Economics 101
Homework #1
Fall 2014
Due 09/18/2014 in lecture

Directions: The homework will be collected in a box before the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Please remember the section number for the section **you are registered,** because you will need that number when you submit exams and homework. Late homework will not be accepted so make plans ahead of time. Good luck!

Your homework reflects you: please make sure that you submit a neat, organized, and legible set of answers!

PART I: Math Review

Remember to show all of your work. Also remember that calculators are not permitted on exams, so you should try these manually.

1. *Deriving equations*

a. Line A passes through the two points (6, 4) and (4, 6).

Find the equation of Line A.

What are the x-intercept and y-intercept of Line A?

Answer: We first find the slope of line A. Recall the slope of a straight line is given by the formula: Slope = (change in the value of y) / (change in the value of x). So slope = (6-4) / (4 – 6) = -1. Recall that the slope – intercept form of a straight line is y = mx + b, where m is the slope and b is the intercept of the line on the y-axis. So given a point on the line and the slope, one can find the value of the y-intercept. Let us use the point (6, 4), so, 4 = (-1) (6) + b or, b = 10. So the equation of Line A in the slope-intercept form is y = -x + 10. The x-intercept of any line can be found by putting y=0 in the equation of the line. The y-intercept of any line can be found by putting x=0 in the equation of the line. So, the x -intercept is 0 = -x + 10, or, x= 10. The y-intercept is y = 0 + 10, or, y = 10.

b. The reciprocal of the slope of Line B is 0.25 and passes through the point (2, 3).

Find its equation and write this equation in slope-intercept form.

Answer: The reciprocal of the slope of Line B is 0.25. So the slope of Line B = 1/ 0.25 = 4. We use the point (2, 3) and the slope value in the slope-intercept form to get the value of the y-intercept. So, 3 = (4)(2) + b or, b = -5.

Hence the equation of Line B in slope-intercept form is $y = 4x - 5$.

c. What is the equation of Line A (from problem 1a) when its y-intercept is reduced by 4 units?

What is the equation of Line B (from problem 1b) when its x-intercept is reduced by 4 units?

Answer: Equation of Line A: $y = -x + 10$. New equation of line A, with the y-intercept reduced by 4 units is $y = - x + (10 – 4)$ or, $y = -x + 6$. Equation of Line B: $y = 4x – 5$. To reduce the x –intercept by 4 units and get the equation of the new line, we first have to write the equation in terms of x, so that we can get the x-intercept in the equation.
$y = 4x – 5$
or, $(1/4)y = x – (5/4)$ [diving by 4 on both sides]
or, $x = (1/4)y + (5/4)$ [Note this is an equation in terms of x, and (5/4) is the x- intercept]

New equation of line B, with the x-intercept reduced by 4 units is
$x = (1/4)y + (5/4 - 4)$
or, $x = (1/4)y + (-11/4)$
This new equation can be written in y-intercept form as
$(1/4)y = x + (11/4)$
$y = 4x + 11$

2. *Playing with two given equations.*

Consider the following equations of two straight lines

i. $y = x/5 + 4$ and $0.2y = 4 - x$

ii. $y = 7x + 1$ and $y = 7x + 2$

Answer the following questions for each of the above 2 sets of equations. Start with the two equations given in (i) and answer the next three questions; then, repeat for the two equations given in (ii).

a. What are the slopes, x-intercept and y-intercept of both lines?

Answer:
a. i. Calculating the slopes.
$y = x/5 + 4$. Here the slope is (1/5) since the equation is given to you in slope-intercept form.
$0.2y = 4 – x$. Let us rewrite this equation so that it is in slope-intercept form (that is, so that y stands alone).
or, $(2/10)y = 4 – x$
or, $(1/5)y = 4 – x$
or, $y = 20 – 5x$
Here the slope is (-5).

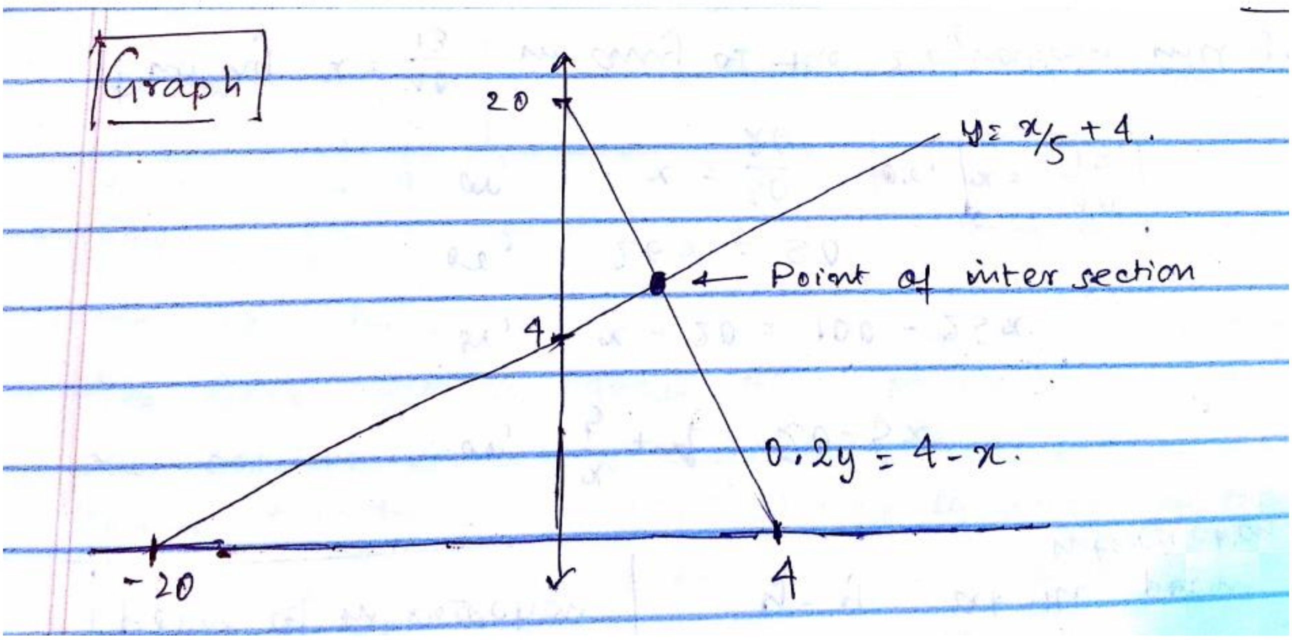
Calculating the x-intercept and the y-intercept
$y = x/5 + 4$.
Putting y=0, we have, $0 = x/5 +4$, or, $x =- 20$. The x-intercept is (-20) for the first equation.
Putting x=0, we have, $y = 0 +4$, or, $y = 4$. The y-intercept is 4 for the first equation.
$0.2y = 4 – x$.
Putting y=0, we have $0= 4 -x$ or, $x = 4$. The x-intercept is 4 for the second equation.
Putting x=0, we have $0.2y = 4 - 0$, or, $y = 20$. The y-intercept is 20 for the second equation.

a. ii. Slopes:

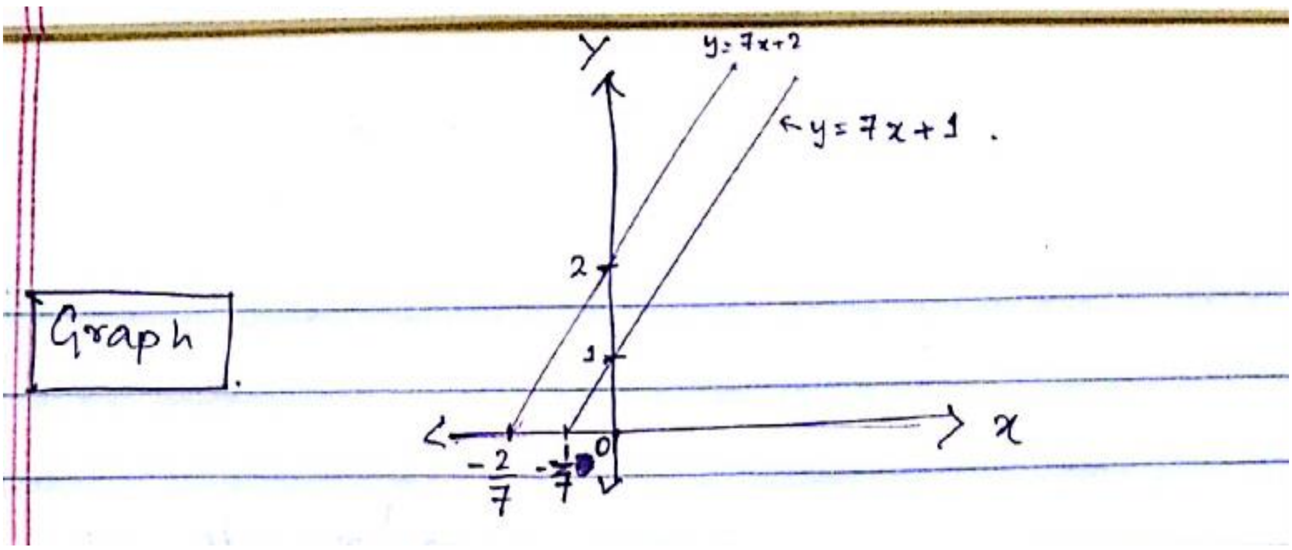
Slope of y = 7x +1 is 7.
Slope of y = 7x +2 is 7.
x-intercept and y-intercept:
The y-intercept of y = 7x +1 is
y = (7)(0) +1
or, y =1
The x-intercept of y = 7x +1 is
0 = 7x +1
or, x = -(1/7)
The y-intercept of y = 7x +2 is
y = (7)(0) + 2
or, y = 2
The x-intercept of y = 7x +2 is
0 = 7x + 2
or, x = (-2/7)

b. Graph the two lines in a single graph measuring y on the vertical axis and x on the horizontal axis.

Answer:
b. i. Graphs of the two straight lines



b. ii. Graph of the two straight lines



c. At which co-ordinate point (x, y) do the two straight lines intersect? [*Hint:* Be careful on this step when you are working with the equations given in (ii.)]

Answer:
c. i. Points of intersection. The point at which the two lines intersect will satisfy the equation of both the lines. Putting y = y, or, x/5 + 4 = 20 – 5x [note that we got y = 20 – 5x , by rewriting the given equation in terms of y , in the solution to part a.]
or, x + 20 = 100 – 25x
or, x = 80/26
or, x = 40/13

Plugging this value of x, in any of the two equations we can get the value of y, of the point of intersection.
Plugging it in y = x/5 + 4, we have
y = (1/5)(40/13) + 4
or, y = (8/13) + 4
or, y = 60/13

So the point of intersection of y = x/5 + 4 and 0.2y = 4 – x, is (40/13, 60/13)

c. ii. Clearly from the graph it can be seen that the two lines never intersect. This is because the two lines have the same slope and are hence parallel. Parallel straight lines never intersect at any point. So, the lines y = 7x +1 and y = 7x +2 being parallel, do not intersect at any point.

3. *Adding equations*

Consider the following two equations

$$2y = 6 - x \text{ and } 3y = 18 - 2x$$

a. What are the intercepts on the X-axis and the Y- axis for these two equations?

Answer:
a. x-intercept and y-intercept.

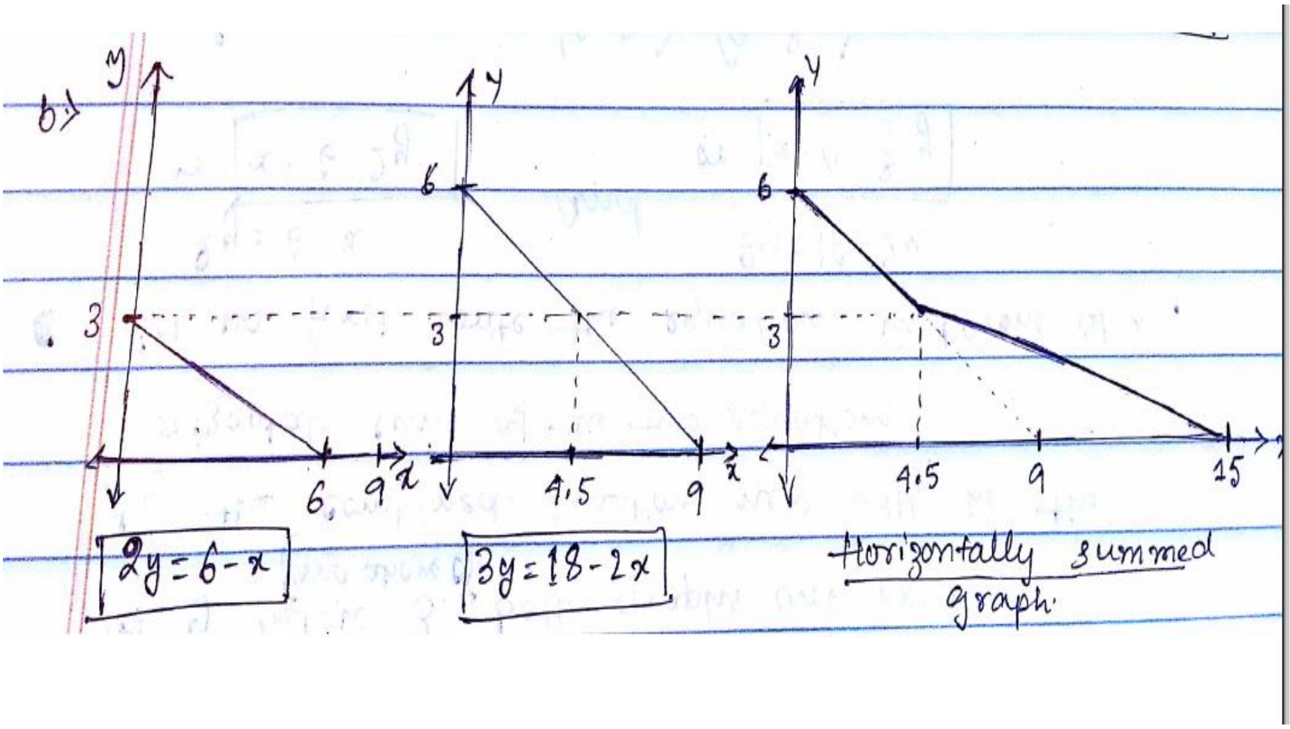
The two given lines are
2y = 6-x and 3y = 18 -2x

The x-intercept of 2y = 6 - x is
(2)(0) = 6 -x, or, x = 6.
The y-intercept of 2y = 6 - x is
2y = 6 – 0, or, y = 3.

The x-intercept of 3y = 18 - 2x is
(3)(0) = 18 – 2x, or, 2x = 18, or, x = 9.
The y-intercept of 3y = 18 - 2x is
3y = 18 – (2)(0), or, 3y =18, or, y =6.

b. Draw the graph of the two straight lines, and show graphically the graph of what you get when you add these two straight lines horizontally. Use three graphs: in the first graph draw the first line, in the second graph draw the second line, and in the third graph draw the graph that results when you add the two lines together horizontally. **In your three graphs consider only the values found in the first quadrant of your coordinate graph (that is, only consider the positive values of x and y when graphing).**

Answer:
b. Graph of the two straight lines and the horizontally summed line



c. What is the equation for the line(s) you drew in the third graph? (Hint: If you have more than one line you will need to provide a range of values for each equation.)

Answer:
c. EXPLANATION:
Note that, for all values of y in between 3 and 6, the first equation (or, graph of it) does not exist (considering only positive values of x and y). Hence in the combined graph, until the point (4.5, 3) the equation of the combined graph will be the same as that of 3y = 18 -2x. (4.5 is the value of x,

when y takes the value of 3, on the line 3y = 18 -2x. If you look at the x-axis, for all values of x in between 0 and 4.5, the equation of the combined graph is 3y = 18 -2x.)
Now for all values of y below 3 and above 0, you will need to sum together horizontally both equations. So for these values of y, the combined equation will be the horizontal summation of the two given equations.

Since we are summing horizontally, we have to rewrite the equations in terms of x.
So, $2y = 6 – x$ can be rewritten as $x = 6 – 2y$.
and, $3y = 18 – 2x$, can be rewritten as $x = 9 – (3/2)y$.

So, for all values of y below 3and above 0, the equation of the combined graph is
$x = (6 – 2y) + (9 – (3/2)y)$
or, $x = 15 – (7/2)y$

Hence, the equation of the combined graph is written as follows, -

For values of y such that, $3 \leq y \leq 6$, the equation of the combined graph is $x = 9 – (3/2)y$
And for values of y such that, $0 \leq y \leq 3$, the equation of the combined graph is $x = 15 – (7/2)y$

Part II: Opportunity Costs, Absolute Advantage, Comparative Advantage, Production Possibility Frontier (PPF)

4. Schmidt and Alex are employed as Librarians in College Library. At the end of each day, they have to deal with a number of returning books, ***B***, and laptops, ***L***. Their work consists of scanning the code bar stuck to either book or laptop, so that each item is discharged.

How quickly they can scan each item is given as below:

	# of books scanned in per minute	# of laptops scanned in per minute
Schmidt	10	20
Alex	20	30

According to the information above, answer the following questions (a)-(e):

a. For Schmidt, what is the opportunity cost of scanning 1 book (remember that opportunity cost is measured in terms of the good that is given up)? What is opportunity cost of scanning 1 laptop for him? For both answers provide the units of measurement. Now, answer the same two questions for Alex.

   Answer: Schmidt in one minute can scan twice as many laptops as books: the opportunity cost of scanning 1 book is therefore the 2 scanned laptops he must give up. The opportunity cost for Schmidt of scanning 1 laptop is the ½ scanned books he must give up. Alex in one minute can scan either 20 books or 30 laptops: therefore the opportunity cost of scanning 1 book equals 1.5 laptops, and the opportunity cost of scanning 1 laptop is 2/3 books.

b. Who has the comparative advantage in scanning books? Who has the comparative advantage in scanning laptops? In your answer provide a verbal explanation.

Answer: From part (a) we know Schmidt and Alex's respective opportunity costs for scanning books and laptops. We also know that the individual (or country) who incurs lower opportunity cost of producing something must have the comparative in producing that item. According to the answer to (a), Alex has the comparative advantage in scanning books (1.5 scanned laptops versus 2 scanned laptops), and Schmidt has the comparative advantage in scanning laptops (1/2 scanned books versus 2/3 scanned books).

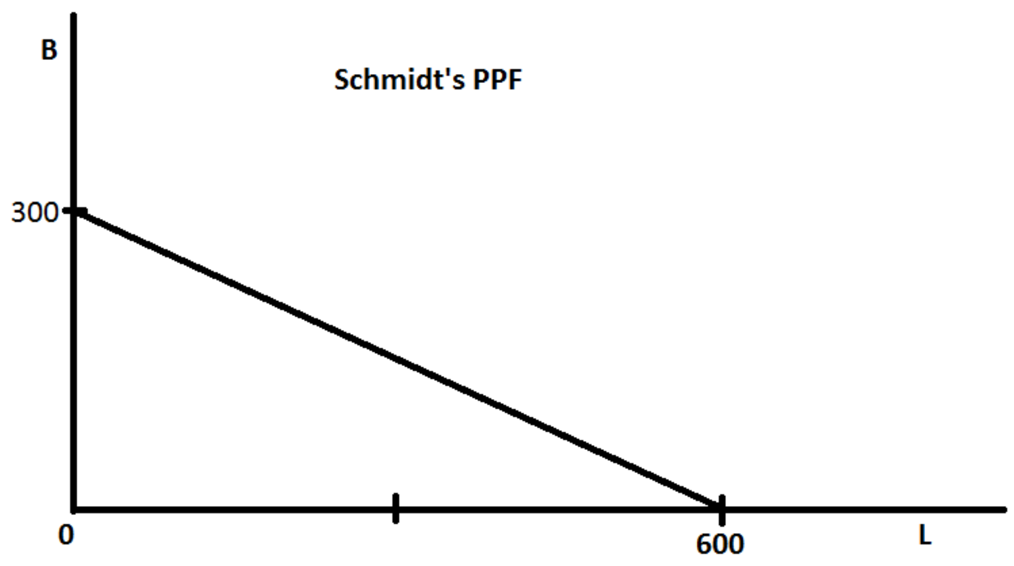
c. Suppose Schmidt and Alex decide to specialize in the production of these two goods and then trade with one another to get the work done. If they do this what is the acceptable range of trading prices that they will both accept for a scanned book measured in terms of scanned laptops?

Answer: Alex has the comparative advantage in scanning books, when exchanging scanning books for scanning laptops, he is willing to accept an exchange rate equal to or greater than 2/3 scanned laptop for each book he scans. For Schmidt who has the comparative advantage in scanning laptops, he is willing to accept a trading price equal to or less than 2 scanned laptops for one scanned book. In sum, the acceptable trading range for one scanned book should be within the interval [3/2 scanned laptops, 2 scanned laptops].

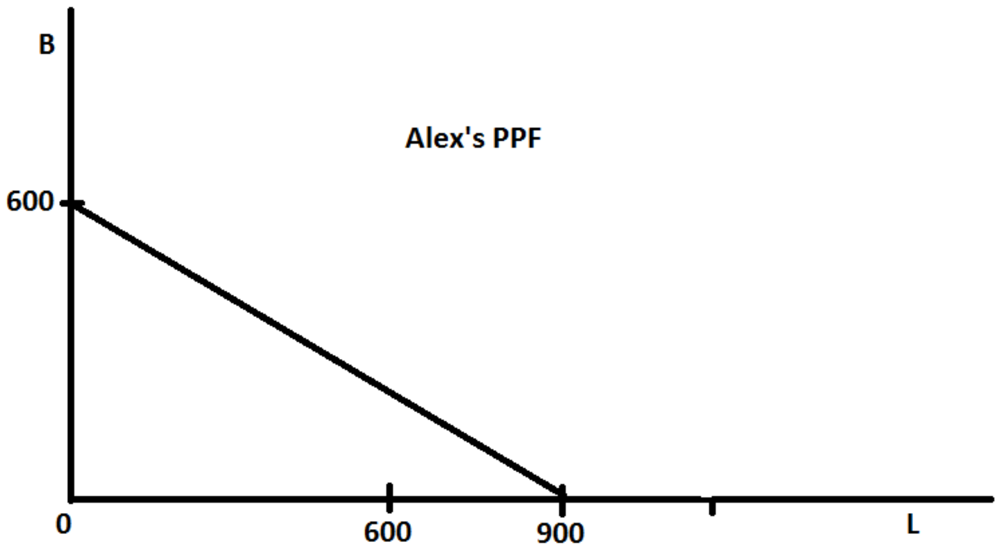
Suppose today Schmidt and Alex go to College Library very late so that they both have only 30 minutes to do their scanning work. Assume this constraint is true for questions (d) through (e).

d. In two separate graphs draw the PPFs of Schmidt and Alex measuring scanned books (B) on the vertical axis and scanned laptops (L) on the horizontal axis. Write an equation for Schmidt's PPF and one for Alex's PPF in slope-intercept form and where you have assumed that **each** of these individuals have a total of 30 minutes time to scan either books or laptops. Label your graphs carefully and completely!

Answer: For Schmidt, the PPF equation is B = 300 – (1/2)L:

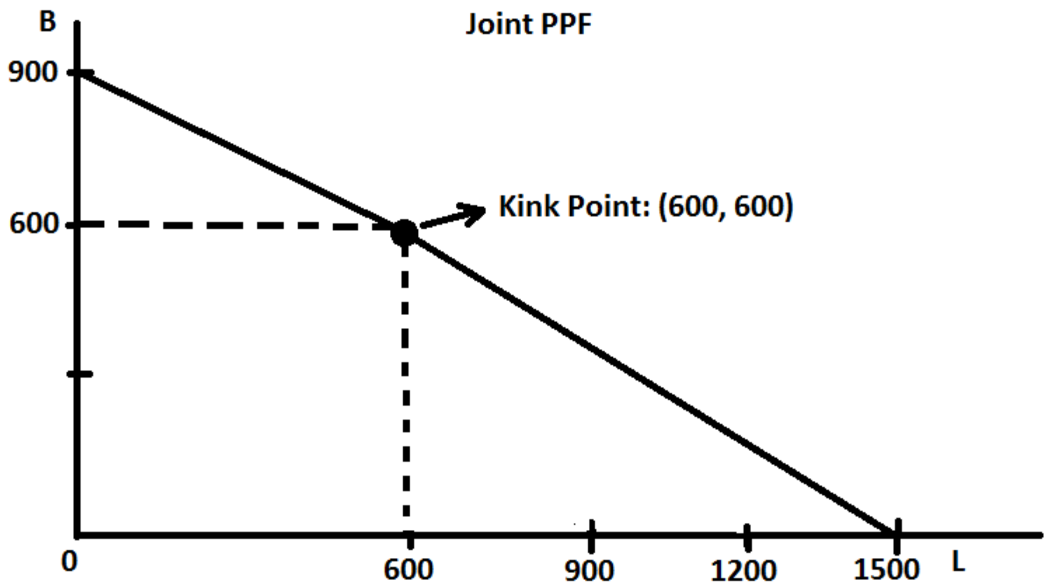


For Alex, the PPF equation is B = 600 – (2/3)L:



e. Draw a graph of the joint PPF between Schmidt and Alex and provide the coordinate values (L, B) for the kink point. Measure scanned books (B) on the vertical axis and scanned laptops (L) on the horizontal axis.

Answer: The joint PPF equation is: B = 900 - 0.5L, if L is within the domain [0 scanned laptops, 600 scanned laptops]; and B = 1000 – (2/3)L, if L is within the domain [600 scanned laptops, 1500 scanned laptops]. And the kink point is (600 scanned laptops, 600 scanned books).



5. Suppose there are two counties in a southern state, ***East*** and ***West***. Both counties could produce sweaters (***S***) and woolly hats (***H***) by weavers. Each county employs 10 weavers per year, and each weaver works for 2000 hours per year. Both East and West have linear production possibility frontiers in the production of sweaters and hats. By using up all the labour hours in a whole year,

East could reach an infinite number of possible production bundles including these two: (3000 woolly hats, 1000 sweaters), or (5000 woolly hats, 0 sweaters); and ***West*** could reach an infinite number of possible production bundles including these twos: (4000 woolly hats, 500 sweaters), or (5000 woolly hats, 250 sweaters). Initially, county East and West are blocked by a giant mountain so that no trade takes place between them.

According to the given information, answer the following questions (a)-(e):

a. What is the opportunity cost of each product for each county? Remember that each county faces a linear tradeoff (i.e. that the tradeoff is constant–the opportunity cost of producing the first sweater is the same as the opportunity cost of producing the second sweater and so forth). Fill out the table below.

County	Opportunity Cost of a Sweater	Opportunity Cost of a Woolly Hat
East		
West		

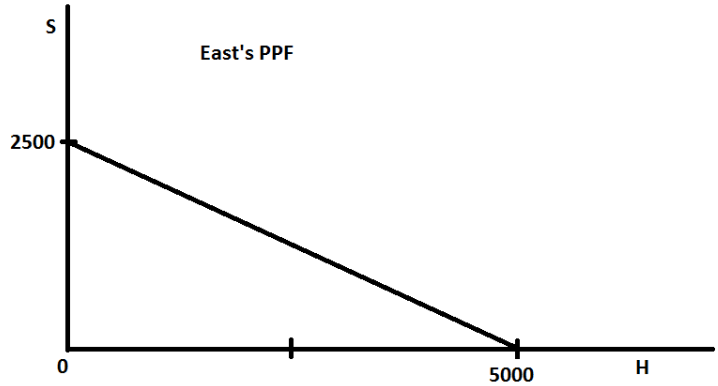
Answer:

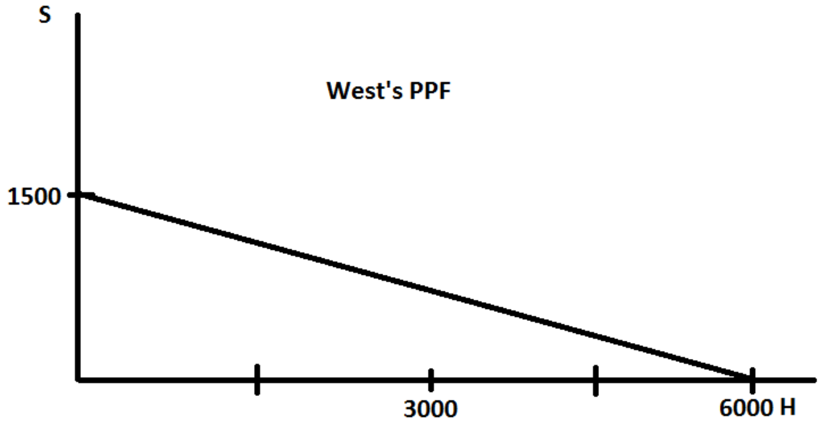
County	Opportunity Cost of a Sweater	Opportunity Cost of a Woolly Hat
East	2 woolly hats	½ sweaters
West	4 woolly hats	¼ sweaters

b. Which county has the comparative advantage in producing woolly hats? Which county has the comparative advantage in producing sweaters?
Answer: according to the table in answer (a), West has the comparative advantage in producing woolly hats, and East has the comparative advantage in producing sweaters.

c. Let ***S*** and ***H*** be the number of piece of sweater and woolly hat produced, respectively. Give the equations for the PPF of each county. (Measure S on the vertical axis and H on the horizontal axis)
Answer: the PPF for East is S = 2500 – (1/2)H, and the PPF for West is S = 1500 – (1/4)H.

d. From (c), draw on two separate graphs the PPF of each county with S measured on the vertical axis and H measured on the horizontal axis. Make sure your graphs are completely and carefully labelled. Which county has the absolute advantage in producing woolly hats? Which county has the absolute advantage in producing sweaters?
Answer:





From the graphs and your answer to (c), it is easy to see that if East only produces sweaters, it would be able to produce 2500 sweaters in all; and if it only produces woolly hats, it would be able to produce 5000 hats in all. And the numbers for West are 1500 sweaters and 6000 hats. Comparing these numbers we can conclude that East has the absolute advantage in producing sweaters and West has the absolute advantage in producing woolly hats.

e. In each county, how many labor hours are needed to produce a woolly hat? And how many labor hours are needed to produce a sweater? Fill out the table below.

County	Labor hours Needed to Produce a Woolly Hat	Labor hours Needed to Produce a Sweater
East		
West		

Answer: From the given information we know that the total labor hours each county has is 2000*10=20,000 hours of labor. Then, if we also use our answer from (d), we get:

County	Labour hours Needed to Produce a Piece of Woolly Hat	Labour hours Needed to Produce a Piece of Sweater
East	20,000 hours of labor/5000 hats = 4 hours of labor per hat	20,000 hours of labor/2500 sweaters = 8 hours of labor per sweater

West	20,000 hours of labor/6000 hats = 10/3 hours of labor per hat	20,000 hours of labor/1500 sweaters = 40/3 hours of labor per sweater

f. Suppose now that a tunnel has been successfully constructed through the mountain, thereby making it possible for East and West to trade and transport goods by train. Regardless of transportation cost, what is the acceptable range of trading prices for one sweater? Please explain your answer.

Answer: since West has the comparative advantage in producing woolly hats, it would be willing to buy a sweater from East for a price that is equal to or less than 4 hats since the opportunity cost for West of producing 1 sweater is 4 hats. Since East has the comparative advantage in producing sweaters, it is willing to trade one sweater for a price that is equal to or greater than 2 hats since the opportunity cost for East of producing 1 sweater is 2 hats. Therefore, the trading range of acceptable prices for one sweater will be between 2 hats and 4 hats.

g. If the two counties cooperate in the production of woolly hats and sweaters, then which County should specialize in producing each good? Draw the (joint) PPF with ***S*** and ***H*** on the vertical and horizontal axes, respectively. Clearly mark the kink point and the two end points on the PPF.

Answer: Since West has the comparative advantage in producing woolly hats, it should specialize in producing woolly hats, and East should specialize in producing sweaters.
The joint PPF equation is:
S = 4000 – (1/4)H, if H is within the domain [0 hats, 6000 hats]; and S = 5500 – (1/2)H, if H is within the domain [6000 hats, 11000 hats].
And the kink point is (6000 hats, 2500 sweaters).