

**Directions:** The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

**Please realize that you are essentially creating “your brand” when you submit this homework. Do you want your homework to convey that you are competent, careful, and professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!**

### Part I: Math Review

Remember to show all of your work. Also remember that calculators are not permitted on exams, so you should try these by hand.

1. The price of a dozen eggs was \$1.50 in 2012.
  - a. In 2013, the price decreases to \$1.47. What is the percentage change from 2012 to 2013?

First we find the price change, subtracting the original price from the new,  
 $1.47 - 1.50 = -0.03$ , and then we divide by the base.  
 $-0.03 / 1.50 = -0.02$  (Shunning calculators, note  $30/15 = 2$  and then adjust decimals)  
Convert to a percentage by multiplying by 100, so we have a 2% decrease.

- b. Egg economists forecast that egg prices will rise 20% from 2012 to 2014. What is the anticipated price of eggs in 2014?

We find 20% of 1.50,  $0.2 \times 1.50 = 0.30$ . Next, add 0.30 to 1.50 to get a forecasted price of \$1.80.

- c. The price of an 18-pack of eggs is projected to be \$2.40 in 2014. Suppose you're writing some long-term egg contracts subject to the eventual 2014 market prices and you want to get the most eggs per dollar. Find and compare the eggs per dollar price for the dozen and 18-packs in 2014 given the projected prices.

18 pack:  $18 / 2.40 = 3 / 0.4 = 7.5$  eggs per dollar  
12 pack:  $12 / 1.80 = 2 / 0.3 = 6.6667$  eggs per dollar

2. On June 14, 2000, the Indiana Pacers lost to the Los Angeles Lakers in game four of the NBA finals, giving the Lakers a 3-1 edge on the series. Shaquille O'Neal made 10 free throws on 17 attempts. Reggie Miller made 11 free throws on 12 attempts. How many more free throws would Shaq have to make in a row to match Reggie's percentage?

Note that though this question speaks in terms of percentages, the question can be approached more simply by leaving everything in terms of fractions. We can convert this to an exercise in algebra. We want to find  $x$ , the number of additional shots made so that the following is true.

$$\begin{aligned}(10 + x) / (17 + x) &\geq 11 / 12 \\ 12(10 + x) &\geq 11(17 + x) \\ 120 + 12x &\geq 187 + 11x \\ x &\geq 67\end{aligned}$$

3. Consider the line given by equation  $y = 50 + 4x$ .
- a. Does this line intersect the line with equation  $12x = 150 - 3y$ ? Where?

Converting the second equation to  $y = mx + b$  format, we have  $y = 50 - 4x$ . We can add the two equations,

$$y = 50 + 4x$$

$$+ y = 50 - 4x$$

$$2y = 100 \quad \text{then } y = 50 \text{ and } x = 0. \text{ So we have an intersection at } (0, 50).$$

- b. Does this line intersect the line with equation  $x = 50 + 4y$ ? Where?

We have two equations,

$$y = 50 + 4x$$

$$4y = x - 50$$

Adding,  $5y = 5x$ , so  $x = y$ .

Substituting,  $-3x = 50$ ,  $x = -50/3 = y$ .

- c. What is the  $x$ -intercept of the original line? The  $y$ -intercept? What is the slope?

The  $x$ -intercept is found by plugging in 0 for  $y$  and solving for  $x$ .

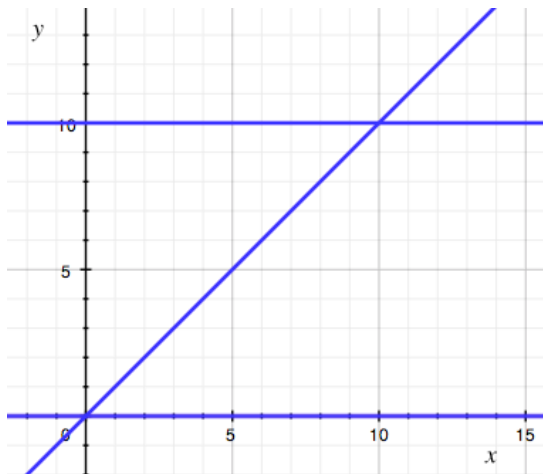
$$y = 0 = 50 + 4x$$

$$-4x = 50$$

$$x = -12.5$$

The  $y$ -intercept is recognized as the constant on the right hand side of the original equation, 50. Verify by plugging in  $x = 0$ . The slope is recognized as the number in place of  $m$  in  $y = mx + b$  format. So our slope is 4.

4. Consider the three lines described by equations  $y = 0$ ,  $y = 10$ , and  $y = x$ .
- a. Graph these lines.



- b. Another line passes through points  $(x, y) = (20, 0)$  and  $(1, 38)$ . Give its equation and add it to your graph.

The slope (rise/run) is  $-38/19 = -2$ . Solve for the  $y$ -intercept using the slope and either of the points.

$$0 = -2(20) + b$$

$$b = 40$$

So we have equation  $y = 40 - 2x$ .



- c. These four lines enclose a trapezoid. Give its area.

The trapezoid has parallel legs of length 20 and 5 units from the graph. Its height is 10 units. Solving for the intersections of these lines gives each corner and is useful in verifying these numbers.

Upper left corner:  $y = x = 10$

Upper right corner:  $y = 40 - 2x = 10$   
 $30 = 2x, x = 15$

Bottom left corner:  $y = x = 0$

Bottom right corner:  $y = 40 - 2x = 0, x = 20$

To find the area of the trapezoid, multiply the height by the average length of the two legs. Area =  $10(25/2)$  = 125 sq. units.

- d. Take the line from part (b) and shift it horizontally 2 units rightward. That is, at every  $y$ -value, add two to the  $x$ -values. Give the new equation.

#### Method 1

Original line  $y = 40 - 2x$ .

Shifting,  $y = 40 - 2(x-2)$ .

$$y = 40 - 2x + 4$$

$$y = 44 - 2x$$

#### Method 2

Original line  $y = 40 - 2x$

Solve for  $x$ , fix  $y$ , and add two.

$$2x = 40 - y$$

$$x = 20 - 0.5y$$

Now, shift the line.

$$x = 22 - 0.5y$$

$$y = 44 - 2x$$

- e. Now, with your newly solved equation, shift it down (vertically) 4 units and give the new equation.

$$y = 44 - 2x$$

$$y = 40 - 2x$$

This is the same equation we started with. Note that a 4 unit vertical shift undid a horizontal shift of 2 units. The vertical/horizontal ratio (4 to 2, or 2 to 1) is exactly our slope.

## Part II: The Economics – Opportunity Cost, Absolute Advantage, Comparative Advantage, Production Possibility Frontier

5. Sage and Jeremy work in Teton Snow Shop. Work in the shop consists of waxing skis or snowboards. Each of them has 10 hours of working time each day. Sage takes 2.5 hours to wax a snowboard, while Jeremy needs 1 hour. It takes Sage 1.25 hours to wax a ski, while Jeremy needs 2 hours. Answer the following questions.
- a. Assuming that shop manager assigns Sage to waxing snowboards, how many boards can he wax during his work shift? What if he is assigned to waxing skis instead?

We know it takes Sage 2.5 hours to wax a snowboard. With 10 hours of time, he can wax at most  $10/2.5 = 4$  snowboards. Similarly, it takes him 1.25 hour to wax a ski; thus, he can wax at most  $10/1.25 = 8$  skis.

- b. Assuming that the shop manager assigns Jeremy to waxing snowboards, how many boards can he wax during his work shift? What if he is assigned to waxing skis instead?

We know it takes Jeremy 1 hour to wax a snowboard. With 10 hours of time, he can wax at most  $10/1 = 10$  snowboards. Similarly, it takes him 2 hours to wax a ski; thus, he can wax at most  $10/2 = 5$  skis.

- c. Fill out the following table.

Employee	Opportunity Cost of Waxing a Snowboard (in Terms of a Ski)	Opportunity Cost of Waxing a Ski (in Terms of a Snowboard)
Sage		
Jeremy		

Employee	Opportunity Cost of Waxing a Snowboard (in Terms of a Ski)	Opportunity Cost of Waxing a Ski (in Terms of a Snowboard)
Sage	$8/4 = 2$ Skis per Snowboard	$4/8 = 0.5$ Snowboard per Ski
Jeremy	$5/10 = 0.5$ Skis per Snowboard	$10/5 = 2$ Snowboards per Ski

The detailed answer is provided for Sage's opportunity cost of waxing a snowboard; the rest of the table follows the same logic. Since Sage is able to either wax 8 snowboards or 4 skis, then to get 4 snowboards waxed, he needs to give up 8 skis; thus, the opportunity cost of waxing 4 snowboards is 8 skis forgone, so 2 skis forgone per 1 snowboard.

- d. Who has the absolute advantage in waxing snowboards? Who has the absolute advantage in waxing skis?

**With the same amount of resources (time)**, Jeremy can wax 10 snowboards, more than the 4 snowboards that Sage can wax in the same amount of time. Thus, Jeremy has absolute advantage in waxing

snowboards. Similarly, **with the same amount of resources**, Sage can wax 8 skis, which is more than the 5 skis done by Jeremy in the same amount of time. Thus, Sage has absolute advantage in waxing skis.

- e. Who has the comparative advantage in waxing snowboards? Who has the comparative advantage in waxing skis?

Comparative advantage asks for the person with lowest opportunity cost in doing something. Clearly Sage has lower opportunity cost of waxing a ski since he has to give up half a snowboard for a ski to be waxed. Jeremy has lower opportunity cost of waxing a snowboard since he has to give up half a ski per snowboard. Thus, Jeremy and Sage have the comparative advantage in waxing a snowboard and a ski, respectively.

- f. Sage has decided to work overtime. His work shift is now 15 hours per day. Does this change your answers to part (d) and (e)?

This is a concept-check question. It does not change the answers in both part (d) and (e). Note that absolute advantage means “**given the same amount of resources**, who can do the most of a particular task.” We have already compared when each person has 10 hours per day in part (a), (b) and (d). Furthermore, the opportunity cost remains the same for both persons, so there is no change to answers in part (e).

- g. The shop manager has given new waxing equipment to Jeremy as a birthday present. It takes Jeremy half as much time to complete each task. Does this change your answers to part (d) and (e)?

It takes Jeremy half as much time to do both tasks, namely, he can wax a snowboard and a ski in 0.5 and 1 hour, respectively. Thus, he can either wax  $10/0.5 = 20$  snowboards or  $10/1 = 10$  skis. Therefore, referring to part (d), not only does Jeremy still have the absolute advantage in waxing snowboards, but he also has the absolute advantage in waxing skis as well. However, since the opportunity cost remains the same, i.e. 2 snowboards per ski and 0.5 ski per snowboard, the answers in part (e) do not change.

6. Production Possibility Frontiers (PPFs) can be used to investigate international trade – trade among countries. You are a trade negotiator of a world leading trade organization. Suppose there are only two countries in the world, North and South. Using crude oil, measured in barrels, as the only input used in the production of gasoline (G) and plastics (P) both countries produce both gasoline measured in gallons and plastics measured in tons. You have the following information about these two countries:
- Currently, each country does not trade with each other, namely, North produces what is needed for her consumption, and so does South. (This is called “autarky.”)
  - Each country is endowed with 1,000 barrels of crude oil per year.
  - The following table provides information about two possible production points (Point A and Point B) for each country. Note that Point A for North is different than Point A for South; Point B for North is different from Point B for South. Assume that both North and South have linear production possibility frontiers with respect to producing gasoline and plastic.

Country	Gasoline (G)		Plastics (P)	
	Production Point A	Production Point B	Production Point A	Production Point B
North	750 Gallons	500 Gallons	100 Tons	200 Tons
South	300 Gallons	400 Gallons	100 Tons	50 Tons

Answer the following questions. (**Hint: Sometimes life does not work forward but backwards!**)

- a. What is the opportunity cost of each good for each country? Assume that each country faces a linear tradeoff (i.e. that the tradeoff is constant—the opportunity cost of producing the first ton of plastic is the

same as the opportunity cost of producing the 200th ton and so forth). Fill out the table and correctly specify the units.

Country	Opportunity Cost of a Gallon of Gasoline	Opportunity Cost of a Ton of Plastics
North		
South		

Country	Opportunity Cost of a Gallon of Gasoline	Opportunity Cost of a Ton of Plastics
North	100/250 = 0.4 Ton of Plastics per Gallon of Gasoline	250/100 = 2.5 Gallons of Gasoline per Ton of Plastics
South	50/100 = 0.5 Ton of Plastics per Gallon of Gasoline	100/50 = 2 Gallons of Gasoline per Ton of Plastics

The detailed solution is provided for the North's opportunity cost of a gallon of gasoline; the rest of the table follows the same logic. From the information given, one can see that the North can either choose 750 gallons of gasoline and 100 tons of plastics or 500 gallons and 200 tons. Hence, to produce 250 gallons more of gasoline, she has to forgo 100 tons of plastics. Thus, the opportunity cost of 250 gallons of gasoline is 100 tons of plastics, so for a gallon of gasoline, the opportunity cost is  $100/250 = 0.4$  ton of plastics.

- b. Which country has the comparative advantage in refining gasoline? Which country has the comparative advantage in producing plastics?

Comparative advantage asks for the country with lowest opportunity cost in doing something. Clearly South has the lower opportunity cost of producing plastics since she has to give up less gasoline for a ton of plastics than North. North has lower opportunity cost of refining gasoline since she has to give up fewer plastics. Thus, North and South have the comparative advantage in refining gasoline and producing plastics, respectively.

- c. Let  $G$  and  $P$  be the number of gallons of gasoline refined and tons of plastics produced, respectively. The equation of this straight line PPF can be written as  $G = a + bP$ , so that  $G$  is on the vertical axis and  $P$  is on the horizontal axis. Give the equations for the PPF of each country. Clearly identify which equation is North's PPF and which equation is South's PPF.

To solve for the PPF equations for each country, apply the method used to solve for the equation of a line.

For North

From the two points  $(P_1, G_1) = (100, 750)$  and  $(P_2, G_2) = (200, 500)$ , the slope is:

$$b = (G_2 - G_1)/(P_2 - P_1) = (500 - 750)/(200 - 100) = -2.5$$

To determine  $G$ -intercept, take the PPF equation with the slope obtained above and any point, here,  $(P_2, G_2) = (200, 500)$ , so:

$$500 = a - 2.5(200), \text{ so } a = 1000$$

Hence, the PPF equation for North is  $G = 1000 - 2.5P$ .

For South

From the two points  $(P_1, G_1) = (100, 300)$  and  $(P_2, G_2) = (50, 400)$ , the slope is:

$$b = (G_2 - G_1)/(P_2 - P_1) = (400 - 300)/(50 - 100) = -2$$

To determine  $G$ -intercept, take the PPF equation with the slope obtained above and any point, here,  $(P_2, G_2) = (50, 400)$ , so:

$$400 = a - 2(50), \text{ so } a = 500$$

Hence, the PPF equation for the North is  $G = 500 - 2P$ .

Notice carefully that the slopes of PPFs represent the opportunity cost of each country for the production of the good measured on the horizontal axis: for example, the opportunity cost of producing 1 ton of plastic for North is 2.5 gallons of gasoline.

- d. What is the maximum amount of each good that each country can produce? Fill out the following table. (Hint: The answers should be obvious following from part (c).)

Country	Maximum Amount of Gasoline Refined	Maximum Amount of Plastic Produced
North		
South		

Country	Maximum Amount of Gasoline Refined	Maximum Amount of Plastic Produced
North	1000 Gallons	400 Tons
South	500 Gallons	250 Tons

From the equations solved in part (c), suppose a country decides to produce no plastics at all, then  $P = 0$ ; therefore, all the resources are devoted to refining gasoline. This gives the maximum amount of gasoline a country can refine. Similarly, if a country decides to refine no gasoline at all, then  $G = 0$ , the maximum amount of plastics produced is attained.

For North

From  $G = 1000 - 2.5P$ , let  $P = 0$ , then  $G = 1000 - 2.5(0) = 1000$ . The maximum amount of gasoline North can refine is 1000 gallons. Let  $G = 0$ , then  $0 = 1000 - 2.5P$ , so  $P = 400$ . The maximum amount of plastics North can produce is 400 tons.

For South

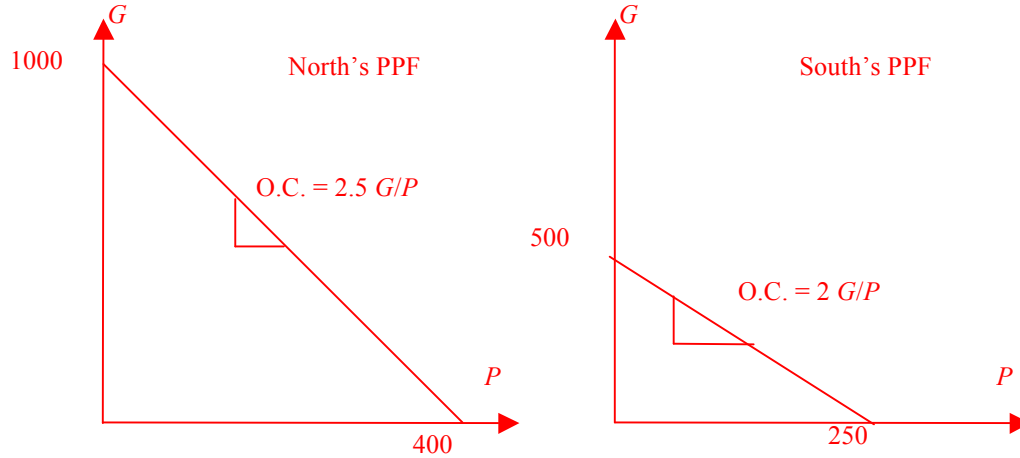
From  $G = 500 - 2P$ , let  $P = 0$ , then  $G = 500 - 2(0) = 500$ . The maximum amount of gasoline South can refine is 500 gallons. Let  $G = 0$ , then  $0 = 500 - 2P$ , so  $P = 250$ . The maximum amount of plastics South can produce is 250 tons.

- e. Which country has the absolute advantage in refining gasoline? Which country has the absolute advantage in producing plastics?

From answers in part (d), it can be observed that **given the same amount of resources**, North can refine more gasoline and produce more plastics compared to South. Thus, North has the absolute advantage in both gasoline refinement and plastics production.

- f. Draw the PPF of each country with  $G$  and  $P$  on the vertical and horizontal axes, respectively. Clearly denote the opportunity cost on each PPF.

One can draw PPFs of both countries from the equations in part (c) or answers from part (d). The slope of the PPF for each country should be measured as gallons of gasoline per tons of plastics.



- g. In each country, how many barrels of crude oil are required to refine a gallon of gasoline? How many barrels of crude oil are required to produce a ton of plastics? Fill out the following table.

Country	Crude Oil Needed to Refine a Gallon of Gasoline	Crude Oil Needed to Produce a Ton of Plastics
North		
South		

Country	Crude Oil Needed to Refine a Gallon of Gasoline	Crude Oil Needed to Produce a Ton of Plastics
North	1 Barrel of Crude Oil per Gallon of Gasoline	2.5 Barrel of Crude Oil per a ton of Plastics
South	2 Barrels of Crude Oil per Gallon of Gasoline	4 Barrels of Crude Oil per a ton of Plastics

This is the question where you need to work backward! The detailed solution is outlined for North, similar logic follows for South. Recall that North can either refine at most 1000 gallons of Gasoline or produce at most 400 tons of plastic. Using 1000 barrels of crude oil, she can refine 1000 gallons of Gasoline, which means that it takes North  $1000/1000 = 1$  barrel of crude oil to refine a gallon of Gasoline. Similarly, with 1000 barrels of crude oil, she can produce 400 tons of Plastics; thus, it takes  $1000/400 = 2.5$  barrels of crude oil to produce a ton of Plastics.

- h. You, as a trade negotiator, are trying to promote international trade policy by opening the borders between North and South. In order to make trade possible, the price of the goods must be right. Suggest the range of possible prices for a gallon of gasoline. Also, what is the range of possible prices for a ton of plastics? Which country should specialize in producing each good?

From part (a), we have established that North has comparative advantage in refining gasoline and South in producing Plastics. Thus, North should specialize in Gasoline refinement and South in Plastics production. North should sell Gasoline to South and vice versa for Plastics.

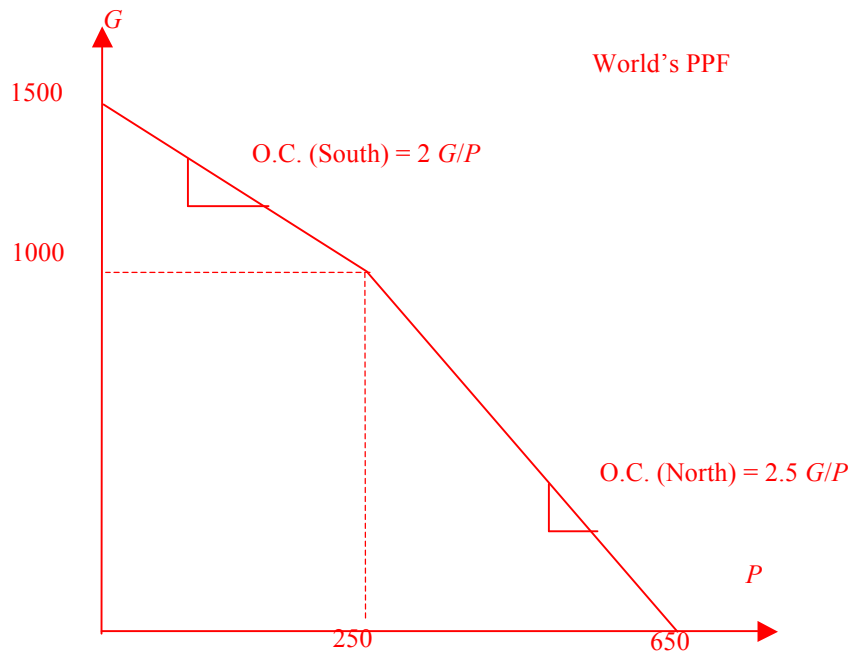


The opportunity cost of refining a gallon of Gasoline for North is 0.4 ton of Plastics, and 0.5 ton for South. Hence, the range of possible prices for a gallon of Gasoline is between 0.4 and 0.5 ton of Plastics. North is not willing to sell a gallon of Gasoline to South if she receives less than 0.4 ton of Plastics. South is not willing to buy Gasoline from North if she has to pay more than 0.5 tons of Plastics per gallon of gasoline; otherwise, the South can simply refine Gasoline herself at lower price.

Similarly, since the opportunity cost of producing a ton of Plastics for South is 2 gallons of Gasoline, and 2.5 gallons of gasoline for North. Hence, the range of possible prices for a ton of Plastics is between 2 and 2.5 gallons of Gasoline.

- i. Draw the (joint) world PPF with  $G$  and  $P$  on the vertical and horizontal axes, respectively. Clearly mark the kink point and the two end points on the PPF.

To draw the world's PPF, one must first find the total amount of Gasoline and Plastics both countries can produce combined. The total amount of Gasoline the world can refine is  $1000 + 500 = 1500$  gallons. The total amount of Plastics the world can produce is  $400 + 250 = 650$  tons. Second, one must find out who produces what. Suppose both countries are producing only Gasoline and no Plastics, to produce the first ton of Plastics, South should be the first country to produce plastic rather than gasoline since South has a lower opportunity cost for producing plastic. South can produce up to 250 tons of Plastics, at this point, the world still has 1000 gallons of Gasoline – contributed by North, and 250 tons of Plastics – contributed by South. To obtain the 251<sup>st</sup> ton of Plastics, North will need to start producing some Plastic (and that means that North will need to reduce its Gasoline production). North can contribute 400 additional tons of Plastics, making 650 tons in total, at which point, no gasoline is refined. Note that the joint PPF contains both horizontal and vertical summation from each country's PPF.



- j. Suppose there is an oil well discovery in the Northern Sea, which is only accessible by North. She is now endowed with 1,500 barrels of crude oil. Does this change the trade policy you have prescribed in part (g)? If yes, prescribe the new policy. If no, why?

The increase in resources for North does not change the opportunity cost of producing Gasoline or Plastics for North. Hence, the “terms of trade” remains the same. It does not change the trade policy.

- k. Suppose there is a technological advancement in the South so that, with the same crude oil input, she can refine twice as much gasoline while producing the same amount of plastics. Does this change the trade policy you have prescribed in part (g)? If yes, prescribe the new policy. If no, why? In your answer assume that the oil well discovery in (j) did not occur.

The opportunity cost for South changes since she can refine twice as much gasoline for the same amount of resource. Hence, South can either refine 1000 gallons of Gasoline or produce 250 tons of Plastics. The opportunity cost for a ton of Plastics in South becomes  $1000/250 = 4$  gallons of Gasoline. The opportunity cost for a gallon of Gasoline in South is  $250/1000 = 0.25$  ton of Plastics. Hence, it is now more costly to produce Plastics in South.

A new trade policy must be implemented. The range of trading prices for a ton of Plastics will now be between 2.5 and 4 gallons of Gasoline, with North selling Plastics to South. The range of trading prices for a gallon of Gasoline will be between 0.25 and 0.4 ton of Plastics, with South selling Gasoline to North.