

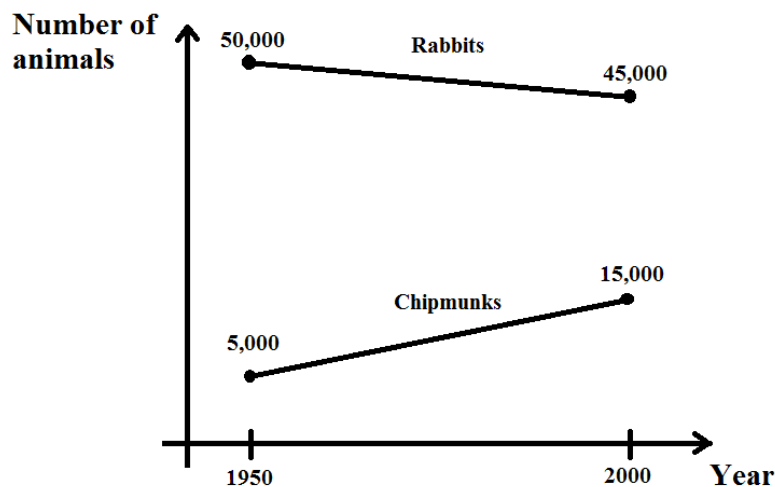
Directions:

- The homework will be collected in a box **before** the lecture.
- Please place **your name, TA name** and **section number** on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time.
- **Show your work.** Good luck!

Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

Part I: Math review

1. Imagine that you are studying how the number of rabbits and chipmunks evolved between 1950 and 2000 in Madison, Wisconsin. You have collected historical data and have summarized it in the following graph:



a) Find an expression for each species that helps you calculate the number of animals of each species between 1950 and 2000 as a function of the year. Use R as a symbol for rabbits, C as the symbol for chipmunks, and Y as the symbol for years. You will be finding two separate equations.

To write the equation for the number of rabbits as a function of the year, start by finding the slope of the line: $\text{slope} = (\text{change in number of rabbits}) / (\text{change in years}) = (45000 - 50000) / (2000 - 1950) = -5000 / 50 = -100$. Then, use the general form of the slope-intercept equation and the value of the slope to find the y intercept:

$$R = -100Y + b$$

Substitute one of the given points into this equation to find the value of the y-intercept, b:

$$50000 = -100(1950) + b$$

$$b = 245000$$

Then, the equation for the number of rabbits (R) in terms of the year (Y) is:

$$R = -100Y + 245000$$

To write the equation for the number of chipmunks, repeat the same steps as before with the two points for chipmunks. Thus, slope = (change in number of chipmunks) / (change in years) = $(15000 - 5000) / (2000 - 1950) = 10000 / 50 = 200$. Using the general form of the slope-intercept equation, the value of the slope and one of the given points to find the y intercept:

$$15000 = 200(2000) + b$$

$$b = -385000$$

Then, the equation for the number of chipmunks in terms of the year (Y) is:

$$C = 200Y - 385000$$

b) For which year was the population of rabbits approximately four times the population of chipmunks?

You are asked to find the value of Y where:

$$R = 4C$$

Using the equations obtained in a) and substituting for rabbits (R) and chipmunks (C) in terms of years (Y), you get that:

$$-100Y + 245000 = 4(200Y - 385000)$$

Passing all of the Y terms to the left side and simplifying the expression you obtain that:

$$900Y = 1785000, \text{ so}$$

$$Y = 1983.33$$

Then, around 1983 the population of rabbits was four times the population of chipmunks.

c) There is data that supports the hypothesis that the linear trends observed between 1950 and 2000 will continue in the future. By which year will both populations be approximately the same?

You are asked to find the value of Y where

$$R = C$$

Using the equations obtained in a) and substituting for rabbits (R) and chipmunks (C) in terms of years (Y), you get that:

$$-100Y + 245000 = 200Y - 385000$$

Passing all of the Y terms to the left side and simplifying the expression you obtain that:

$$300Y = 630000, \text{ so}$$

$$Y = 2100$$

Then, you can estimate that by the year 2100 the population of rabbits will be the same as the population of chipmunks.

2. Imagine that you are planning a trip to Europe and are deciding which clothes to pack. You remember from your high school classes that there exists a linear relationship between Celsius and Fahrenheit degrees. You also recall that 0°C is equal to 32°F , and 20°C is equal to 68°F .

a) Derive the formula that converts temperature expressed in Fahrenheit to Celsius. Show explicitly how you found this formula only considering the data included in the question. Measure Celsius on the vertical axis and provide your equation in slope-intercept form where C is Celsius and F is Fahrenheit.

Using the fact that there exists a linear relationship between Celsius and Fahrenheit degrees, you need to find an equation of the form $C = mF + b$, where C is degrees in Celsius and F is degrees in Fahrenheit. You know two points on that line (32,0) and (68,20). To obtain the y-intercept form, start by finding the slope = (change in Celsius) / (change in Fahrenheit) = $(20-0) / (68-32) = 20 / 36 = 5/9$. Then, use the general form of the slope-intercept equation, the coordinates of one of the points and the value of the slope to find the y intercept:

$$C = mF + b$$

$$0 = 5/9 (32) + b, \text{ so}$$

$$b = -17.7777$$

Then, the formula that converts temperature expressed in Fahrenheit to Celsius is

$$C = (5/9)F - 17.777,$$

where F is the temperature in Fahrenheit and C is the temperature in Celsius.

b) Derive the formula that converts temperature expressed in Celsius to Fahrenheit. (That is, rewrite this formula in X-intercept form where the X- variable is still Fahrenheit (F).) Show your work.

Now, you need to find an expression of F in terms of C. This can be done working with the equation obtained in a). Start with

$$C = (5/9)F - 17.777, \text{ so multiplying both sides by } 9/5 \text{ you get:}$$

$$(9/5) C = F - 32, \text{ so}$$

$$F = (9/5) C + 32$$

where F is the temperature in Fahrenheit and C is the temperature in Celsius.

c) If you know that the weather forecast is 35°C, what is the temperature forecast in Fahrenheit?

Using the formula obtained in b) and plugging in the value of C = 35, you get

$$F = (9/5)(35) + 32 = 95. \text{ So the weather forecast is } 95^{\circ}\text{F. (Hot weather; pack summer clothes).}$$

d) The weather forecast expressed in Celsius unexpectedly decreases by 20% from the expected 35°C. By what percentage does the forecast in Fahrenheit decrease?

Using the formula of percentage change to get the final forecast in Celsius, you get that

$$-20\% = [(Final_forecast - 35) / (35)] (100\%)$$

So, rearranging the equation you get that:

$$Final_forecast = 28^{\circ}\text{C}$$

Using the equation obtained in b) and plugging the value of C = 28, you get that

$$F = (9/5)28 + 32 = 82.4, \text{ so the new weather forecast is } 82.4^{\circ}\text{F.}$$

Using the formula of percentage change again, you can calculate that

$$\%Change = [(82.4 - 95) / 95] (100\%) = -13.26\%$$

e) Does your answer to d) change if the original weather was forecast to be 20°C instead of 35°C? Explain your answer fully.

Yes, it changes. You can check this out by repeating the same steps of c) and d) but now with an initial forecast of 20°C.

Using the data given, 20°C = 68°F. So the initial forecast is 68°F (Mild weather; pack spring clothes).

Using the formula of percentage change, you can get the final forecast again

$$-20\% = [(Final_forecast - (20)) / 20] (100\%)$$

So, rearranging the equation you get that:

$$Final_forecast = 16^\circ C.$$

Using the equation derived in b) and plugging the new forecast of C = 16, you get that

$$F = (9/5)(16) + 32 = 60.8, \text{ so the new weather forecast is } 60.8^\circ F.$$

Using once again the formula of percentage change, you can calculate that

$$\% \text{ Change} = [(60.8 - 68) / 68] (100\%) = -10.59\%.$$

3. Jimmy and Jane are two close, but very competitive friends. They are both taking Math 202 this semester. In their class, the final grade is obtained as follows: the midterm test is worth 25% of the final grade, while the final test is worth 75% of the grade. Jimmy got an 80 on the midterm and a 60 on the final test. Meanwhile, Jane got a grade on the midterm that was 12.5% lower than Jimmy's. What should be the percentage difference in Jane's grade on the final test with respect to Jimmy's, so that she obtains exactly the same final grade as he does?

First, calculate Jimmy's final score (X) using the data given in the problem:

$$X = (0.25) 80 + (0.75) 60 = 65.$$

Then, using the formula of percentage change you can find Jane's grade on the midterm (Y)

$$-12.5\% = [(Y - 80) / (80)] (100\%)$$

Solving the equation, you get that Y = 70.

Then, Jane's final grade can be obtained as $70 (0.25) + Z (0.75)$, where Z is Jane's grade on the final exam. In order to have the same final grades for both friends, you need that

$$70 (0.25) + Z (0.75) = 17.5 + 0.75Z = 65. \text{ So,}$$

$$Z = 63.33 = 190/3 = 63.33.$$

To express the percentage difference of the grades on the final exam, use the formula of percentage change where the final value is 190/3 (Jane's grade) and the initial value is 60 (Jimmy's grade). Thus, $\% \text{ Change} = [(190/3 - 60) / 60] (100\%) = 5.5555\%.$

This percentage change gives us exactly the difference in grades on the final exam so that Jane gets the same final grade as Jimmy.

It is also valid in this problem to use Jane's grade as the initial value and Jimmy's grade as the final value, because there is nothing clear in the context of the problem that determines which value you should use as final value. Then, another valid solution is $\% \text{ Change} = [(60 - 190/3) / 190/3] (100\%) = -5.2632\%.$

Part II: Opportunity cost, absolute advantage, comparative advantage, Production Possibility Frontier (PPF)

4. Mary and Orson own farms on Central Wisconsin. Mary owns Farm 1 comprised of 20 acres of land, while Orson owns Farm 2 comprised of 30 acres of land. The type of soil in each farm is different. For each acre of land assigned to apple production, Mary harvests 5 apples per year while

Orson harvests 8 apples per year. For each acre of land assigned to orange production, Mary harvests 10 oranges per year, while Orson harvests 9 oranges per year. Assume that Mary and Orson can produce either apples or oranges on any acre in any year.

a) Given the above information, what is the maximum amount of apples that Mary and Orson can harvest in a year? What is the maximum amount of oranges that Mary and Orson can harvest in a year?

Mary has a total of 20 acres. Mary can harvest 5 apples in a year for each acre of land devoted to apple trees. Thus, the maximum amount of apples that Mary can harvest in a year is given by $(20 \text{ acres}) (5 \text{ apples/acre}) = 100 \text{ apples}$. In contrast, Orson has a total of 30 acres and can harvest 8 apples for each acre of land devoted to apple trees. Thus, the maximum amount of apples that Orson can harvest in a year is $(30 \text{ acres}) (8 \text{ apples/acre}) = 240 \text{ apples}$. Maximum production of apples is therefore 340 apples per year.

By the same logic, the maximum amount of oranges that Mary can harvest in a year is $(20 \text{ acres}) (10 \text{ oranges/acre}) = 200 \text{ oranges}$. The maximum amount of oranges that Orson can harvest in a year is $(30 \text{ acres}) (9 \text{ oranges/year}) = 270 \text{ oranges}$. Maximum production of oranges is therefore 470 oranges per year.

b) Given the information above, what is Mary's opportunity cost of harvesting one more apple? What is Orson's opportunity cost of harvesting one additional apple? Who has the lowest opportunity cost of producing an additional apple?

To harvest an additional apple, Mary would have to designate an additional $1/5$ acre of land to apple trees. In order to do so, she would have to cut down the production of oranges given by $(1/5 \text{ acres}) (10 \text{ oranges / acre}) = 2 \text{ oranges}$. Thus, Mary's opportunity cost of an additional apple is 2 oranges.

Repeat the same analysis for Orson. To harvest an additional apple, Orson would have to designate an additional $1/8$ acre of land to apple trees. Then, he would have to reduce the production of oranges by $(1/8 \text{ acres}) (9 \text{ oranges/acres}) = 9/8 \text{ oranges}$. Thus, Orson's opportunity cost of an additional apple is $9/8 \text{ oranges}$.

Since $9/8 < 2$, Orson has a lower opportunity cost of producing an additional apple.

c) Who has the absolute advantage in producing apples? Who has the absolute advantage in producing oranges?

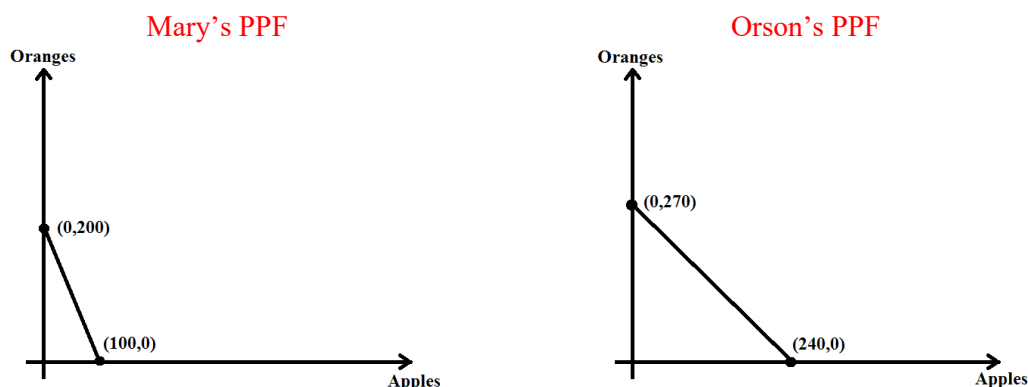
Remember that you have to normalize the level of input to determine which individual has an absolute advantage. With the information included in the setup of the problem, it is easy to do the comparison in terms of who is more productive per acre. For each acre of land assigned to apple production, Mary harvests 5 apples per year while Orson harvests 8 apples per year. Then, Orson has the absolute advantage in apples. For each acre of land assigned to orange production, Mary harvests 10 oranges per year, while Orson harvests 9 oranges per year. Thus, Mary has the absolute advantage in oranges.

d) Draw two separate graphs: on the first graph represent Mary's PPF and on the second represent Orson's PPF. In each graph, measure apples on the X-axis and oranges on the Y-axis.

If apples are graphed on the X-axis, you know two points on the PPF of Mary: (Apples, Oranges) = $(100,0)$ and $(0,200)$. Both of these points are points where she completely specializes. The opportunity

cost of the production of apples for Mary is constant and equal to 2, so you can also conclude that the PPF is linear. For Orson's PPF, use the same logic. You know that points (240,0) and (0,270) are on his PPF and that the trade-off that Orson faces between producing apples and oranges is constant, so the PPF is linear.

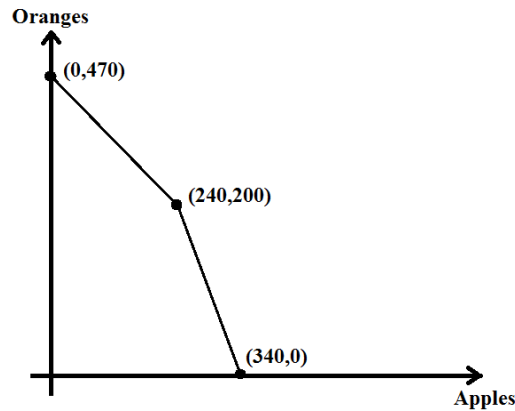
Then the PPFs are as follows:



e) Consider that Mary decides to sell all the land she owns to Orson. This sale doesn't affect the previous levels of productivity of Mary's farm. Draw the new PPF of Orson ("joint PPF"), considering that he now owns both farms. As before, graph in the X-axis the amount of apples and in the Y-axis the amount of oranges. Label clearly the coordinates for any "kink points".

This question is equivalent to the question of finding the joint PPF in an international setup with two countries, but instead you are working with two farms. The easiest approach is to obtain the vertices of the joint PPF. The maximum amount of apples that can be produced in both farms is equal to $100 + 240 = 340$. So the point (340, 0) is part of the joint PPF. Analogously, the maximum amount of oranges that can be produced in both farms is equal to $200 + 270 = 470$, so (0, 470) is on the joint PPF.

The third point you can find in the PPF is where Farm 1 is completely specialized in the production of one fruit, while Farm 2 is completely specialized in the production of the other fruit. Considering the answer in b), you know that in Farm 2 (originally the only farm that was owned by Orson) the opportunity cost of producing apples is less than in Farm 1. Thus, if one farm has to specialize in one fruit and the second farm has to specialize in the other fruit, it is efficient for Orson to harvest apples at Farm 2. And, he would harvest oranges at Farm 1. This gives us the kink point (240, 200) on our joint PPF. With these three points, we get the following graph of the joint PPF:



f) Orson has been hired to sell produce to Whole Foods Market. They have ordered Orson to harvest at least 100 apples and 350 oranges every year. Will Orson be able to satisfy this level of production?

In simple words, this question asks you if the point (100,350) is inside the joint PPF or on the PPF (is it feasible). One easy way to obtain this is to calculate the maximum number of oranges Orson can produce if he harvests 100 apples and the production of the two fruits is assigned efficiently. In mathematical terms, you need to find the y-coordinate of the PPF when $x = 100$.

You are on the left side of the kink point, so you can obtain the y-intercept form equation of the left straight line by using the points (0,470) and (240,200). First, calculate the slope as $m = (\text{change in } y) / (\text{change in } x) = (200 - 470) / (240 - 0) = -270 / 240 = -9/8$. This matches with the opportunity cost of apples for Farm 2. Try to fully understand why this happens. From point 1, $b = 470$. Then $y = (-9/8)x + 470$ is the equation of the left part of the PPF, where x is between $[0, 240]$.

Plugging in the equation of the straight line the value of $x = 100$, you get that $y = (-9/8)(100) + 470 = 357.5$. Thus, Orson can produce at most 357.5 oranges when he produces 100 apples. Since $350 < 357.5$, he can satisfy the minimum level established by Whole Foods Market.

g) (Challenging) Now consider that Orson can decide freely how many apples and oranges to harvest in both farms and that the sale price of apples and oranges is the same. If Orson wants to maximize the amount he earns, how many apples and oranges would he harvest? How would he divide the production of fruits between Farm 1 and Farm 2?

You don't need to know the actual prices of apples and oranges to be able to answer this question. The only important thing is the relative prices of apples and oranges. An easy way to answer this question is to arbitrarily assign a value to the prices. For simplicity, suppose the price of apples = \$1 / apple and the price of oranges = \$1 / orange.

Evaluate now the three points that you know of Orson's joint PPF. If he only produces oranges, he would earn (470 oranges) (\$1 / orange) = \$470 in total. If he only produces apples, he would earn only (340 apples) (\$1 / apple) = \$340. If he uses Farm 1 for the production of apples and Farm 2 for the production of oranges, he would earn (240 oranges) (\$1 / orange) + (200 apples) (\$1 / apple) = \$440 in total, that is less than \$470.

To maximize his revenue given these prices he should only harvest oranges on both farms. He would harvest a total of 470 oranges and earn a total of \$470. To formalize the argument, you can use your answer to b). The opportunity cost of producing apples in Farm 1 was equal to 2 oranges, while the

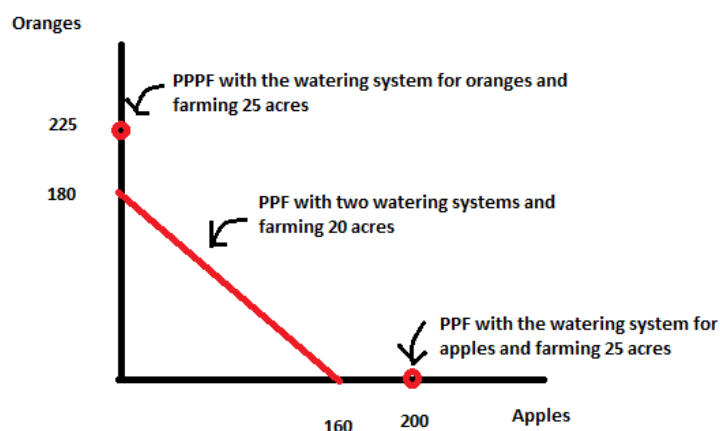
opportunity cost of producing apples in Farm 2 was equal to $9/8$ oranges. Then, in both farms the opportunity costs of apples are higher than what Orson receives for their production. It is then better to only produce oranges.

h) (Even more challenging) Return to the original setup where Orson owned only Farm 2. Draw the PPF of Orson if he now faces the following restriction: In order to start harvesting a particular type of fruit, he needs to install a special watering device that occupies 5 acres of land (no fruit will grow on the land devoted to this water device) and that is specific to the type of tree planted. In this sense, Orson could plant both types of trees at his farm, but then he would need to install two different watering devices (and he would lose 10 acres of land).

To answer this question, get the points of complete specialization first. If Orson only plants apple trees, he would only have to construct one watering device and would have 25 acres to use for apples. The maximum number of apples he can harvest is then given by $(25 \text{ acres}) (8 \text{ apples} / \text{acre}) = 200$ apples. Do the same calculation for oranges. The maximum number of oranges that he can harvest is $(25 \text{ acres}) (9 \text{ oranges} / \text{acre}) = 225$ oranges. Then, the points $(200,0)$ and $(0,225)$ are on the Joint PPF.

But now, the Joint PPF is not given by the straight line that connects these two points. If Orson wants to harvest both types of fruits, he would only have 20 available acres to plant trees since he will need to set aside ten acres for the two watering systems. You can get the rest of the Joint PPF by connecting the extreme points $(160,0)$ and $(0,180)$, that correspond to the maximum amount of apples and oranges that Orson could harvest if he had only 20 available acres for fruit production. Orson could still produce at the extreme points $(160, 0)$ and $(0, 180)$ but he would have the flexibility of switching his production from only oranges to only apples or some combination of the two fruits. So, effectively, we are talking about three different levels of resources here:

- i) One watering system for oranges and no ability to produce apples, and 25 acres of land
 - ii) One watering system for apples and no ability to produce oranges, and 25 acres of land
 - iii) Two watering systems allowing Orson to produce both apples and oranges on 20 acres of land
- So, we really have three PPFs! Two that are simply a single point (options i) and ii)) and one that is a line expressed by the equation: $\text{Oranges} = 180 - (9/8)\text{Apples}$. Here's the graph:



5. Professor Kelly's analysis of available hours in a week has left you thinking about how to boost your performance in the two classes (Economics and Calculus) that you are taking this semester. You face the following restrictions:

- You only have 20 hours to spend studying for these two classes.
- The grade range used for these two classes goes from 0 to 100.
- If you don't study at all for a class, you'll get a 0.
- The first 10 hours that you spend studying for each of the classes has an important effect on your grade. On the one hand, you estimate that for every hour that you spend on Economics, you'll observe an increase of 7 points in your grade in that class. On the other hand, for every hour that you spend in Calculus, you'll observe an increase of 6 points in your grade in that class.
- If you spend more than 10 hours on a single class, diminishing returns kick in. You estimate that one additional hour of studying after the 10-hour limit will result in an increase of 3 points in your Economics grade and an increase of 4 points in your Calculus grade.

a) If you plan to study Economics for 12 hours and Calculus for 8 hours, what would be the opportunity cost of studying Economics one additional hour in terms of your Calculus grade? What would be the opportunity cost of obtaining one additional point in Economics in terms of your Calculus grade? [Hint: you might find it helpful to organize this information in a table like the following:

Hours Studying Econ	Grade in Econ	Hours Studying Calculus	Grade in Calculus
0	0	20	100
1	7	19	96
...

If you study Economics for one additional hour, you would only spend 7 hours studying Calculus. Then your grade in Calculus would decrease from $(8 \text{ hours})(6 \text{ points in Calculus/hour}) = 48$ points in Calculus to $(7 \text{ hours})(6 \text{ points in Calculus/hour}) = 42$ points in Calculus. Thus, the opportunity cost of studying Economics for one additional hour is 6 points in your Calculus grade, if you were studying Economics 12 hours and Calculus 8 hours initially.

From passing from 12 to 13 hours allotted to Economics, you are gaining 3 points in Economics and losing 6 points in Calculus. Then, the opportunity cost of obtaining only one more point in Economics is 2 points in Calculus, if you are initially at the point corresponding to 12 hours of Economics study and 8 hours of Calculus study.

b) If you plan to study Economics for 8 hours and Calculus for 12 hours, what would be the opportunity cost of studying Economics one additional hour in terms of your Calculus grade? What would be the opportunity cost of obtaining one additional point in Economics in terms of your Calculus grade?

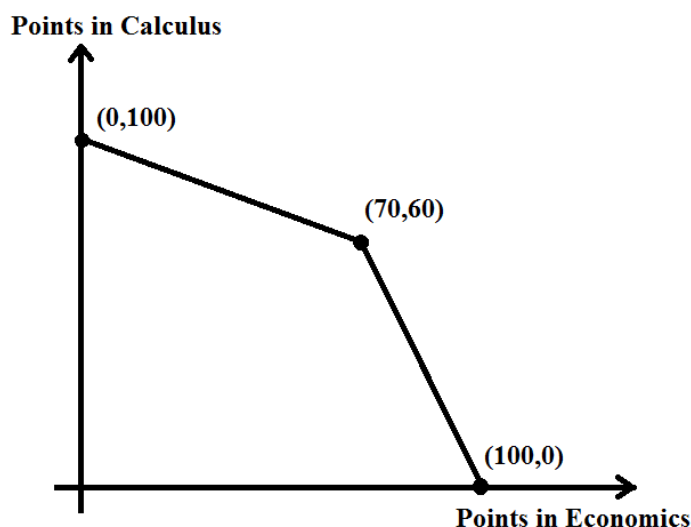
If you increase the time to study Economics from 8 to 9 hours, then you would have to reduce your time allotted to Calculus from 12 hours to 11 hours. Then, your grade in Calculus would decrease from $(10 \text{ hours})(6 \text{ points in Calculus/hour}) + (2 \text{ hours})(4 \text{ points in Calculus/hour}) = 68$ points in Calculus to $(10 \text{ hours})(6 \text{ points in Calculus/hour}) + (1 \text{ hour})(4 \text{ points in Calculus/hour}) = 64$ points in Calculus. Thus, the opportunity cost of studying Economics one additional hours is equal to 4 points in your Calculus grade, if you were studying Economics for 8 hours and Calculus for 12 hours initially.

In passing from 8 to 9 hours allotted to Economics, you are gaining 7 points in Economics and losing 4 points in Calculus. Then, the opportunity cost of obtaining only one more point in Economics is $4/7$ points in Calculus, if you are initially at the point corresponding to 8 hours of Economics study and 12 hours of Calculus study.

c) Graph a PPF for the grades you can obtain this semester. On the X-axis, graph your final grade (Points in Economics) in Economics and on the Y-Axis, graph your final grade (Points in Calculus) in Calculus. (Hint: The PPF has a “kink point”. Try to use your answers to a) and b) to identify this point. Label clearly the coordinates for this point.)

To graph the PPF, you can first find the points of complete specialization in your studies. If you spend all of the 20 hours studying Economics you could get $(10 \text{ hours})(7 \text{ points in Economics/hour}) + (10 \text{ hours})(3 \text{ points in Economics/hour}) = 100$ points in Economics. So the point $(100,0)$ is on the PPF. If you spend all of the 20 hours studying Calculus, you could get $(10 \text{ hours})(6 \text{ points in Calculus/hour}) + (10 \text{ hours})(4 \text{ points in Calculus/hour}) = 100$ points in Calculus. So the point $(0,100)$ is on the PPF.

The “kink point” is given by the grades that you get when you study 10 hours for each class. These grades are 70 in Economics and 60 in Mathematics. Thus, the “kink point” is $(70,60)$. To the left of this point, the opportunity cost of one additional point in Economics is equal to $4/7$ points in Calculus using the results in a). To the right of this point, the opportunity cost of one additional point in Economics is equal to 2 points in Calculus using the results in b).



d) Find a mathematical expression for the grade you can obtain in Calculus, in terms of your Economics grade (This will be a set of equations representing your "Grade PPF").

You need an equation for the straight line where $x < 70$, and an equation for the straight line where $x > 70$. You have two points from both straight lines, so calculate the slope of both lines with the formula $m = (\text{change in } y)/(\text{change in } x)$.

For Line 1 that is the segment of the joint PPF to the left of $(70,60)$, $m = (100-60) / (0-70) = -4/7$. For Line 2 that is the segment of the joint PPF to the right of $(70,60)$, $m = (0-60) / (100 - 70) = -2$. These

are the same as the opportunity costs of obtaining one additional point in Economics in terms of your Calculus grade. Try to fully understand why this is true.

For Line 1, you already have the y-intercept that is equal to 100. So the equation for Line 1 is $y = -(4/7)x + 100$. For Line 2, you need to calculate the y-intercept. Use the value of the slope and one point on Line 2 to get that $60 = -2(70) + b$. So $b = 200$. Thus, the equation for Line 2 is $y = -2x + 200$.

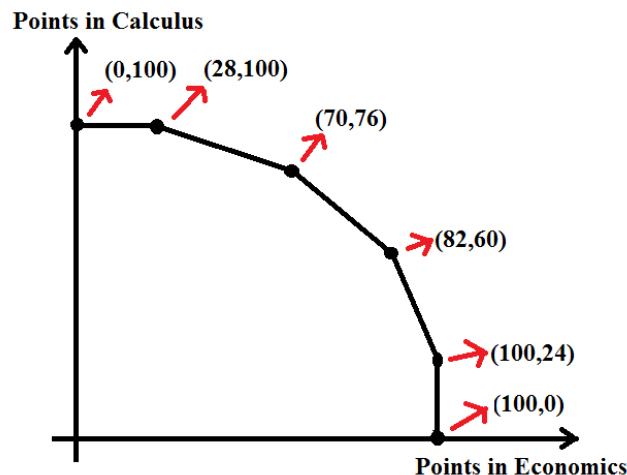
Hence, the mathematical expression for your grade in Calculus (y), in terms of your Economics grade (x) is $y = -(4/7)x + 100$ if x is between $[0,70]$ and $y = -2x + 200$, if x is between $[70,100]$.

e) You need a 65 in each of the subjects to get a B. Can you get a B in both Economics and Calculus?

No, you can't. Fix your Economics grade as $x = 65$, and use the formula obtained in d) to find the highest grade you can get in Calculus given this Economics grade. Plugging in the value of $x = 65$ into the equation $y = -(4/7)x + 100$, you get that $y = 62.86$. If you want a 65 in Economics, then you can only get a 62.86 in Calculus. So you can't get a B in both classes.

f) (Optional – Very challenging) Suppose you realize that you have instead 24 hours instead of 20 hours, graph the PPF for the grades. (Hint 1: Consider the grade range carefully. If you study more than 20 hours for each of the subjects, you will still get a 100; Hint 2: You have more than one “kink point” now. Hint 3: You might find it helpful to construct a table similar to the one you made earlier!)

The trickiest part of this question is not the range, but that now you can face five different opportunity costs for an additional point in your Economics grade. The PPF looks like this:



How can you identify all these different segments? Start from the segment on the left. You could be studying 24 hours of Calculus and getting a 0 in Economics and a 100 in Calculus. But if you reduce the hours allotted to Calculus up to 20 hours, you could earn a higher grade in Economics and still score a 100 in Calculus. Thus, in the segment with endpoints $(0,100)$ and $(28,100)$ the opportunity cost of getting one more point in Economics is 0. This explains why the slope of this segment is 0.

Things change when you move to the left of point $(28,100)$, or when you start studying Economics for more than 4 hours. Now, the decrease of hours allotted to Calculus starts to have an effect on your Calculus grade. On the segment with endpoints $(28,100)$ and $(70,76)$, where you are studying

Economics between 4 and 10 hours, you have to sacrifice 4 points in Calculus to earn 7 points in Economics. The opportunity cost of an additional point in Economics in this segment is equal to $4/7$.

In the segment with endpoints (70,76) and (82,60), you are allotting between 10 and 14 hours to Economics (and between 10 and 14 hours to Calculus). On this range, you sacrifice 4 points of your Calculus grade to obtain 3 more points in Economics. The opportunity cost of an additional point in Economics in this third segment is equal to $4/3$.

In the segment with endpoints (82,60) and (100,24), you are allotting between 14 hours and 20 hours to Economics (and between 4 hours and 10 hours to Calculus). On this range, you sacrifice 6 points of your Calculus grade to obtain 3 more points in Economics. Thus, the opportunity cost of an additional point in Economics in this fourth segment is equal to 2.

Finally, if you are spending 20 or more hours on Economics and you decide to increase even more the hours allotted to the subject, this will not affect the Economics grade but will decrease your grade in Calculus up to the point (100,0). Intuitively, you get a segment that is vertical because you can't increase your grade above 100.