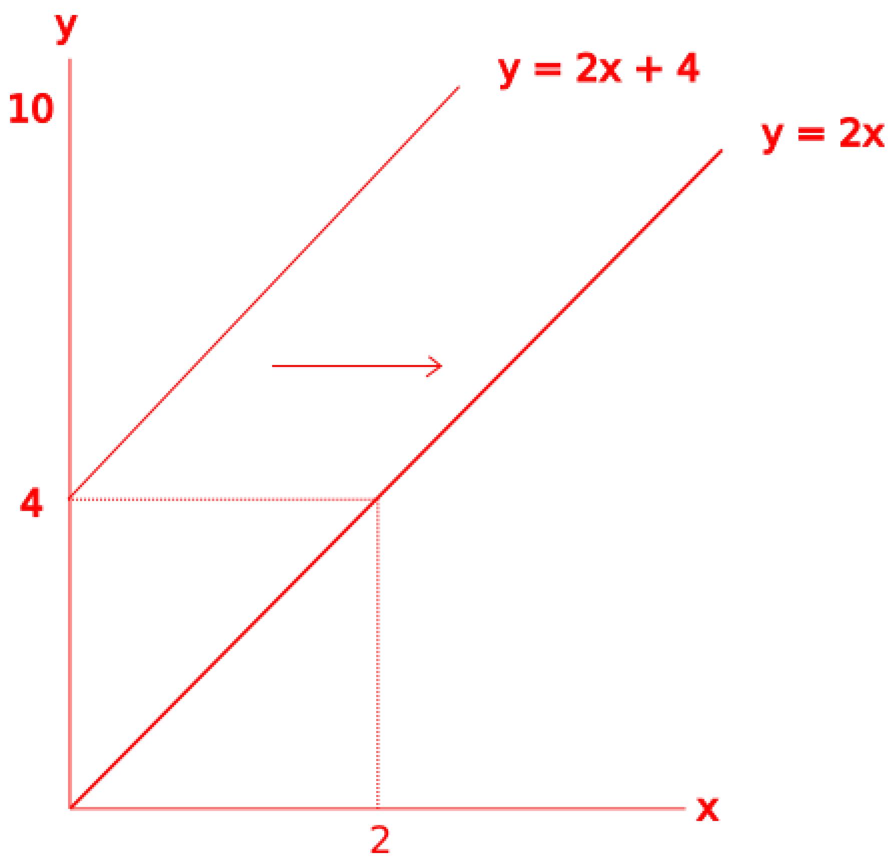
Solution:



b. Plot and find the new equation A' assuming that equation (A) shifted out 2 units horizontally at every y value. What vertical shift results in the same new equation A'?

Solution: There are at least two ways to solve this: (1) Work the algebra, then plot, or (2) plot, then back out the algebra from there.

To solve algebraically, we can rearrange the equation into x-intercept form as $x = y/2 – 2$. Then a 2 unit horizontal shift outward corresponds to increasing the x-intercept by 2. Thus we get the equation $x = y/2$, or equivalently, $y = 2x$. From this, we can see that the vertical intercept was reduced by 4, so an outward 2 unit horizontal shift corresponds to a 4 unit downward vertical shift.



c. Find the intersection of lines (A) and (B), ignoring the shift from the previous part.

Solution: Start by being careful to rearrange either (A) or (B) so both are in the same form. Here I have rearranged equation B so that it is in slope intercept form

(1/4)y = (5/2) – x

y = 10 – 4x

Then, we set both equations equal to one another:

2x + 4 = 10 – 4x

6x = 6

x = 1

Then use this x value and either equation to find the value of y:

y = 2x + 4 = 2(1) + 4 = 6 or

y = 10 – 4x = 10 – 4(1) = 6

(x, y) = (1, 6)

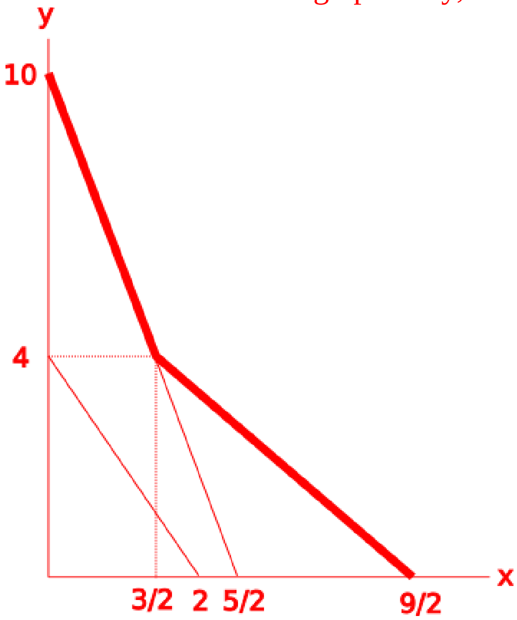
d. Suppose we shift equation (B) by c units vertically down. Find an algebraic expression that expresses the intersection of (A) and this shifted version of (B) in terms of the size of the shift, c.

Solution: Following the method from part (b), we can express the shifted version of (B) by y = -4x + 10 - c. Using this, we can repeat the previous part:

$$2x + 4 = -4x + 10 - c$$
$$6x = 6 - c$$
$$x = 1 - (1/6)c, \; y = 6 - (1/3)c$$

e. Suppose these equations stop at the edge of the first quadrant. Graph and find the algebraic expression for the horizontal sum of equation B and the equation y = -2x + 4.

Solution: We solve this graphically, in the usual fashion:



From this, we can find the equation of each segment separately. For x < 3/2, the horizontal sum is simply equation B. Past x = 3/2, the equation is that of the lower line which can be found in the usual way resulting in y = (-4/3)x + 6.

For those struggling with the math here is the "usual way":
we know two points on the lower line: (x, y) = (3/2, 4) and (9/2, 0). Use these two points to find the slope: slope = (change in y)/(change in x) = -4/3. Use the general slope intercept form and one of these known points to find the y-intercept of this line. Thus,

$y = mx + b$

$y = (-4/3)x + b$

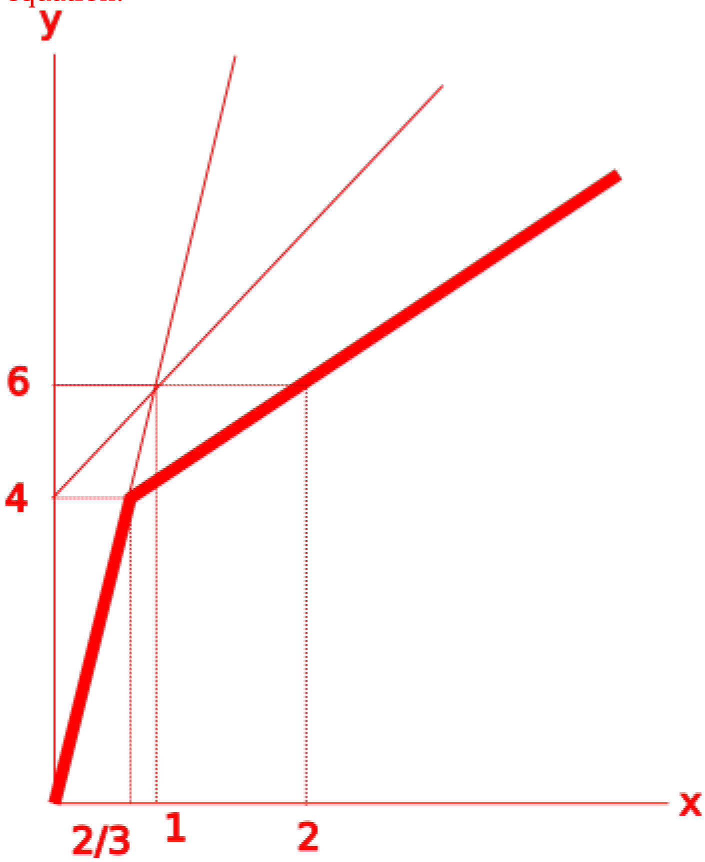
$0 = (-4/3)(9/2) + b$

$b = 6$

Hence, the equation is y = (-4/3)x + 6.

f. Using only the values in the first quadrant, graph and find the horizontal sum of equation A and the equation y = 6x.

Solution: We repeat the above procedure. To find the equation for the upper part of the line, we plug in y = 6 into both equations, finding x = 1 for both equations. (We chose y = 6 since it was easy, but you could have chosen any y value greater than 4 and gotten the x values for each equation and then added these x values together to get the new x value for your chosen y value.) Adding these, we get a second point on the horizontal sum that we can use to graph and find the equation:



From the plot we find that the horizontal sum can be expressed as y = 6x for x < 2/3, and y = (3/2)x + 3 for x > 2/3.

2) Last year, Alice earned $36,000 from her job. During the same year, she saved $9,000.

a. What was Alice's savings rate last year (measured as a percent)? Show your work.

Solution: Alice saves $9000 out of $36000 which is 1/4 or 0.25. Multiplying by 100 to convert this number to a percent: her savings rate is 25%.

b. Suppose Alice wishes to maintain that savings rate this year during which time she made $40,000. How much money should she save this year? Show your work.

Solution: In order to maintain the same rate, she must save 0.25 * 40000 = $10,000.

Suppose Alice decides to move some of savings out of her mattress and into an interest-bearing account. She deposits $10,000 in the account at the beginning of the year. When Alice checks the account at the end of the year, she finds she now has $10,500 in her account.

c. What was the rate of return on this account? That is, what interest rate did she earn over the course of this year when she deposited her savings. Assume that we are not doing anything fancy here: just a simple interest rate (or rate of return).

Solution: the interest rate on this account solves $10500 = (1 + r) * 10000$ where r is the interest rate. Solving for r we find that $r = 0.05$, or equivalently 5%.

d. If she does not deposit any more money and simply lets interest accrue, how much money will be in her account after 3 years? (You may want to use a calculator for this problem). Assume that each year she earns the same annual interest on the account. Keep this calculation simple!

Solution: We can express the money in her account after 3 years as $A = 1.05^3 * 10000 =$ \$11576.30. To see this:
First year Alice earns: 10,000(1 + .05) = 10,500
Second year Alice earns: 10,500(1 + .05) = 10,000(1 + .05)(1 + .05)
Third year Alice earns: 10,000(1 + .05)(1 + .05)(1 + .05)

For this next question, it is recommended you do some research on compound interest (the Wikipedia article on the topic is a good place to start), if you aren't already familiar with the topic. A calculator may be useful for these last two parts.

Suppose Alice decides to put some money in a new account that compounds twice per year. At the beginning of the year, before the first compounding, she deposits \$10,000. At the end of the year, she finds \$11,000 in the account.

e. What was the interest rate on this account?

Solution: The interest rate solves the following equation:

$$11000 = 10000 * (1 + r/2)^2$$

With the help of a calculator, we can find that the interest rate is approximately 9.8%.

Part II: Opportunity Costs, Absolute vs. Comparative Advantage, and Production Possibility Frontiers

3) Alice has found herself on a desert island and must gather supplies to survive. Alice has 8 hours of useful stamina that she can use towards gathering firewood or coconuts. She finds that she can gather 1 bundle of firewood every two hours, or 6 coconuts every 4 hours.

a. In terms of a number of coconuts, what is the opportunity cost of 1 bundle of wood? What is the opportunity cost of 12 coconuts in terms of bundles of wood?

Solution: In the amount of time it takes to gather a bundle of firewood, Alice could have gathered (1/2) * 6 = 3 coconuts. Thus, the opportunity cost of one bundle of firewood is 3 coconuts. The opportunity cost of 12 coconuts is 4 bundles of firewood: Alice can gather 12 coconuts in 8 hours. If she were to gather firewood instead during these 8 hours then she would be able to gather 4 bundles of firewood: hence, when she chooses to gather 12 coconuts she is giving up 4 bundles of firewood.

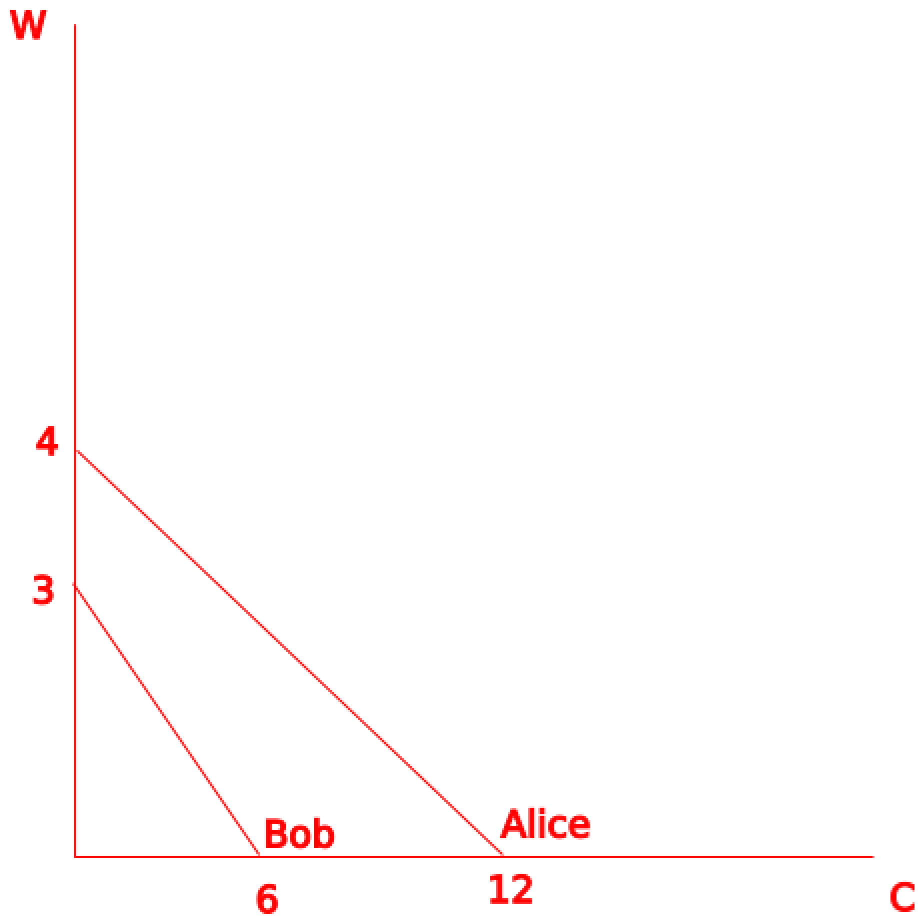
b. Draw a plot of and give an algebraic expression for Alice's production possibility frontier (PPF). Measure coconuts on the horizontal axis and bundles of wood on the vertical axis.

Solution: Denoting coconuts by C and firewood by W, we can find the equation for the PPF by graphing directly, or by noting that if all time is spent gathering wood, she can gather 4 bundles of wood, so the W intercept is 4. Because the opportunity cost of a bundle of wood is 3 coconuts, the coefficient on C will be -1/3. Thus we have $W = (-1/3)C + 4$, or equivalently $C = -3W + 12$. (Alice's PPF is plotted below in the solution to part (c).)

Bob just crash landed on the island too. He doesn't have quite the stamina that Alice does so he can only work 6 hours a day. He finds that he can gather a bundle of wood in 2 hours or 1 coconut per hour.

c. Repeat parts (a) and (b) for Bob.

Solution: Each bundle of firewood takes as long as gathering 2 coconuts, so the opportunity cost of one bundle of firewood is 2 coconuts. The opportunity cost of 1 coconut is ½ bundle of firewood. Repeating the procedure from before, we find Bob's PPF can be expressed by either $W = (-1/2)C + 3$ or $C = -2W + 6$.



d. Who has the absolute advantage in the production of firewood and coconuts, respectively? (In this case, we will say a person has the *absolute advantage* in the production of a good if he or she produces more when devoting all available resources to the production of that good, so compare Alice spending all 8 of her hours on each with Bob spending all 6 of his hours.)

Who has the comparative advantage in the production of firewood and coconuts, respectively?

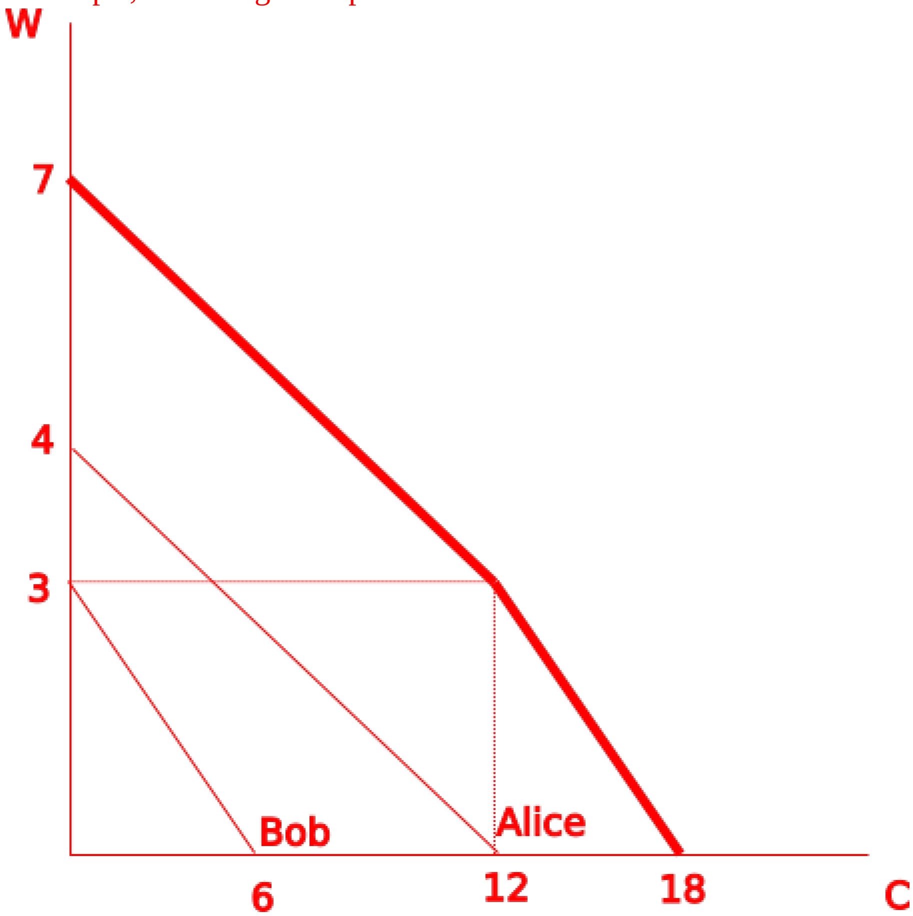
Solution: Alice has absolute advantage in the production of both goods since she can out-gather Bob in both if she devotes all her time to one or the other. Bob has comparative advantage in gathering firewood since each bundle of wood only costs him 2 coconuts compared to 3 for Alice. Alice then has comparative advantage in coconuts since she can gather 3 coconuts at a cost of 1 bundle of wood, but Bob can only gather 2 coconuts at a cost of 1 bundle of wood.

e. At most how many coconuts would Bob be willing to pay Alice for a bundle of wood? What is the minimum number of coconuts Bob would accept to sell a bundle of wood? Would this trade ever occur? Why or why not?

Solution: Bob is willing to pay at most 2 coconuts per bundle of wood (any more and he would be better off gathering wood himself), but Alice would have to be paid at least 3 coconuts per bundle of wood (any less, and she would be willing to gather coconuts herself). Given this, this trade would never happen. Any trades would be Alice paying Bob coconuts in return for bundles of wood.

f. Plot the joint PPF for Alice and Bob if they pool their resources. Give an algebraic expression for this joint PPF.

Solution: By the usual method, the kink point is at C = 12 and W = 3 where each is spending all of his or her time producing the good in which he or she has comparative advantage. Adding the intercepts, we then get the plot:



Solving for the equations in the usual fashion we get W = (-1/3)C + 7 if C < 12, and W = (-1/2)C + 36 for C > 12. Or equivalently C = -2W + 18 for W < 3 and C = -3W + 21 for W > 3.

4) Allison is taking her econ 101 exam. The exam is composed of 40 binary-choice problems and 40 multiple-choice problems. She has 100 minutes to solve the exam. Binary-choice problems are all the same difficulty and can be correctly solved at a rate of 1 per minute. Multiple-choice problems have a mix of difficulties, but can, on average, be correctly solved at a rate of 1 every 4 minutes.

a. Can Allison solve the entire exam in the allotted time? How much time would she need?

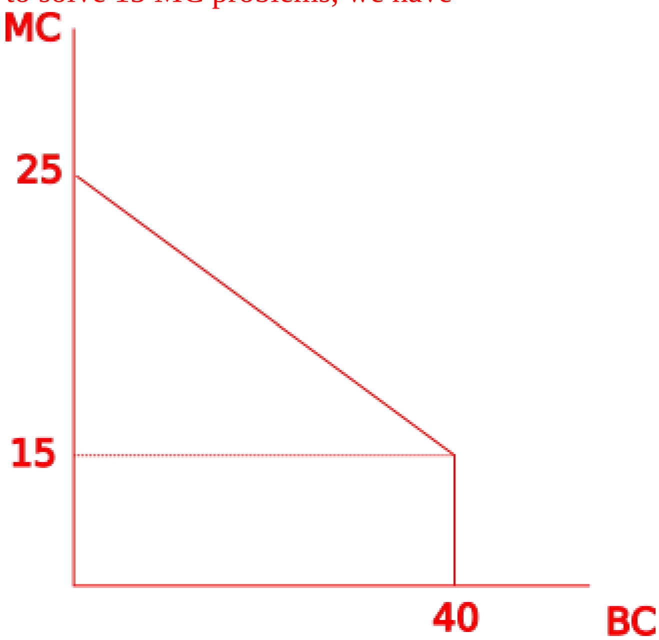
Solution: No. It takes 40 minutes to solve all the BC problems, and another 160 minutes to solve all the MC problems, so 200 minutes total.

b. In terms of a number of binary-choice problems answered, what is the opportunity cost of 1 multiple-choice problem answered.

Solution: Every MC problem takes as long as 4 BC problem to solve, so the opportunity cost of answering one multiple choice question is 4 BC problems.

c. Carefully plot Allison's PPF. Measure MC questions on the vertical axis and BC questions on the horizontal axis. When you draw this graph remember that Allison only gets 100 minutes to do the exam.

Solution: Being careful to note that Allison can solve all the BC problems with enough time left to solve 15 MC problems, we have



Suppose each correct binary-choice problem is worth 1 point and each correct multiple-choice problem is worth 3 points.

d. What is Allison's optimal test-taking strategy? That is, given the exam format and the amount of time she is given to do the exam, what test-taking strategy will give her the best outcome? Explain. What will be her score from following this strategy?

Solution: Since each MC problem is worth 3 BC problems, but each MC problem takes her as much time as 4 BC problems, Allison, should solve all the BC problems, before using the remaining time on MC problems (she will produce at the kink on her PPF). Her score from this strategy will be 40 + 15 * 3 = 85.

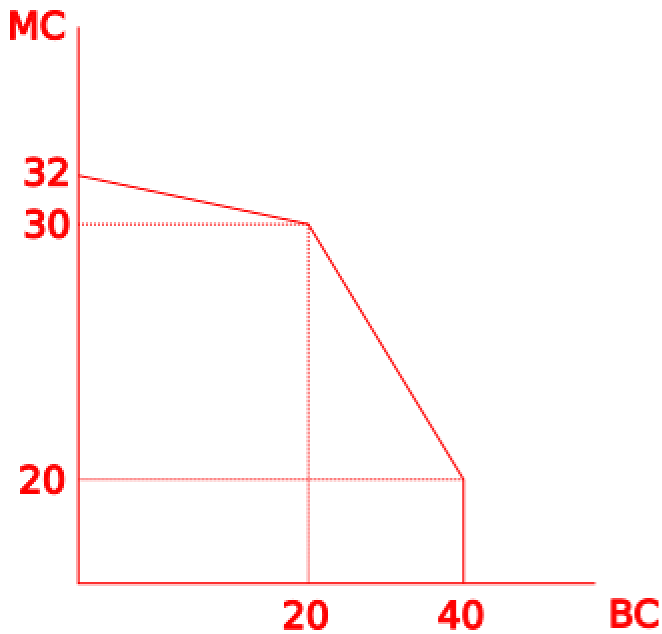
Now suppose Allison can spend 20 minutes at the start of the exam (leaving her 80 minutes to solve the exam) to read through and label questions as 'easy' or 'hard.' Binary-choice questions are all the same difficulty, and they can be correctly solved at a rate of one BC question per minute. In contrast, 30 of the multiple-choice problems are easy and take only 2 minutes per MC question to correctly solve while the remaining 10 MC questions take 10 minutes per MC question to correctly solve.

e. Not including the originally invested 20 minutes, how much time would Allison now need to solve the entire exam?

Solution: On average, multiple choice problems still take 4 minutes each ((3/4) * 2 + (1/4) * 10 = 4) so it would still take 200 minutes to solve the entire exam (not including the 20 extra minutes spent reading through it first).

f. Plot Allison's PPF supposing she invests the 20 minutes reading through the exam first. (*Hint: What is the opportunity cost of solving a hard multiple-choice problem vs an easy one? Which type of multiple-choice problem should she solve first?)*

Solution: This problem is very similar to solving for a joint-PPF. Easy questions have a lower opportunity cost, so Allison should not solve any hard questions unless she has already solved all of the easy ones. Taking this into account, we can plot the PPF.
If Allison starts with the easy BC questions, she can do 40 of them in 40 minutes leaving her 40 minutes to devote to the MC questions. She starts with the easy MC questions and solves 20 of them. She is out of time. So we get the point (BC, MC) = (40, 20) in the graph. If Allison wants to answer more than 20 MC questions then she will need to answer fewer BC questions. For each easy MC question she answers she must give up answering 2 BC questions. She can answer the 30 easy MC questions in 60 minutes and that leaves her 20 minutes in which she can answer 20 BC questions. Thus, we get the kink point (BC, MC) = (20, 30). If Allison wants to answer more than the 30 easy MC questions, then she will need more time: 30 easy MC questions required 60 minutes and with the remaining 20 minutes she can answer 2 of the hard MC questions. She will not answer any BC with this test-taking strategy. She will be at the point (BC, MC) = (0, 32).



g. What is Allison's optimal strategy if she invests the 20 minutes in analyzing the difficulty of the exam? What is her score from following this strategy? Should she invest the 20 minutes?

Solution: Now easy MC problems have a higher return than BC problems, since they are worth three times as much but only take twice as long. Hard MC problems are still not worth it. Thus, she should first solve all 30 easy MC problems, then use the remaining time to solve as many BC problems as possible. Thus she will produce at the upper kink of her PPF (30 MC (all easy) and 20 BC). This results in a score of 110, which is higher than her score without the investment in analyzing the difficulty of the exam.