“Bid Shopping” in Procurement Auctions with Subcontracting

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Abstract

We analyze the equilibrium effects of “bid shopping” – a contractor soliciting a subcontractor bid for part of a project prior to a procurement auction, then showing that bid to a competing subcontractor in an attempt to secure a lower price. Such conduct is widely criticized as unethical by professional organizations, and has been the target of legislation at both the federal and state level, but is widespread in procurement auctions in many places. We find that in equilibrium, a winning contractor’s practice of shopping her subcontractor’s bid to other subcontractors who have already submitted bids is welfare-decreasing, while shopping bids to new subcontractors who have not yet bid can be welfare-increasing, particularly when subcontractors’ bid preparation costs are sufficiently high.

Keywords: procurement, subcontracting
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1 Introduction

A large volume of construction, engineering and industrial projects, such as buildings, utilities, roadways and bridges, is acquired by the public sector through procurement. For example, in 2018, U.S. federal, state and local government spending on construction alone was over 300 billion dollars.[1] Contracts for procurement are usually put up for bid in an auction. Indeed, to protect the public interest against overcharging, and to prevent abuses such as fraud or favoritism, governmental agencies are typically required to solicit bids and award the prime contract to the lowest bidder. Procurement is also common in the private sector, for example for the construction of factories and office buildings, and similar concerns often motivate businesses to solicit competing bids for procurement.

Prime contractors – general contracting firms who bid for large government projects – are often unable to, or not licensed to, perform all the work on a project themselves, and therefore solicit the services of subcontractors. In construction, for example, work on concrete, steel, roofing, drywall, mechanical, electrical and plumbing is usually contracted out. On large projects, 40 to 100 subcontractors could be involved in construction (Stipanowich, 1998, p. 474), typically performing between 60% to 90% of the work (Hinze and Tracey, 1994, p. 274 and GAO, 2015, p. 1). In an effort to secure the lowest possible cost and thereby increase the likelihood they'll be awarded the project, prime contractors typically put the subcontracted portion of the work up for bid, soliciting competing bids from subcontractors ("sub-bids") before the main procurement auction ("prime auction") occurs.

In the U.S. and many other jurisdictions, however, an asymmetry exists in the legal treatment of these pre-auction subcontracting arrangements. Since the prime contractor relies on the subcontractor's proposal in preparing his bid, the subcontractor's proposal is viewed as a binding commitment, and the subcontractor can be held to honor it (Drennan v. Star Paving Co. (1958)), even if it contained a mistake. On the other hand, U.S. courts have consistently held that since the prime contractor cannot accept a subcontractor's proposal "unless and until it knows that its offer has been accepted by the owner" (the government or private entity holding the prime auction), the winning prime contractor is *not* bound to use the subcontractors it relied on in preparing its bid (Gregory and Travers, pp. 30-31). Thus, a winning prime contractor can try to lower its costs by disclosing existing subcontractor bids to competing subcontractors to see if they can do the work at a lower price, a practice called "bid shopping" – or demanding subcontractors lower their sub-bids under the threat of replacing them, a practice called "bid chopping" or "bid chiseling."

Bid shopping could potentially occur either before or after the auction. In fact, in response to the threat of pre-auction bid shopping, many subcontractors do not submit their sub-bids until very close to the deadline for bids in the prime auction.[2] There are no such obvious "countermeasures" available for post-auction bid shopping, however – attempts to disallow it explicitly by contract have failed legally.[3] A prime contractor could commit voluntarily to honor his lowest pre-auction sub-bids, but might hesitate because as we discuss

[1]Federal government spending on construction was $21 Bn in 2018, and state and local government spending was $285 Bn. US Census Bureau, Construction Spending, Nov. 1, 2019.

[2]Korman et al., 1992, p. 26 contains a vivid description of sub-bids arriving in the minutes before the prime auction deadline, which we excerpt in the next section.

[3]For example, West Construction v Florida Blacktop (Fla. 4th Dist. Ct. App., 2012) concerned a subcontractor whose sub-bid contained the provision, "In the event the "buyer" in any way uses Florida Blacktop, Inc.'s bid ... such action(s) shall in all instances constitute acceptance of Florida Blacktop, Inc.'s bid and shall create a binding contract between the parties consistent with the bid documents." The prime contractor did indeed integrate Florida's bid into its own proposal, won the prime contract, then showed Florida's bid to a competing subcontractor and subcontracted with them at a lower price; the court ruled Florida had no recourse, because unilateral attempts to bind the general contractor are invalid unless the general agrees in advance (Smith, Currie and Hancock LLP, 2013).

below, there are legitimate business reasons for not always working with the lowest bidder.

While there is some uncertainty over the exact prevalence of bid shopping and other related practices, we'll show evidence in the next section that they are widespread. Survey evidence suggests bid shopping is commonplace.[4] Litigation about bid shopping is common (Goldberg, 2011, p. 552). Attempts to introduce legislation to curb bid shopping date back to 1932 (Schueller, 1960, p. 504); since 2000, almost every Congress has had legislation before it to effectively end bid shopping in federal procurement, only to see it die in chambers (more detail below). Twenty-nine U.S. states have passed laws meant to halt bid shopping, and other countermeasures (such as the use of "bid depositories," discussed below) have been tried as well. And professional organizations involved in construction in the U.S. strongly condemn bid shopping, going so far as to call the practice unethical.[5]

But while there have been efforts to stop bid shopping through contract clauses, bid depositories, legislation, and professional codes of conduct, no one to our knowledge has properly analyzed the actual effects of bid shopping in equilibrium. These effects are not obvious. A report by the U.S. Senate in 1955 claimed that "As long as subcontractors will not submit their final price prior to the award of the prime contract because of bid shopping after the award, the Government cannot get the full benefit of the low competitive price" (U.S. Senate Committee on the Judiciary, 1955, p. 8). But if prime contractors anticipate that post-award bid shopping will lower their costs, in equilibrium they will base their prime bid on that lower expected cost; if bid shopping leads to lower actual costs, at least some of those savings would likely be passed on to the procurer. On the other hand, fear of bid shopping, and anticipation of the need to lower their bid ex post, could lead subcontractors to inflate their pre-auction sub-bids, and could lower their willingness to participate at all.[6] The net effect of bid shopping on social surplus, on the likelihood of successful procurement, and on the price paid by the procurer are not obvious at a glance.

This, then, is the goal of our paper: to understand the equilibrium effects of bid shopping. We introduce a tractable theoretical model of pre-auction subcontracting and bidding. Our model considers a procurement auction for a project with a piece that must be subcontracted. A fixed number of prime contractors are eligible to bid in the prime auction, and each prime contractor has its own set of subcontractors it can subcontract with. For tractability, prime contractors have identical costs to complete the non-subcontracted part of the project; subcontractors have independent private costs to complete the subcontracted piece, as well as a common cost to prepare a bid.

We use this model to study the equilibrium effect of permitting post-auction bid shopping, i.e., of switching from a regime where no prime contractor would bid-shop to a regime where every prime contractor, should they win, would approach one or more additional subcontractors. We find that the equilibrium effects of bid shopping depend on who the winning prime contractor will approach for a new bid.

[4] Gregory and Travers (2010, p.29) report on a 2008 survey of U.S. and Canadian construction contractors, which found that 80% of respondents knew of others who has engaged in bid shopping or bid pedding, with "an amazing 32%" admitting they had done so themselves; Thurnell and Lee (2009) found that in a 2009 survey of Auckland subcontractors, "all participants indicated that bid shopping in the electirical and mechnical services trades occurred in at least 30% of all tenders for which quotations were submitted."

[5] For example, 1995 guidelines issued jointly by the Association of General Contractors of America, the American Subcontractors Association and the Associated Specialty Contractors refer to bid shopping and bid peddling as "abhorrent business practices that threaten the integrity of the competitive bidding system" and goes on to say that "The bid amount of one competitor should not be divulged to another before the award of the subcontract or order, nor should it be used by the contractor to secure a lower proposal from another bidder on that project (bid shopping)," later calling the practice "unethical." Similarly, the Code of Ethics of the American Society of Professional Estimators prohibits professional estimators from participating in bid shopping, stating that "this practice is unethical, unfair and is in direct violation of this Code of Ethics."

[6] The 1955 Senate report found that 75% of electrical contractors surveyed avoid sub-bidding on Federal construction jobs, of which "93% gave the prevalence of bid shopping as their reason" (U.S. Senate Committee on the Judiciary, 1955, p. 7).

- If the winning prime contractor will shop her lowest subcontractor bid to her other subcontractors *who have already bid*, this practice always decreases total surplus.

- If the winning prime contractor will shop her lowest subcontractor bid to a *new* subcontractor who hasn't already bid, this practice can either increase or decrease total surplus.

  – If the "new" subcontractor is truly new to the game – would not have bid in a counterfactual setting without bid shopping – then bid shopping increases total surplus. This is the only possibility when bid preparation costs are sufficiently high, as a subcontractor who could bid pre-auction never finds it profitable to wait to bid after the auction.

  – If bid preparation costs are low and bid shopping does not attract new subcontractors to the game who would not otherwise have bid, then bid shopping decreases total surplus.

The distributional effects of bid shopping similarly depend on where the winning prime contractor will go for a new bid:

- If she will approach a subcontractor who already bid, then bid shopping may typically benefit both prime contractors and subcontractors in equilibrium, at the expense of the procurer, although we do not have a general result

- If she will approach a new subcontractor who would not have bid in the absence of bid shopping, bid shopping benefits the procurer at the expense of the existing subcontractors

- If she will approach a new subcontractor who otherwise would have bid pre-auction, bid shopping benefits subcontractors, but the effect on the procurer is ambiguous.

Our results, then, suggest that who wins and who loses from bid shopping, and whether the gains outweigh the losses, depends very much on the details of how bid shopping occurs.

In addition, bid shopping may raise concerns that are outside of our model. Some fear that post-award competition leads subcontractors to skimp on quality in an attempt to offer a price low enough to win the work.[7] Bid shopping also introduces the possibility of ex post regret by the winning prime contractor – if she bids based on expected cost, she might win, then fail to lower her cost, and wind up with negative profit from completing the project ex post. If this might lead to attempts at renegotiation or breach which were costly to the procurer, this could be a concern; and we show that two obvious candidate policies to prevent it would undo much of the benefit of bid shopping.

The punch line of our paper, therefore, is that if we want to know whether bid shopping is "good or bad," it's complicated, and requires a detailed understanding of the environment, but the effects need not always be "pernicious." If prime contractors only shop their subcontractors' bids to other subcontractors who have already bid, the overall welfare effect is negative. But if prime contractors shop bids to new subcontractors who have not yet bid, then the overall welfare effect is often, though not always, positive. In particular, if subcontractors' bid preparation costs are large and bid shopping allows for bids from subcontractors who

[7] In enacting the California Subletting and Subcontracting Fair Practices Act (1941,§4101), the California legislature noted, "The Legislature finds that the practices of bid shopping and bid peddling in connection with the construction, alteration, and repair of public improvements often result in poor quality of material and workmanship to the detriment of the public, deprive the public of the full benefits of fair competition among prime contractors and subcontractors, and lead to insolvencies, loss of wages to employees, and other evils."

otherwise could not have bid, then it increases welfare, reduces the procurer's costs, and can even increase the likelihood of successful procurement. One might still worry about its distributional effects (or for the solvency of subcontractors or their willingness to participate), its effect on quality and reliability, or the moral hazard it might introduce for winning prime contractors who fail to lower their costs ex post – but it is not the case that bid shopping must consistently make procurement less likely or more costly for procurers.

The rest of our paper proceeds as follows. Section 2 gives a very brief overview of the economic literature on procurement auctions and subcontracting. Section 3 lays out the legal and historical background on bid shopping. Section 4 introduces our model, and characterizes equilibrium play and some comparative statics and preliminary results. Section 5 compares welfare and equilibrium payoffs with bid shopping to those without. Section 6 considers several extensions to our baseline model. Section 7 concludes. Proofs omitted from the text are in the Appendix.

2 Literature on Procurement Auctions

There is a substantial economic literature on procurement auctions. Since none of it deals directly with bid shopping, we give only a brief overview, focusing on issues related to subcontracting.

One branch of literature deals with how prime contractors choose a subcontractor. Wambach (2009) and Nakabayashi (2011) both consider a theoretical model where each prime contractor holds either a first-price or second-price "upstream auction" among a group of potential subcontractors, prior to the "downstream" procurement auction. In this setting, revenue equivalence breaks down: when prime contractors hold first-price auctions, subcontractors have an added incentive to bid more aggressively to help the prime contractor win the prime auction; this incentive vanishes with second-price auctions, since the winning subcontractor's bid does not set the price. Both papers show that first-price upstream auctions therefore lead to lower prices. They also lead to greater efficiency – with second-price upstream auctions, the prime contractor with the lowest-cost subcontractor won't always win the prime auction. Watanabe and Nakabayashi (2011) find that these theoretical predictions hold in a lab experiment: first-price upstream auctions lead to more aggressive bidding than if there was no downstream auction to follow, and lead to greater efficiency than second-price upstream auctions; second-price auctions, however, lead to higher prime contractor profits.

A few papers, mostly empirical, consider the effect of auction design choices in procurement. Branzoli and Decarolis (2015) compare two auction formats used in Italian procurement, "average bid auctions" – where the bidder closest to the average bid wins – and first-price auctions. They find that the switch from average bid to first-price auctions reduced both entry and subcontracting. (Average bid auctions function like lotteries for a windfall profit, inducing entry even by firms that are not particularly efficient, who then have a stronger incentive to subcontract should they win; first-price auctions limit entry to the more competitive firms, who have less need to subcontract.) Moretti and Valbonesi (2015) compare Italian firms who are fully qualified to complete a project themselves (and can therefore choose whether to subcontract) with firms who are not fully qualified (and are therefore required to subcontract); they find that after controlling for observables, the firms who must subcontract bid higher prices. Marechal and Morand (2003) offer some theoretical analysis on the trade-offs between ex ante and ex post subcontracting (before and after the prime auction), in a setting where there is post-award renegotiation via change orders. Balat, Komarova and Krasnokutskaya (2017) offers insight on the California highway procurement market, and propose a framework for understanding the empirical impact of a choice of ex ante or ex post subcontracting.

A few papers consider the effect of allowing or disallowing subcontracting. Gale, Hausch and Stegeman (2000) consider two firms with non-constant marginal costs, who compete in a sequence of procurement auctions, and can subcontract to each other. With increasing marginal costs, the ability to subcontract increases the firms' profits ex post; however, it also leads to more aggressive bidding, and may hurt the firms ex ante. Marion (2015) similarly considers horizontal subcontracting – firms that compete in the prime auction but also subcontract to other bidders. Theoretically, this can soften competition but also increase efficiency. Analyzing data from California highway construction auctions, he finds evidence that horizontal subcontracting increases efficiency but has a negligible effect on procurement costs. Jeziorski and Krasnokutskaya (2016) model sequential auctions, where the ability to subcontract helps firms manage their capacity constraints. Calibrating a model to the California highway procurement market, they estimate that subcontracting allows a 12% reduction in procurement costs and a 20% increase in the number of projects procured.

A significant strand of literature, starting with Bajari and Tadelis (2001), considers the fact that procurement contracts are incomplete, and subject to post-award revision and renegotiation. Bajari and Tadelis (2001) show that "cost-plus" contracts are preferable to fixed-price contracts when the project is more complex. Bajari, Houghton and Tadelis (2014) use highway paving data to estimate the magnitude of "adaptation costs," and find they account for 8-14% of the winning bid. Bajari, McMillan and Tadelis (2008) examine private sector data to understand the trade-offs between procurement auctions and direct negotiations. Tadelis (2012) finds that the added flexibility in private sector procurement, relative to the restrictions put on public agencies, offers efficiency advantages. De Silva et al. (2017) examine highway procurement data from before and after a policy shift limiting contract revisions, and find that the policy change reduced project costs and improved on-time and on-budget performance. Miller (2014) looks at the impact of contract incompleteness and ex post revisions on both subcontracting and bidding behavior. Gil and Marion (2013) focus on the importance of ongoing relationships between contractors and subcontractors.

A number of empirical papers examine other issues related to procurement auctions. Lewis and Bajari (2011) consider procurement auctions that use scoring rules to favor faster completion of projects, and find this can increase welfare by 22% of the contract's value. Porter and Zona (1993) study bidder collusion in procurement auctions. Krasnokutskaya and Seim (2011) study the impact of "bid preference programs" designed to favor small businesses, domestic firms, or other targeted bidders. Bolotnyy and Vasserman (2019) study scaling auctions, where suppliers submit unit price bids for components of a project, and find this format offers substantial savings. Finally, Li and Zheng (2009), Krasnokutskaya (2011), Balat (2017), and Somaini (2020) use data from procurement auctions to develop or showcase general methodological advances for empirical analysis of auctions, developing tools to account for endogenous entry, unobserved heterogeneity, interdependent costs, or intertemporally linked costs in a dynamic setting.

3 Background on Bid Shopping

3.1 Background – The Subcontracting Process

As noted above, many large construction projects are procured through auctions, and large projects often involve subcontractors doing a substantial part of the work. Prime contractors want to know their costs before bidding in the prime auction, so they seek sub-bids from subcontractors before the auction.

The bid procedure for subcontracts involves two phases. In the first phase, the prime contractor invites subcontract bids, makes available the specifications for the subcontracts, and selects a deadline for the submission of subcontract bids. This deadline is often set just hours before the general contractor's bid is due (Closen and Weiland, 1980, p. 274). The second or "project buyout" phase occurs only if the prime contractor wins the procurement auction, and consists of the time between the contract award to the prime contractor and the awarding of the subcontracts (Zwick and Miller, 2004, p. 245). During the buyout phase, the prime contractor meticulously reviews the subcontractor bids for any holes or double coverage, and reanalyzes them in relation to the entire project. Indeed, time constraints during the bidding phase leave room for errors, which need to be corrected to ensure a smooth operation of the entire project. Buyout is also the time when purchase orders for materials are issued.

Both the subcontractor bidding stage and the buyout stage present the prime contractor with an opportunity to engage in "bid shopping," the disclosure of one subcontractor's bid to another subcontractor in an effort to secure a lower bid. The following passage vividly describes this process during the subcontractor bidding stage:

> Normally, the twenty-four hour period preceding the prime bid deadline is one of great activity, with the subcontractors and suppliers making the rounds of the contractors who are still preparing their bids. By this time the first quotations have generally become known and the quoted prices are often revised downward at the last minute, with the prime contractors revising their bids accordingly.[8]

Indeed, the fear of being bid shopped during the pre award stage has provoked a strategic reaction by subcontractors, who often hold their bids until the last few minutes before the end of the bid period. By doing so, subcontractors keep their bid confidential, and limit the opportunity for the prime contractor to communicate with a competing subcontractor and extract a lower price. This phenomenon is reminiscent of "sniping," the practice of last minute bidding on Ebay that stems from bidders' fear of being overbid (Roth and Ockenfels, 2002). As a consequence, bid day can be nerve wracking and chaotic, as witnessed in the following description (Korman et al., 1992, p. 26):

> It's 1:51 p.m. at Swinerton & Walberg Co.'s San Francisco office, and the first telephone price quote from an electrical subcontractor arrives. Then another eight electrical subs call in rapid succession. Nothing unusual here – the general contractor's bid is due at 2 p.m. In the two hours before the deadline, seven Swinerton & Walberg staff members have fielded about 500 phone calls related to the bid to build a new high school near Fresno. Another 13 compare quotes and scopes until there are only two minutes left. Then numbers are relayed by phone to an employee near the owner's office. He scribbles them down and runs inside to have the bid stamped – one minute before it's due.

The hectic pace of bid day inevitably leads to mistakes and oversights that have to be corrected during the buyout stage. The very nature of the buyout process thus requires communication between the prime contractor and his subcontractors, setting the stage for post award bid shopping. Once the general contractor is awarded the prime contract, he is in a strong bargaining position, giving him an opportunity to pressure

[8] Janke Constr. Co. v. Vulcan Materials Co., 527 F.2d 772, 775-76 (7th Cir. 1976), cited in Closen and Weiland (1980, p. 576).

competing subcontractors to offer a price below the bid of the winning subcontractor.[9] One example of bid shopping is documented in McCandlish Electric, Inc. v. Will Construction Co. (2001). Will Construction was awarded a public contract for renovation on the wastewater treatment plant of the city of Leavenworth, Washington. In the submission of its winning bid, Will relied on McCandlish, who had been the lowest bidder for an electrical subcontract, and in accordance with State law, listed McCandlish as the electrical subcontractor in its bid to the City. Will accepted the City contract, but subsequently asked the City to allow it to substitute Calvert Technologies, since Calvert could perform the electrical work at a lower price. In its finding of fact, the Court determined that Will had engaged in bid shopping.[10]

Alternatively, upon being awarded the prime contract, the general contractor may pressure the winning subcontractor into lowering its original bid by threatening to subcontract the work to a third party (Oertly, 1975, p. 564). This practice is variously known as "bid chopping" or "bid chiseling." Losing subcontractors who learn the bid price of the winning subcontractor may undercut it in an attempt to secure the job from a prime contractor who is awarded the main contract. This process is known as "bid peddling." For most purposes, bid peddling is simply a response of competing subcontractors to the bid shopping activity of a general, and bid shopping and peddling may be treated as one (Lambert, 1970, p. 395).

3.2 Legal Treatment in the United States

What makes bid shopping feasible is that U.S. courts have always held that a subcontractor's bid is only an offer to provide the stated work at this price, and that no actual contract between the two parties is formed until formal acceptance of the subcontractor's bid, which can only happen after the prime contractor becomes the winning bidder on the project. Thus, starting when a subcontractor submits a bid, continuing when they "win" the pre-auction competition for the subcontract and the prime contractor uses that bid to compute and submit his primary bid, and through the time when he wins the primary auction, there is no contractual relation between the subcontractor and the prime contractor. Such a relation exists only after the latter finally accepts the subcontractor's bid (Closen and Weiland, 1980, pp. 565-566; Loulakis and Santiago, 1997).

This absence of contractual liability would seem to put prime contractors at risk, since subcontractors would appear able to withdraw their bid without penalty at any time prior to formal acceptance of their bid. Thus, upon being awarded the project, the prime contractor would find himself unable to complete the project at the anticipated price. Recognizing the possibility of these adverse consequences, starting with Drennan v. Star Paving Co. (1958), courts have relied upon a legal doctrine of "promissory estoppel," viewing the general contractor's use of the winning subcontractor's bid in formulating his bid on the prime as reasonable detrimental reliance. As a consequence, such a subcontractor can be held to his bid, effectively making bid withdrawal impossible (Lambert, 1970, p. 392). This is true even if the subcontractor made a

[9] Opportunistic behavior by the main contractor can happen even if there is no bidding for the subcontract. When the Air Force undertook a project to establish installations for detecting and determining the direction and yield of nuclear detonations in North America, General Electic partnered with Air Technology as a subcontractor in submitting a proposal to the Air Force. Air Technologies had considerable experience in the detection of EM radiation, and developped an electromagnetic sensory subsystem for the proposal. The Air Force then sent out a request for proposal to thirty-six companies, including GE. When GE was awarded the contract, and manned with the knowledge it obtained from Air Technologies, it invited competition on the subcontract for the sensor (Air Technology Corp. v. General Electric Co., 199 N.E.2d 538 (Mass. 1964)).

[10] McCandlish Elec. v. Will Constr. Co., 107 Wn. App. 85, 94 (2001).

mistake in his sub-bid.[11,12]

Subcontractors often invest significant time in preparing their bids, particularly in mechanical specialties like plumbing, heating, air conditioning and electrical (Schueller, 1960, p. 501). Also, since winning subcontractors can be held to perform on their bid, capacity limitations may prevent them from bidding on other projects, foregoing potential profits elsewhere (Closen and Weiland, 1980, p. 580). However, these costs are not recognized as detrimental reliance, as U.S. courts have universally ruled that upon award of the project the prime contractor is not bound to use the lowest bid subcontractor, even if he used the sub-bid in the calculation of his prime bid, and even if he listed this subcontractor in his proposal (Gregory and Travers, 2010, pp. 30-31).[13] The reason for this disparate treatment stems from contract law: the general contractor cannot accept the subcontractor's offer (bid) "unless and until it knows that its offer has been accepted by the owner" (Gregory and Travers, pp. 30-31).

3.3 Prevalence in the United States

While there is some controversy surrounding the prevalence of the practice,[14] there is clear evidence that bid shopping, and the associated phenomena of bid chopping and bid peddling, are widespread, and that concerns about these practices are first-order. Bid shopping has been, and continues to be, the subject of numerous legal cases brought by both prime contractors and subcontractors – Goldberg (2011, p. 552) makes reference to "more that 60 Drennan-style cases." Attempts to introduce anti-bid-shopping legislation at the federal level date back to 1932 (Schueller, 1960, p. 504). As we'll discuss below, concerns about the effects of bid shopping have led to industry and legislative attempts to curb the practice in nearly every U.S. state, and the passage of state legislation in more than half of them.

Survey research indicates that bid shopping takes place regularly (Arditi and Chotibhongs (2005), Thurnell and Lee (2009)). In a 2008 survey of U.S. and Canadian construction contractors, 80% of respondents knew of others who had engaged in bid shopping or bid peddling, and "an amazing 32% admit(ted) that 'they have bid shopped or peddled themselves' " (Gregory and Travers, 2010, p.29). A survey conducted by the Construction Management Association of America (FMI/CMAA, 2004) identifies bid shopping as one of the most critical issues perceived by owners, architects, engineers, construction managers, general contractors and subcontractors.

Finally, all the professional organizations involved in construction in the US strongly condemn bid shopping, going so far as to call the practice unethical. The Association of General Contractors of America (AGC) is "resolutely opposed" to bid shopping and bid peddling, and in its 1995 guidelines, issued jointly with the American Subcontractors Association (ASA) and the Associated Specialty Contractors, called their use "abhorrent business practices that threaten the integrity of the competitive bidding system." More specifically, the guidelines state:

> The bid amount of one competitor should not be divulged to another before the award of the subcontract or order, nor should it be used by the contractor to secure a lower proposal from

[11] For example, in Weitz Company v. Hands, Inc (2016, 294 Neb. 215.), a plumbing subcontractor who refused to perform on his winning subbid of $2,430,000 was found liable, and made to pay $242,000 in damages.

[12] For more details on promissary estoppel and the exact conditions for it to apply see Goldberg (2014, p.1073), Kovars and Schollaert (2006) and Marston et al (2013).

[13] For a review of some of these cases, see Oertly (1975, pp. 557-558).

[14] For example, in a 2015 report to the US Senate, the Government Account Office states that is was "unable to determine if bid shopping occurs or does not occur on federal construction projects" and that it "could not find evidence of bid shopping in the contract files we reviewed."

another bidder on that project (bid shopping). Neither should the subcontractor or supplier request information from the contractor regarding any sub-bid in order to submit a lower proposal on that project (bid peddling).

The preparation of bids, proposals, submissions or design-build documents is the result of professional consideration which is the intellectual property of the preparer, and so any such information should be considered proprietary. It is unethical to disclose to others, any information that is provided with an expectation that such information will be kept confidential.

The Code of Ethics of the American Society of Professional Estimators (ASPE) prohibits professional estimators from participating in bid shopping, stating that "this practice is unethical, unfair and is in direct violation of this Code of Ethics." The American Institute of Constructors (AIC) credentialing process includes a means of disciplining construction professionals who engage in unethical conduct. Its code of ethics specifically addresses unethical and deceptive practices, including bid peddling and bid shopping. Even some contractors have explicitly banned the practice: "many large general and specialty contractors have developed their own codes of ethics, which their employees are expected to sign" (Mojica and Clarke, 2007, p. 2).

3.4 Legal Treatment and Prevalence in Other Countries

Bid shopping is not limited to procurement in the United States. Court cases reveal that in Canada, *pre*-award bid shopping, and the associated phenomenon of last minute bidding, are prevalent.[15] Post-award bid shopping, however, has effectively been ruled out by a pair of influential court cases. Since R. v. Ron Engineering (1981), Canadian courts have interpreted a general contractor's request for bids to be a contract offer, and the submission of bids by contractors to be an acceptance of that offer, making subcontractors' bids irrevocable, even if they contain an error (Henley, 1991, p. 390). And Ron Engineering also established that when a general contractor uses a subcontractor's bid in his prime bid and names the subcontractor in his bid, a second contract is formed, binding the prime contractor to use this subcontractor if he is awarded the project, so either party is entitled to damages if the other party reneges (Henley, 1991, p. 385).[16] However, the existence of a contract, and hence its breach, is hard to prove if the main contractor does not name the winning subcontractor in his bid, and courts were reluctant to find so,[17] leaving the door ajar for post-award bid shopping. Interestingly, in its best practices guidelines the Canadian Construction Association urges main contractors to name subcontractors in their prime bids (CCA, 2015). Like its US counterpart, this professional organization also condemns bid shopping and bid peddling (CCA, 2008, p. 3):

All contractors (Prime Contractors and Subcontractors) should not seek nor accept information concerning a competitor's bid prior to bid closing, nor should they attempt to modify prices after

[15] In Gloge v. Northern (1986), Gloge telephoned its bid to Northern only minutes before bid closing, in order to prevent the general contractor from bid shopping among competing subcontractors (Blom, 1987, p. 213) . In Fred Welsh v. B.G.M.(1996) a critical element of Welsh's bid was withheld from BGM until ten minutes before the bid close, in an effort to prevent BGM from bid shopping (Pattison, 2004, p. 738) .

[16] Thus in Peddlesden v. Lidell (1981), Peddlesden submitted a bid which was used by Lidell in his subsequent bid to the owner. When the owner accepted this bid, Lidell did not wish to use Peddlesden, and substituted another subcontractor. Peddlesden sued and was awarded damages (Henley, 1991, p. 389).

[17] In Ron Brown v Johanson (1990), Brown was the lowest bidder on a mechanical subcontract. When Johanson was awarded the project, he used his own forces to do the mechanical work. Brown sued, arguing that Johanson had used his bid. However, because Brown was not named in Johanson's prime bid, the court refused to find for Brown (Henley, 1991, p. 393).

the bid closing. They should avoid any activity that could be construed as bid shopping or bid peddling.

Further confusing things, even after Ron Engineering, courts in some provinces, primarily Alberta and Ontario, ruled that neither the naming of a subcontractor nor carrying its bid requires the winning general contractor to use that subcontractor (Revay, 1997, p. 3), allowing bid shopping to continue. It was not until a second Supreme Court of Canada ruling in Naylor Group v. Ellis Don (2002) that both the prime contractors who seek bids and the subcontractors who bid in the prime auction became in a legally binding relationship. From then on, unless the owner (the procurer) objects, the winning general contractor is contractually bound to use their named subcontractor, and that subcontractor is legally bound to perform the work at their bid price. Post-auction bid shopping in Canada therefore became illegal.

Other Commonwealth countries, including England, Australia and New Zealand, follow the Canadian legal model in which two contracts are formed.[18] The first contract forms when bidders submit their offers to the main contractor, and is termed the Process Contract (Mullan, 2012, p. 177). The second contract forms when the main contractor is awarded the project, and is termed the Substantive Contract (Carnie, 2016). Yet case law in these countries is far less developed than in the US or Canada, and it is unclear how courts would handle bid shopping cases. Some commentators have speculated that New Zealand courts would take a dim view on bid shopping,[19] but there is currently no case law to support this stand. Indeed, survey evidence indicates that bid shopping is rampant in New Zealand.[20] In Australia, there is no legislation to prevent bid shopping, and codes of ethics are designed to curb its practice (May et al., 2001, p. 252). All jurisdictions have their own codes, which generally condemn bid shopping. For example, the New South Wales Government Code of Practice for Procurement contains the following passage:[21]

> Principals should not use tender negotiations as an opportunity to trade-off one tenderer's prices against other tenderers' prices in order to obtain lower prices. This practice, known as "bid shopping," is prohibited.

Like their U.S. counterparts, professional organizations take a dim view on bid shopping. For example, in 2009, the Master Building Association of Australia's Code of Ethics contained the following provisions (Uher and Davenport, 2009, pp. 210-211):

> No members shall make known the tender of any subcontractor or supplier before the closing of tenders. It is equally improper for a subcontractor or supplier to disclose his tender to another subcontractor or supplier prior to the closing of tenders ... No member shall attempt to solicit a contract after the closing date of tenders to the disadvantage of a lower tenderer ... Members shall treat all quotations from suppliers of materials and subcontractors as strictly confidential and shall not reveal competitors' prices to each other or to any architect or engineer ... A member

[18] The defining case for England and Wales is Blackpool & Fylde Aero Club Ltd v Blackpool Borough Council (1990), for Australia it is Hughes Aircraft Systems International v Airservices Australia (1997), and for New Zealand it is Pratt Contractors Ltd v Palmerston North City Council (1995).

[19] Carnie (2016) states that "where an owner ... participates in bid shopping, it might well be found to have breached the terms of the process contract, notwithstanding the existence of a privilege clause."

[20] Thurnell and Lee (2009, p. 1163) found that "bid shopping takes place regularly and is a matter of much concern to subcontractors."

[21] An identical passage is contained in the Victoria's Best Practice Guide for Tendering and Contract Management (2008, p. 26). The Guide also states that it "equally applies to the relationship that principal contractors have with their sub-contractors" (p. 8).

who invites a quotation from a supplier or subcontractor and incorporates such price in his tender shall, if the tender is accepted, award the work to the firm who supplied the quotation.

However, despite the presence of codes of ethics and tendering, bid shopping is actively employed by general contractors (Uher and Davenport, 2009, p. 211). Indeed, extensive survey evidence by Uher and Runeson (1985), May et al. (2001) and London (2005) indicates that bid shopping is endemic to the Australian construction industry, and bid shopping has been implicated in causing high degrees of insolvency among small Australian subcontractors (Coggins et al., 2016, p. 44).

3.5 Specific Concerns about Bid Shopping

In condemning bid shopping, contract awarding agencies, associations of general contractors, federations of subcontractors, other professional organizations, and legislatures note a variety of concerns about the practice. One often cited is that it is an attempt by general contractors to enrich themselves at the expense of the other stakeholders. For example, Lambert (1970, p. 395) states that "Any price reductions gained through the use of post-award bid shopping by the general will be of no benefit to the awarding authority." J.C.C., Jr. (1967) concurs, arguing that "the awarding authority receives no benefit since it has already agreed to pay the general a fixed sum."[22]

While correct ex-post, such a view is incomplete, because it ignores any ex-ante endogenous responses of the contracting parties to the presence of bid shopping. Indeed, anticipating a lower post-award cost, the general contractor may submit a lower bid for the prime contract, thereby benefiting the awarding authority. Likewise, in anticipation of being bid-shopped, subcontractors may inflate their pre-auction bids (Gregory and Travers, 2010, p. 32). All else equal, the latter would negatively affect the surplus of the general contractor and the awarding authority alike. In addition, concerns about being bid shopped, and thereby losing bid preparation costs and foregoing profits on other contracts they could have competed for instead, may reduce the number of subcontractors willing to bid on a job. Such a reduced participation rate would raise the general contractors' costs, raising the expected cost to the procuring agency. The welfare effects of bid shopping are therefore complex, and the goal of our paper is to shed some light on this issue.

Another concern often raised relates to quality of the subcontracted work. Armed with the leverage of being awarded the contract, and in an effort to increase profits, a general contractor will approach competing subcontractors during the buyout stage to obtain price concessions; to keep (or win) the job, subcontractors will be tempted to offer or accept substitution of materials or find other ways to cut corners. This is often done under the guise of 'value engineering,' and results in a specification and workmanship slide that ultimately harms the owner, who gets less value for his money and suffers the consequences of eventual failures. In effect, the general contractor "places a profit squeeze on the subcontractors, impairing their incentive and ability to perform to their best, and possibly precipitating bankruptcy."[23] In enacting the California Subletting and Subcontracting Fair Practices Act (1941, §4101), the California legislature explicitly recognized these dangers:

The Legislature finds that the practices of bid shopping and bid peddling in connection with

[22]The perspective that the only party gaining from bid shopping is the prime contractor engaging in the practice is still prevalent today. For example, the article "Why Bid Shopping Hurts The Masonry Industry," (Masonry Magazine, 2017) states that "the only one who benefits from bid shopping is the (general) contractor" and "There is no benefit seen by the public or the client."

[23]Southern Cal. Acoustics Co. v. C.V. Holder, 71 Cal.2d 719, fn. 7.

> the construction, alteration, and repair of public improvements often result in poor quality of material and workmanship to the detriment of the public, deprive the public of the full benefits of fair competition among prime contractors and subcontractors, and lead to insolvencies, loss of wages to employees, and other evils.

It should be noted that the concern with subcontractor insolvency is distinct from the problem of low subcontractor bid participation rates. Low participation rates stem from the subcontractor's desire to cover the fixed cost of bid preparation, whereas bankruptcy occurs because of an inability to cover the fixed overhead of machinery and the central office. The concern over bankruptcy is echoed by Baltz (1997, p. 24), who states that bid shopping "creates lower profit margins for subcontractors," the long-term effect of which may be "a reduction in the number of subcontractors... with a resultant decrease in competition for subcontract work and higher costs."

A final misgiving with bid shopping or bid peddling is that bid-shopped subcontractors must incur the cost of bid preparation, which others can free ride upon. Thus opportunistic subcontractors may refrain from preparing their own bids, essentially waiting until the buyout stage to undercut the lowest bidding subcontractor, and avoid bid preparation costs by relying on the estimating prowess of others (Baltz, 1997, p. 22).

3.6 Attempts to Curb Bid Shopping in the U.S.

Worries about the effects of bid shopping have led to various attempts to curb the practice by industry participants as well as state and federal legislatures.

Contractual Terms

One method that has consistently run into obstacles is contractual provisions. For example, in 2013 the ASA released an updated edition of its popular ASA Subcontractor Bid Proposal, a standard form and accompanying instructions that subcontractors can download from the ASA website and use in their bid proposals (Construction Business Owner, 2013). The revision included a new clause designed to deter general contractors from bid shopping:

> Subcontractor has devoted time, money, and resources toward preparing this bid in exchange for Customer's express agreement that the parties shall have a binding contract consistent with the terms of this bid proposal and Customer unconditionally and irrevocably accepts this bid proposal if it (A) in any way uses or relies on the bid proposal or information therein to prepare "Customer's bid" for the project at issue and Customer is awarded a contract for the work; or (B) divulges the bid or any information therein to others competing with Subcontractor for the work.

However, such unilateral attempts to create a binding contract have not held up in court. For example, in its bid proposal to general contractor West Construction for a public construction project, a Florida asphalt paving subcontractor, Florida Blacktop, included the following provision:[24]

[24] West Construction, Inc. v. Florida Blacktop, Fla. 4th Dist. Ct. App., 2012

> In the event the "buyer" in any way uses Florida Blacktop, Inc.'s bid ... such action(s) shall in all instances constitute acceptance of Florida Blacktop, Inc.'s bid and shall create a binding contract between the parties consistent with the bid documents.

West integrated Florida's bid into its own proposal, was awarded the prime contract, and furnished the owner with a list of proposed subcontractors that identified Florida as the asphalt paving subcontractor. Subsequently, West divulged Florida's bid to a third party paving subcontractor and awarded it the work. Florida had no recourse, because West never signed a formal contract with Florida; the court ruled that unilateral attempts to bind the general contractor are invalid, unless the general agrees in advance (Smith, Currie and Hancock LLP, 2013).

On the other hand, if prior to contract award the general contractor were to guarantee the subcontract would go to the lowest bidder, e.g. by signing a letter of intent conditioned only on receipt of the award, then a conditional bilateral contract would be formed and be enforceable (Closen and Weiland, 1980, p. 600).[25] However, a general contractor may be reluctant to commit to rules or otherwise engage in actions that create a conditional bilateral contract, as there are legitimate business reason for not working with the lowest bidder. Because of the hectic pace at which the sub-bids come in right before the general contractor must submit his bid, there are often gaps in the contract with various subcontractors, and the low bidder may turn out to be insufficiently qualified, or not to have the required office personnel or labor force to do the job adequately and in a timely fashion. Making such a deal with an ex-ante trustworthy subcontractor avoids this peril, but deprives the general of legitimate competition that may be needed to secure the prime contract (J.C.C., 1967, p. 1746). For this reason, general contractors will likely be advised by their construction attorneys not to make such promises before or at the time subcontractors submit their bids (Closen and Weiland, 1980, p. 600).

Bid Depositories

With contractual clauses proving ineffective in stopping bid shopping, one might expect industry efforts at self regulation to emerge. One form this has taken is the creation of "bid depositories." A bid depository is a facility created and run by a trade association of construction subcontractors, a bank, or an independent agency, which "receives bids from the subcontractors for the supplying of construction services or supplies and presents those bids *en masse* to the general contractors who intend to bid for the prime contract on a public or private construction job" (Orrick, 1967, p. 520). Use of the registry's services is typically open to all subcontractors and all prime contractors, regardless of their membership status in the registry, provided they agree to abide by its rules. Sanctions for violating those rules include suspension from registry services, expulsion, and circulation of the offender's name (Stewart, 1989, p. 34).

Operation of the registry starts once the awarding authority has announced that it is soliciting bids on a project. The depository then compiles and makes public a list of the prime contractors who indicate a wish to use its services. At their discretion, subcontractors may then deliver sealed bids for each listed prime

[25] For example, in AROK Construction Co. v. Indian Construction Services (174 Ariz. 291, 1993), prior to submitting its prime bid, Indian Construction successfully convinced AROK to reduce its sub-bid in exchange for a promise to award AROK the subcontract should its prime bid be successful. After winning the prime contract, Indian bid shopped and gave the job to a different subcontractor. AROK sued and the court found in its favor, ruling that Indian's agreement had created a conditional bilateral contract (Kovars, J. and M. Schollaert, 2006, p.10). Similarly, an electrical subcontractor was able to recover lost profits on two subcontracts where it was the lowest bidder, and had been made oral promises by the general contractor that "if we get the job, so do you" (Guarantee Electrical Construction Co. of St. Louis v. HS Construction Company, Mo. App. 2002).

contractor they wish to engage with, up to a deadline which is usually set three to four hours prior to the prime bid closing time. Once received by the registry, subcontractor bids may not be amended or withdrawn, or may be withdrawn only under penalty of a fine.

Depository rules typically require that subcontractors using the registry not submit any non-registered bids. Similarly, any prime contractor using the registry may not accept any bids from any subcontractor who did not submit a timely registered bid (Orrick, 1967, pp. 521-522).[26] In effect, this segments the market into two separate competing pools of prime contractors and subcontractors: those who use the registry and those who do not (Stewart, 1989, p. 35). Within the depository pool, bid shopping is thereby effectively ruled out. Indeed, pre-award bid shopping is precluded because the depository rules require subcontractors to submit definitive bids; post-award bid shopping is ruled out because the prime contractors participating in the depository must use depository sub-bids in their prime bids.

Bid depositories have operated in the U.S. since at least the 1930's (Schueller, 1960, p. 528), and at one time were very successful, with more than 100 depositories in operation in the early 1980's (Kaskell, 1981, p. 1). In some sense, they worked too well, and therefore came under antitrust scrutiny. Early cases focused on blatantly anticompetitive practices such as bid rigging, allocation of markets, boycotts and coercion.[27,28] Such practices are restraints of trade, and therefore per se violations of the Sherman Act; the depository institution itself was not on trial, as it merely acted as "an ancillary device to the primary conspiracy" (Stewart, 1989, p. 36). Starting in 1958 with the Bakersfield case,[29] however, state and federal litigation started attacking rules central to the operation of bid depositories, declaring illegal requirements that prime contractors using the depository may not accept outside bids or must report such bids to the registry, stipulations that preclude subcontractors using the depository from submitting bids to outside general contractors, or rules that prohibit generals from accepting subcontractor bids after the deadline of the depository. As a consequence, bid depositories became ineffective in combatting bid shopping or bid peddling, and currently there are only two bid depositories left operating within the United States, the Builders Bid Service of Utah and the Maine Construction Bid Depository.[30] These depositories do not prevent bid shopping, but still have some advantage in that they make bid shopping more apparent, as the bids are public (Carr, 2013, p. 2).

In Canada, bid depositories were formed in the 1950's by the Canadian Construction Association and key trade contractor associations, such as mechanical, electrical and masonry (Read, Kelly and Worthington, 2008, p. 27). The process is initiated by owners (procurers), who may elect to conduct the tender process through the bid depository. After owners submit full project plans to the depository, interested subcontractors then identify their work requirements, and submit their bids prior to a deadline set several days before the end of the prime contractor competition. The depository forwards the subcontractor bids to the owner and the general contractors who expressed an interest in bidding for the project (Coggins, Teng and Rameezdeen, 2016, p. 44). After reviewing the subcontractor bids, prime contractors 'nominate' their subtrades, i.e. select the subcontractors they wish to engage in each trade, and identify these parties

[26] Some depositories allow prime contractors to accept bids from outside subcontractors, but only if they register those bids in the depository themselves. One such example is the Electrical Bid Registration Service of Memphis (Stewart 1989, p. 34).

[27] Schueller (1960, pp. 528-530) contains an exhaustive list of 28 civil or criminal bid depository cases brought by the Department of Justice over the twenty year period 1939-1959.

[28] Two examples of such flagrant violations are the refusal of union labor to subcontractors who do not use the depository (Schueller, 1960, p. 508), and subcontractors eliminating competing bids by paying the competitor's fine for withdrawing his bid (Ducker, 1969, p. 759).

[29] United States v. Bakersfield Associated Plumbing Contractors, U.S. District Court, S.D. California, 1958.

[30] See http://www.buildersbidservice.com and https://agcmaine.org/about/maine-construction-bid-depository/.

in their bids to the owner. The winning prime contractor then must employ the named subcontractors in the construction project (Read, Kelly and Worthington, 2008, p. 27). The Canadian bid depository system effectively precludes all bid shopping after the close of the subcontractor auction. Indeed, Canadian courts have found the winning prime contractor liable for implementing the rules of the depository, in particular the employment of the named subcontractors at their deposited bids. Owners, because of their knowledge of this information and their duty of fair treatment and good faith, are also potentially liable, and have been dragged into court by subcontractors that were bid shopped.[31] Ever since the Canadian Supreme Court decision in Naylor Group v. Ellis Don (2002), bid shopping has been illegal, making it questionable whether the owner should use the bid depository system at all (Read, Kelly and Worthington, 2008, p. 146). As a consequence, bid depositories in Alberta, Ontario, Manitoba and Ottawa effectively ceased operation due to lack of use.[32] Today, bid depositories continue to operate in British Columbia, Saskatchewan and Quebec. In the latter province, use of the bid depository is mandatory for any project whose cost exceeds $20,000.[33]

Federal and State Legislation

The failure of contractual and self-regulatory attempts to curb bid shopping in the U.S. has prompted various attempts at legislative intervention. As early as 1932, bills were introduced in Congress mandating that prime contractors name the subcontractors to be used in their bids to the awarding agency (Schueller, 1960, p. 504). Since 2000, nearly every Congress has had legislation before it making such bid listing mandatory, only to see it die in chambers.[34] Thus, there is essentially no federal law protecting subcontractors from bid shopping. At one time, the Department of the Interior and the General Services Administration required subcontractor bid listing, but they stopped the practice in 1975 and 1983, respectively (GAO, 2015, p. 17). The only protection available to subcontractors comes from the revised Small Business Administration rules implementing the Small Business Job Act of 2010. These rules require that for covered contracts (those that require the submission of a subcontracting plan and whose value exceed a minimal threshold), the prime contractor must notify the contracting officer whenever it does not employ the small business subcontractor used in preparing its bid proposal.[35] Under the current Federal Acquisition Rules, a prime contractor who fails to make a "good faith" effort to use said subcontractor can be found to have materially breached its contract, and such failure can be used in the future to evaluate the prime's contract performance.[36]

State legislatures, in contrast, have had considerable success enacting statutes that require listing of subcontractors on public projects. The first such statute was introduced by North Carolina in 1925, and there has been a steady growth of states requiring bid listing since. By 1993, eighteen states already had some sort of listing requirement. Our comprehensive search reveals that as of 2020, that list has grown to twenty-two states, shown in Table 1.[37] A feature common to subcontractor listing laws is that the prime contractor

[31]Twin City Mechanical v. Bradsil, Ontario Court of Justice (1996) and Ken Toby v. B.C. Building Corporation, British Columbia Court of Justice (1997).

[32]See Versace (2008) and Read, Kelly and Worthington (2008, p. 28).

[33]Today, more than 5000 contractors are users of the electronic bid transmission system of the Bureau des Soumissions Déposées du Québec (BSDQ), which reaches an annual volume of more than 4500 projects, attracting some 40,000 bids (BSDQ, Rapport Annuel 2019, p. 4).

[34]Representative Paul Kanjorski introduced "Construction Quality Assurance Acts" in each of the 106th to the 111th Congress (HR4012 (2000), HR1859 (2001), HR1348 (2003), HR2834 (2005), HR3854 (2007) and HR3492 (2009)), Rep. Carolyn Maloney intruduced bills under the same name during the 112th and 113th Congress (HR1778 (2011), HR1942 (2013)), and Rep. Scott Peters introduced "Stop Unfair Bid Shopping Acts" in the 115th and 116th Congress (HR7221 (2018), HR5247 (2019)).

[35]Federal Register, Document No. 2013-16967, 78FR 42391-42406.

[36]Federal Acquisition Rules, paragraph 19.704, "Subcontracting Plan Requirements," sections a-12 and a-13

[37]In addition, the state of Wisconsin has a listing requirement for public work by municipalities.

must identify the names of the chosen subcontractors, their bids, and the work to be performed. It usually also includes a procedure for substituting a listed subcontractor, with the awarding authority's approval, for specified reasons. Bid listing laws are not foolproof in preventing bid shopping, however. While the majority of courts have upheld the listed subcontractor's right to perform the subcontract unless statutory reasons for substitution exist, a minority have ruled otherwise (Gregory and Travers, 2010, p. 33). For example, the Supreme Court of Kentucky found that the listing of a subcontractor's name and bid in a general's bid is solely for the protection of the awarding authority, and does not establish a contractual relationship between the named subcontractor and the general contractor (Goldberg, 2011, p. 570). State listing statutes may also limit the listing of subcontractors to certain trades only. For example, Kansas requires only the listing of electrical and mechanical subcontractors.[38] Some listing statutes also leave wiggle room for post-award bid shopping, because they do not require the bid list to be submitted until a specified period after the selection of the winning general contractor. For example, Alaska gives the winning prime five days after its selection to submit its list.[39] Finally, bid listing can be undermined by general contractors who auction off a position on the list to "subcontractors who are willing to perform the work for less than the lowest subbid received" (Daus and Ruprecht, 1990, p. 6).

Since bid listing offers limited protection against bid shopping, other states have come up with alternative remedies. One procurement method that has gained some acceptance is that of "separate specifications" or "multiple primes." Under this system, major subcontracts must be bid separately from the general contract. The oldest such law dates back to 1913, when Pennsylvania enacted its Separations Act, which requires separate first price auctions for plumbing, heating, ventilation, and electrical work in the erection, construction, or alteration of public buildings.[40] New York followed in 1921 with its famous Wicks Law, which mandates separate bidding for plumbing, ventilation and electrical in any building project exceeding a threshold.[41] North Dakota is the only other state where multiple primes is currently mandatory, while Ohio and Texas permit the system but do not require it. Separate specifications make bid shopping impossible, but have been criticized for being costly. Indeed, the system puts more oversight onus on the awarding agency, requiring expertise in dealing with each of the separate trades, as the general contractor has no contractual responsibility to coordinate their work with the specialists. Furthermore, generals now may find themselves working with subcontractors they have no familiarity or working relationship with. Indeed, concerns about the increased costs associated with the multiple prime system led one commentator to exclaim that Wicks Law is "the poster child of inefficiency in public procurement" (New York City Bar, 2008, p. 5). Preoccupation with inefficiency has led to repeated attempts to repeal or weaken Wicks Law,[42] which so far have failed. Empirical studies comparing the single and multiple prime system have focused on direct procurement costs, and thereby partly measure the effect of eliminating bid shopping, and have obtained somewhat mixed findings (Rojas, 2008).

The state of Massachusetts has a procurement system that avoids potential inefficiencies associated with the separate prime system, yet still guards against bid shopping, whether pre- or post-award. Under its

[38] Kansas Statutes Annotated § 75-3741(b)(2) (2018).

[39] Alaska Statutes § 36.30.115 (2018).

[40] Act of May 1, 1913, (P.L. 155, No. 104), as amended by 53 P.S.§1003 (Municipal) and 71 P.S. §1618 (State).

[41] Wicks Law is a collective reference to three separate laws, one that applies to the State (N.Y. State Fin. Law § 135), one that applies to public housing (Public Housing Law, Section 151-a), and one that applies to municipalities and other political subdivisions (General Municipal Law, Section 101).

[42] See e.g. New York Times, "Pro & Con: Wicks Law; Is Inefficiency Built Into Public Construction?" April 3, 1988.

Table 1: Current regulation of bid shopping by U.S. states

No regulation (21 states)	**Bid listing requirement (22 states)**	**Other regulations (7 states)**
Alabama	Alaska**	*Multiple primes (required)*
Arizona	Arkansas	New York
Colorado	California	North Dakota
Georgia	Connecticut	Pennsylvania
Indiana	Delaware	
Kentucky*	Florida	*Multiple primes (permitted)*
Louisiana	Hawaii	Ohio
Maine	Idaho	Texas
Maryland	Illinois	
Michigan	Iowa	*Filed sub-bids*
Minnesota	Kansas	Massachusetts
Mississippi	Nevada	Wisconsin***
Missouri	New Jersey	
Montana	New Mexico	
Nebraska	North Carolina	
New Hampshire	Oregon	
Oklahoma	Rhode Island	
South Dakota	South Carolina	
Vermont	Tennessee	
Virginia	Utah	
Wyoming	Washington	
	West Viginia	

* Kentucky requires bid listing, but court has ruled this does not create a binding contract
** Alaska: winning prime contractor has five days after selection to submit a subcontractor list
*** Wisconsin also has a bid listing requirement for public work by municipalities

"filed sub-bid system," bidding occurs in two stages.[43] First, subcontractors in each of eighteen trades must submit their bids to the awarding authority. Subcontractor bids may be restricted to certain contractors only, or may be unrestricted. At the end of this stage, the awarding authority compiles a list of all sub-bids received, and makes it available to all general contractors who have expressed interest. In a second stage, general contractors then submit their bids, for every trade including the name of the filed sub-bidder to be used in the work. The winning general must select its listed subcontractors to perform the work at the prices included in its bid. Generals are not required to use the lowest filed sub-bid in the preparation of their own bids. Indeed, at least one court case has ruled that being the lowest filed sub-bidder does not entitle one to the job.[44] However, the awarding authority may request that the general substitute the lowest filed sub-bidder. Massachusetts' filed sub-bid statute dates back to 1939, and for a long time was the only such system. In 2013, Wisconsin became the second state with a filed sub-bid system, but in contrast to Massachusetts, its statute requires prime contractors to use the winning subcontractors in each of the auctions for mechanical, electrical and plumbing work.[45]

[43] Massachusetts General Law, Part I, Title XXI, Chapter 149, Section 44F.
[44] Interstate Engineering Corp. v. City of Fitchburg, 367 Mass. 751 (1975), 329 N.E.2d 128
[45] The 2013 biennial budget (Wisconsin Act 20) amended Wisconsin Statute Wis. § 16.855 (Construction Project Contracts)

State statutes only govern public contracts, leaving subcontractors vulnerable in dealing with private owners as well as federal agencies. Furthermore, as Table 1 shows, only 29 U.S. states offer some form of protection from bid shopping on state contracts, meaning bid shopping is legal in auctions for state contracts in the other 21 states.

4 Model and Equilibrium

4.1 Terminology

With this background in mind, our aim in the next sections is to understand the equilibrium effects of bid shopping. If subcontractors and prime contractors are aware that bid shopping will occur, how does this change their behavior? And on balance, how do the outcomes that result compare to outcomes if bid shopping were impossible?

First, a note on terminology. In our setting, there are potentially two auctions: the main procurement auction perhaps being run by a government entity, and a general contractor's solicitation of competing bids from subcontractors, which can also take the form of an auction. To avoid confusion, we will typically refer to the former as the *prime auction*. Whenever we speak of subcontractor bids, or *sub-bids*, these are in the latter – bids a subcontractor submits to a prime contractor, which help determine the prime contractor's cost (and therefore her bid in the prime auction). As we'll see, however, under the assumptions of our model, a subcontractor's bid will often mechanically determine a prime contractor's bid, and it is therefore sometimes convenient to ignore the prime contractors and imagine the subcontractors competing directly against each other. This is a modeling abstraction, and hopefully does not cause confusion.

4.2 Model

While bid shopping can occur either before or after the prime auction occurs and the project is awarded, we focus on the latter case, post-award bid shopping. This is largely because submitting sub-bids at the last minute is a well-established and fairly effective countermeasure to pre-award bid shopping, but does not prevent post-award bid shopping. Further, "bid listing" legislation targets post-award bid shopping; as noted above, such legislation is in effect in roughly half of U.S. states, creating appealing variation for future empirical work once we know "what to look for."

A few key features we want in our model are that subcontractors face a significant cost to prepare a bid; subcontractors who are shown another subcontractor's bid may face a smaller bid preparation cost; and prime contractors can solicit multiple sub-bids before the prime auction, and may be able to solicit additional ones after. As we'll show, this (along with some simplifying assumptions) leads to a rich yet tractable framework that will help us to understand the equilibrium effects of bid shopping.

We will first introduce a benchmark model without bid shopping, then show two different ways we can modify it to incorporate bid shopping. We then establish equilibrium play for each model, and examine how each type of bid shopping changes equilibrium outcomes and payoffs.

from a multiple primes system to a filed sub-bid system.

Benchmark Model Without Bid Shopping

Our model has three types of players: a *procurer*, some *prime contractors*, and some *subcontractors*.

The procurer – say, a government agency – has a single project to be completed. He is committed to holding a second-price auction, with reserve price r. The procurer values completion of the project at v, and will receive payoff $v - p$ from successful completion of the project at price p, or payoff 0 if there are no bids in the prime auction and the project is not procured.

There are $m > 1$ *prime contractors* eligible to bid in the prime auction. One part of the project cannot be completed by any of the prime contractors, and must therefore be subcontracted. (Think of a plumbing or electrical component of a construction project, which the winning general contractor will outsource to a specialist firm.) The prime contractors face identical costs to perform the portion of the project that is not subcontracted; we normalize this cost to 0.[46] The winning prime contractor will therefore get surplus equal to the payment she receives from the procurer (either the second-lowest bid or the reserve price) minus the payment she makes to her subcontractor; losing prime contractors get payoff 0.

Finally, there is a set of *subcontractors* able to perform the portion of the project the prime contractors cannot. For concreteness, we will assume each subcontractor is associated with a single prime contractor, i.e., the prime contractors have non-overlapping sets of possible subcontractors, but this is not essential.[47] Let $n_i \geq 1$ be the number of subcontractors associated with prime contractor i, and $n = \sum_{i=1}^{m} n_i$ the total number of subcontractors. Subcontractors have independent private costs y_j to perform the subcontracted portion of the project; these costs are independent draws from a distribution F. Subcontractors face an additional cost c to prepare a sub-bid, and are assumed to *know their project cost* y_j when deciding whether or not to incur the cost c and prepare a bid.

For clarity, we will use female pronouns for prime contractors and male pronouns for subcontractors. Prior to the prime auction, each prime contractor solicits competing bids from her subcontractors to determine her cost, effectively holding a first-price auction among them and paying her lowest-bidding subcontractor his bid if she wins the contract. In the absence of bid shopping, the timing of the game is as follows:

1. Subcontractors learn their cost realizations y_j, and decide whether to incur the cost c and bid. Those who do, submit their bid to their associated prime contractor.

2. Those prime contractors who received a sub-bid, bid in the prime auction.

3. The low bid wins, and the contract is awarded; the procurer pays the winning prime contractor the second-lowest bid or (if there is no competing bid) the reserve price, she pays her lowest-cost subcontractor his sub-bid, they complete the project, and payoffs are realized.

We highlight one of our key modeling assumptions: that subcontractors know their private costs y_j when deciding whether to bid. While a real-life subcontractor might not know the exact cost to complete the project before speccing it out in detail, they are likely to know whether their costs are idiosyncratically higher or lower than a typical competitor's, based on their capacity, other commitments, physical location,

[46]Since the prime auction is a second-price auction, any prime contractor costs would simply shift their bids up by that amount.

[47]If the lowest-cost subcontractor submits bids to more than one prime contractor, then the lowest-cost prime contractors will have identical costs and will compete away all the surplus; this would affect the split of surplus between the winning prime contractor and the procurer, but not the bidding behavior of the subcontractors or the total surplus achieved in equilibrium. If every subcontractor who bids is assumed to submit bids to multiple prime contractors, then equilibrium outcomes would also be identical if the prime auction were a first-price rather than a second-price auction.

and so on. (We can think of a richer model where subcontractor costs are a mix of private and common, and imagine that the private component is known before entry and the common component is learned after entry; we are confident the results would be similar.) In addition, if subcontractors did not know their costs prior to entry, they would play a mixed-strategy equilibrium at the entry stage, earning zero expected profit. Since much of our interest is the impact of bid shopping on subcontractor profits, a modeling choice that fixed their profits at zero was unappealing. Of course, a richer model where subcontractors had some information about their costs and learned more upon entry would be most realistic, but too complex to work with.

We make some additional, mostly technical assumptions:

- The private cost distribution F is strictly increasing and differentiable on compact support $[\underline{y}, \overline{y}] \subset \mathbb{R}_+$, and admits a density function $f = F'$ which is continuous on the compact support.
- All players are risk-neutral, and all details of the environment are common knowledge among the players.
- Prime contractors play the dominant-strategy equilibrium of bidding their costs in the prime auction.
- To avoid trivialities, $v > c + \underline{y}$ and $r > c + \underline{y}$.

While we don't need it to understand equilibrium play, many of our welfare results will focus on the case where $r \leq v$. (Without bid shopping, as in Samuelson (1985), social surplus is maximized at $r = v$ and procurer surplus is maximized at $r < v$.)

Model with Bid Shopping to Insiders

We consider two different ways to extend the model to incorporate post-auction bid shopping, reflecting two possibilities for who the prime contractor might approach for a post-auction bid.

The first possibility is that the prime contractor might shop her best sub-bid to one of her subcontractors who already submitted a higher bid. Suppose a prime contractor receives pre-auction subcontractor bids of \$10,000, \$11,000, and \$14,000. After winning the prime auction, she shows the \$10,000 bid to the subcontractor who bid \$11,000, and asks if he can do the job for less than that. If his true cost is below \$10,000, he might indeed undercut the lower sub-bid. The prime contractor could then return to the original low bidder, show him the new lower sub-bid of, say \$9,750, and ask if he would like to lower his own bid, then repeat.

We call this a model of bid shopping to *insiders*. As a slight abstraction, we will assume that after the prime auction, the winning prime contractor holds her own second-price auction among her own subcontractors who had submitted pre-auction bids.

We assume the subcontractors are aware this will happen after the auction, and account for it in their bidding strategy. And crucially, if the game has a monotone equilibrium, even though the actual bid shopping occurs after the prime auction, the prime contractor will know her cost exactly at the time of bidding, since she will correctly infer each subcontractor's cost level from his initial sub-bid, and correctly foresee the cost reduction she will be able to get.

Model with Bid Shopping to Outsiders

The second possibility is that the winning prime contractor will instead shop her best sub-bid to an *outsider* – a subcontractor who has not already bid. Under this possibility, in addition to the n subcontractors who learn their costs and potentially submit sub-bids to their respective prime contractors before the auction, there are an additional $k \geq 1$ subcontractors who do *not* consider bidding pre-auction, and who could potentially be approached after the auction.[48] Some of these might be subcontractors who were not available pre-auction. (For example, a subcontractor might have already bid on a different job and not had the capacity to do both; after learning he didn't get the other job, he decides to bid on this one.) Alternatively, some of these might be subcontractors who could have bid pre-auction, but chose to delay strategically, planning to instead be available to bid after the prime auction. That is, the possibility of bid shopping might allow new subcontractors into the game who would not otherwise have been able to participate; and it also might divert some subcontractors who could have bid anyway from pre- to post-auction bidding. We allow for both of these; in the latter case, as we formalize later, we will assume that the subcontractor chose to wait to bid post-auction because it offered him higher expected profits; and to avoid signaling issues, we will assume that he made that decision before learning his cost.

We assume all subcontractors (pre- and post-auction) have costs y_j drawn from the same distribution F. As before, pre-auction subcontractors face a cost of c to prepare a sub-bid; post-auction subcontractors face a possibly lower bid preparation cost of αc, where $\alpha \leq 1$ reflects the fact that seeing a competitor's bid may allow a subcontractor to save time and effort preparing his own.[49]

With bid shopping, like without, the pre-auction subcontractors learn their private costs and decide whether to bid; those who do, submit sub-bids to their associated prime contractors. Each prime contractor who received a sub-bid calculates her expected cost (taking into account the possibility of receiving a lower sub-bid after the prime auction) and bids in the prime auction if her expected cost is below r; the auction occurs, and the winner is chosen. The game then continues:

1. The winning prime contractor chooses *one* post-auction subcontractor at random, shows him the lowest pre-auction sub-bid she received, and invites him to enter and bid against it

2. The new subcontractor forms beliefs about the cost y_j of the subcontractor who submitted that bid, and decides whether to incur the cost αc to bid against the incumbent.

    - If he does, the incumbent (pre-auction) and new (post-auction) subcontractors bid against each other in a second-price auction.
    - If he does not, the winning prime contractor sticks with the incumbent (pre-auction) subcontractor and pays him his pre-auction bid.

3. The procurer pays the winning prime contractor the second-lowest bid or the reserve price, the winning prime contractor pays her lowest-bidding subcontractor his bid, they execute the project, and payoffs are realized.

[48] Another possibility would be for the winning prime contractor to approach one of her subcontractors who *could have* bid pre-auction but chose not to. As we'll see, however, our model of bid shopping to outsiders has a symmetric, monotone equilibrium, so any sub who learned his cost pre-auction and chose not to bid would reveal himself to have cost y_j above the entry threshold, and therefore higher than the cost of any subcontractor who chose to bid. Since we assume post-auction bid preparation costs are still nonzero, such a subcontractor would not want to bid after the auction, anticipating that he would be outbid by the incumbent; so the winning prime contractor would have no reason to approach him for a bid.

[49] Multiple sources note the concern that bid shopping allows late subcontractors to free-ride on others' bid preparation efforts.

We assume that prime contractors who do not receive a pre-auction bid have expected ex post cost above r, and therefore do not bid in the prime auction.

4.3 Equilibrium

Equilibrium Without Bid Shopping

Next, we consider equilibrium play, beginning with the benchmark model without bid shopping. With prime contractors' construction costs normalized to zero, each prime contractor's cost is equal to the lowest subcontractor bid she received, which is what she bids in the prime auction. The lowest subcontractor bid therefore determines the winning prime contractor. The subcontractors, then, are effectively just in competition with each other, in an n-bidder first-price auction with a common entry cost. Since the subcontractors know their costs when they make entry decisions, the game among the subcontractors has a symmetric equilibrium where each subcontractor enters if his cost is below a common threshold level y_n^*.

Proposition 1 *Define y_n^* as the unique solution to*

$$(r - y_n^*)(1 - F(y_n^*))^{n-1} \quad = \quad c \tag{1}$$

and define a function $\beta_{n,0} : [\underline{y}, y_n^*] \to [0, r]$ by

$$\beta_{n,0}(y) \quad = \quad y + \frac{1}{(1 - F(y))^{n-1}} \left(c + \int_y^{y_n^*} (1 - F(s))^{n-1} ds \right) \tag{2}$$

The game with $n \geq 2$ subcontractors and no bid shopping has a unique[50] *symmetric equilibrium, in which subcontractors with costs $y > y_n^*$ do not bid, and subcontractors with costs $y \leq y_n^*$ bid $\beta_{n,0}(y)$.*

Proofs of all results are in the appendix. The intuition for Equation 1 is that the marginal entrant will win only if the other $n-1$ subcontractors don't bid, which happens with probability $(1 - F(y_n^*))^{n-1}$; will bid the reserve, and thus earn surplus $r - y_n^*$ when he wins; and must exactly cover his bid preparation cost with his expected ex post surplus. Since $(r - y)(1 - F(y))^{n-1}$ is decreasing in y, y_n^* is uniquely determined. The equilibrium bid function then follows from the equilibrium allocation (the lowest-cost subcontractor wins) and the envelope theorem. Note that y_n^* is increasing in r and decreasing in n and c, approaching $\underline{y}$ as $n \to \infty$ and $\min\{r, \overline{y}\}$ as $c \to 0$.

Since the equilibrium is symmetric, total surplus is straightforward to calculate. Ex post surplus is 0 if no subcontractors bid, and $v - y - \tilde{n}c$ if $\tilde{n} > 0$ subcontractors bid and the project is completed by a subcontractor with cost y. In equilibrium, each subcontractor bids with probability $F(y_n^*)$, and the project is completed by the lowest-cost subcontractor, provided the lowest cost is below y_n^*. Integrating $v - y$ times the density function of the lowest of n independent draws from F allows us to calculate expected surplus, and from there to derive a couple of useful comparative statics:

Corollary 1 *In the game without bid shopping,*

[50] Uniqueness is up to the actions of subcontractors with cost realization exactly y_n^*, who are indifferent between entering (and bidding $\beta_{n,0}(y_n^*) = r$) and staying out.

1. *Total surplus is*

$$W_{n,0} = \int_{\underline{y}}^{y_n^*} (v-s)n(1-F(s))^{n-1} f(s)ds - nF(y_n^*)c \quad (3)$$

2. *Total surplus is maximized at* $r = v$

3. *Total surplus can either increase or decrease in the number of subcontractors* n

Increasing the number of subcontractors has a complex effect on welfare. Holding the participation threshold fixed, it stochastically lowers the cost with which the project is implemented, but raises aggregate bid preparation costs. It also discourages participation, which decreases surplus whenever $r \leq v$. Which effect dominates – and therefore whether total surplus rises or falls when a subcontractor is added – depends on the cost distribution F and the magnitude of bid preparation costs c. For example, for $r \leq v$, total surplus increases in n if c is sufficiently small, but decreases in n if $f(\underline{y}) > \frac{1}{r-\underline{y}}$ and c is sufficiently large.

For completeness, without bid shopping, we can write the ex ante expected surplus of a subcontractor, a representative prime contractor, and the procurer, respectively, as

$$U_{n,0} = \int_{\underline{y}}^{y_n^*} F(s)(1-F(s))^{n-1} ds,$$

$$\Pi_i = \int_{\underline{y}}^{y_n^*} (1-(1-F(s))^{n_i})(1-F(s))^{n-n_i} \beta'_{n,0}(s)ds,$$

and

$$PS = (v-r)(1-(1-F(y_n^*))^n) + \int_{\underline{y}}^{y_n^*} \left(1-(1-F(s))^n - \sum_i (1-(1-F(s))^{n_i})(1-F(s))^{n-n_i}\right) \beta'_{n,0}(s)ds$$

The calculations are explained in the appendix. The expression for subcontractor surplus follows from the envelope theorem. The expressions for prime contractor and procurer surplus are standard expressions for payoffs in an (asymmetric) second-price procurement auction where bidder i's cost is drawn from the distribution $G_i(x) = 1-(1-F(\beta_{n,0}^{-1}(x)))^{n_i}$, followed by some simplification and a change of variable to $s = \beta_{n,0}^{-1}(x)$.

Equilibrium with Bid Shopping to Insiders

Next, we consider bid shopping to insiders – after the auction, the winning prime contractor returns to her subcontractors who have already bid, and uses competition among them to lower her cost. As noted above, abstracting away from the prime going back and forth as her subcontractors undercut each others' bids incrementally, we assume she runs a second-price auction among them. Since her subcontractors have already done the work of scoping out the project, they have no reason to stop competing until the price reaches their actual cost level. In equilibrium, then, the prime contractor will pay the lesser of two prices: the original *bid* of her lowest-cost subcontractor, and (assuming two or more entered) the actual *cost* of her second-lowest.

For tractability, for this model, we assume the prime contractors each have the same number of subcontractors, i.e., n_i is the same for all prime contractors. Our first result is that bid shopping to insiders does not change the entry threshold:

Proposition 2 *Assume n_i is the same for all prime contractors. In the game with bid shopping to insiders...*

1. *A symmetric, monotone equilibrium exists.*
2. *In any symmetric monotone equilibrium, the entry threshold is the same as in the game without bid shopping.*

The proof of equilibrium existence closely follows the logic of Athey (2001) – using a fixed point theorem to establish symmetric monotone equilibria for versions of the game with increasingly fine discrete strategy spaces, and then showing that the limit of these strategies is an equilibrium of the continuous game. The intuition for the second result is as follows. With bid shopping to insiders, the marginal subcontractor – the subcontractor with the highest cost level to enter – will still only win if he is the only entrant. (If his own prime contractor receives another bid, it will be from a subcontractor with lower costs, who he can't undercut post-auction. If his own prime contractor does not receive another bid, she'll only win the auction if none of her competitors receive a bid.) Thus, whatever is the entry threshold y^*, the marginal entrant will again bid the reserve price, and receive expected surplus of $(r - y^*)(1 - F(y^*))^{n-1} - c$ from entering. As before, the marginal entrant's payoff must be zero in equilibrium, so y^* must be the same as the entry threshold without bid shopping.

The symmetric equilibrium is tricky to fully characterize. For subcontractors with costs close to y^*, equilibrium bids are above y^*, and these bids therefore only compete "directly" against rival prime contractors who received just one subcontractor bid. Subcontractors with lower costs, however, may bid below y^*, meaning their bid can compete both against prime contractors who received a single subcontractor bid, and against prime contractors who received more than one (and whose cost is therefore based on the second-lowest *cost* among their subs, rather than the lowest *bid*). For this reason, below y^*, there can be gaps in the support of the bid function.[51] Nonetheless, Proposition 2 ensures existence of a symmetric monotone equilibrium, and that entry is the same as in the game without bid shopping; in the next section, this will allow us to derive clean welfare results for this model.

Equilibrium with Bid Shopping to Outsiders

Finally, we consider bid shopping to outsiders. Recall there are n "pre-auction subcontractors" – subcontractors who learn their valuations pre-auction and choose whether to bid – and an additional k "post-auction subcontractors," one of whom will be approached after the prime auction by the winner.[52] In equilibrium, the pre-auction subcontractors' bid function will be strictly monotonic, so by seeing the pre-auction bid, the post-auction subcontractor will have point beliefs $\hat{y}$ about the incumbent's cost level. Given his own cost level y_0 and bid preparation cost αc, the post-auction subcontractor's optimal strategy is straightforward:

[51] Normally in a first-price procurement auction, gaps are impossible because if no bidders bid within an interval $(\underline{b}, \overline{b})$, bidders whose equilibrium strategy called for them to bid $\underline{b}$ would deviate upwards to $\overline{b} - \epsilon$, winning just as often but at a higher price. But in our game, even if no subcontractors submit pre-auction *bids* in a range $(\underline{b}, \overline{b})$, if the density of subcontractor *costs* is high enough in that range, then the density of rival prime contractor costs in that range will be high, so subcontractors could still prefer bidding below that range to within it.

[52] The assumption that she will approach only one new subcontractor for a post-auction bid significantly simplifies the analysis; in a later section, we will relax this assumption. The assumption that she will find a new subcontractor for sure, rather than with probability less than one, is not essential. An earlier version of this model allowed post-auction subcontractors to have costs drawn from a different distribution than pre-auction subcontractors, and our equilibrium characterization and main results were identical; this different distribution could have a point mass at $+\infty$ to reflect a positive chance the prime contractor does not find a new subcontractor to bid.

- if $y_0 < \hat{y} - \alpha c$, enter and bid, expecting the incumbent to bid down to $\hat{y}$ leading to surplus $\hat{y} - y_0 - \alpha c > 0$
- if $y_0 > \hat{y} - \alpha c$, do not enter, as entering would lead to profits of $\max\{0, \hat{y} - y_0\} - \alpha c < 0$

From the point of view of a prime contractor who plans to bid-shop, then, upon receiving a low pre-auction bid of b from a subcontractor whose bid reveals him to have cost y, her expected ex post cost is

$$C(b, y) \quad = \quad (1 - F(y - \alpha c))b + F(y - \alpha c)y \tag{4}$$

since she will pay the incumbent's bid b if $y_0 > y - \alpha c$ and pay the incumbent's cost level y (to the post-auction subcontractor) if $y_0 < y - \alpha c$.

As in the case without bid shopping, there will be a symmetric monotone equilibrium with an entry cutoff above which subcontractors will choose not to bid. Since the prime contractor's expected cost is below the pre-auction sub-bid she receives, the marginal entrant can bid above the reserve; specifically, if he has cost y^*, the highest sub-bid b^* that would induce a prime contractor to bid is defined by $C(b^*, y^*) = r$. Since the marginal pre-auction subcontractor will only win if both the other pre-auction subcontractors *and the post-auction subcontractor* have costs too high to enter, the entry threshold with bid shopping will satisfy

$$(b^* - y^*)\,(1 - F(y^*))^{n-1}(1 - F(y^* - \alpha c)) \quad = \quad c \tag{5}$$

The expression defining b^*, $r = (1 - F(y^* - \alpha c))b^* + F(y^* - \alpha c)y^*$, implies (subtracting y^* from each side) that $r - y^* = (1 - F(y^* - \alpha c))(b^* - y^*)$, so Equation 5 simplifies to

$$(r - y^*)(1 - F(y^*))^{n-1} \quad = \quad c$$

This is identical to Equation 1, the analogous expression in the absence of bid shopping, so the entry threshold is again the same! (Fixing the number of pre-auction subcontractors, bid shopping to outsiders causes the marginal pre-auction entrant to win less often, but allows him to bid higher and therefore earn more surplus when he does win; the two effects happen to exactly offset each other, so the entry threshold is unchanged.) This leads to the following result:

Proposition 3 *Let y_n^* continue to refer to the solution to Equation 1 (the entry threshold without bid shopping). In the game with $n \geq 2$ pre- and $k \geq 1$ post-auction subcontractors, there is a symmetric equilibrium with participation threshold y_n^*; subcontractors with costs $y \leq y_n^*$ bid*

$$\beta_{n,k}(y) \quad \equiv \quad y + \frac{c + \int_y^{y_n^*}(1 - F(s))^{n-1}(1 - F(s - \alpha c))ds}{(1 - F(y))^{n-1}(1 - F(y - \alpha c))} \tag{6}$$

and subcontractors with costs $y > y_n^*$ do not bid.

As we discuss later, this is not the only symmetric equilibrium, but it is the only one that survives a common refinement, and the one we find most plausible. Given the choice of this equilibrium, we can easily compare subcontractor bids with and without bid shopping:

Corollary 2 *Fix the number of pre- and post-auction subcontractors $n \geq 2$ and $k \geq 1$.*

1. *$\beta_{n,k}(y) - \beta_{n,0}(y)$ is positive for y close to y_n^*, negative for y close to $\underline{y}$, and crosses zero once from below*

2. *For any cost realization y, a prime contractor's expected cost with bid shopping upon receiving the bid $\beta_{n,k}(y)$ is lower than $\beta_{n,0}(y)$*

That is, holding fixed the number of pre-auction subcontractors, the threat of bid shopping can cause a particular pre-auction sub-bid to be either higher or lower; but even when a pre-auction subcontractor bids higher, when combined with the prime contractor's expected post-auction cost reduction, the combined effect is always to lower the prime contractor's expected cost.

An immediate implication of Corollary 2 part 2 is therefore that *if* bid shopping does not alter the number of pre-auction subcontractors, it always benefits the procurer: the likelihood of successful procurement is the same, while the price paid is lower (for each realization of subcontractor costs). However, one effect of bid shopping to outsiders is that one or more subcontractors might wait to bid after the auction who otherwise would have bid before; this would effectively change the value of n, making the comparison of equilibrium with and without bid shopping more complicated. We consider this possibility in the next section, where we seek to understand the welfare effects of bid shopping.

5 Welfare Effects of Bid Shopping

In this section, we examine the welfare effects of bid shopping – that is, how equilibrium outcomes with bid shopping compare to equilibrium outcomes in the comparable setting without bid shopping.

5.1 Bid Shopping to Insiders

Effect on Total Surplus

We showed above that when bid shopping is to insiders – subcontractors who had already submitted losing bids – the equilibrium entry threshold y_n^* is the same as without bid shopping. This leads to a very clean result on the effect of bid shopping on total surplus:

Theorem 1 *Any symmetric monotone equilibrium of the game with bid shopping to insiders gives lower total surplus than the game without bid shopping.*

The intuition is as follows. Since bid shopping does not change the entry threshold (Proposition 2), each possible realization of subcontractor costs yields the same set of subcontractors choosing to bid (and incurring the bid preparation cost) with and without bid shopping. However, conditional on entry, the game without bid shopping ensures that the work is done by the lowest-cost subcontractor, while the game with bid shopping does not.[53] Thus, in expectation, bid shopping to insiders must lower total surplus.

[53] Suppose that in equilibrium, subcontractors bid 150% of their cost; and suppose prime contractor 1 receives pre-auction subcontractor bids of \$18,000 and \$21,000, while prime contractor 2 receives a lone sub-bid of \$15,000. With just one interested subcontractor, prime 2 will have no way to reduce her cost through bid shopping, and bids \$15,000 in the prime auction. Prime 1, on the other hand, infers that her subcontractors have costs of \$12,000 and \$14,000, and that by bid shopping she can reduce her cost to \$14,000 if she wins the prime auction; she therefore bids \$14,000. This leads to prime 1 winning, and the work being done by her lower-cost subcontractor at cost \$12,000, despite prime 2's subcontractor having lower cost of \$10,000. If there were just one prime contractor ($m = 1$), bid shopping could not distort the allocation away from the efficient one in this way, and (by Revenue Equivalence) bid shopping to insiders would have no effet on expected payoffs.

Distributional Effects

Due to the complexity of equilibrium (as discussed above), we do not have general results on the distributional effects of bid shopping to insiders. In one specific example (based on uniform F) we solved numerically and analyzed by simulation, bid shopping to insiders led to higher expected payoffs for both subcontractors and prime contractors, but significantly lower expected surplus for the procurer. While we are not sure these distributional effects always hold, we do have some intuition for why they are not surprising, and might be "typical."

First, consider prime contractors. Without bid shopping, a prime contractor's cost is the lowest sub-bid she has received. With bid shopping, however, her cost could be either her lowest sub-bid (in the event she receives only one, or it is particularly low), *or* the second-lowest cost among her subcontractors (in the event she receives multiple bids and successfully bid-shops). This means the distribution of possible prime contractor costs extends all the way down to the bottom of the support of the subcontractor cost distribution. In addition, since subcontractors' pre-auction bids are competing against fewer other subcontractors on the margin (since they can still compete post-auction with other subs bidding to the same prime), subcontractors' pre-auction bids are higher with bid shopping than without, at least for subs with costs close to the entry threshold. Together, these lead to a "spreading out" of prime contractor costs under bid shopping. Since the winning prime contractor's surplus is the difference between her own cost and the lowest cost among her rivals, this spreading out of costs is likely to benefit the prime contractors in expectation.

Next, consider subcontractors. We found above that bid shopping does not change the entry threshold for subcontractors. In addition, by symmetry, each subcontractor is equally likely to win, so bid shopping does not change a subcontractor's ex ante chance of winning. It does, however, change a sub's chance of winning conditional on his cost realization, since with bid shopping, the lowest-cost subcontractor will no longer always win. (See Footnote 53 for an example.) In particular, bid shopping will increase a subcontractor's chance of winning when he is "weak" (has cost close to y_n^*), but decrease his chance of winning when he is "strong" (has very low cost), without altering his chance of winning overall. These changes will typically increase his ex ante expected payoff if the cost distribution F is log-concave like many standard distributions, but decrease his expected payoff if F is log-convex.[54]

Finally, we explained above that bid shopping to insiders does not change the amount of entry, but does in a sense "spread out" prime contractor costs (and therefore their bids in the prime auction). The effect this has on the procurer is ambiguous. However, Theorem 1 tells us that total surplus must be lower with bid shopping; so whenever bid shopping benefits both prime contractors and subcontractors, it must reduce the surplus of the procurer.

[54]Let $\Pr(win|y)$ denote a subcontractor's equilibrium probability of being awarded the subcontract given his cost y. Standard envelope theorem arguments let us write a sub's ex ante expected surplus as $U = \int_{\underline{y}}^{y_n^*} F(y) \Pr(win|y) dy = \int_{\underline{y}}^{y_n^*} \frac{F(y)}{f(y)} \Pr(win|y) f(y) dy$. If F is log-concave, $\frac{F(y)}{f(y)}$ is increasing, so increasing $\Pr(win|y)$ for y high (when $\Pr(win|y)f(y)$ receives more weight in the integral) and decreasing it for y low (when $\Pr(win|y)f(y)$ receives less weight) is likely to increase the overall expression, increasing a sub's expected surplus; if F is log-convex and $\frac{F(y)}{f(y)}$ decreasing, the reverse holds.

5.2 Bid Shoppping to Outsiders

Who Are the Outsiders?

Next, we turn to the model of bid shopping to outsiders – subcontractors who have not already bid. To understand the welfare effects of this type of bid shopping, we first need to determine the right counterfactual to compare to. Suppose an environment with bid shopping has 10 pre-auction subcontractors and two post-auction subcontractors. Should this be compared to a setting without bid shopping and 12 subcontractors? 10? 11? This depends on "who the outsiders are," or on why they waited until after the auction to bid – whether they were unavailable before the auction, or chose to delay strategically. That is, the possibility of bid shopping might allow new subcontractors into the game who would not otherwise have been able to bid; but it also might cause some subcontractors who could have bid anyway, to wait and only bid after the auction. We refer to this as *diversion* – one or more subcontractors being diverted from pre-auction to post-auction bidding.

To allow for either possibility, we imagine the environment consists of $N_1 \geq 0$ subcontractors who can *only* bid pre-auction, $N_2 \geq 0$ who could bid *either* pre- or post-auction, and $N_3 \geq 0$ who can only bid post-auction. If bid shopping is not allowed, the $N_1 + N_2$ subcontractors in the first two groups will all bid pre-auction. If bid shopping is allowed, the N_2 subcontractors in the second group will each decide (based on ex ante payoffs) whether to be "pre-auction subcontractors" or "post-auction subcontractors," leading to a pool of $n \geq N_1$ potential pre-auction bidders and $k \geq N_3$ potential post-auction bidders, with $n + k = N_1 + N_2 + N_3$.

We think of N_1, N_2, and N_3 as underlying primitives, and n and k (the number of pre- and post-auction subcontractors when bid shopping is allowed) as endogenous objects derived from them. However, empirically, one can never hope to observe N_1, N_2, or N_3; one would instead observe n and k (if bid shopping is allowed), or $N_1 + N_2$ (if it is not). We would like to know what these observables allow us to say about the implications of permitting (or banning) bid shopping.

To do this, we will make a revealed preference-style assumption about subcontractors. We will assume that any subcontractor who would be available to bid pre-auction[55] if bid shopping were not allowed, *could* also bid pre-auction when bid shopping is allowed; so if he instead chooses to wait and bid post-auction, this must give him weakly higher expected surplus. That is, if bid shopping diverts any subcontractors from pre- to post-auction bidding, post-auction bidding must be at least as good for them, given when their competitors are choosing to bid. And we assume this calculation is done ex ante – in expectation over his possible cost realizations – but knowing how many other subcontractors will be available to bid pre- and post-auction.

To formalize this, let $U^{pre}_{n,k}$ and $U^{post}_{n,k}$ be the expected surplus of each pre-auction and post-auction subcontractor, respectively, in a setting with n pre- and k post-auction subs. (When $k = 0$, this corresponds to the case without bid shopping.) We make the following reduced-form assumption about how the games with and without bid shopping relate to each other:

Assumption 1 *Suppose that in the presence of bid shopping there are n pre- and k post-auction subcontractors. Then in the absence of bid shopping there would be $n + j$ subcontractors for some $0 \leq j \leq k$, and*

$$U^{post}_{n,k} \quad \geq \quad U^{pre}_{n+1,k-1} \tag{7}$$

[55] That is, would learn his cost y, and bid in the event it's below the entry threshold.

if $j > 0$.

That is, *if* there are fewer subcontractors competing pre-auction with bid shopping – because one or more have been diverted to post-auction bidding – then those subcontractors who were diverted must be getting weakly higher expected surplus than they would get by switching to pre-auction bidding. Assumption 1 is a necessary (but not sufficient) condition for (n, k) to result from a pure strategy equilibrium of a game in which the N_2 "flexible" subcontractors each choose whether to bid pre- or post-auction, either simultaneously or sequentially.[56] Note also that, since we assume that n and k are common knowledge when subcontractors make their entry and bidding decisions, we are implicitly assuming that if a pre-auction subcontractor switched and became a post-auction subcontractor (or vice versa), the other subcontractors would be aware of this switch.[57]

Under Assumption 1, in our motivating example above with 10 pre- and 2 post-auction subcontractors, there are three possibilities to consider to understand the welfare impact that bid shopping could be having. Define $W_{n,k}$ as expected total surplus.

1. One possibility is that neither post-auction subcontractor would be able to bid pre-auction, so if bid shopping were eliminated, only the 10 pre-auction subcontractors would be available to bid. In that case, the relevant welfare question is whether $W_{10,2}$ is greater or less than $W_{10,0}$.

2. A second possibility is that *one* of the post-auction subcontractors would be able to bid pre-auction and one would not. The relevant welfare question is whether $W_{10,2}$ is greater or less than $W_{11,0}$; and we can make use of the fact that $U^{post}_{10,2} \geq U^{pre}_{11,1}$.

3. The third possibility is that both post-auction subcontractors could bid pre-auction – both are bidding post-auction "by choice," and would bid pre-auction if bid shopping were eliminated. In this case, the relevant welfare question is whether $W_{10,2}$ is greater or less than $W_{12,0}$, and we can again use the fact that $U^{post}_{10,2} \geq U^{pre}_{11,1}$.

Overview of Welfare Results

The welfare impact of bid shopping depends in part on whether it diverts any existing subcontractors away from pre-auction bidding, or only introduces new post-auction subcontractors who otherwise would not have bid. To preview our results, it's useful to think of the welfare effect of bid shopping as having two components,

$$W_{n,k} - W_{n+j,0} \quad = \quad (W_{n,k} - W_{n,0}) + (W_{n,0} - W_{n+j,0})$$

The first term, $W_{n,k} - W_{n,0}$, is the effect of allowing the winning prime contractor to solicit another bid after the auction, holding fixed the number of pre-auction subcontractors; we label this the *competition effect.* The second term, $W_{n,0} - W_{n+j,0}$, is the effect of reducing the number of (pre-auction) subcontractors

[56] In the simultaneous-move game, equilibrium requires $U^{post}_{n,k} \geq U^{pre}_{n+1,k-1}$ if $j > 0$ and $U^{pre}_{n,k} \geq U^{post}_{n-1,k+1}$ if $j < N_2$. In the sequential game, consider the *last* of the N_2 subcontractors to choose to bid post-auction on the equilibrium path. By assumption, $k-1$ other subcontractors are already committed to post-auction bidding. By choosing post-auction bidding, he expects to receive $U^{post}_{n,k}$. If he deviated and chose pre-auction bidding, he would receive $U^{pre}_{n+1-z,k-1+z}$, where $z \geq 0$ is the number of subs after him who would choose post-auction bidding after his deviation. Since $U^{pre}_{n,k}$ is decreasing in n and constant in k, this is at least $U^{pre}_{n+1,k-1}$, so (7) is necessary, though not sufficient, for equilibrium in the sequential game as well.

[57] For consistency with the rest of the model, we assume a post-auction subcontractor who switched to pre-auction bidding would submit a bid to just one prime contractor, but as noted above, this assumption is not essential.

in the absence of bid shopping; we label this the *diversion effect*. Corollary 1 part 3 established that in general, the diversion effect can have either sign. Proposition 4 below will establish that the competition effect is always positive. This means that the overall welfare effect must be positive if $j = 0$ (there is no equilibrium diversion); Theorem 2 below will give sufficient conditions for this to be the case. Similarly, the overall welfare effect will be positive whenever the diversion effect is positive (sufficient conditions are given in Theorem 3 below), or smaller in magnitude than the competition effect (Theorem 4 below). On the other hand, when c is sufficiently small, the diversion effect is negative; and if, in addition, $j \geq 2$ (at least two existing subcontractors are diverted to post-auction bidding), the diversion effect outweighs the competition effect, making the overall welfare effect of bid shopping negative (Theorems 5 and 6 below).

Welfare Effects Without Diversion

The easiest case to analyze is when there is no diversion effect: bid shopping allows new subcontractors to bid who could not have bid otherwise, but does not divert any existing subcontractors from pre-auction bidding. In that case, the welfare effect can only be positive.

Proposition 4 *If $k \geq 1$, then for any r, $W_{n,k} \geq W_{n,0}$, with strict inequality if $y_n^* - \alpha c > \underline{y}$.*

The best intuition for this result comes from considering the combined externality imposed on the other players by the k post-auction subcontractors being added to the game. As noted above (Proposition 3), the presence of post-auction subcontractors does not alter the entry threshold y_n^*, and therefore does not affect whether the procurer successfully buys the project. Their presence does affect the price paid – the equilibrium bids of the pre-auction subcontractors, and of the prime contractors – but these are transfers, and do not affect the combined surplus of the "pre-auction players" (the procurer, the winning prime contractor, and the pre-auction subcontractors). In the event a post-auction subcontractor does replace the pre-auction low bidder, by assumption, he does so at a price which exactly matches the cost y of the subcontractor he replaces; as a result, the cost reduction to the prime contractor, $\beta_{n,k}(y) - y$, is exactly the same as the profit lost by the incumbent subcontractor. As a result, the net externality imposed by the addition of the post-auction subcontractors to the game without bid shopping is zero; so total surplus increases by the ex ante expected profits of the post-auction subs.

In the presence of bid shopping (when $k \geq 1$), we can calculate subcontractors' ex ante expected surplus as[58]

$$U_{n,k}^{pre} = \int_{\underline{y}}^{y_n^*} F(s)(1-F(s))^{n-1}(1-F(s-\alpha c))ds$$

and

$$U_{n,k}^{post} = \frac{1}{k}\int_{\underline{y}}^{y_n^*} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y_n^*))^n\right]ds$$

In certain cases, we can show that diversion cannot be profitable to a subcontractor, and thus cannot occur under Assumption 1. In these cases, then, Proposition 4 implies bid shopping to outsiders must increase total surplus. Recall that we assume $c < r - \underline{y}$ so that the game is not trivial; thus, when we say "c sufficiently large," we mean c sufficiently close to $r - \underline{y}$, not unboundedly large.

Proposition 5 *Fix n and the model primitives F, α, and r.*

[58]We established earlier that in the absence of bid shopping, $U_{n,0}^{pre} = \int_{\underline{y}}^{y_n^*} F(s)(1-F(s))^{n-1}ds$.

1. *For any $k \geq 1$, if c is sufficiently large, $U^{post}_{n,k} < U^{pre}_{n+1,k-1}$.*

2. *If k is sufficiently large, $U^{post}_{n,k} < U^{pre}_{n+1,k-1}$.*

Thus, if subcontractors' bid preparation costs are sufficiently high, it is never profitable for a subcontractor who could bid pre-auction to wait and bid post-auction. Similarly, if there are already sufficiently many other subcontractors planning to bid post-auction, it's never profitable for a subcontractor to wait. This leads to:

Theorem 2 *Under Assumption 1, if c is sufficiently large or if N_3 is sufficiently large (bid shopping introduces sufficiently many new post-auction subcontractors), then there is no equilibrium diversion, and bid shopping to outsiders increases total surplus.*

In both these cases, no subcontractors would choose to wait to bid post-auction voluntarily, so bid shopping leaves the number of pre-auction subcontractors unchanged, guaranteeing (by Proposition 4) that bid shopping increases total surplus.

Welfare Effects With Diversion

Bid shopping must also increase surplus when the diversion effect is positive – when $W_{n,0} > W_{n+j,0}$, so reducing the number of pre-auction subcontractors increases surplus on its own, even before accounting for the competition effect. One setting where this is the case is when y^*_n is close to $\underline{y}$ and F has sufficiently high hazard rate at the bottom of its support. This leads to the following result:

Theorem 3 *If $f(\underline{y}) > \frac{1}{r-\underline{y}}$, $r \leq v$, and n (the number of pre-auction subcontractors when bid shopping is allowed) is sufficiently large, $W_{n,0} > W_{n+j,0}$, and bid shopping to outsiders therefore increases total surplus.*

Another case where we can often characterize the welfare effect of bid shopping is when exactly one subcontractor is diverted from pre- to post-auction bidding. First, two preliminary results:

Proposition 6 1. *If F has increasing hazard rate, then $U^{post}_{n,k} \geq U^{pre}_{n+1,k-1}$ implies $U^{post}_{n,1} \geq U^{pre}_{n+1,0}$.*

2. *If $r = v$, then $U^{post}_{n,1} \geq U^{pre}_{n+1,0}$ implies $W_{n,1} > W_{n+1,0}$.*

Part 1 says that if a subcontractor can gain from waiting to bid post-auction, he would also gain from waiting if there were no other post-auction subcontractors; interestingly, this does not always hold when F does not have an increasing hazard rate.[59] To prove part 2, we show that when $r = v$ and there are no other post-auction subcontractors, a single pre-auction sub switching from pre- to post-auction bidding imposes a positive net externality on the other players in the game. This implies that if his own expected payoff goes up ($U^{post}_{n,1} \geq U^{pre}_{n+1,0}$), total surplus goes up.

Proposition 6 implies the following:

Theorem 4 *Suppose $r = v$ and Assumption 1 holds. If F has increasing hazard rate, then if bid shopping causes exactly one subcontractor to divert from pre- to post-auction bidding, it increases total surplus.*

[59] Example 1 below offers a setting where $U^{post}_{2,2} > U^{pre}_{3,1}$ and $U^{post}_{2,1} < U^{pre}_{3,0}$. That said, the conclusion of Proposition 6 part 1 also holds under the weaker assumption that the hazard rate of F not decrease too dramatically. The result also holds if $F(y^*_n) \leq \frac{1}{2}$, i.e., if bid preparation costs (or the number of subcontractors) are sufficiently high that on average, fewer than half the available subcontractors submit bids.

Even if F does not have an increasing hazard rate, the second part of Proposition 6 still holds, which implies that if bid shopping causes one subcontractor to divert to post-auction bidding and does not attract any new subcontractors ($N_3 = 0$), then bid shopping increases total surplus. This means that if there is exactly one post-auction subcontractor when bid shopping is allowed, total surplus must be higher with bid shopping than without: either the post-auction subcontractor was diverted from pre-auction bidding (in which case Proposition 6 part 2 applies), or he was not (in which case Proposition 4 applies). Example 2 in the appendix (following the proof of Proposition 6) offers an example of the former case, where bid shopping does not change the overall number of subcontractors in the game, but leads a single subcontractor to switch from pre- to post-auction bidding.[60] Since $r = v$ is welfare-maximizing in the absence of bid shopping, Theorem 4 implies that bid shopping with either $r = v$ or with r set at the social optimum gives higher total surplus than the same environment without bid shopping and *any* reserve price.[61]

When can bid shopping be welfare-decreasing?

Theorem 2 established that if c is sufficiently large, or if bid shopping attracts sufficiently many new subcontractors to the game, then bid shopping can only increase total surplus. However, we next show that at the opposite extreme – if c is small and $N_3 = 0$ (bid shopping does not attract any new subcontractors) – bid shopping can only decrease total surplus.

Theorem 5 *Suppose $r \leq v$ and Assumption 1 holds. If c is sufficiently small and bid shopping does not attract any new subcontractors, then if bid shopping has any effect, it reduces total surplus.*

The key step of the proof is to show that if c is sufficiently small, $U^{post}_{n,1} < U^{pre}_{n+1,0}$, which makes it impossible under Assumption 1 for bid shopping to divert exactly one subcontractor. This means that if bid shopping does not attract new subcontractors, it either has no effect, or it diverts more than one subcontractor from pre- to post-auction bidding. When c is close to zero, however, total surplus is effectively just $v - y_{min}$, where y_{min} is the realized cost of the lowest-cost subcontractor who gets to bid. With bid shopping, just one of the k post-auction subcontractors gets an opportunity to bid, so y_{min} is the lowest of $n + 1$ independent draws from F. If all the post-auction subcontractors were diverted from pre-auction bidding, then without bid shopping, y_{min} would be the lowest of $n + k$ independent draws; with $k \geq 2$, y_{min} is therefore lower in expectation without bid shopping.

To show that Theorem 5 is not vacuous and to build intuition, we next present an example with small c where two subcontractors find it profitable to wait and bid after the auction, reducing total surplus.[62]

Example 1 *There are four subcontractors, each of whom could bid pre- or post-auction if bid shopping is allowed. The procurer's valuation and the reserve price are both* 1.1, *and the cost distribution has support* $[0, 1]$ *and cumulative distribution*

$$F(y) \quad = \quad \frac{1}{1 + \frac{2}{3}\left(\frac{1-y}{y}\right)^{\epsilon}}$$

[60] The same example illustrates that Proposition 6 part 2 can require $r = v$; when v is significantly higher than r, diversion is still profitable for one subcontractor, but bid shopping now decreases total surplus.

[61] The probability of a failed auction, $(1 - F(y^*_n))^n$, can be either increasing or decreasing in n, depending on the details of the cost distribution F. The conclusion of Proposition 6 part 2 holds for any $r < v$ if $(1 - F(y^*_{n+1}))^{n+1} \geq (1 - F(y^*_n))^n$, but can fail if $(1 - F(y^*_{n+1}))^{n+1} < (1 - F(y^*_n))^n$, i.e., if reducing the number of pre-auction subcontractors by one reduces the likelihood of successful procurement. A sufficient condition for $(1 - F(y^*_{n+1}))^{n+1} \geq (1 - F(y^*_n))^n$ is $\frac{f(y)}{1-F(y)} \geq \frac{1}{r-y}$ for $y \in (y^*_{n+1}, y^*_n)$.

[62] In the example, $U^{post}_{2,2} \geq U^{pre}_{3,1}$; but since $c \approx 0$, $U^{post}_{2,1} < U^{pre}_{3,0}$. This does not contradict part 1 of Proposition 6 because the example does not have an increasing hazard rate.

We consider a double limit, first taking c to zero and then taking ϵ to zero. As c goes to zero, y_n^* and $F(y_n^*)$ both go to 1 for any n.[63] *As ϵ goes to zero, for any $y \in (0,1)$, $F(y) \to 0.6$, so in the limit, F is a bimodal distribution putting weight 0.6 on $y = 0$ and weight 0.4 on $y = 1$.*

In this setting under Assumption 1, bid shopping could result in two of the four subcontractors choosing not to bid pre-auction and instead waiting to be approached by the winning prime contractor after the auction. If it does, total surplus is lower than if bid shopping were banned and all four subcontractors bid pre-auction.

We say above that bid shopping *could*, not *does*, lead to two subcontractors waiting to bid post-auction, because there are multiple equilibria in the "subcontractors choose when to bid" game. Since $c \approx 0$, Theorem 5 implies $U^{pre}_{4,0} > U^{post}_{3,1}$, so it is also plausible that all four subcontractors would choose to bid pre-auction, in which case allowing bid shopping in this setting would have no effect.

In this example, since two subcontractors wait to bid post-auction, it must be that $U^{post}_{2,2} \geq U^{pre}_{3,1}$. The calculations are in the appendix, but the intuition for why is this. With $c \approx 0$ and $r > \overline{y}$, $F(y^*) \approx 1$ and the auction never fails. With or without bid shopping, as $\epsilon \to 0$, pre-auction subcontractors with high costs $y = 1$ bid 1, and pre-auction subcontractors with low costs $y = 0$ mix on a range whose support extends up to 1. (If they did not, then low-cost subs who planned to bid at the top of their range would have a profitable upward deviation.) With two pre- and two post-auction subs, each post-auction sub knows he'll be selected to bid with probability one-half, and will then earn surplus of 1 if both pre-auction subcontractors had costs of 1 and he has costs of 0; so his expected surplus is $\frac{1}{2} \cdot 0.6 \cdot 0.4^2 = 0.048$. If instead there were three pre- and one post-auction sub, we can calculate a pre-auction sub's expected surplus from the case where he bids just under 1 if he has low cost; in this case, he'll earn surplus of 1 if he has cost $y_j = 0$ and the other two pre-auction subs *and* the post-auction sub all have costs $y_j = 1$, which occurs with probability $0.6 \cdot 0.4^3 = 0.0384$. Thus, $U^{post}_{2,2} > U^{pre}_{3,1}$, and two post-auction subcontractors is consistent with Assumption 1. As for total surplus, with $c \to 0$, total surplus is simply the expected value of $v - y_{min}$, where y_{min} is the lowest cost realization among the bidders who get to bid; since with bid shopping, only three of the four subcontractors bid, the expected value of y_{min} is higher, and total surplus is therefore lower with bid shopping.

Given part 2 of Proposition 6, for bid shopping to be surplus-decreasing under Assumption 1, there must be at least two post-auction subcontractors, at least one of whom could have bid pre-auction, which requires $U^{post}_{n,k} \geq U^{pre}_{n+1,k-1}$ with $k \geq 2$. This is easiest to satisfy with $k = 2$ and $\alpha = 0$, so in general, a necessary condition for this is

$$\frac{1}{2}\int_{\underline{y}}^{y_n^*} F(s)\left[(1-F(s))^n - (1-F(y_n^*))^n\right] ds \quad > \quad \int_{\underline{y}}^{y_{n+1}^*} F(s)(1-F(s))^{n+1} ds \tag{8}$$

Example 1 satisfies this condition because in the limit, $y_n^* = y_{n+1}^*$, so the regions of integration of the two integrals are the same; $1 - F(y_n^*) = 0$, so the term being subtracted on the left vanishes; and $1 - F(s) = 0.4 < \frac{1}{2}$ on the entire range of integration, so the additional $(1 - F(s))$ term in the integrand on the right is smaller than the $\frac{1}{2}$ multiplying the left side.

More generally, we should expect (8) to be satisfied if $F(y_n^*)$ is close to 1 (again making the two regions of integration similar and the term $(1 - F(y_n^*))^n$ subtracted on the left-hand side small) and F is sufficiently concave (so that it's above $\frac{1}{2}$ for a lot of its support). The former holds when c is small and $r \geq \overline{y}$, as

[63] Since $c \to 0$, $\alpha c \to 0$, so the value of $\alpha \in (0, 1]$ is irrelevant.

y_n^* would then be close to $\overline{y}$. In addition, as discussed above following Theorem 5, when c is sufficiently small and bid shopping reduces the number of pre-auction subcontractors by at least two, it decreases total surplus. This leads to the following result:

Theorem 6 *Suppose $r \geq \overline{y}$, $N_2 \geq 2$, and $N_3 = 0$ – bid shopping does not allow in any new subcontractors, but at least two subcontractors can choose whether to bid pre- or post-auction. Under Assumption 1, bid shopping to outsiders can reduce total surplus if c is sufficiently small and $\log f$ decreases sufficiently quickly ($(\log f(y))' < -\lambda$ for λ sufficiently large).*

Once again, we say bid shopping *can*, not *does*, reduce total surplus. Since c is small, $U_{n,0}^{pre} > U_{n-1,1}^{post}$, so while one possible outcome under Assumption 1 is for two or more subcontractors to divert to post-auction bidding, reducing surplus, another possible outcome is for no subcontractors to divert, in which case the availability of bid shopping has no effect.

Distributional Effects of Bid Shopping to Outsiders

The distributional effects of bid shopping to outsiders depend on whether or not it reduces the number of pre-auction subcontractors.

Theorem 7 *When bid shopping is to outsiders...*

1. *If bid shopping does not change the number of pre-auction subcontractors, then it reduces the expected surplus of each pre-auction subcontractor and increases the surplus of the procurer.*
2. *If bid shopping reduces the number of pre-auction subcontractors, then it increases the expected surplus of each remaining pre-auction subcontractor, and can increase or decrease the surplus of the procurer.*
3. *Whether or not bid shopping reduces the number of pre-auction subcontractors, it can either increase or decrease prime contractors' profits.*

For the first result – on bid shopping that does not change the number of pre-auction subcontractors – we established above that in that case, bid shopping does not change the entry threshold for subcontractors to bid, and therefore does not change the likelihood the prime auction is successful. With bid shopping, the pre-auction subcontractors face greater competition than without, and therefore earn lower expected surplus. (Pre-auction subcontractors with costs right at the entry threshold y_n^* earn zero surplus either with or without bid shopping; subcontractors with costs below that earn strictly lower surplus, due to a lower equilibrium likelihood of winning.) The procurer, on the other hand, benefits from each prime contractor having lower expected costs (Corollary 2 part 2) and therefore bidding lower.

For the second result – on bid shopping that reduces the number of pre-auction subcontractors – the remaining pre-auction subs are now facing reduced pre-auction but increased post-auction competition. By assumption, they only face one additional subcontractor post-auction, and at least one fewer subcontractor pre-auction; even if they swap just one for one, they benefit from facing this last competing subcontractor post-auction, since (assuming $\alpha > 0$) the post-auction subcontractor will not enter when his cost is only slightly below the incumbent's, increasing the pre-auction subs' chances of winning. The procurer's surplus, however, depends on how the subcontractors are distributed among prime contractors without bid shopping and how this changes with bid shopping, and it is straightforward – even in otherwise identical environments – to generate examples where bid shopping can either benefit or harm the procurer.

For the third result, prime contractors' profits similarly depend on the distribution of subcontractors across prime contractors and other details of the environment, and can similarly either increase or decrease with bid shopping. When bid shopping does not reduce the number of pre-auction subcontractors, it reduces each prime contractor's expected cost for each realization of subcontractor costs (Corollary 2 part 2); each prime benefits from the reduction in her own costs, but is hurt by the reduction in her opponents' costs. When c is sufficiently high (or r sufficiently low), the net effect is positive: with a low enough entry threshold for subcontractors, most of a prime contractor's profits come from auctions where she is the only one to bid, so the benefit from the reduction in her own costs is larger than the harm from the reduction in her opponents'. When entry costs are low, however, prime contractor surplus often decreases with bid shopping – and it can either increase or decrease when bid shopping causes a reduction in pre-auction subcontractors.

We can also consider the effect of a reduction in post-auction subcontractors' bid preparation costs:

Proposition 7 *A decrease in α (the fraction of bid preparation costs borne by post-auction subcontractors) increases total surplus, procurer surplus, and the surplus of post-auction subcontractors, and decreases the surplus of pre-auction subcontractors.*

Unsurprisingly, post-auction subcontractors benefit directly from having low bid preparation costs, which also lowers procurer's costs, while pre-auction subcontractors suffer from the increased competition. Since bid shopping increases total surplus by the post-auction subs' surplus, total surplus also rises more when post-auction subcontractors have low bid preparation costs; the effect on prime contractors is ambiguous.

6 Bid Shopping to Outsiders – Complications and Extensions

In this section, we consider a number of complications and extensions to the model of bid shopping to outsiders.

6.1 Endogenous Reserve Price

Our results so far have compared bid shopping to no-bid-shopping at the same reserve price. We can also consider what would happen if the procurer adjusts the reserve price to maximize his expected surplus.

For this extension, we need to make a simplifying change to our model. We saw above that total surplus, and the expected surplus of each subcontractor, depend on the number of pre-auction subcontractors but not how those subcontractors are allocated among the prime contractors; the division of the remaining surplus between the prime contractors and the procurer, however, does depend on this. We also noted above (in and around footnote 47) that our assumption that each subcontractor submits a bid to just one prime contractor is made for concreteness, but that most of our results would be identical if some subs submitted bids to more than one prime contractor. For this section, it is more expedient to assume that *every* subcontractor submits bids to at least *two* prime contractors. This means the two lowest-cost primes will have identical costs, and the procurer will receive all surplus not captured by subcontractors. We will also make two additional technical assumptions.

Assumption 2 *(i) Each subcontractor submits bids to at least two prime contractors; (ii) F is such that $y + \frac{F(y)}{f(y)}$ is strictly increasing; and (iii) $\alpha \approx 0$.*

Note that a sufficient (but not necessary) condition for (ii) is for F to be log-concave, which many commonly-used distributions are. Under these assumptions, we can characterize how a surplus-maximizing procurer would set the reserve price:

Proposition 8 *Under Assumption 2,*

1. *In the absence of bid shopping, procurer surplus with n subcontractors is single-peaked in r and maximized at the solution to*

$$r = v - \frac{F(y_n^*)}{f(y_n^*)} \tag{9}$$

2. *In the presence of bid shopping to outsiders, procurer surplus with n pre-auction and k post-auction subcontractors is single-peaked in r and maximized at the solution to*

$$r = v - (1 - F(y_n^*))\frac{F(y_n^*)}{f(y_n^*)} \tag{10}$$

We saw earlier that without bid shopping, total surplus is maximized at $r = v$. With bid shopping, the socially optimal reserve price is *above* v:

Proposition 9 *In the absence of bid shopping, total surplus is single-peaked in r and maximized at $r = v$. In the presence of bid shopping to outsiders, total surplus is single-peaked in r and maximized at the solution to*

$$r = v + \int_{\underline{y}+\alpha c}^{y_n^*} F(s - \alpha c)ds$$

Since prices (transfers) are welfare-neutral, the reserve price affects total surplus only through its impact on the entry threshold y_n^*. Intuition for Proposition 9 therefore comes from thinking about the externality caused when the marginal subcontractor – one with costs $y = y_n^*$ – chooses to enter. Since he'll make the highest equilibrium bid, his presence only affects the final allocation when he's the only entrant – in which case his entry causes the procurer to earn $v - r$ instead of 0, and the prime contractor to earn 0 in expectation. The externality is therefore proportional to $v - r$, so entry is below the efficient level when $r < v$ (and therefore total surplus is increasing in r), and efficient when $r = v$. With bid shopping, however, the marginal entrant imposes an additional positive externality: by allowing the auction to succeed when he's the sole entrant, he also creates positive surplus for the post-auction subcontractor, who earns (in expectation) $\int_{\underline{y}}^{y_n^* - \alpha c}(y_n^* - y_0)dF(y_0)$. Simplifying, the externality is proportional to $v - r + \int_{\underline{y}+\alpha c}^{y_n^*} F(s - \alpha c)ds$, so surplus is increasing in r as long as this term is positive.

Proposition 9 implies that both with and without bid shopping, total surplus is increasing in the reserve price as long as $r \leq v$. Proposition 8 established that the procurer's optimal reserve is always below v. Together, these make the welfare effect of a change in reserve in the relevant range easy to understand, leading to the following results.

Theorem 8 *Under Assumption 2, if the procurer is setting the reserve price to maximize his expected surplus,*

1. *If bid shopping to outsiders does not change the number of pre-auction subcontractors, it leads to an increase in the reserve price, which further increases total surplus beyond the initial gain from bid shopping.*

2. *Suppose bid shopping to outsiders reduces the number of pre-auction subcontractors from $n+j$ to n. Let r^*_{n+j} denote the procurer-optimal reserve price without bid shopping and with $n+j$ subs, and $y^*_n(r^*_{n+j})$ and $y^*_{n+j}(r^*_{n+j})$ the entry thresholds at that reserve price.*

   If
   $$(1 - F(y^*_n(r^*_{n+j}))) \frac{F(y^*_n(r^*_{n+j}))}{f(y^*_n(r^*_{n+j}))} \quad < \quad \frac{F(y^*_{n+j}(r^*_{n+j}))}{f(y^*_{n+j}(r^*_{n+j}))} \tag{11}$$
   then bid shopping leads to an increase in the reserve price, which increases total surplus (relative to bid shopping at the old reserve price).

   If Equation (11) *does not hold, then bid shopping leads to a decrease in the reserve price, which decreases total surplus.*

Since $y^*_n > y^*_{n+j}$, Equation (11) will hold – and bid shopping will therefore lead to a higher reserve price, increasing total surplus – if $\frac{F(y)}{f(y)}$ is decreasing or nearly constant in the relevant range, which holds if F is locally log-convex or "not very log-concave." It also holds if $F(y^*_n(r^*_{n+j}))$ is high – i.e., if the bid preparation cost is low enough for "most" subcontractors to enter – or if y^* does not change much in response to a change in n.

The main takeaway from Theorem 8 is that an endogenously-set reserve price does not reverse the other welfare results. When bid shopping to outsiders does not cause any existing subcontractors to switch from pre- to post-auction bidding, Proposition 4 said that with fixed reserve price, total welfare increases with bid shopping; Theorem 8 part 1 says that with the reserve adjusted to maximize the procurer's surplus, total welfare increases even more. When bid shopping does lead some existing subcontractors to wait until after the prime auction to consider bidding, the welfare effect with a fixed reserve price can go either way depending on primitives, and the added effect of an endogenously-adjusting reserve price can similarly go either way.

6.2 Equilibrium Multiplicity

Proposition 9 established that when bid shopping is allowed and $r \leq v$, pre-auction entry is below the efficient level. We should therefore expect changes that increase the entry threshold to typically increase total surplus on the margin, and changes that decrease the entry threshold to decrease total surplus. This intuition helps with a number of other extensions.

The results on bid shopping to outsiders in Sections 4 and 5 are based on the symmetric equilibrium where the entry threshold is the same as without bid shopping. In that equilibrium, when a subcontractor has costs just below the entry threshold, he "inflates" his bid to be substantially above r, knowing that anticipating bid shopping, the prime contractor will still calculate her expected cost to be below r and bid. This depends, of course, on the prime contractor's beliefs about the cost of the subcontractor who submitted the bid (and her beliefs about the beliefs of the post-auction subcontractor, whose willingness to enter she depends upon to lower her cost). This suggests that by changing beliefs, we might be able to alter the equilibrium.

Fix n, and suppress the n subscript on entry thresholds. Consider a different possible equilibrium. This time, prime contractors ignore subcontractor bids above r. Specifically, they believe that any bid above r must come from a subcontractor with actual costs $y \geq r$, and therefore that by soliciting just one additional bid, there is no hope of reducing costs below r; expected costs are then above the reserve price, and the prime contractor will rationally not bid.[64]

If subcontractors have no reason to enter unless they will bid r or less, there is again a symmetric equilibrium, but the participation threshold y_* must now satisfy

$$(r - y_*)(1 - F(y_*))^{n-1}(1 - F(y_* - \alpha c)) \quad = \quad c$$

since the marginal entrant cannot bid more than r, and will only win when the other $n-1$ pre-auction subcontractors and the post-auction subcontractor all choose not to enter. By inspection, $y_* < y^*$. Assuming a symmetric, monotone equilibrium, we can again use the Envelope Theorem to calculate subcontractors' expected surplus at each cost level, use it to infer their bid function, and verify this is an equilibrium. As noted above, total surplus with bid shopping is increasing in the entry threshold in the relevant range; so this new equilibrium will have lower total surplus than the "good" one considered earlier.

Not only are there symmetric equilibria with entry thresholds y_* and y^*; there is a continuum of symmetric equilibria with every threshold in between.

Proposition 10 *With bid shopping to outsiders, there is a continuum of symmetric equilibria, characterized by entry thresholds $\hat{y} \in [y_*, y^*]$. Among these equilibria, total surplus is increasing in the entry threshold. In some environments, all of these equilibria give total surplus higher than the game without bid shopping and the same number of pre-auction subcontractors; in other environments, some of these equilibria do not.*

In the appendix, we show that when F is the uniform distribution on $[0, 1]$, $r = v = 1$, and $\alpha = 0$, there is a cutoff c^* such that bid shopping increases total surplus in all symmetric equilibria when $c < c^*$, but decreases total surplus in some symmetric equilibria when $c > c^*$.

That said, we have chosen to focus on results based on the "best" equilibrium with bid shopping, in part because these other equilibria strike us as less realistic. Consider the problem from the point of view of a prime contractor facing an off-equilibrium-path bid very slightly higher than the highest anticipated equilibrium-path bid. To "deter" such a deviation, her equilibrium strategy must be to not bid if that is the best sub-bid she receives, which requires her to believe that bid gives her an expected cost above r. To believe that, she must either believe such a deviation came from a fairly high-cost subcontractor (so that even with bid shopping her expected cost is still too high to bid), or believe it came from a fairly low-cost subcontractor (so that post-auction subcontractors won't want to enter and she won't be able to lower her cost). But high-cost subcontractors would not be able to cover their entry cost even if their deviation led the prime contractor to bid; and low-cost subcontractors earn high enough equilibrium profits to make such a deviation unprofitable even if it worked. If the prime contractor's beliefs are restricted to the types of subcontractors who could actually benefit from such a deviation, she should likely want to bid; in which case this would be a profitable deviation for certain types.

We can formalize this intuition by appealing to the Never a Weak Best Response refinement of Kohlberg

[64] Off-equilibrium-path bids could also be deterred by beliefs that they must come from a very *low*-cost sub: this would deter entry by post-auction subcontractors, preventing the winning prime contractor from reducing her cost.

and Mertens (1986):[65]

Proposition 11 *With bid shopping to outsiders, the only symmetric equilibrium satisfying the NWBR refinement is the one with entry cutoff* y_n^*.

Formally, the NWBR refinement says that at any off-equilibrium-path event (here, a pre-auction bid above the anticipated range), *if* there are subsequent beliefs that would rationalize this as a weak best-response for any type of player, then the off-path beliefs must put all their weight on such types. In this case, for symmetric equilibria with entry thresholds $\hat{y} < y_n^*$, there exist off-path bids b for which some beliefs would make b a weak best-response for subcontractors with cost level $\hat{y}$, but would not make b a strictly profitable deviation for anyone; and there are no beliefs which would make b a weak best-response for subcontractors with any other cost level without making b strictly profitable for someone. As a result, any equilibrium satisfying NWBR would have to assign beliefs $y = \hat{y}$ in response to the deviation b; but these beliefs would make b a strictly profitable deviation for subcontractors with costs close to $\hat{y}$. On the other hand, in the y_n^* equilibrium, no such problem exists, as the marginal type y_n^* cannot raise his bid and still induce a bid from the prime contractor.

6.3 Soliciting Post-Auction Bids from Multiple Outsiders

Our model of bid shopping to outsiders assumes the winning prime contractor will solicit exactly one additional sub-bid after the auction, which led to the result that when bid shopping does not change the number of pre-auction subcontractors, it does not change the entry threshold y_n^*. This occurred because, *conditional* on the winning prime contractor switching subcontractors after the auction, the price she pays is exactly y_{min}, the actual cost of the incumbent subcontractor she is replacing. This led to the marginal subcontractor's inflation of his bid (padding his profits when he wins to counteract the times he is replaced post-auction) and the prime contractor's deflation of that bid (to account for her anticipated post-auction cost reduction) exactly counteracting each other, at least for the marginal subcontractor with cost $y = y_n^*$.

Next, we consider what would happen if the winning prime contractor solicited *multiple* post-auction bids. Specifically, suppose she could simultaneously approach two or more new subcontractors and ask them to bid against the incumbent (and each other). For simplicity, consider the limit $\alpha \to 0$. If none of the new subcontractors she approaches have costs below the incumbent's cost y_{min} (as revealed by his bid), none of them will bid and the price remains the incumbent's pre-auction bid. If one new subcontractor has cost below y_{min}, as before, he will enter, and the incumbent will compete down to his cost level y_{min}. If multiple new subcontractors have costs below y_{min}, they will compete with each other down to the second-lowest cost. Thus, *conditional on* at least one new subcontractor entering, the prime contractor's expected cost is strictly less than y_{min}; let's call it $y_{min} - \delta$. If we let "replace" be shorthand for the times the prime contractor receives a new post-auction bid and replaces the subcontractor who had submitted the low pre-auction bid, we can write her expected cost at the time of the auction, given a low bid b by a subcontractor whose cost

[65] Our game is not a standard signaling game, as it has multiple senders (every pre-auction subcontractor) and multiple receivers (the general contractor receiving the pre-auction bid, and the subcontractor she will bid shop with after the auction), so application of refinements requires some care. The analysis is further complicated by the fact that senders of different types do not agree on the ranking of joint mixed best responses of the receivers, a monotonicity property that is crucial for the equivalence of many standard refinements (Cho and Sobel, 1990).

she infers to be y, as

$$(1-\Pr(\text{replace}))E(p|\text{no replace}) + \Pr(\text{replace})E(p|\text{replace}) = (1-\Pr(\text{replace}))b + \Pr(\text{replace})(y-\delta)$$

since she'll pay the incumbent his original bid when she fails to replace him. As before, the marginal entrant will bid high enough to make this expected cost r, so if we let y^* be the equilibrium entry threshold and $b(y^*)$ the corresponding pre-auction bid,

$$r \quad = \quad (1-\Pr(\text{replace}))b(y^*) + \Pr(\text{replace})(y^*-\delta)$$

Subtracting y^* from each side gives

$$r - y^* + \Pr(\text{replace})\delta \quad = \quad (1-\Pr(\text{replace}))(b(y^*)-y^*)$$

The marginal subcontractor's expected payoff gross of entry cost is the probability he's the only pre-auction bidder, times the probability he isn't replaced post-auction, times the surplus when he wins the job, which is

$$(1-F(y^*))^{n-1}(1-\Pr(\text{replace}))(b(y^*)-y^*) \quad = \quad (1-F(y^*))^{n-1}\left(r-y^*+\Pr(\text{replace})\delta\right)$$

Setting this equal to c gives $y^* > y_n^*$ – the entry threshold is now *higher* than without bid shopping. This means that when multiple outsiders will be approached for post-auction bids, bid shopping leads to *more* pre-auction participation, and increases total surplus by *more than* the expected surplus of the post-auction subcontractors. (We formalize this result in the appendix.)

It's at first surprising that relative to the simple case analyzed earlier, the threat of *more competitive* post-auction bid shopping induces *greater* pre-auction entry. The reason for this is that rather than paying exactly y whenever she replaces the incumbent subcontractor with a new one, the winning prime contractor now gets an added cost reduction (to costs below y) in some states of the world; and this occurs *in states of the world where the incumbent wasn't going to receive the contract anyway.* The prime contractor therefore receives an added cost reduction that the incumbent subcontractor "doesn't pay for"; this allows pre-auction subcontractors to inflate their bids further and still induce the prime contractor to bid, allowing subcontractors to enter at a higher cost level.

This shows it's possible for bid shopping to *increase* the probability of procurement. If $r < v$, this can potentially *increase* the combined surplus of the pre-auction players (procurer, prime contractors, and pre-auction subcontractors) above what it was with a single outsider being approached (or without bid shopping), and therefore increase total surplus by more than the expected surplus of the post-auction subcontractors.

On the other hand, if bid shopping imposes any additional costs, then the opposite can happen. If the presence of bid shopping imposes any costs on either the winning prime contractor (such as search costs or reputational costs) or the incumbent subcontractor who might be replaced (such as opportunity costs of not being able to bid on other jobs but still not knowing whether he'll win this one), this leads to a reduction in the entry threshold to a level below y_n^*, reducing – or possibly reversing – the welfare gains associated with bid shopping. We flesh out such a result in the appendix.

6.4 Ex Post Regret by the Prime Auction Winner

One other concern with bid shopping to outsiders is that, by introducing uncertainty in prime contractors' costs at the time of the prime auction, it introduces the possibility of ex post regret by the winner. Expecting to bid shop if she wins, a prime contractor (rationally) bids her expected cost, which is less than the lowest sub-bid she's received so far. If she narrowly wins the prime auction, then fails to improve her cost through bid shopping, the payment from the procurer will be less than her cost to complete the project – ex post, she will be facing a loss.

If contracts are ironclad, this isn't a problem – the prime contractor maximized her expected profit, and has to live with an unlucky draw ex post. (If prime contractors are risk averse, this risk would raise their bids some, but wouldn't fundamentally change anything.) In some settings, however, prime contractors facing a loss might attempt to get out of the contract – either to walk away entirely, or to renegotiate terms with the procurer. Since this could impose risk, delays, or added costs on the procurer, it's something procurers might reasonably worry about.

Exploring this topic thoroughly would require a different model, but our model can show that two obvious potential responses to this concern would likely backfire, or at least undermine much of the possible benefit of bid shopping. Consider our baseline model of bid shopping to outsiders, with one post-auction subcontractor being approached for a bid, and consider two candidate rules for the prime auction:

1. Bid shopping is allowed, but prime contractors may only bid if they've received a subcontractor bid of r or less; if they have, they can bid however they want.

2. Bid shopping is allowed, but prime contractors cannot bid less than the lowest sub-bid they've received before the auction.

Our result is that both of these rules lower pre-auction participation down to the level of the "worst" symmetric equilibrium considered earlier. Let the "y_* equilibrium" refer to the symmetric equilibrium with the lowest entry threshold described earlier, which therefore (per Proposition 10) has the lowest total surplus of the symmetric equilibria.

Proposition 12 *With bid shopping to outsiders, under either of the two rules described above, there is a unique symmetric equilibrium, which has pre-auction entry threshold y_*.*

1. *Under Rule 1, equilibrium payoffs are those of the y_* equilibrium.*

2. *Under Rule 2, total surplus is the same as in the y_* equilibrium; equilibrium payoffs are the same as in the y_* equilibrium* except *for a transfer of surplus from the procurer to the prime contractors.*

Combining this with Proposition 10, then, total surplus under either Rule 1 or Rule 2 is lower than in the absence of these rules, and could be higher or lower than total surplus without bid shopping, depending on the environment.

The intuition for Proposition 12 is as follows. Rule 1 effectively rules out pre-auction subcontractor bids above r, since they won't induce the prime contractor to bid. This has the same effect as deterring bids above r via off-equilibrium-path beliefs, and therefore effectively forces the players into the worst symmetric equilibrium of the simple-bid-shopping game, in which sub-bids above r were ignored.

As for Rule 2, relative to Rule 1, it does not change subcontractor bidding at all – subcontractors still enter with costs $y \leq y_*$, and bid exactly as before, since this is required by incentive compatibility and the Envelope Theorem. All that changes is that prime contractors bid their lowest-yet-received bid, rather than their expected cost, and therefore receive an extra windfall profit in the event bid shopping is successful. Thus, Rule 2 would *cause* an outcome decried in the U.S. Senate report in 1955, but seemingly at odds with equilibrium play – that subcontractors would inflate their bids in response to bid shopping, but that the procurer would not get the benefit of the lower prices realized ex post.[66]

Overall, then, relative to ordinary bid shopping, Rule 1 selects the "worst" symmetric equilibrium (the one with the lowest total surplus, which may be higher or lower than total surplus without bid shopping); and relative to Rule 1, Rule 2 leaves total surplus unchanged, just shifts surplus from the procurer to the winning prime contractor. If breach or post-auction renegotiation is a concern with bid shopping, then, rules of this sort seem a poor way to handle it.

7 Conclusion

The presence of bid shopping affects private and public procurement in several ways. In anticipation of being bid shopped, subcontractors will alter their bids to the general contractors. Equally, anticipating their ability to bid shop after being awarded the procurement, general contractors will modify their pre-award bids to the procurer. Bid shopping can also affect the participation rate, the threshold level of the job cost beyond which a subcontractor no longer finds it worthwhile to incur the up-front cost to prepare a bid.

In addition, bid shopping introduces general equilibrium effects that influence the distribution of surplus across the various stakeholders. For example, while bid shopping raises a general contractor's profit by lowering her expected cost, the stronger competition she faces from other generals will lower her expected revenue from receiving the contract. Finally, bid shopping can affect the identity of the prime contractor that wins the auction, and therefore the efficiency with which the project is executed.

In view of the disparate legal treatment of bid shopping in federal, state, local and private procurement, perhaps the single most important theoretical question to be resolved is whether a ban on bid shopping would be socially beneficial. Our results reveal that the answer to this question depends very much on the *type of subcontractor* the project is being bid shopped to: a sub who was unable to participate pre-auction, a sub who could have participated pre-auction but was drawn by the higher profitability of bid peddling or bid shopping, or a sub who bid in the prime's subcontracting auction. In the first case, bid shopping unambiguously increases welfare; in the second case the welfare effect is generally ambiguous, but under plausible conditions is positive; and in the third case the welfare effect is unambiguously negative.

A key tool in demonstrating this ranking is the following surprising and rather unanticipated result of our paper: bid shopping that does not divert subcontractors from the pre-auction competition does *not* alter the participation level. The invariance of the participation threshold has two immediate consequences: (1) bid shopping does not affect the likelihood with which the project will be implemented; and (2) when bid shopping is successful, the welfare effect depends on whether the project is carried out at lower or higher cost. If the winning prime contractor successfully bid shops to an outsider, the project is carried out at lower

[66] "Present bidding procedures cause the subcontractor submitting a bid "to bid so high that he, the subcontractor, can still come down and get the job." ...As long as subcontractors will not submit their final price prior to the award of the prime contract because of bid shopping after the award, the Government cannot get the full benefit of the low competitive price" (United States Senate Committee on the Judiciary, 1955, p. 8).

cost, so bid shopping enhances efficiency. In fact, since the post-auction subcontractor earns exactly the difference between his cost and that of the winning pre-auction subcontractor, the expected gain in surplus from bid shopping equals the expected profit of the post auction subcontractor. In this case the private and social incentives coincide. If instead prime contractors approach insiders, then the lowest cost prime will not always be the one with the lowest cost subcontractor. This happens whenever the second lowest cost sub of a competing prime beats both the bid of this lowest cost subcontractor and the second lowest cost sub of the latter's prime. Bid shopping then results in the project being carried out at higher cost, thereby lowering welfare.

When bid shopping leads one or more subcontractors to forego submitting a pre-auction bid in favor of waiting to bid post-award, matters are more complicated, because the entry threshold for the remaining subcontractors is higher under bid shopping. This difference in entry thresholds affects both the incentive for subcontractors to wait, and the associated change in welfare. Nevertheless, provided waiting to bid post-award pays, welfare will tend to increase. To see why, recall that bid shopping to an outsider raises total surplus by the ex-ante expected profits of this sub. Since outsiders and insiders are ex-ante identical, the same statement remains true if the post auction sub is instead an insider who switched roles. Thus bid shopping will increase welfare if the expected profits of the subcontractor who switched exceed the change in surplus from adding an extra pre-auction sub in the game without bid shopping. Thus whenever adding a sub to this game lowers surplus, or does not increase it too much, diversion of a sub from pre- to post-auction competition will be socially beneficial.

The crucial question from a policy standpoint is then whether or not most bid shopping or bid peddling involves insiders, i.e. subs who already participated in the prime contractor's subcontractor auction. Evidence on this question is hard to come by. Nevertheless, while some commentators recognize the possibility of a prime contractor approaching an outsider,[67] or even of an insider switching from pre-award bidding to post award peddling,[68] the majority emphasize bid shopping to insiders.[69] The impression that this type of bid shopping is the norm is confirmed by several legal cases,[70] though there appear to be exceptions.[71] On the whole, our research therefore lends support to the view that bid-shopping is more typically socially harmful.

Against this background, it is perhaps a bit surprising that we find that subcontractors are ex ante likely to be better off from bid shopping to insiders. This view is in sharp contrast to the official position of

[67]For example, John Stewart (1989) argues that "The general contractor's ability to extract price concessions after the award is made for a particular job will be limited by the existence of other work available to subcontractors."

[68]Frank Baltz (1997, p.23) states, "some subcontractors will refrain from preparing their own bids for the contract work, but will instead wait until the bid shopping begins and then submit bids that undercut the original bidder," referring to this practice as bid peddling.

[69]Zwick and Miller (2004, p. 247) state that during the buy-out stage "a few subcontractors who are not the apparent low bidders, may try to become the low bidder," Degn and Miller (2003, p. 48) contend that in the post award stage "the (prime) contractor returns to the subcontractors and attempts to further chisel down their bid prices by using the incorporated subcontractor's bid as a negotiating tool.," and Hinze and Tracey (1994, p. 277) state that "The general contractor... may be tempted to approach one of the subcontractors with a proposal in which the subcontractor is asked to reduce the bid that was submitted to the general contractor."

[70]Gregory and Travers (2010, p. 32) quote Sheet Metal Employers' Association v. Giordano, in which the court stated "the (general contractor) may then proceed to play one bidder against the other," and Complete General Construction Co. v.. Kard Welding, Inc. (2009), in which the subcontractor "noted that after the prime contract award, the general contractor had continued to "negotiate" bids submitted by other structural steel subcontractors (Ibid., p. 34).

[71]For example, in Vila v. Posen (2012 WL 4093545, Fla. App. 2 Dist.), the court noted that "Vila and Posen entered into the Subcontract on July 15, 2008. After Vila began preparations for performance of the Subcontract, Posen asked Vila to lower the price to which the parties had already agreed. Vila refused to lower its price, and Posen obtained a lower price proposal from another *potential* subcontractor."

subcontractor, contractor and other professional organizations, the stance of most legal scholars, as well as the legal treatment of bid shopping in many states, who see bid shopping as harmful to subcontractors, even going as far as worrying that it might "precipitate their bankruptcy" (Southern Cal. Acoustics Co. v. C.V. Holder, Inc.).[72]

An important source of this discrepancy in perspectives is that unlike economists, legal scholars and professionals do not have access to the tools of economic modeling, and therefore do not correctly anticipate all the equilibrium effects. Lambert (1970, p. 395), for example, notes: "Subcontractors will pad their initial bids in order to make further reductions during the post-award negotiations." This view is held nearly universally by all commentators.[73] An equally strongly held position among legals scholars is that bid shopping only benefits the prime contractor, at the detriment of her subs and the owner. Again quoting Lambert (1970, p. 394-395):[74] "Bid shopping by the general will be of no benefit to the awarding authority. The price of the contract has already been set, and the general's purpose is to simply drive down his own cost, increasing his profit at the expense of the subcontractor." Both of these statements ignore the complex calculus that determines a subcontractor's optimal bid. We cannot argue with the fact that a prime contractor who bid shops receives a pure ex-post transfer, the difference between the lowest pre-award sub bid and the price she can negotiate with an alternate sub when she wins the contract. However, a shrewd prime will anticipate this price reduction and use it to bid more aggressively in the owner's auction. This contractor behavior in turn allows high cost subcontractors to pad their bids in order to raise their margins in case they win (indeed, the marginal bidder type bids above the reserve r). On the other hand, if post-award competition comes from outsiders, low cost contractors lower their bids simply because they face more competition (and because it deters post-auction subcontractors from sinking the bid preparation cost). Despite these differences in pre-auction response to the possibility of being bid shopped, all subcontractors have a lower probability of winning. Application of the envelope theorem then shows that all subcontractor types are harmed by bid shopping.

When post-award competition comes from insiders, subs do not face any extra additional competition, so they have no incentive to lower their pre-auction bids on this account. In fact, subcontractors will raise their pre-auction bids, as these bids compete with fewer subcontractors on the margin, i.e. only determine their payoffs in the event that they face no post auction competition. The possibility of having to lower this bid due to post-auction bid shopping does however complicate the overall effect on the subs' expected payoff. What comes to our rescue is the envelope theorem, which states that the derivative of this expected payoff w.r.t. type equals the likelihood with which this type wins the auction. Bid shopping to insiders does not change a subcontractor's ex-ante chance of winning the auction, so it cannot be that *all* contractor types gain or lose from its practice. A subcontractor's chance of winning *conditional* on his realized cost is affected by bid shopping, as the lowest cost subcontractor will then not always win the auction. Our research indicates that low cost subcontractors are then less likely to win the auction, and high cost subcontractors are more likely to win the auction. As a consequence, low cost subcontractors are harmed by bid shopping, while high cost subcontractors gain. On average, bid shopping will likely benefit subs provided the distribution of subcontractor costs is log-concave.

In closing, let us observe that legal scholars and courts have at times embraced the pro-competitive

[72]Cited in Closen and Weiland (1980, p. 580).

[73]See e.g. Dorweiler and Becker (1993, p. 320), Baltz (1997, p. 24), Degn and Miller (2003, p. 50), Gregory (2010, p. 9) and Gregory and Travers (2010, p. 32).

[74]See also Shueller (1960, p. 500), J.C.C., Jr. (1967, p. 1724), Barrett (1972, p. 776), Gregory and Travers (2010, p. 29).

aspects of bid shopping. For example, in the famous Oakland-Alameda bid depository case the court opined, "... it is readily apparent that the practice of 'bid peddling' is illustrative of open price competition in its purest form..."[75] Our research supports the view that bid shopping can indeed be socially beneficial: this happens when outsiders are permitted to compete post-auction while they were unable or unwilling to do so in the pre-auction stage. However, as surveyed above, the record suggests that this form of bid shopping is not the most prevalent one. Thus, measures to curb the practice may well be called for, albeit for reasons entirely distinct from the 'evils' envisaged by those who oppose it.

[75]Cited in Stewart (1989, p.37). Goldberg (2011, p. 542) writes "... these (bid shopping and bid peddling) are the public bidding counterpart to the haggling that takes place in private contracting ..."

APPENDIX A – proofs and calculations not in text

A.1 Proof of Proposition 1

Given our assumption that prime contractors will bid their costs, we verify this is an equilibrium by ruling out deviations by subcontractors. Note that since $c = (r - y_n^*)(1 - F(y_n^*))^{n-1}$, $\beta_{n,0}(y_n^*) = r$, so any deviation above the range of $\beta_{n,0}(\cdot)$ cannot induce a general contractor to bid; usual arguments rule out deviations to bids below $\beta_{n,0}(\underline{y})$.

Since $\beta_{n,0}$ is continuous and we've ruled out deviations above and below its range, any plausible deviation is to the equilibrium bid of some type of subcontractor y'. A subcontractor with cost y who bids $\beta_{n,0}(y')$ earns expected payoff

$$U(y, \beta_{n,0}(y')) \quad = \quad (1 - F(y'))^{n-1} \left(\beta_{n,0}(y') - y\right) - c$$

Plugging in (2) and simplifying,

$$U(y, \beta_{n,0}(y')) \quad = \quad (1 - F(y'))^{n-1}(y' - y) + \int_{y'}^{y_n^*} (1 - F(s))^{n-1} ds$$

Differentiating and simplifying,

$$\frac{\partial}{\partial y'} U(y, \beta_{n,0}(y')) \quad = \quad (n - 1)(1 - F(y'))^{n-2} f(y')(y - y')$$

so U is increasing in y' when $y' < y$, decreasing when $y' > y$.

For $y \leq y_n^*$, then, $U(y, \beta_{n,0}(y)) > U(y, \beta_{n,0}(y'))$ for all $y' \neq y$, and $U(y, \beta_{n,0}(y)) \geq U(y, \beta_{n,0}(y_n^*)) = U(y, r) \geq U(y_n^*, r) = 0$, with equality only when $y = y_n^*$, so entering and bidding $\beta_{n,0}(y)$ is a best-response. For $y > y_n^*$ and any $y' \leq y_n^*$, $U(y, \beta_{n,0}(y')) \leq U(y, \beta_{n,0}(y_n^*)) = U(y, r) < U(y_n^*, r) = 0$, so not bidding is a best-response. Thus, the strategies described in Proposition 1 are an equilibrium.

That this is the *only* symmetric equilibrium follows from the following:

1. Any equilibrium must use weakly monotone strategies – for the usual reason that a subcontractor's expected payoff has strictly decreasing differences in his cost and his probability of winning

2. No symmetric equilibrium bid function can be flat over a range of cost types – for the usual reason that either the highest- or lowest-cost types would benefit from an infinitesimal deviation upward or downward, respectively

3. Any symmetric equilibrium bid function must have range $[\underline{b}, r]$ for some $\underline{b}$ – i.e., it must go all the way up to r (otherwise subcontractor types bidding the highest equilibrium bid would benefit from bidding r instead), and it must not have holes (otherwise bidders bidding "just below" a gap in the range should deviate to the top of the gap)

4. Any symmetric equilibrium bid function must have an entry threshold that excludes some bidders – since otherwise, given points 1 and 2 above, the highest-cost subcontractors would never win but would still incur the entry cost

5. Any symmetric equilibrium must have the entry threshold y_n^* in Proposition 1 – if not, then either marginal entrants are strictly profitable (in which case some excluded types could deviate and bid r for positive surplus), or strictly unprofitable (in which case some types are earning negative payoff)

6. Any symmetric equilibrium must therefore use the bid function in Proposition 1 – which follows from the Envelope Theorem and the equilibrium allocation rule (which is known for any symmetric monotone equilibrium given its entry threshold) □

A.2 Proof of Corollary 1

We can calculate expected total surplus as follows. If no subcontractor enters, the project is not procured and total surplus is 0. If any subcontractor enters, the project is procured, and total surplus is $v - y - \tilde{n}c$, where $\tilde{n}$ is the number of subcontractors who entered and y is the actual cost of the subcontractor who does the work.[76] In equilibrium, if all subcontractors have costs above y_n^*, none enter; otherwise, the work is done by the subcontractor with the lowest cost. The CDF of the lowest of n independent draws from F is $1 - (1 - F(s))^n$ which has density $n(1 - F(s))^{n-1} f(s)$; and since each subcontractor enters if his cost is below y_n^*, the expected value of $\tilde{n}$ is $nF(y_n^*)$. We can therefore calculate total surplus as

$$W_{n,0} = \int_{\underline{y}}^{y_n^*} (v - s) n (1 - F(s))^{n-1} f(s) ds - nF(y_n^*)c$$

The reserve price r does not appear explicitly in the statement for $W_{n,0}$, but enters through its impact on y_n^*, which is increasing in r. Thus,

$$\begin{aligned} \frac{dW_{n,0}}{dr} &= \frac{\partial W_{n,0}}{\partial y_n^*} \frac{\partial y_n^*}{\partial r} = \left[(v - y_n^*) n (1 - F(y_n^*))^{n-1} f(y_n^*) - n f(y_n^*) c \right] \cdot \frac{\partial y_n^*}{\partial r} \\ &\propto n f(y_n^*) \left[(v - y_n^*)(1 - F(y_n^*))^{n-1} - c \right] \\ &= n f(y_n^*) \left[(v - y_n^*)(1 - F(y_n^*))^{n-1} - (r - y^*)(1 - F(y_n^*))^{n-1} \right] \\ &= n f(y_n^*)(1 - F(y_n^*))^{n-1}(v - r) \end{aligned}$$

Total surplus is therefore increasing in r when $r < v$, decreasing when $r > v$, therefore maximized at $r = v$.

As for the last result – that the effect of n on total surplus can go either way – let F be the uniform distribution on $[0, 1]$ and calculate total surplus numerically. Table 2 shows two examples, one where increasing n from 3 to 4 increases total surplus, one where it decreases total surplus. (Somewhat more generally, the proof of Theorem 3 shows that $W_{n,0}$ is decreasing in n when $r \leq v$, $f(\underline{y}) > \frac{1}{r - \underline{y}}$, and c is sufficiently large. On the other hand, it's straightforward to show that $W_{n,0}$ is increasing in n when c is sufficiently close to 0.) □

Table 2: The effect of n on total surplus is ambiguous

setting	(1)	(2)	(3)	(4)
r and v	0.5	0.5	10	10
c	0.1	0.1	0.3	0.3
n	3	4	3	4
y_n^*	0.2974	0.2566	0.8192	0.6819
surplus	0.1514	0.1684	8.9587	8.8870

A.3 Calculation of ex ante surplus without bid shopping

By usual envelope theorem arguments, if $U(y)$ is a subcontractor's expected payoff given cost y, then $U'(y)$ must equal his probability of winning in equilibrium, which is the probability the other $n - 1$ subcontractors have costs higher than his own. Since $U(y_n^*) = 0$, $U(y) = \int_y^{y_n^*} (1 - F(s))^{n-1} ds$ for $y \leq y_n^*$ (and 0 for $y > y_n^*$). Taking the expectation with respect to y and simplifying gives the expression for U.

[76] The procurer's surplus is $v - P$, where P is the price paid to the winning prime contractor; the winning prime contractor's surplus is $P - b$, where b is his payment to the subcontractor performing the job; the winning subcontractor earns $b - y - c$ net of entry costs; and the $\tilde{n} - 1$ subcontractors who bid and lost each earn a surplus of $-c$.

For prime contractor surplus, define $G_i(x) = 1 - (1 - F(\beta_{n,0}^{-1}(x)))^{n_i}$; this is the probability that at least one of prime contractor i's n_i subcontractors has cost below $\beta_{n,0}^{-1}(x)$, therefore the probability one of prime i's subs bids below x, and therefore the probability prime i has cost below x when competing in the prime auction. Letting $G_{-i} = 1 - (1 - F(\beta_{n,0}^{-1}(x)))^{n-n_i}$ similarly be the distribution of the cost of prime i's lowest-cost rival, prime i's expected payoff given cost x is

$$\Pi_i(x) \quad = \quad (r-x)(1-G_{-i}(r)) + \int_x^r (z-x)dG_{-i}(z) \quad = \quad \int_x^r (1-G_{-i}(z))dz$$

after integration by parts. Taking the expectation over $x \sim G_i$ gives

$$\Pi_i \quad = \quad \int_{\underline{b}}^r G_i(z)(1-G_{-i}(z))ds$$

where $\underline{b} = \beta_{n,0}(\underline{y})$. Plugging back in the expressions for G_i and G_{-i}, and then changing variables to $s = \beta_{n,0}^{-1}(z)$, gives the expression in the text.

Finally, for procurer surplus, define $G^1(x) = 1 - \prod_i(1 - G_i(x))$ and $G^2(x) = 1 - \prod_i(1 - G_i(x)) - \sum_i G_i(x) \prod_{j \neq i}(1 - G_j(x))$ as the CDFs of the lowest- and second-lowest prime contractor costs. The procurer's expected surplus is

$$PS \quad = \quad (v-r)(G^1(r) - G^2(r)) + \int_{\underline{b}}^r (v-s)dG^2(x)$$

Substituting back in for G^1 and G^2 and then for G_i, and then simplifying and changing variables, gives the expression in the text.

A.4 Proof of Proposition 2

The proof of existence of a symmetric, monotone equilibrium closely follows the logic of Athey (2001). The full proof is available upon request; the rough outline is as follows:

1. We first consider a "constrained game" where subcontractors with costs above y_n^* are not allowed to bid, and subcontractors must bid "truthfully" during post-auction bid shopping. After we establish existence of a symmetric monotone equilibrium for the constrained game, we show that any symmetric monotone equilibrium must have entry by all subcontractors with costs below y_n^*, and therefore that not entering is a best-response for those with costs above y_n^*; and since post-auction competition is a second-price auction, truthful bidding is a best-response.

2. To establish existence of a symmetric monotone equilibrium of the constrained game, we follow the logic of Athey (2001). We consider a sequence of discrete games played on successively finer grids of allowed bids; associate nondecreasing bid functions with nondecreasing sequences of "jump points" at which a bidder's bid function increases to the next allowed bid; and use the properties of our game to show that the mapping from monotone strategies for all other subcontractors to monotone best-responses for the last subcontractor satisfies the requirements of Kakutani's fixed point theorem, yielding a symmetric monotone equilibrium for each discrete game.

3. We then show that a sequence of equilibrium bid strategies in finer and finer discrete games has a convergent subsequence, and use the structure of our game to show that its limit is a best-response to itself in the continuous game, establishing existence of a symmetric monotone equilibrium in the (constrained) continuous game.

The second part of Proposition 2 – that any symmetric monotone equilibrium must use the same entry threshold y_n^* as the game without bid shopping – is explained in the text: a marginal entrant still knows he'll only win if nobody else enters, and will bid r, and must cover his entry costs in expectation, so the entry threshold will still be given by $(r - y^*)(1 - F(y^*))^{n-1} = c$. □

A.5 Proof of Proposition 3

First, we show that in any symmetric monotone equilibrium with entry threshold y_n^*, the bid function must be (6). By the envelope theorem, the expected payoff of a pre-auction subcontractor with cost $y < y_n^*$ must be

$$U(y) \quad = \quad \int_y^{y_n^*} (1 - F(s))^{n-1}(1 - F(s - \alpha c))ds.$$

A pre-auction subcontractor's expected payoff from submitting the low pre-auction bid ("winning" the auction) is $(1 - F(y - \alpha c))(\beta(y) - y)$, so that

$$(1 - F(y))^{n-1}(1 - F(y - \alpha c))(\beta(y) - y) - c \quad = \quad \int_y^{y^*} (1 - F(s))^{n-1}(1 - F(s - \alpha c))ds$$

Solving for $\beta(y)$ yields (6). Differentiating w.r.t. y, we can verify that this bid function is strictly increasing in y.

Faced with this bid function, a general contractor whose lowest sub-bid is from a subcontractor with cost y, bids

$$C(\beta_{n,k}(y), y) \quad = \quad F(y - \alpha c)y + (1 - F(y - \alpha c))\beta_{n,k}(y) \quad = \quad y + \frac{c + \int_y^{y_n^*} (1 - F(s))^{n-1}(1 - F(s - \alpha c))ds}{(1 - F(y))^{n-1}}$$

in the prime auction. Differentiating $C(\beta_{n,k}(y), y)$ with respect to y, we can verify that this is indeed strictly increasing. Furthermore, if the lowest sub-bid comes from a subcontractor with cost y_n^*, the general contractor will bid

$$C(\beta_{n,k}(y_n^*), y_n^*) \quad = \quad y_n^* + \frac{c}{(1 - F(y_n^*))^{n-1}} \quad = \quad y_n^* + (r - y_n^*) \quad = \quad r,$$

justifying the choice of the entry threshold y_n^*.

It remains to be shown that a subcontractor with cost y cannot profitably deviate by submitting a bid different from $\beta_{n,k}(y)$.

First, consider a bid $\beta_{n,k}(y')$ with $y' \leq y + \alpha c$. Then he will win whenever all his pre-auction opponents have costs above y', and the post-auction subcontractor has cost above $y' - \alpha c$ (and therefore doesn't enter), giving expected payoff

$$\begin{aligned} U(y, \beta_{n,k}(y')) \quad &= \quad (1 - F(y'))^{n-1}(1 - F(y' - \alpha c)) \left(\beta_{n,k}(y') - y\right) \\ &= \quad (1 - F(y'))^{n-1}(1 - F(y' - \alpha c)) \left(\beta_{n,k}(y') - y'\right) + (1 - F(y'))^{n-1}(1 - F(y' - \alpha c)) \left(y' - y\right) \\ &= \quad c + \int_{y'}^{y_n^*} (1 - F(s))^{n-1}(1 - F(s - \alpha c))ds + (1 - F(y'))^{n-1}(1 - F(y' - \alpha c)) \left(y' - y\right) \end{aligned}$$

Differentiating with respect to y' gives

$$\begin{aligned} \frac{d}{dy'}U(y, \beta_{n,k}(y')) \quad &= \quad -(1 - F(y'))^{n-1}(1 - F(y' - \alpha c)) + (1 - F(y'))^{n-1}(1 - F(y' - \alpha c)) \\ &\quad + \left(\frac{d}{dy'}\left[(1 - F(y'))^{n-1}(1 - F(y' - \alpha c))\right]\right)(y' - y) \end{aligned}$$

The first two terms cancel, and since $(1 - F(y'))^{n-1}(1 - F(y' - \alpha c))$ is decreasing in y', the deviation payoff is proportional to $y - y'$, hence single peaked in y' over the interval $[\underline{y}, y + \alpha c]$, attaining a maximum at $y' = y$.

Next, consider a deviation to a bid $\beta_{n,k}(y')$ for $y' \in (y + \alpha c, y^*]$. This case is slightly different, because the deviating sub will sometimes let in a post-auction entrant that he can still beat post-auction. This will happen whenever the late entrant has cost between y and $y' - \alpha c$, so the expected payoff from deviating over this range is

$$\begin{aligned} U(y, \beta_{n,k}(y')) &= (1-F(y'))^{n-1}(1-F(y'-\alpha c))\left(\beta_{n,k}(y') - y\right) + (1-F(y'))^{n-1}\int_y^{y'-\alpha c}(s-y)dF(s) \\ &= c + \int_{y'}^{y_n^*}(1-F(s))^{n-1}(1-F(s-\alpha c))ds + (1-F(y'))^{n-1}(1-F(y'-\alpha c))\left(y'-y\right) \\ &+ \ (1-F(y'))^{n-1}\int_y^{y'-\alpha c}(s-y)dF(s) \end{aligned}$$

Differentiating with respect to y' and then simplifying,

$$\begin{aligned} \frac{dU(y,\beta_{n,k}(y'))}{dy'} &= -(n-1)(1-F(y'))^{n-2}f(y')(1-F(y'-\alpha c))(y'-y) \\ &\quad -(n-1)(1-F(y'))^{n-2}f(y')\int_y^{y'-\alpha c}(s-y)dF(s) - (1-F(y'))^{n-1}f(y'-\alpha c)\alpha c \end{aligned}$$

which is unambiguously negative for $y' \geq y + \alpha c$. So if a sub with cost y wanted to bid at least $\beta_{n,k}(y+\alpha c)$, he would prefer to bid exactly $\beta_{n,k}(y+\alpha c)$, which we already know does worse than $\beta_{n,k}(y)$.

Any subcontractor bids above $\beta_{n,k}(y_n^*)$ are ignored by the general contractor, rationalized by beliefs that that they come from a subcontractor with costs at least y_n^* and would therefore give expected cost more than r. Any bid below $\beta_{n,k}(\underline{y})$ is dominated by $\beta_{n,k}(\underline{y})$, as this equilibrium bid is already revealing the subcontractor to have the lowest cost he could have and thus deterring entry by the maximal set of post-auction subcontractors; no beliefs would lead a post-auction subcontractor to enter less often unless the pre-auction subcontractor bid below $\underline{y}$, which would lead to negative payoffs. □

A.6 Proof of Corollary 2

For $k > 0$, let $\Delta(y) = \beta_{n,k}(y) - \beta_{n,0}(y)$. From (2) and (6),

$$\begin{aligned} \Delta(y) &= \frac{c + \int_y^{y_n^*}(1-F(s))^{n-1}(1-F(s-\alpha c))ds}{(1-F(y))^{n-1}(1-F(y-\alpha c))} - \frac{c+\int_y^{y_n^*}(1-F(s))^{n-1}ds}{(1-F(y))^{n-1}} \\ &= \frac{cF(y-\alpha c) + \int_y^{y_n^*}(1-F(s))^{n-1}\left[F(y-\alpha c) - F(s-\alpha c)\right]ds}{(1-F(y))^{n-1}(1-F(y-\alpha c))} \end{aligned}$$

Plugging in $y = y_n^*$ gives

$$\Delta(y_n^*) = \frac{cF(y_n^* - \alpha c)}{(1-F(y_n^*))^{n-1}(1-F(y_n^*-\alpha c))} > 0$$

On the other hand, if $F(y - \alpha c) = 0$,

$$\Delta(y) = \frac{-\int_y^{y_n^*}(1-F(s))^{n-1}F(s-\alpha c)ds}{(1-F(y))^{n-1}} < 0$$

To see it crosses zero only once, note that $\Delta(y) \geq 0$ implies $F(y - \alpha c) > 0$, meaning $y > \underline{y} + \alpha c$; so the derivative of the numerator with respect to y is

$$f(y - \alpha c) \left[c + \int_y^{y_n^*} (1 - F(s))^{n-1} ds \right] \quad > \quad 0$$

Whenever $\Delta(y) = 0$, then, its numerator is strictly increasing, so it crosses zero at most once, from below.

For part 2, with bid shopping, the prime contractor has a realized cost of y whenever the new subcontractor enters, and a cost of $\beta_{n,k}(y)$ otherwise. The prime contractor's expected cost is therefore

$$\begin{aligned}(1 - F(y - \alpha c))\beta_{n,k}(y) + F(y - \alpha c)y \quad &= \quad y + \frac{c + \int_y^{y_n^*} (1 - F(s))^{n-1}(1 - F(s - \alpha c))ds}{(1 - F(y))^{n-1}} \\ &\leq \quad y + \frac{c + \int_y^{y_n^*} (1 - F(s))^{n-1} ds}{(1 - F(y))^{n-1}} \quad = \quad \beta_{n,0}(y),\end{aligned}$$

with strict inequality as long as $F(y_n^* - \alpha c) > 0$. □

A.7 Proof of Theorem 1

The result is explained in the text. Given Proposition 2, any symmetric monotone equilibrium has the same entry threshold as the game without bid shopping, and therefore the same entry decisions at each realization of subcontractor costs. Without bid shopping, given entry, the allocation is efficient (the lowest-cost sub performs the work). With bid shopping, this is not always true: a prime contractor with two bids from subs with moderate costs may have a lower cost than a prime with a bid from a single sub with lower costs, so the prime contractor associated with the lowest-cost sub will not always win the prime auction, and the lowest-cost sub will therefore not always do the work.

A.8 Proof of Proposition 4

As before, let y_{min} be the lowest of the pre-auction subcontractors' realized costs, and $\tilde{n}$ the number of pre-auction cost realizations below y_n^*. We noted above that without bid shopping, the total realized surplus is $\mathbf{1}_{y_{min} \leq y_n^*}(v - y_{min}) - \tilde{n}c$. With simple bid shopping, this is also the combined surplus realized by the procurer, the general contractors, and the pre-auction subcontractors, i.e., the surplus realized by everyone but the post-auction subcontractors. This is because with bid shopping, either the lowest-bidding pre-auction subcontractor is not replaced (in which case ex post surplus is the same as without bid shopping), or he is replaced, in which case the procurer earns $v - P$, the winning prime contractor earns $P - y_{min}$, and $\tilde{n}$ pre-auction subcontractors earn $-c$, for combined payoffs $v - y_{min} - \tilde{n}c$. (To put it another way, when the incumbent subcontractor is replaced after the auction, since the new subcontractor gets paid the incumbent's cost y_{min}, the incumbent's loss $\beta_{n,k}(y_{min}) - y_{min}$ is exactly equal to the prime contractor's gain.) Combined expected surplus of everyone but the post-auction subcontractors is therefore the same with and without bid shopping, and total surplus therefore increases by the expected surplus of the post-auction subcontractors.

To calculate this surplus, and thus the increase in surplus due to bid shopping, fix y_{min}. It's easiest if we think of all prime contractors using the same post-auction subcontractor; the surplus we calculate will in fact be the combined ex ante surplus of all the post-auction subcontractors.

If $y_{min} > y_n^*$, the auction fails, and there is no surplus available to the post-auction subcontractor. If $y_{min} \leq y_n^*$, the auction succeeds, and the winning contractor approaches the post-auction sub, who has cost $y_0 \sim F$. If $y_0 > y_{min} - \alpha c$, this subcontractor will choose not to enter (correctly inferring y_{min} from the equilibrium bid and knowing he can't recover his entry cost by bidding against the incumbent). If $y_0 < y_{min} - \alpha c$, on the other hand, the post-auction subcontractor will enter and win, earning $y_{min} - y_0 - \alpha c$.

Taking the expectation over y_0, the post-auction subcontractor's expected surplus given y_{min} is

$$\int_{\underline{y}}^{y_{min}-\alpha c} (y_{min} - s - \alpha c) dF(s) \quad = \quad \int_{\underline{y}}^{y_{min}-\alpha c} F(s) ds$$

(the latter by integration by parts). Taking the expectation over y_{min}, whose CDF is $1-(1-F(\cdot))^n$, we can write the ex ante surplus of the post-auction subcontractors – and thus the increase in total surplus from bid shopping – as

$$\Delta W \quad = \quad \int_{\underline{y}}^{y_n^*} \left[\int_{\underline{y}}^{y_{min}-\alpha c} F(s) ds \right] n(1-F(y_{min}))^{n-1} f(y_{min}) dy_{min} \tag{12}$$

A.9 Proof of Proposition 5

Part 1

We want to show that for c sufficiently close to $r - \underline{y}$ and $k \geq 1$, $U_{n,k}^{post} < U_{n+1,k-1}^{pre}$. Note that $U_{n+1,k-1}^{pre} \geq U_{n+1,1}^{pre}$, since $U_{n+1,\ell}^{pre}$ does not depend on ℓ for $\ell \geq 1$, and is strictly higher for $\ell = 0$. Thus,

$$\lim_{c \to r-\underline{y}} \frac{U_{n,k}^{post}}{U_{n+1,k-1}^{pre}} \quad \leq \quad \lim_{c \to r-\underline{y}} \frac{U_{n,k}^{post}}{U_{n+1,1}^{pre}} \quad = \quad \lim_{c \to r-\underline{y}} \frac{\frac{1}{k}\int_{\underline{y}+\alpha c}^{y_n^*} [(1-F(s))^n - (1-F(y_n^*))^n] F(s-\alpha c) ds}{\int_{\underline{y}}^{y_{n+1}^*} F(s)(1-F(s))^n (1-F(s-\alpha c)) ds}$$

$$\leq \quad \frac{1}{k} \lim_{c \to r-\underline{y}} \frac{\int_{\underline{y}}^{y_n^*} [(1-F(s))^n - (1-F(y_n^*))^n] F(s) ds}{\int_{\underline{y}}^{y_{n+1}^*} F(s)(1-F(s))^{n+1} ds}$$

In the limit, y_n^* and y_{n+1}^* both go to $\underline{y}$ and so the numerator and the denominator go to zero, so we apply L'Hopital's rule:

$$\lim_{c \to r-\underline{y}} \frac{U_{n,k}^{post}}{U_{n+1,k-1}^{pre}} \quad \leq \quad \frac{1}{k} \lim_{c \to r-\underline{y}} \frac{\frac{d}{dc}\left(\int_{\underline{y}}^{y_n^*} [(1-F(s))^n - (1-F(y_n^*))^n] F(s) ds \right)}{\frac{d}{dc}\left(\int_{\underline{y}}^{y_{n+1}^*} F(s)(1-F(s))^{n+1} ds \right)}$$

$$= \quad \frac{1}{k} \lim_{c \to r-\underline{y}} \frac{\left(\int_{\underline{y}}^{y_n^*} n(1-F(y_n^*))^{n-1} f(y_n^*) F(s) ds \right) \frac{dy_n^*}{dc}}{\left(F(y_{n+1}^*)(1-F(y_{n+1}^*))^{n+1} \right) \frac{dy_{n+1}^*}{dc}}$$

(Note the effect of y_n^* in the upper limit of integration in the numerator is zero, since the integrand evaluated at y_n^* is zero; the derivative of the top term comes from differentiating the $(1-F(y_n^*))^n$ term inside the integral.) We can calculate dy_n^*/dc from $(r-y_n^*)(1-F(y_n^*))^{n-1} - c = 0$ using the Implicit Function Theorem, giving

$$\frac{\partial y_n^*}{\partial c} \quad = \quad -\frac{-1}{-(1-F(y_n^*))^{n-1} - (r-y_n^*)(n-1)(1-F(y_n^*))^{n-2} f(y_n^*)}$$

$$= \quad -\frac{1}{(1-F(y_n^*))^{n-1} + (n-1)\frac{f(y_n^*)}{1-F(y_n^*)} c}$$

and therefore

$$\lim_{c \to r-\underline{y}} \frac{\frac{dy_n^*}{dc}}{\frac{dy_{n+1}^*}{dc}} \quad = \quad \lim_{c \to r-\underline{y}} \frac{(1-F(y_{n+1}^*))^n + n\frac{f(y_{n+1}^*)}{1-F(y_{n+1}^*)} c}{(1-F(y_n^*))^{n-1} + (n-1)\frac{f(y_n^*)}{1-F(y_n^*)} c} \quad = \quad \frac{1+nf(\underline{y})c}{1+(n-1)f(\underline{y})c} \quad \in \quad \left[1, \frac{n}{n-1} \right]$$

Since $1 - F(y_n^*)$ and $1 - F(y_{n+1}^*)$ both go to 1, this leaves us with

$$\lim_{c \to r - \underline{y}} \frac{U_{n,k}^{post}}{U_{n+1,k-1}^{pre}} \quad \leq \quad \frac{1}{k} \lim_{c \to r - \underline{y}} \frac{n f(y_n^*) \int_{\underline{y}}^{y_n^*} F(s) ds}{F(y_{n+1}^*)} \frac{n}{n-1}$$

Since F is increasing, $\int_{\underline{y}}^{y_n^*} F(s) ds \leq (y_n^* - \underline{y}) F(y_n^*)$, so

$$\lim_{c \to r - \underline{y}} \frac{U_{n,k}^{post}}{U_{n+1,k-1}^{pre}} \quad \leq \quad \frac{n}{k} \frac{n}{n-1} \lim_{c \to r - \underline{y}} f(y_n^*)(y_n^* - \underline{y}) \frac{F(y_n^*)}{F(y_{n+1}^*)}$$

Of course, since $y_n^* \to \underline{y}$, $f(y_n^*) \to f(\underline{y}) < \infty$ and $y_n^* - \underline{y} \to 0$, so it suffices to show that $\frac{F(y_n^*)}{F(y_{n+1}^*)}$ remains finite as $c \to r - \underline{y}$ and its numerator and denominator both go to zero. For this final step, note first that

$$(r - y_n^*)(1 - F(y_n^*))^{n-1} \quad = \quad c \quad = \quad (r - y_{n+1}^*)(1 - F(y_{n+1}^*))^n$$

Since $y_n^* \geq y_{n+1}^*$ implies $r - y_n^* \leq r - y_{n+1}^*$, this requires $(1 - F(y_n^*))^{n-1} \geq (1 - F(y_{n+1}^*))^n$, or

$$F(y_n^*) \quad \leq \quad 1 - (1 - F(y_{n+1}^*))^{\frac{n}{n-1}},$$

whence

$$\frac{F(y_n^*)}{F(y_{n+1}^*)} \quad \leq \quad \frac{1 - (1 - F(y_{n+1}^*))^{\frac{n}{n-1}}}{F(y_{n+1}^*)}$$

As $c \to r - \underline{y}$, $F(y_{n+1}^*) \to 0$, so we can calculate the limit of the right-hand side as

$$\lim_{s \to 0} \frac{1 - (1 - s)^{\frac{n}{n-1}}}{s} \quad = \quad \lim_{s \to 0} \frac{\frac{n}{n-1}(1 - s)^{\frac{1}{n-1}}}{1} \quad = \quad \frac{n}{n-1}$$

(again by L'Hopital's Rule). This means that

$$\lim_{c \to r - \underline{y}} \frac{U_{n,k}^{post}}{U_{n+1,k-1}^{pre}} \quad \leq \quad \frac{n}{k} \left(\frac{n}{n-1} \right)^2 f(\underline{y}) \lim_{c \to r - \underline{y}} (y_n^* - \underline{y}) \quad = \quad 0$$

Thus, for $k \geq 1$, $U_{n,k}^{post} / U_{n+1,k-1}^{pre}$ goes to 0 as c goes to $r - \underline{y}$, and is therefore less than 1 (which is all we needed) for c sufficiently large.

Part 2

We know that

$$\frac{U_{n,k}^{post}}{U_{n+1,k-1}^{pre}} \quad = \quad \frac{\frac{1}{k} \int_{\underline{y}+\alpha c}^{y_n^*} F(s - \alpha c)[(1 - F(s))^n - (1 - F(y_n^*))^n] ds}{\int_{\underline{y}}^{y_{n+1}^*} F(s)(1 - F(s))^n (1 - F(s - \alpha c)) ds}$$

so if

$$k \quad > \quad \frac{\int_{\underline{y}+\alpha c}^{y_n^*} F(s - \alpha c)[(1 - F(s))^n - (1 - F(y_n^*))^n] ds}{\int_{\underline{y}}^{y_{n+1}^*} F(s)(1 - F(s))^n (1 - F(s - \alpha c)) ds}$$

then $U_{n,k}^{post} / U_{n+1,k-1}^{pre} < 1$, or $U_{n,k}^{post} < U_{n+1,k-1}^{pre}$.

A.10 Proof of Theorem 2

For part 1, Proposition 5 says that if c is sufficiently large, then $U^{post}_{n,k} < U^{pre}_{n+1,k-1}$ for $k \geq 1$; under Assumption 1, then, if c is sufficiently large, $j = 0$ (the number of pre-auction subcontractors is the same with and without bid shopping), and therefore bid shopping increases total surplus.

For part 2, calculate

$$z = \max_{n \in \{N_1, N_1+1, \ldots, N_1+N_2\}} \frac{\int_{\underline{y}+\alpha c}^{y^*_n} F(s-\alpha c)[(1-F(s))^n - (1-F(y^*_n))^n]ds}{\int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n(1-F(s-\alpha c))ds}$$

If $N_3 > \frac{1}{z}$, then for any $k \geq N_3$, $U^{post}_{n,k} < U^{pre}_{n+1,k-1}$., so under Assumption 1, no subcontractors who could bid pre-auction would choose to bid post-auction. As a result, the number of pre-auction subcontractors is unaffected by bid shopping, so by Proposition 4, bid shopping increases total surplus. □

A.11 Proof of Theorem 3

We showed above that $W_{n,k} > W_{n,0}$; if $W_{n,0} > W_{n+j,0}$ for $j \in \{1, 2, \ldots, k\}$, this will establish that $W_{n,k} > W_{n+j,0}$, i.e., that bid shopping increases total surplus.

We can think of n impacting $W_{n,0}$ in two ways: through the "direct" effect of n on equation 3 (the expression for $W_{n,0}$), and indirectly through its effect on y^*_n. For both, we can treat n as a continuous variable, and calculate

$$\frac{dW_{n,0}}{dn} = \frac{\partial W_{n,0}}{\partial n} + \frac{\partial W_{n,0}}{\partial y^*_n}\frac{dy^*_n}{dn}$$

Since y^*_n is defined by $(r - y^*_n)(1 - F(y^*_n))^{n-1} = c$, the implicit function theorem gives

$$\frac{dy^*_n}{dn} = -\frac{(r-y^*_n)(1-F(y^*_n))^{n-1}\log(1-F(y^*_n))}{-(1-F(y^*_n))^{n-1} - (r-y^*_n)(n-1)(1-F(y^*_n))^{n-2}f(y^*_n)} = -\frac{-\log(1-F(y^*_n))}{\frac{1}{r-y^*_n} + (n-1)\frac{f(y^*_n)}{1-F(y^*_n)}}$$

Integrating equation 3 by parts, we can write

$$W_{n,0} = v - \underline{y} - (v-y^*_n)(1-F(y^*_n))^n - \int_{\underline{y}}^{y^*_n}(1-F(s))^n ds - cnF(y^*_n)$$

from which we can calculate

$$\frac{dW_{n,0}}{dn} = -(v-y^*_n)(1-F(y^*_n))^n\log(1-F(y^*_n)) - \int_{\underline{y}}^{y^*_n}(1-F(s))^n\log(1-F(s))ds - cF(y^*_n)$$

$$+nf(y^*_n)(1-F(y^*_n))^{n-1}(v-r)\left[-\frac{-\log(1-F(y^*_n))}{\frac{1}{r-y^*_n} + (n-1)\frac{f(y^*_n)}{1-F(y^*_n)}}\right]$$

Now, if $v = r$ then the second line vanishes, and we can rewrite this as

$$\left.\frac{dW_{n,0}}{dn}\right|_{v=r} = -c(1-F(y^*_n))\log(1-F(y^*_n)) - \int_{\underline{y}}^{y^*_n}(1-F(s))^n\log(1-F(s))ds - cF(y^*_n)$$

As y^*_n goes to $\underline{y}$ (either because c goes to $r - \underline{y}$ or because n gets large), $F(y^*_n)$ and $\log(1-F(y^*_n))$ go to zero, as does the region of integration, so $dW_{n,0}/dn$ goes to zero; but we can check its sign as y^*_n approaches $\underline{y}$ by calculating its derivative w.r.t. y_n^*, which is

$$\begin{aligned}
\frac{d}{y_n^*}\left(\frac{dW_{n,0}}{dn}\right) &= cf(y_n^*)\log(1-F(y_n^*)) + c(1-F(y_n^*))\frac{f(y_n^*)}{1-F(y_n^*)} - (1-F(y_n^*))^n \log(1-F(y_n^*)) - cf(y_n^*) \\
&= cf(y_n^*)\log(1-F(y_n^*)) - \frac{c}{r-y_n^*}(1-F(y_n^*)\log(1-F(y_n^*)) \\
&= -c\log(1-F(y_n^*))(1-F(y_n^*))\left[-\frac{f(y_n^*)}{1-F(y_n^*)} + \frac{1}{r-y_n^*}\right]
\end{aligned}$$

so for y_n^* close to $\underline{y}$ and $r = v$, $W_{n,0}$ is decreasing in n if $\frac{f(y_n^*)}{1-F(y_n^*)} > \frac{1}{r-y_n^*}$, or $f(\underline{y}) > \frac{1}{r-\underline{y}}$.

When $r < v$, on the other hand, the second line of $dW_{n,0}/dn$ does not vanish, and we can write the whole expression as

$$\begin{aligned}
\frac{dW_{n,0}}{dn} &= -(r-y_n^*)(1-F(y_n^*))^n\log(1-F(y_n^*)) - \int_{\underline{y}}^{y_n^*}(1-F(s))^n\log(1-F(s))ds - cF(y_n^*) \\
&\quad -(v-r)(1-F(y_n^*))^n\log(1-F(y_n^*)) + nf(y_n^*)(1-F(y_n^*))^{n-1}(v-r)\left[-\frac{-\log(1-F(y_n^*))}{\frac{1}{r-y_n^*}+(n-1)\frac{f(y_n^*)}{1-F(y_n^*)}}\right] \\
&\leq -(r-y_n^*)(1-F(y_n^*))^n\log(1-F(y_n^*)) - (y_n^*-\underline{y})\log(1-F(y_n^*)) - cF(y_n^*) \\
&\quad -(v-r)(1-F(y_n^*))^n\log(1-F(y_n^*)) + nf(y_n^*)(1-F(y_n^*))^{n-1}(v-r)\left[-\frac{-\log(1-F(y_n^*))}{\frac{1}{r-y_n^*}+(n-1)\frac{f(y_n^*)}{1-F(y_n^*)}}\right] \\
&= -\log(1-F(y_n^*))\Bigg[c(1-F(y_n^*)) + (y_n^*-\underline{y}) - c\frac{F(y_n^*)}{-\log(1-F(y_n^*))} \\
&\qquad +(v-r)(1-F(y_n^*))^n\left(1-\frac{n\frac{f(y_n^*)}{1-F(y_n^*)}}{\frac{1}{r-y_n^*}+(n-1)\frac{f(y_n^*)}{1-F(y_n^*)}}\right)\Bigg]
\end{aligned}$$

As y_n^* goes to $\underline{y}$ and $F(y_n^*)$ goes to zero, the first line inside the square brackets vanishes, but the second line does not, and is negative if

$$\frac{n\frac{f(y_n^*)}{1-F(y_n^*)}}{\frac{1}{r-y_n^*}+(n-1)\frac{f(y_n^*)}{1-F(y_n^*)}} > 1$$

which is when $\frac{f(y_n^*)}{1-F(y_n^*)} > \frac{1}{r-y_n^*}$; so once again, when y_n^* is close to $\underline{y}$, $W_{n,0}$ is decreasing in n when $\frac{f(y_n^*)}{1-F(y_n^*)} > \frac{1}{r-y_n^*}$. As noted at the start, if $W_{n,0}$ is decreasing in n, then $W_{n,k} > W_{n,0} \geq W_{n+j,0}$ and bid shopping increases surplus. □

A.12 Proof of Proposition 6

Part 1

We want to show that $U^{post}_{n,k} \geq U^{pre}_{n+1,k-1}$ implies $U^{post}_{n,1} \geq U^{pre}_{n+1,0}$. Note that

$$U^{pre}_{n,0} = \int_{\underline{y}}^{y^*_n} F(s)(1-F(s))^{n-1} ds$$

$$U^{pre}_{n,k} = \int_{\underline{y}}^{y^*_n} F(s)(1-F(s))^{n-1}(1-F(s-\alpha c)) ds$$

$$U^{post}_{n,k} = \frac{1}{k}\int_{\underline{y}+\alpha c}^{y^*_n} \left[(1-F(s))^n - (1-F(y^*_n))^n\right] F(s-\alpha c) ds$$

Since $U^{post}_{n,k}$ is decreasing in k (for $k \geq 1$) and $U^{pre}_{n+1,k-1}$ does not depend on k (for $k \geq 2$), it suffices to show the result for $k = 2$, meaning

$$\frac{1}{2}\int_{\underline{y}}^{y^*_n} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y^*_n))^n\right] ds \quad \geq \quad \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n(1-F(s-\alpha c)) ds$$

implies

$$\int_{\underline{y}}^{y^*_n} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y^*_n))^n\right] ds \quad \geq \quad \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n ds$$

so it obviously suffices if

$$2\int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n(1-F(s-\alpha c)) ds \quad \geq \quad \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n ds$$

Since the left-hand side is increasing in α, if this holds for $\alpha = 0$, it holds for every α, so it suffices if

$$\int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n(2-2F(s)) ds \quad \geq \quad \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n ds$$

or

$$\star \quad \equiv \quad \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n(1-2F(s)) ds \quad \geq \quad 0$$

Now, if $F(y^*_{n+1}) \leq \frac{1}{2}$, the integrand is everywhere positive and we're done; if not, let $y_m = F^{-1}(\frac{1}{2})$, and split the integral into two pieces, an integral over $[\underline{y}, y_m]$ and an integral over $[y_m, y^*_{n+1}]$. At the same time, multiply and divide the integrand by $f(s)$ and separate one of the $1 - F(s)$ terms, giving

$$\star \quad = \quad \int_{\underline{y}}^{y_m} F(s)(1-F(s))^{n-1}(1-2F(s))\frac{1-F(s)}{f(s)} f(s) ds + \int_{y_m}^{y^*_{n+1}} F(s)(1-F(s))^{n-1}(1-2F(s))\frac{1-F(s)}{f(s)} f(s) ds$$

Note that the integrand is positive in the first integral and negative in the second. If F has increasing hazard rate, $\frac{1-F(y)}{f(y)}$ is decreasing, so it's weakly greater than $\frac{1-F(y_m)}{f(y_m)}$ everywhere in the first integral and weakly less than that everywhere in the second integral, so

$$\star \quad \geq \quad \frac{1-F(y_m)}{f(y_m)} \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^{n-1}(1-2F(s)) f(s) ds$$

(since the positive part is bigger and the negative part is smaller). Next, let $z = 1 - F(s)$, so that $dz = -f(s)ds$, and rewrite the integral as

$$\star \quad \geq \quad \frac{1 - F(y_m)}{f(y_m)} \int_{1-F^{-1}(y^*_{n+1})}^{1} (1-z)z^{n-1}(2z-1)dz$$

Since the integrand is negative for $z < \frac{1}{2}$ and positive for $z > \frac{1}{2}$, if the integral is positive on $[0,1]$, it's positive on $[x,1]$ for every x; so it suffices if

$$\int_0^1 (1-z)z^{n-1}(2z-1)dz \quad \geq \quad 0$$

We can compute this integral to be

$$\int_0^1 (1-z)z^{n-1}(2z-1)dz \quad = \quad \frac{n-2}{(n+2)(n+1)n} \quad \geq \quad 0$$

giving the result.

Part 1

The expected total surplus expressions $W_{n,0}$ (equation (3) in the body) and $W_{n,k} = W_{n,0} + \Delta W$ (equation (12) above) can be simplified to

$$W_{n,0} \quad = \quad v - \underline{y} - (v - y^*_n)(1 - F(y^*_n))^n - \int_{\underline{y}}^{y^*_n} (1 - F(s))^n ds - nF(y^*_n)c$$

$$W_{n,k} \quad = \quad W_{n,0} + \int_{\underline{y}+\alpha c}^{y^*_n} \left[(1 - F(s))^n - (1 - F(y^*_n))^n\right] F(s - \alpha c)ds$$

Now, starting in a setting with $n + 1$ pre-auction subcontractors and no bid shopping, let Δ denote the combined change in everyone *else's* expected surplus when a single pre-auction subcontractor switches to post-auction bidding. In the latter case, as established above, the total surplus of everyone else is $W_{n,0}$; in the former case, it was $W_{n+1,0} - U^{pre}_{n+1,0}$, so the change in everyone else's combined surplus is

$$\begin{aligned}
\Delta \quad &\equiv \quad W_{n,0} - (W_{n+1,0} - U^{pre}_{n+1,0}) \\
&= \quad v - \underline{y} - (v - y^*_n)(1 - F(y^*_n))^n - \int_{\underline{y}}^{y^*_n} (1 - F(s))^n ds - nF(y^*_n)c \\
&\quad + \int_{\underline{y}}^{y^*_{n+1}} F(s)(1 - F(s))^n ds \\
&\quad -(v - \underline{y}) + (v - y^*_{n+1})(1 - F(y^*_{n+1}))^{n+1} + \int_{\underline{y}}^{y^*_{n+1}} (1 - F(s))^{n+1} ds + (n + 1)F(y^*_{n+1})c
\end{aligned}$$

Now, we...

- cancel the $v - \underline{y}$ and $-(v - \underline{y})$ terms
- split each $(v - y^*)$ term into $(v - r) + (r - y^*)$ and group the two resulting $(v - r)$ terms
- combine $\int_{\underline{y}}^{y^*_{n+1}} F(s)(1 - F(s))^n ds + \int_{\underline{y}}^{y^*_{n+1}} (1 - F(s))^{n+1} ds$ into $\int (1 - F(s))^n ds$
- split $(r - y^*_{n+1})(1 - F(y^*_{n+1}))^{n+1}$ into $(r - y^*_{n+1})(1 - F(y^*_{n+1}))^n(1 - F(y^*_{n+1})) = (r - y^*_{n+1})(1 - F(y^*_{n+1}))^n - F(y^*_{n+1})c$, and combine the $-F(y^*_{n+1})c$ with $(n + 1)F(y^*_{n+1})c$

leaving us with

$$\begin{aligned}\Delta &= (v-r)\left[(1-F(y^*_{n+1}))^{n+1}-(1-F(y^*_n))^n\right] \\ &\quad -(r-y^*_n)(1-F(y^*_n))^n - \int_{\underline{y}}^{y^*_n}(1-F(s))^n ds - nF(y^*_n)c \\ &\quad +(r-y^*_{n+1})(1-F(y^*_{n+1}))^n + \int_{\underline{y}}^{y^*_{n+1}}(1-F(s))^n ds + nF(y^*_{n+1})c\end{aligned}$$

The last two lines are the function $(r-x)(1-F(x))^n + \int_{\underline{y}}^{x}(1-F(s))^n ds + nF(x)c$, evaluated at y^*_n (with a negative sign) and y^*_{n+1}. Writing the difference as the integral of the derivative of that function,

$$\begin{aligned}\Delta &= (v-r)\left[(1-F(y^*_{n+1}))^{n+1}-(1-F(y^*_n))^n\right] \\ &\quad -\int_{y^*_{n+1}}^{y^*_n}\frac{d}{dx}\left[(r-x)(1-F(x))^n + \int_{\underline{y}}^{x}(1-F(s))^n ds + nF(x)c\right]dx \\ &= (v-r)\left[(1-F(y^*_{n+1}))^{n+1}-(1-F(y^*_n))^n\right] \\ &\quad -\int_{y^*_{n+1}}^{y^*_n}\left[-(1-F(x))^n - n(r-x)(1-F(x))^{n-1}f(x) + (1-F(x))^n + nf(x)c\right]dx \\ &= (v-r)\left[(1-F(y^*_{n+1}))^{n+1}-(1-F(y^*_n))^n\right] \\ &\quad +\int_{y^*_{n+1}}^{y^*_n} n\left[(r-x)(1-F(x))^{n-1}-c\right]f(x)dx\end{aligned}$$

Since $(r-x)(1-F(x))^{n-1}$ is decreasing in x, and is equal to c at $x=y^*_n$, the integrand in the second line is everywhere positive, so the integral is positive. Thus, $\Delta \geq 0$ whenever the first line is nonnegative, which holds when $r=v$ (or when $r<v$ and $(1-F(y^*_{n+1}))^{n+1} \geq (1-F(y^*_n))^n$).

This establishes that when $r=v$, $W_{n,0} > W_{n+1,0} - U^{pre}_{n+1,0}$. If in addition, $U^{post}_{n,1} \geq U^{pre}_{n+1,0}$, then $W_{n,0} + U^{post}_{n,1} \geq W_{n+1,0}$; since the left-hand side is $W_{n,1}$, the result follows.

□

A.13 Example 2

To illustrate that it is sometimes profitable for a single subcontractor to wait until after the auction to bid (and therefore that Proposition 6 is not vacuous), but also that Proposition 6 can fail with $r<v$, consider the following setting.

Example 2 *Suppose there are 5 subcontractors, with costs uniformly distributed on* $[0,1]$ except *that all valuations between* 0 *and* 0.05 *are replaced by a point mass at* 0.025.[77] *Let* $r=1$, *and* $c=0.8$. *When there is no bid shopping and all 5 subs bid pre-auction, the entry threshold* y^* *must be below* 0.05, *which means that subs with costs* $y=0.025$ *mix between entering and not entering, and subs with costs of* 0.05 *and above never enter. The entry probability is such that* $(1-0.025)(1-\Pr(entry))^4 = 0.8$, *which means each sub enters with probability approximately* 0.048. *On the other hand, when there are four pre- and one post-auction subcontractor, the entry threshold is* 0.054, *so all subs with costs* $y=0.025$ *and a very few with costs above* 0.05 *enter.*

[77] More formally, let F be the distribution whose density function is $\frac{0.05}{2\epsilon}$ on $[0.025-\epsilon, 0.025+\epsilon]$, 1 on $[0.05,1]$, and 0 elsewhere, and consider the limit $\epsilon \to 0$.

With $n = 5$ and no bid shopping, subs get 0 expected surplus; with $n = 4$ and bid shopping, the post-auction sub knows with some small probability, he will have cost $y_0 = 0.025$ and the pre-auction entrant will have cost above 0.05. So as long as αc is below 0.025, $U^{post}_{4,1} > 0 = U^{pre}_{5,0}$, making it profitable for one subcontractor to wait to bid post-auction. We can also calculate that bid shopping reduces the probability of a sale – to 20%, from about 22% without bid shopping. If $v = 1$, bid shopping must increase total surplus; it does this by reducing the expected number of subcontractors incurring the entry cost, which outweighs the reduction in gross surplus from a lower probability of sale. However, if v is, say, 1.5, then the reduction in sales costs more in surplus, and bid shopping would then reduce total surplus.

A.14 Proof of Theorem 4

Under Assumption 1, if one subcontractor is diverted to post-auction bidding, $U^{post}_{n,k} \geq U^{pre}_{n+1,k-1}$. Proposition 6 part 1 then implies $U^{post}_{n,1} \geq U^{pre}_{n+1,0}$. Proposition 6 part 2 then implies $W_{n,1} > W_{n+1,0}$. Since $W_{n,k}$ does not depend on k for $k \geq 1$, this means $W_{n,k} > W_{n+1,0}$, so bid shopping that diverts one subcontractor increases total surplus.

A.15 Proof of Theorem 5

We first want to show that for c sufficiently small, $U^{post}_{n,1} < U^{pre}_{n+1,0}$. As noted in the body,

$$U^{post}_{n,1} = \int_{\underline{y}}^{y^*_n} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y^*_n))^n\right] ds$$

and

$$U^{pre}_{n+1,0} = \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n ds$$

Define $\Delta = U^{post}_{n,1} - U^{pre}_{n+1,0}$, which we want to show is strictly negative at c very small but positive. As $c \to 0$, note that y^*_n and y^*_{n+1} both go to r if $r < \overline{y}$, and to $\overline{y}$ otherwise. We treat the two cases separately.

First, if $r < \overline{y}$, then

$$\begin{aligned}\lim_{c\to 0}\Delta &= \int_{\underline{y}}^{r} F(s)\left[(1-F(s))^n - (1-F(r))^n\right] ds - \int_{\underline{y}}^{r} F(s)(1-F(s))^n ds \\ &= -\int_{\underline{y}}^{r} F(s)(1-F(r))^n ds \quad < \quad 0\end{aligned}$$

On the other hand, if $r > \overline{y}$, then $(1-F(y^*_n))^n \to 0$ as $c \to 0$, and therefore $\Delta \to 0$. Thinking of Δ as a function of c, then, we can calculate its sign for small c by calculating the sign of its derivative at 0. Differentiating,

$$\begin{aligned}\Delta'(c) &= \frac{d}{dc}\left[\int_{\underline{y}}^{y^*_n} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y^*_n))^n\right] ds - \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n ds\right] \\ &= -\alpha\int_{\underline{y}}^{y^*_n} f(s-\alpha c)\left[(1-F(s))^n - (1-F(y^*_n))^n\right] ds \\ &\quad + \frac{dy^*_n}{dc}F(y^*_n-\alpha c)\left[(1-F(y^*_n))^n - (1-F(y^*_n))^n\right] - \frac{dy^*_{n+1}}{dc}F(y^*_{n+1})(1-F(y^*_{n+1}))^n\end{aligned}$$

Now, y^*_n is defined by $(r-y^*_n)(1-F(y^*_n))^{n-1} = c$, and so differentiating both sides,

$$\frac{dy^*_n}{dc}\left[-(1-F(y^*_n))^{n-1} - (r-y^*_n)(n-1)(1-F(y^*_n))^{n-2}f(y^*_n)\right] = 1$$

and therefore

$$\frac{dy_n^*}{dc}(1-F(y_n^*))^{n-2} \quad = \quad -\frac{1}{1-F(y_n^*)+(r-y_n^*)(n-1)f(y_n^*)}$$

As $c \to 0$ and $y_n^* \to \overline{y}$ (and $F(y_n^*)$ to 1), then, the left-hand side converges to $\frac{1}{(r-\overline{y})(n-1)f(\overline{y})}$, which is finite. Thus, since $1-F(y_n^*) \to 0$, $(1-F(y_n^*))^n \frac{dy_n^*}{dc} \to 0$, and likewise $\frac{dy_{n-1}^*}{dc}(1-F(y_{n+1}^*))^n$. This means that the second line in $\Delta'(c)$ above vanishes as $c \to 0$, and therefore that

$$\begin{aligned}\lim_{c\to 0} \Delta'(c) \quad &= \quad \lim_{c\to 0}\left(-\alpha \int_{\underline{y}}^{y_n^*} f(s-\alpha c)\left[(1-F(s))^n-(1-F(y_n^*))^n\right] ds\right) \\ &= \quad -\alpha \int_{\underline{y}}^{\overline{y}} f(s)(1-F(s))^n ds \quad < \quad 0\end{aligned}$$

Since $\Delta(0)=0$, this implies that $\Delta(c)$ is strictly negative for c small but positive, as claimed.

Thus, for c sufficiently small, $U_{n,1}^{post} < U_{n+1,0}^{pre}$. Under Assumption 1, then, if $N_3 = 0$ (no "new" subcontractors), bid shopping can't divert exactly one subcontractor to post-auction bidding. This means it either diverts none (in which case it has no effect), or diverts more than one. In the latter case, we argue it strictly decreases total surplus when c is sufficiently small. This is because as c goes to zero, incurred bid preparation costs vanish, and total surplus approaches $v - y_{min}$, where y_{min} is the lowest cost realization among the subcontractors who bid. Without bid shopping, all n subs bid, so y_{min} is the lowest of n independent draws from F. With bid shopping, if $k > 1$ subs wait until after the auction and only one of them is approached for a bid, then only $n-k+1 < n$ subs have an opportunity to bid, so y_{min} is the lowest of $n-k+1$ independent draws from F, and therefore stochastically higher than without bid shopping; so expected total surplus is lower.

A.16 Calculations for Example 1

First, note that as $c \to 0$, $F(y_n^*) \to 1$. (Since $(r-y_n^*)(1-F(y_n^*))^{n-1} = c$ and $r - y_n^*$ is bounded below by $r - \overline{y} = 0.1$, $1-F(y_n^*)$ must go to 0 as c does.) So

$$\begin{aligned}\lim_{\epsilon\to 0}\lim_{c\to 0} U_{2,2}^{post} \quad &= \quad \lim_{\epsilon\to 0}\lim_{c\to 0} \frac{1}{2}\int_{\alpha c}^{y_2^*} F(s-\alpha c)\left[(1-F(s))^2-(1-F(y_n^*))^2\right] ds \\ &= \quad \lim_{\epsilon\to 0} \frac{1}{2}\int_0^1 F(s)\left[(1-F(s))^2-0\right] ds \\ &= \quad \lim_{\epsilon\to 0} \frac{1}{2}\int_0^1 0.6\cdot 0.4^2 ds \quad = \quad 0.048\end{aligned}$$

the last line because $F(s) \to 0.6$ for every $s \in (0,1)$ as $\epsilon \to 0$. Similarly,

$$\begin{aligned}\lim_{\epsilon\to 0}\lim_{c\to 0} U_{3,1}^{pre} \quad &= \quad \lim_{\epsilon\to 0}\lim_{c\to 0}\int_0^{y_3^*} F(s)(1-F(s))^2(1-F(s-\alpha c))ds \\ &= \quad \lim_{\epsilon\to 0}\int_0^1 F(s)(1-F(s))^3 ds \\ &= \quad \int_0^1 0.6\cdot 0.4^3 ds \quad = \quad 0.0384\end{aligned}$$

confirming that (in the double limit) Assumption 1 holds.

As for total surplus, with $c = 0$ in the limit and entry occurring with probability 1, total surplus is $1.1 - 1$ when all subcontractors who bid have cost $y_j = 1$, and 1.1 when at least one subcontractor has cost $y_j = 0$ and bids. The probability of some subcontractor having cost $y_j = 0$ and having an opportunity to bid is $1 - 0.4^4$ without bid shopping, but $1 - 0.4^3$ with bid shopping, since only one of the two post-auction subcontractors (plus the two pre-auction subcontractors) will bid.

A.17 Proof of Theorem 6

For bid shopping to divert at least two subcontractors, we need $U^{post}_{n,2} \geq U^{pre}_{n+1,1}$, where n is the number of pre-auction subcontractors in the presence of bid shopping. As long as this holds, the rest of the result is automatic – reducing the number of pre-auction subcontractors by two (or more) and introducing bid shopping is always welfare-decreasing when $c \approx 0$, so all we need is for it to be possible. So we need to show

$$\frac{1}{2}\int_{\underline{y}}^{y^*_n} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y^*_n))^n\right] ds \quad \geq \quad \int_{\underline{y}}^{y^*_{n+1}} F(s)(1-F(s))^n(1-F(s-\alpha c))ds$$

Since we want this to hold for c sufficiently small, it suffices if it holds strictly in the limit $c \to 0$; with $r \geq \overline{y}$, this means $y^*_n, y^*_{n+1} \to \overline{y}$ and $(1-F(y^*_n))^n \to 0$ as $c \to 0$, so it suffices if

$$\frac{1}{2}\int_{\underline{y}}^{\overline{y}} F(s)(1-F(s))^n ds \quad > \quad \int_{\underline{y}}^{\overline{y}} F(s)(1-F(s))^{n+1} ds$$

or

$$\int_{\underline{y}}^{\overline{y}} F(s)(1-F(s))^n(2F(s)-1)ds \quad > \quad 0$$

We'll call the left-hand side of this last expression $\star$.

Now, let $y_x = F^{-1}(x)$, and break the integral into three pieces, writing it as

$$\begin{aligned}\star \quad = \quad & \int_{\underline{y}}^{y_{1/2}} F(s)(1-F(s))^n(2F(s)-1)\frac{1}{f(s)}f(s)ds + \int_{y_{1/2}}^{y_{3/4}} F(s)(1-F(s))^n(2F(s)-1)\frac{1}{f(s)}f(s)ds \\ & + \int_{y_{3/4}}^{\overline{y}} F(s)(1-F(s))^n(2F(s)-1)\frac{1}{f(s)}f(s)ds\end{aligned}$$

The second integral is positive; so if f is decreasing as assumed,

$$\begin{aligned}\star \quad > \quad & \int_{\underline{y}}^{y_{1/2}} F(s)(1-F(s))^n(2F(s)-1)\frac{1}{f(y_{1/2})}f(s)ds + \int_{y_{3/4}}^{\overline{y}} F(s)(1-F(s))^n(2F(s)-1)\frac{1}{f(y_{3/4})}f(s)ds \\ = \quad & \frac{1}{f(y_{1/2})}\int_0^{\frac{1}{2}} z(1-z)^n(2z-1)dz + \frac{1}{f(y_{3/4})}\int_{\frac{3}{4}}^1 z(1-z)^n(2z-1)dz\end{aligned}$$

Since the first integral is negative and the second is positive, and neither integral depends on the distribution F, the entire expression is positive if $f(y_{3/4})/f(y_{1/2})$ is sufficiently small. Now,

$$\log\left(\frac{f(y_{3/4})}{f(y_{1/2})}\right) \quad = \quad \log f(y_{3/4}) - \log f(y_{1/2}) \quad = \quad \int_{\frac{1}{2}}^{\frac{3}{4}} \left(f(F^{-1}(z))\right)' dz \quad = \quad \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{f'(F^{-1}(z))}{f(F^{-1}(z))}dz$$

so if $\frac{f'}{f} < -\lambda$ everywhere, $\frac{f(y_{3/4})}{f(y_{1/2})} < \exp\left(-\frac{1}{4}\lambda\right)$. So if λ is sufficiently large, $\frac{f(y_{3/4})}{f(y_{1/2})}$ is small, making $\star > 0$ and therefore $U^{post}_{n,2} \geq U^{pre}_{n+1,1}$ for c sufficiently small, which proves the result.

A.18 Proof of Theorem 7

Part 1

For $k > 0$,

$$U^{pre}_{n,0} = \int_{\underline{y}}^{y^*_n} F(s)(1-F(s))^{n-1} ds$$

$$> \int_{\underline{y}}^{y^*_n} F(s)(1-F(s))^{n-1}(1-F(s-\alpha c)) ds = U^{pre}_{n,k}$$

giving the result for subcontractors. For the procurer, Corollary 2 established that for each realization of subcontractor's costs, each prime contractor has lower costs in the presence of bid shopping than in its absence; so each prime contractor bids lower, increasing the procurer's surplus.

Part 2

The result for subcontractors is because for $j \geq 1$,

$$\begin{aligned}
U^{pre}_{n,k} &= \int_{\underline{y}}^{y^*_n} F(s)(1-F(s))^{n-1}(1-F(s-\alpha c)) ds \\
&\geq \int_{\underline{y}}^{y^*_n} F(s)(1-F(s))^{n} ds \\
&\geq \int_{\underline{y}}^{y^*_{n+j}} F(s)(1-F(s))^{n+j-1} ds \\
&= U^{pre}_{n+j,0}
\end{aligned}$$

since $1 - F(s-\alpha c) \geq 1 - F(s)$ and $y^*_n > y^*_{n+j}$. For the procurer, Example 1 leads to illustrations of cases where bid shopping either increases or decreases procurer surplus. In the absence of bid shopping, taking the limit $c \to 0$ but with $\epsilon > 0$, subcontractors bid $\beta_{4,0}(y) \to y + \frac{1}{(1-F(y))^3}\int_y^1 (1-F(s))^3 ds$. If we let G denote the CDF of a given subcontractor's bids, then $G(b) = F(\beta^{-1}_{4,0}(b))$, and we can calculate G from $G^{-1}(z) = \beta_{4,0}(F^{-1}(z))$. $F(y) = 1/\left(1 + \frac{2}{3}(\frac{1-y}{y})^\epsilon\right)$ implies $F^{-1}(z) = 1/\left(1 + (\frac{3}{2}(\frac{1}{z}-1))^{1/\epsilon}\right)$, and so

$$\beta_{4,0}(F^{-1}(z)) = F^{-1}(z) + \frac{1}{(1-z)^3}\int_{F^{-1}(z)}^1 (1-F(s))^3 ds$$

As $\epsilon \to 0$, $F(s) \to \frac{3}{5}$ for $s \in (0,1)$, and $F^{-1}(z) \to 0$ for $z < 0.6$, and $\to 1$ for $z > 0.6$. Thus, as $\epsilon \to 0$,

$$\beta_{4,0}(F^{-1}(z)) \to F^{-1}(z) + \frac{1}{(1-z)^3}\left(\frac{2}{5}\right)^3 (1-F^{-1}(z)) \to \begin{cases} \frac{0.4^3}{(1-z)^3} & for \quad z < 0.6 \\ 1 & for \quad z > 0.6 \end{cases}$$

Inverting $\beta_{4,0} \circ F^{-1}$ implies that the CDF of each subcontractor's sub-bids, in the absence of bid shopping, is

$$G(y) = 1 - 0.4y^{-1/3}$$

for $y \in (0.4^3, 1)$, with a point mass of weight 0.4 at 1. That is, in the limit $\epsilon \to 0$ where F puts mass 0.6 on $y = 0$ and 0.4 on $y = 1$, subcontractors with costs $y = 1$ all bid 1, while subcontractors with costs $y = 0$ mix on the interval $[0.4^3, 1]$ to give the combined CDF above.

This means that if each prime contractor has two subcontractors bidding, the CDF of each prime's lower bid (and hence her cost in the absence of bid shopping) is $1 - (1 - G(\cdot))^2 = 1 - 0.4^2 z^{-2/3}$. Since the prime auction is a second-price auction, the procurer pays the higher of the two primes' bids, and the CDF of the procurer's payment is therefore

$$H_{2,2,no\ shop}(z) \quad = \quad \left(1 - 0.4^2 z^{-2/3}\right)^2$$

On the other hand, if one prime contractor has one subcontractor and the other has three, the CDFs of the two primes' lowest sub-bids are $G(\cdot)$ and $1-(1-G(\cdot))^3$, and the CDF of the procurer's payment is their product,

$$H_{1,3,no\ shop}(z) \quad = \quad \left(1 - 0.4 z^{-1/3}\right)\left(1 - 0.4^3 z^{-1}\right)$$

With bid shopping and one pre-auction sub per prime, as $c \to 0$ but $\epsilon > 0$, pre-auction sub-bids are $\beta_{2,2}(y) \to y + \frac{1}{(1-F(y))^2}\int_y^1 (1-F(s))^2 ds$, and a prime contractor's expected cost is

$$C(\beta_{2,2}(y), y) \quad = \quad (1-F(y-\alpha c))\beta_{2,2}(y) + F(y-\alpha c)y \quad \to \quad y + \frac{1}{1-F(y)}\int_y^1 (1-F(s))^2 ds$$

so if we this time let J denote the CDF of each prime contractor's interim expected cost (and therefore bid),

$$J^{-1}(z) \quad = \quad C(\beta_{2,2}(F^{-1}(z), F^{-1}(z))) \quad \to \quad F^{-1}(z) + \frac{1}{1-z}(1-F^{-1}(z))\left(\frac{2}{5}\right)^2$$

and therefore

$$J^{-1}(z) \quad \to \quad \begin{cases} \dfrac{0.4^2}{1-z} & for \quad z < 0.6 \\ 1 & for \quad z > 0.6 \end{cases}$$

giving $J(z) = 1 - 0.4^2 z^{-1}$. Since the procurer pays the higher of the two primes' bids (equal to their interim expected costs), the CDF of his payment is

$$H_{1,1,shop}(z) \quad = \quad (1 - 0.4^2 z^{-1})^2$$

Now, the procurer's cost is always weakly below 1; $z \leq 1$ implies $(1-0.4^2 z^{-2/3})^2 \geq (1-0.4^2 z^{-1})^2$, and therefore $H_{2,2,no\ shop} \leq_{FOSD} H_{1,1,shop}$ – with no bid shopping and two subcontractors linked to each prime, the procurer pays less (in a first-order stochastic dominance sense), and therefore earns higher surplus. On the other hand, $H_{1,3,no\ shop}$ and $H_{1,1,shop}$ are not ranked by first-order stochastic dominance, but we can calculate that when one prime has one subcontractor and one has three, the procurer pays *less* on average when there is bid shopping. Thus, bid shopping can either benefit or harm the procurer.

Part 3

The same example can establish that prime contractor surplus can either increase or decrease with bid shopping. In this example, if there were two prime contractors, each with two subcontractors, and bid shopping caused one of each of their subcontractors to wait to bid post-auction, we can calculate numerically that prime contractor surplus would decrease from 0.345 to 0.317. On the other hand, if there were four prime contractors, each with one subcontractor, and bid shopping caused two of the primes to miss out on the auction completely because their subcontractors were waiting to bid post-auction, bid shopping would cause prime contractor surplus to increase from 0.243 to 0.317.

When bid shopping does not change the number of pre-auction subcontractors, prime contractor profits can likewise either increase or decrease with bid shopping. Consider an environment where F is the uniform distribution on $[0,1]$, $r = 1$, $\alpha \approx 0$, and there are three prime contractors, each with three pre-auction subcontractors. If bid shopping does not change the number of pre-auction subcontractors, just introduces a new subcontractor the winning prime can shop her bid to, we can calculate that when $c = 0.002$, bid shopping reduces each prime contractor's expected surplus from 0.0513 to 0.0501; and when $c = 0.075$, bid shopping increases each prime contractor's expected surplus from 0.1003 to 0.1015.[78]

A.19 Proof of Proposition 7

Note that α does not influence y_n^* or $W_{n,0}$. Post-auction subcontractors' surplus

$$U_{n,k}^{post} = \frac{1}{k}\int_{\underline{y}}^{y_n^*} F(s-\alpha c)\left[(1-F(s))^n - (1-F(y_n^*))^n\right] ds$$

are transparently decreasing in α, and therefore so is total surplus $W_{n,k} = W_{n,0} + kU_{n,k}^{post}$. Pre-auction subcontractors' surplus

$$U_{n,k}^{pre} = \int_{\underline{y}}^{y_n^*} F(s)(1-F(s))^{n-1}(1-F(s-\alpha c))ds$$

is similarly increasing in α. Finally, Corollary 2 established that a prime contractor's expected cost at the time of the prime auction is

$$(1-F(y-\alpha c))\beta_{n,k}(y) + F(y-\alpha c)y = y + \frac{c + \int_y^{y_n^*}(1-F(s))^{n-1}(1-F(s-\alpha c))ds}{(1-F(y))^{n-1}}$$

when her lowest-cost pre-auction subcontractor's cost is y; since this is increasing in α, each prime contractor's bid (for each realization of subcontractor costs) will be higher with higher α, so procurer costs will be as well.

[78]Expected prime contractor surplus (either with or without bid shopping) can be written as

$$\Pi_i = \int_{\underline{y}}^{y_n^*} \left(1-(1-F(s))^{n_i}\right)(1-F(s))^{n-n_i} C'(s)ds$$

where $C(s)$ is the expected cost in equilibrium of a prime contractor whose lowest-cost (pre-auction) sub has cost realization s – so $C(s) = \beta_{n,0}(s)$ without bid shopping, and $C(s) = (1-F(s-\alpha c))\beta_{n,k}(s) + F(s-\alpha c)s$ with bid shopping. Plugging in the equilibrium bid functions, after lots of simplification, we find

$$\Pi_i^{shop} - \Pi_i^{noshop} = \int_{\underline{y}}^{y_n^*} F(s)F(s-\alpha c)\left[-(n-n_i)\frac{\sum_{j=1}^{n_i-1}(1-F(s))^{n-1-j}}{n_i-1} + n(1-F(s))^{n-1}\right] ds$$

If either c is large enough or r low enough such that $1-F(y_n^*) > \left(\frac{n-n_i}{n}\right)^{1/(n_i-1)}$, then the term in square brackets is positive over the entire integrand, in which case prime contractor surplus is higher with bid shopping. On the other hand, if F is the uniform distribution on $[0,1]$, $r \geq 1$, and $n_k \geq 2$ for each subcontractor k, we can laboriously calculate the integral in the limit as $c \to 0$ and find

$$\lim_{c\to 0}\left(\Pi_i^{shop} - \Pi_i^{noshop}\right) = \frac{1}{n_i-1}\left[\frac{1}{n-n_i+1} - \frac{2}{n-n_i+2} + \frac{n^2+nn_i-2n_i}{n(n+1)(n+2)}\right] < 0$$

so prime contractor surplus is lower with bid shopping for c sufficiently small.

A.20 Proof of Proposition 8

In the absence of bid shopping, we can calculate expected procurer surplus as the expected value of $v - b$, where b is the equilibrium bid of the subcontractor with the lowest cost. This is

$$\begin{aligned} \Pi_{n,0} &= \int_{\underline{y}}^{y_n^*} (v - \beta_{n,0}(y)) n(1-F(y))^{n-1} f(y) dy \\ &= \int_{\underline{y}}^{y_n^*} \left(v - y - \frac{1}{(1-F(y))^{n-1}} \left(c + \int_y^{y_n^*} (1-F(s))^{n-1} ds \right) \right) n(1-F(y))^{n-1} f(y) dy \\ &= n \int_{\underline{y}}^{y_n^*} \left((v-y)(1-F(y))^{n-1} - c - \int_y^{y_n^*} (1-F(s))^{n-1} ds \right) f(y) dy \end{aligned}$$

Differentiating,

$$\begin{aligned} \frac{d\Pi_{n,0}}{dr} &= \left[n \left((v - y_n^*)(1-F(y_n^*))^{n-1} - c \right) f(y_n^*) - n \int_{\underline{y}}^{y_n^*} (1-F(y_n^*))^{n-1} f(y) dy \right] \frac{dy_n^*}{dr} \\ &= n(1-F(y_n^*))^{n-1} f(y_n^*) \left[v - r - \frac{F(y_n^*)}{f(y_n^*)} \right] \frac{dy_n^*}{dr} \end{aligned}$$

Since y_n^* is defined by $\log(r - y_n^*) + (n-1)\log(1-F(y_n^*)) - \log c = 0$,

$$\frac{dy_n^*}{dr} = -\frac{\frac{1}{r-y_n^*}}{-\frac{1}{r-y_n^*} - (n-1)\frac{f(y_n^*)}{1-F(y_n^*)}} = \frac{\frac{1}{r-y_n^*}}{\frac{1}{r-y_n^*} + (n-1)\frac{f(y_n^*)}{1-F(y_n^*)}} \in (0,1)$$

and therefore

$$\begin{aligned} \frac{d}{dr}\left(r + \frac{F(y_n^*)}{f(y_n^*)} \right) &= 1 + \frac{dy_n^*}{dr} \frac{d}{dy_n^*} \left(\frac{F(y_n^*)}{f(y_n^*)} \right) \\ &\propto \frac{1}{\frac{dy_n^*}{dr}} + \frac{d}{dy_n^*} \left(\frac{F(y_n^*)}{f(y_n^*)} \right) \\ &> 1 + \frac{d}{dy_n^*} \left(\frac{F(y_n^*)}{f(y_n^*)} \right) \\ &= \frac{d}{dy} \left(y + \frac{F(y)}{f(y)} \right) \bigg|_{y=y_n^*} \end{aligned}$$

so if F is regular, $r + \frac{F(y_n^*)}{f(y_n^*)}$ is increasing, and therefore $\Pi_{n,0}$ is single-peaked in r and maximized at the solution to $r + \frac{F(y_n^*)}{f(y_n^*)} = v$. (Note that $\Pi_{n,0}$ must have an interior maximizer: at $r = \underline{y} + c$, $y_n^* = \underline{y}$ and $\Pi_{n,0} = 0$, while $\Pi_{n,0} > 0$ for any $r \in (\underline{y} + c, v]$; and it's easy to see that $d\Pi_{n,0}/dr < 0$ for $r > v$.)

For part 2, we can calculate procurer's surplus as the expected value of $v - B$, where B is now the bid submitted by each prime contractor receiving the lowest-cost subcontractor's bid. As calculated in the proof of Corollary 2, this is

$$(1 - F(y - \alpha c))\beta_{n,k}(y) + F(y - \alpha c) y = y + \frac{c + \int_y^{y_n^*} (1-F(s))^{n-1}(1 - F(s - \alpha c)) ds}{(1-F(y))^{n-1}}$$

and therefore

$$\Pi_{n,k} = \int_{\underline{y}}^{y_n^*} \left(v - y - \frac{c + \int_y^{y_n^*} (1-F(s))^{n-1}(1-F(s-\alpha c))ds}{(1-F(y))^{n-1}} \right) n(1-F(y))^{n-1} f(y)dy$$

$$= n \int_{\underline{y}}^{y_n^*} \left((v-y)(1-F(y))^{n-1} - c - \int_y^{y_n^*} (1-F(s))^{n-1}(1-F(s-\alpha c))ds \right) f(y)dy$$

Once again, r enters only through y_n^*, and

$$\frac{d\Pi_{n,k}}{dr} = \left[n\left((v-y_n^*)(1-F(y_n^*))^{n-1} - c\right) f(y_n^*) - n\int_{\underline{y}}^{y_n^*} \left((1-F(y_n^*))^{n-1}(1-F(y_n^*-\alpha c))\right) f(y)dy \right] \frac{dy_n^*}{dr}$$

$$= n(1-F(y_n^*))^{n-1} \left[v - r - (1-F(y_n^*-\alpha c)) \frac{F(y_n^*)}{f(y_n^*)} \right] \frac{dy_n^*}{dr}$$

So, if $r + (1-F(y_n^*-\alpha c))\frac{F(y_n^*)}{f(y_n^*)}$ is increasing in r, $\Pi_{n,k}$ is single-peaked and maximized where $r + (1-F(y_n^*-\alpha c))\frac{F(y_n^*)}{f(y_n^*)} = v$. To see when this holds, note

$$\frac{d}{dr}\left(r + (1-F(y_n^*-\alpha c))\frac{F(y_n^*)}{f(y_n^*)} \right) = 1 + \left(-f(y_n^*-\alpha c)\frac{F(y_n^*)}{f(y_n^*)} + (1-F(y_n^*-\alpha c))\frac{d}{dy_n^*}\frac{F(y_n^*)}{f(y_n^*)} \right) \frac{dy_n^*}{dr}$$

In the limit $\alpha \to 0$ (as assumed in Assumption 2), this is

$$\begin{aligned}
\frac{d}{dr}\left(r + (1-F(y_n^*-\alpha c))\frac{F(y_n^*)}{f(y_n^*)} \right) &= 1 + \left(-f(y_n^*)\frac{F(y_n^*)}{f(y_n^*)} + (1-F(y_n^*))\frac{d}{dy_n^*}\frac{F(y_n^*)}{f(y_n^*)} \right) \frac{dy_n^*}{dr} \\
&\propto \frac{1}{\frac{dy_n^*}{dr}} - F(y_n^*) + (1-F(y_n^*))\frac{d}{dy_n^*}\frac{F(y_n^*)}{f(y_n^*)} \\
&> (1-F(y_n^*))\left(1 + \frac{d}{dy_n^*}\frac{F(y_n^*)}{f(y_n^*)} \right) \\
&= (1-F(y_n^*))\frac{d}{dy}\left(1 + \frac{F(y)}{f(y)} \right)\bigg|_{y=y_n^*}
\end{aligned}$$

which again is positive by Assumption 2, giving the result.

A.21 Proof of Proposition 9

Recall that r affects total surplus only through y^*, which is increasing in r. The proof of Corollary 1 showed that

$$\frac{\partial W_{n,0}}{\partial y_n^*} = nf(y_n^*)(1-F(y_n^*))^{n-1}(v-r)$$

which is positive for $r < v$ and negative at $r > v$, giving the first result. For the second, note that

$$W_{n,k} = W_{n,0} + \int_{\underline{y}+\alpha c}^{y_n^*} \left[(1-F(s))^n - (1-F(y_n^*))^n\right] F(s-\alpha c)ds$$

and therefore

$$\frac{\partial W_{n,k}}{\partial y_n^*} = nf(y_n^*)(1-F(y_n^*))^{n-1}(v-r) + \int_{\underline{y}+\alpha c}^{y_n^*} n(1-F(y_n^*))^{n-1} f(y_n^*) F(s-\alpha c) ds$$

$$= n(1-F(y_n^*))^{n-1} f(y_n^*) \left[v - r + \int_{\underline{y}+\alpha c}^{y_n^*} F(s-\alpha c) ds \right]$$

We established above that $\frac{dy_n^*}{dr} < 1$; as a result,

$$\frac{d}{dr}\left(r - \int_{\underline{y}+\alpha c}^{y_n^*} F(s-\alpha c) ds \right) = 1 - \frac{dy_n^*}{dr} F(y_n^* - \alpha c) > 0$$

and so $v = r - \int_{\underline{y}+\alpha c}^{y_n^*} F(s-\alpha c) ds$ has a unique solution which maximizes $W_{n,k}$, and $\frac{dW_{n,k}}{dr}$ is positive below that solution and negative above it, hence single-peaked.

A.22 Proof of Theorem 8

For part 1, let r_n^* denote the procurer-optimal reserve without bid shopping, and y_n^* the resulting entry threshold; and let $r_{n,k}^*$ denote the procurer-optimal reserve with bid shopping. Note that

$$r_n^* + (1-F(y_n^*))\frac{F(y_n^*)}{f(y_n^*)} < r_n^* + \frac{F(y_n^*)}{f(y_n^*)} = v$$

(the latter by Equation 9). Since $r + (1-F(y_n^*))\frac{F(y_n^*)}{f(y_n^*)}$ is increasing in r (shown above), and must equal v at $r = r_{n,k}^*$, we must have $r_{n,k}^* > r_n^*$, but also $r_{n,k}^* < v$. By Proposition 9, total surplus is strictly increasing in r for r below v, giving the result.

For the second result, r_{n+j}^* is now the optimal reserve without bid shopping, and let $y_n^*(\cdot)$ and $y_{n+j}^*(\cdot)$ the entry thresholds at that reserve level. By Equation 9,

$$r_{n+j}^* + \frac{F(y_{n+j}^*(r_{n+j}^*))}{f(y_{n+j}^*(r_{n+j}^*))} = v$$

If Equation 11, holds, then,

$$r_{n+j}^* + (1-F(y_n^*(r_{n+j}^*)))\frac{F(y_n^*(r_{n+j}^*))}{f(y_n^*(r_{n+j}^*))} < v$$

Again, $r + (1-F(y_n^*))\frac{F(y_n^*)}{f(y_n^*)}$ is increasing in r, and equals v at $r_{n,k}^*$, so $r_{n,k}^* > r_{n+j}^*$. Once again, $r_{n,k}^* < v$, so since total surplus is increasing in r below v, the result follows. If Equation 11 does not hold, then the opposite: $r_{n,k}^*$ must be lower than r_{n+j}^*, lowering total surplus below where it would be with bid shopping but at the old reserve price.

A.23 Proof of Proposition 10 (multiple equilibria)

Symmetric equilibrium with general $\hat{y} \in [y_*, y^*]$

Pick $\hat{y} \in [y_*, y^*]$, and define a function $\beta_{\hat{y}} : [\underline{y}, \hat{y}] \to \mathbb{R}_+$ by

$$\beta_{\hat{y}}(y) = y + \frac{c + \int_y^{\hat{y}} (1-F(s))^{n-1}(1-F(s-\alpha c)) ds}{(1-F(y))^{n-1}(1-F(y-\alpha c))}$$

We claim that subcontractors bidding $\beta_{\hat{y}}(y)$ for $y \leq \hat{y}$, and staying out for $y > \hat{y}$, is an equilibrium, accompanied by appropriate off-equilibrium-path beliefs about the cost of a subcontractor submitting an off-equilibrium-path bid $b > \hat{b} \equiv \beta_{\hat{y}}(\hat{y})$, which will be specified below.

Note that $\hat{b}$, the equilibrium bid of a subcontractor with cost $\hat{y}$, satisfies

$$(\hat{b} - \hat{y})(1 - F(\hat{y}))^{n-1}(1 - F(\hat{y} - \alpha c)) \quad = \quad c$$

so a subcontractor with cost $\hat{y}$ is indifferent between bidding $\hat{b}$ and staying out. A subcontractor with costs $y > \hat{y}$ would lose money by bidding $\hat{b}$, and a subcontractor with costs $y < \hat{y}$ would make money.

A prime contractor will always want to bid after receiving a sub-bid of $\hat{b}$ or less. To see this, note that $\hat{y} < y^*$ implies $(r - \hat{y})(1 - F(\hat{y}))^{n-1} > c$. It follows that

$$\begin{aligned} C(\hat{b}, \hat{y}) &= (1 - F(\hat{y} - \alpha c))\hat{b} + F(\hat{y} - \alpha c)\hat{y} \\ &= \hat{y} + \frac{c}{(1 - F(\hat{y}))^{n-1}} \quad < \quad \hat{y} + (r - \hat{y}) \quad = \quad r \end{aligned}$$

so any sub-bid of $\hat{b}$ or less makes it profitable for the general contractor to bid.

Next, we rule out deviations to bids $b > \hat{b}$. By inspection, $\hat{b}$ is increasing in $\hat{y}$, and y_* is defined such that $\beta_{y_*}(y_*) = r$. Thus, $\hat{b} \geq r$; so if the prime contractor does not expect to reduce her costs (at least in expectation) via bid shopping, a sub-bid above $\hat{b}$ will not induce her to bid (and such a sub-bid will therefore not be a profitable deviation). If we assign beliefs that any off-equilibrium-path beliefs must come a subcontractor with costs $y < \underline{y} + \alpha c$, then following an off-equilibrium-path belief, no post-auction subcontractor will be willing to enter, and the prime contractor will be unable to reduce her costs, making her unwilling to bid.

As in previous proofs, bids below $\beta_{\hat{y}}(\underline{y})$ are dominated by $\beta_{\hat{y}}(\underline{y})$, and $\beta_{\hat{y}}$ is continuous, so there are no holes in its range, so we only need to consider deviations to a different type's equilibrium bid; the proof that such deviations are unprofitable mirrors the proof of Proposition 3.

Ranking the symmetric equilibria by total surplus

With simple bid shopping, the symmetric equilibrium with entry threshold $\hat{y}$ has expected total surplus

$$W_{\hat{y}} \quad = \quad \int_{\underline{y}}^{\hat{y}} \left[v - y + \int_0^{y - \alpha c} F(s) ds \right] n(1 - F(y))^{n-1} f(y) dy - ncF(\hat{y})$$

(As before, if y_{min} is the lowest cost realization among the pre-auction subcontractors, $v - y_{min}$ is the combined ex post surplus of the procurer, prime contractors, and pre-auction subs, and $\int_0^{y_{min} - \alpha c} F(s) ds$ the interim expected surplus of the chosen post-auction subcontractor (fixing y_{min} but not the post-auction sub's cost); integrating over the distribution of y_{min}, and subtracting the bid preparation costs of the pre-auction subs who enter, gives $W_{\hat{y}}$.) Differentiating, and then substituting $(r - y^*)(1 - F(y^*))^{N-1}$ for c, gives

$$\begin{aligned} \frac{dW_{\hat{y}}}{d\hat{y}} &= \left[v - \hat{y} + \int_0^{\hat{y} - \alpha c} F(s) ds \right] n(1 - F(\hat{y}))^{n-1} f(\hat{y}) - ncf(\hat{y}) \\ &= nf(\hat{y}) \left((1 - F(\hat{y}))^{n-1} \left[v - \hat{y} + \int_0^{\hat{y} - \alpha c} F(s) ds \right] - (r - y^*)(1 - F(y^*))^{N-1} \right) \end{aligned}$$

Noting that $y^* \geq \hat{y}$ and $(1-F(y^*))^{n-1} \leq (1-F(\hat{y}))^{n-1}$, then,

$$\begin{aligned}\frac{dW_{\hat{y}}}{d\hat{y}} &\geq nf(\hat{y})\left((1-F(\hat{y}))^{n-1}\left[v-\hat{y}+\int_0^{\hat{y}-\alpha c}F(s)ds\right]-(r-\hat{y})(1-F(\hat{y}))^{n-1}\right)\\ &= n(1-F(\hat{y}))^{n-1}f(\hat{y})\left[v-r+\int_0^{\hat{y}-\alpha c}F(s)ds\right]\end{aligned}$$

For $r \leq v$, this is positive, so total surplus increases in the entry threshold, giving the ranking of symmetric equilibria.

These are all the symmetric equilibria

It's worth noting that these are all the symmetric equilibria (modulo the actions of the threshold types). The logic is similar to the proof of uniqueness; any symmetric equilibrium must be weakly monotonic, then strictly monotonic because mass points in bid distributions can't exist; once the entry threshold is chosen, bidders at the threshold must earn zero profits (or else bidders above the threshold would enter), and the bid function is pinned down by the envelope theorem (incentive compatibility). An entry threshold lower than y_* would mean $\beta_{\hat{y}}(\hat{y}) < r$, which would allow for profitable deviations since an off-equilibrium-path bid below r would still induce a prime contractor bid (since no beliefs could imply ex post costs above the pre-auction sub-bid); and an entry threshold higher than y^* would induce costs above r at the threshold.

A.24 Total surplus can increase or decrease in the "worst" equilibrium

We note in the text (immediately after Proposition 10) that in an example based on a uniform distribution, even the "worst" equilibrium with bid shopping gives greater total surplus than no-bid-shopping when c is low, but not when c is high. To see this, let F be the uniform distribution on $[0,1]$, $r = v = 1$, and $\alpha = 0$. (This happens to be a knife-edge case where the entry threshold y_n^* has a simple closed-form expression.) Without bid shopping, expected total surplus is

$$\begin{aligned}W_{noshop} &= \textstyle\int_0^{y_n^*}(v-s)n(1-F(s))^{n-1}f(s)ds - nF(y_n^*)c\\ &= \textstyle\int_0^{y_n^*} n(1-s)^n ds - ny_n^*c\\ &= \tfrac{n}{n+1}\left(1-(1-y_n^*)^{n+1}\right) - ny_n^*c\end{aligned}$$

and

$$c \quad = \quad (r-y_n^*)(1-F(y_n^*))^{n-1} \quad = \quad (1-y_n^*)^n \quad \longrightarrow \quad y_n^* \quad = \quad 1-c^{\frac{1}{n}}$$

giving

$$W_{noshop} \quad = \quad \frac{n}{n+1}\left(1-c^{\frac{n+1}{n}}\right) - nc\left(1-c^{\frac{1}{n}}\right)$$

With simple bid shopping with entry threshold $\hat{y}$, total surplus is

$$\begin{aligned}W_{shop} &= \textstyle\int_0^{\hat{y}}\left[v-y+\int_0^{y-\alpha c}F(s)ds\right]n(1-F(y))^{n-1}f(y)dy - ncF(\hat{y})\\ &= \textstyle\int_0^{\hat{y}}\left[1-y+\int_0^{y} sds\right]n(1-y)^{n-1}dy - nc\hat{y}\\ &= \textstyle\int_0^{\hat{y}}\left[1-y+\frac{1}{2}y^2\right]n(1-y)^{n-1}dy - nc\hat{y}\end{aligned}$$

This is easiest to calculate by changing variables to $t = 1 - y$, giving

$$\begin{aligned} W_{shop} &= \int_{1-\hat{y}}^{1} \left[t + \tfrac{1}{2}(1-t)^2\right] nt^{n-1} dt - nc\hat{y} \\ &= \int_{1-\hat{y}}^{1} \left[\tfrac{1}{2} + \tfrac{1}{2}t^2\right] nt^{n-1} dt - nc\hat{y} \\ &= \tfrac{n}{2}\left[\tfrac{t^n}{n} + \tfrac{t^{n+2}}{n+2}\right]\Big|_{t=1-\hat{y}}^{1} - nc\hat{y} \\ &= \tfrac{1}{2}\left(1-(1-\hat{y})^n\right) + \tfrac{n}{2(n+2)}\left(1-(1-\hat{y})^{n+2}\right) - nc\hat{y} \end{aligned}$$

We can calculate y_*, the lowest equilibrium entry threshold, as

$$c \quad = \quad (r - y_*)(1-F(y_*))^{n-1}(1-F(y_* - \alpha c)) \quad = \quad (1-y_*)^{n+1} \qquad \longrightarrow \qquad y_* \quad = \quad 1 - c^{\frac{1}{n+1}}$$

and plug this into W_{shop} to calculate the expected surplus of the "worst" symmetric equilibrium with bid shopping. After some simplification, this gives

$$W_{shop}^{y_*} - W_{noshop} \quad = \quad \frac{1}{(n+2)(n+1)} - \frac{1}{2}c^{\frac{n}{n+1}} + \frac{n(2n+3)}{2(n+2)}c^{\frac{n+2}{n+1}} - \frac{n^2}{n+1}c^{\frac{n+1}{n}}$$

If we call the right-hand side $\phi(c)$, it's easy to see that $\phi(0) = \frac{1}{(n+2)(n+1)} > 0$, and straightforward to calculate that $\phi(1) = 0$. With a bit more work, we can show there's a cutoff c^* such that $\phi(c)$ is positive for $c < c^*$ and negative for $c \in (c^*, 1)$. To do this, we first show there's a different cutoff $c_0 \in (0,1)$ with $\phi'(c) < 0$ for $c \in (0, c_0)$ and $\phi'(c) > 0$ for $c \in (c_0, 1)$: we calculate

$$\begin{aligned} c\phi'(c) &= -\frac{n}{2(n+1)}c^{\frac{n}{n+1}} + \frac{n(2n+3)}{2(n+1)}c^{\frac{n+2}{n+1}} - nc^{\frac{n+1}{n}} \\ &= c^{\frac{n+1}{n}}\left[-\frac{n}{2(n+1)}c^{-\frac{2n+1}{n(n+1)}} + \frac{n(2n+3)}{2(n+1)}c^{-\frac{1}{n(n+1)}} - n\right] \end{aligned}$$

Let $x = c^{-\frac{1}{n(n+1)}}$, so that $x \in [1, \infty)$ and the sign of $\phi'(c)$ equals the sign of

$$\psi(x) \quad = \quad -\frac{n}{2(n+1)}x^{2n+1} + \frac{n(2n+3)}{2(n+1)}x - n$$

Note that $\psi(1) = 0$, $\psi(x) \to -\infty$ as $x \to \infty$, $\psi'(1) = \frac{2n}{2(n+1)} > 0$, and $\psi''(x) < 0$ for all $x \geq 1$; this means there's a cutoff x_0 with $\psi(x) > 0$ for $x \in (0, x_0)$ and $\psi(x) < 0$ for $x > x_0$. Letting $c_0 = x_0^{-n(n+1)}$, we get $\phi'(c) < 0$ for $c \in (0, c_0)$ and $\phi'(c) > 0$ for $c \in (c_0, 1)$; along with $\phi(0) > 0$ and $\phi(1) = 0$, implies there's a different cutoff c^* such that $\phi(c)$ is positive below c^* and negative above it. (We found numerically that for values of n between 2 and 10,000, this cutoff c^* is between 0.16 and 0.18.)

A.25 Proof of Proposition 11 (NWBR)

Fix a symmetric equilibrium as described in Proposition 10, and let $\hat{y} \in [y_*, y^*)$ be the entry threshold; we will show the equilibrium fails NWBR. Define

$$b_{\max} \quad = \quad \frac{r - F(\hat{y} - \alpha c)\hat{y}}{1 - F(\hat{y} - \alpha c)},$$

and consider any off-equilibrium-path bid $b \in (\beta_{\hat{y}}(\hat{y}), b_{\max})$. Let $\mu(y')$ denote the distribution over bidder types describing the common beliefs of a general contractor and her outside subcontractor should the general submit a bid in the prime auction after receiving the pre-auction bid b. We will show that the only belief μ satisfying the NWBR criterion puts all weight on the deviator having cost $\hat{y}$, and that with this belief the deviation becomes strictly profitable for subcontractors with costs near $\hat{y}$.

Our proof has three steps. First, for the NWBR criterion to have any bite, we must establish that there is some PBE supporting the equilibrium path described in the proof of Proposition 10 in which bidding b is a weak best response for some subcontractor type. We do this by showing that there exists a belief μ that puts weight only on types $\underline{y}$ and $\hat{y}$ such that bidding b is weakly profitable to type $\hat{y}$ and strictly unprofitable for all other types. The second step of the proof establishes that in any PBE supporting the equilibrium path, it is never a weak best response for any bidder type $y \neq \hat{y}$ to submit the bid b. The final step shows that when beliefs following the bid b are concentrated on $\hat{y}$ then the deviation is strictly profitable to a sub with cost $\hat{y}$.

As a preliminary, let us describe the optimal strategies for the post auction subcontractor and the general contractor following receipt of the deviating bid b when μ are the beliefs about the type of subcontractor who made it. A post-auction subcontractor with cost y_0 will enter and bid if

$$\int_{y' \geq y_0} (y' - y_0) d\mu(y') \quad \geq \quad \alpha c$$

Define y_m as

$$y_m \quad = \quad \max\left\{ y_0 : \int_{y' \geq y_0} (y' - y_0) d\mu(y') \quad \geq \quad \alpha c \right\}$$

if the set on the right is nonempty, and $y_m = \underline{y}$ otherwise. Then entry will occur whenever the post-auction subcontractor has cost below y_m. The prime contractor will then have expected cost

$$C \quad = \quad (1 - F(y_m))b + \int_{y_0 \leq y_m} \int_{y'} \max\{y', y_0\} d\mu(y') dF(y_0)$$

and will enter and bid C as long as $C < r$, and may randomize in entry if $C = r$.

Step 1: *There exist beliefs μ that make b a weak best response for a pre-auction sub with cost $\widehat{y}$ and a strictly unprofitable deviation for subs of cost $y \neq \widehat{y}$.*

We specify beliefs that put weight $1 - \omega$ on $y = \underline{y}$ and weight ω on $y = \hat{y}$, where ω is chosen such that the prime contractor's expected cost equals r. The prime contractor then mixes between entry and non-entry so as to set the expected payoff from this deviation to zero for a subcontractor of type $\hat{y}$. We then show that it is strictly unprofitable for any other type of subcontractor to deviate to the bid b.

Observe that if $\omega = 1$ then the prime contractor's expected cost is strictly below r. Indeed, with these beliefs the entry threshold for the post-auction contractor is $y_m = \hat{y} - \alpha c$, so we have $C = (1 - F(\hat{y} - \alpha c))b + F(\hat{y} - \alpha c)\hat{y} < r$, as $b < b_{\max}$ and the left-hand side would be equal to r at $b = b_{\max}$. Equally, if $\omega = 0$ the prime contractor's expected cost is strictly above r. Indeed, with those beliefs the post auction subcontractor can recover his entry cost αc only if his cost is $y_0 \leq \underline{y} - \alpha c$, which is impossible, so the prime contractor's expected cost is $b > r$. As ω changes, y_m changes continuously, and since F is continuous, so does C; so a value of ω exists giving the prime contractor expected cost r.

Now consider a pre-auction subcontractor with cost $\hat{y}$. With the specified beliefs we have $y_m < \hat{y} - \alpha c$, since otherwise the marginal post-auction entrant would never cover his entry cost. A pre-auction sub with cost $\hat{y}$ thus knows that when he gets bid-shopped he'll lose. His expected payoff is therefore

$$\Pr(bid)(1 - F(\hat{y}))^{n-1}(1 - F(y_m))(b - \hat{y}) - c \tag{13}$$

where $\Pr(bid)$ is the probability with which his prime contractor chooses to enter and bid r following receipt of the bid of b. When $\Pr(bid) = 1$ expression (13) is strictly positive, since it's at least $(1 - F(\hat{y}))^{n-1}(1 - F(\hat{y} - \alpha c))(b - \hat{y}) - c$ which is strictly positive since $b > \beta_{\hat{y}}(\hat{y})$. Of course when $\Pr(bid) = 0$ expression (13) is strictly negative. By setting

$$\Pr(bid) \quad = \quad \frac{c}{(1-F(\hat{y}))^{n-1}(1-F(y_m))(b-\hat{y})} \tag{14}$$

we ensure the deviating bid b gives a pre-auction sub with cost $\hat{y}$ a payoff of zero, making it a weak best response for him.

Next, we show that bidding b *strictly lowers the payoff* of a pre-action subcontractor with cost $y \neq \hat{y}$. When $y > \hat{y}$ this is obvious, for y wins less often than $\hat{y}$ and receives a lower margin whenever he does so. Bidding b thus results in a strictly negative payoff for him.

Consider therefore a sub with cost $y < \hat{y}$. His expected payoff from deviating to bid b is

$$U_D(y) \quad = \quad \Pr(bid)(1-F(\hat{y}))^{n-1}\left[(1-F(y_m))(b-y) + \int_{\min\{y,y_m\}}^{y_m}(y_0-y)dF(y_0)\right] - c \tag{15}$$

so his gain from deviation is given by

$$\Delta(y) \quad = \quad U_D(y) - \int_y^{\hat{y}}(1-F(s))^{n-1}(1-F(s-\alpha c))ds \tag{16}$$

Differentiating, when $y \geq y_m$ we get

$$\begin{aligned}\Delta'(y) &= -\Pr(bid)(1-F(\hat{y}))^{n-1}(1-F(y_m)) + (1-F(y))^{n-1}(1-F(y-\alpha c)) \\ &= -\frac{c}{b-\hat{y}} + (1-F(y))^{n-1}(1-F(y-\alpha c)) \\ &> -\frac{c}{\beta_{\hat{y}}(\hat{y})-\hat{y}} + (1-F(\hat{y}))^{N-1}(1-F(\hat{y}-\alpha c)) \quad = \quad 0\end{aligned} \tag{17}$$

(where the second equality follows from (14), the inequality because $b > \beta_{\hat{y}}(\hat{y})$ and $y < \hat{y}$, and the final equality because upon multiplication by $\beta_{\hat{y}}(\hat{y}) - \hat{y}$ this is the equilibrium payoff of the marginal pre-auction entrant) and when $y < y_m$ we get

$$\Delta'(y) = -\Pr(bid)(1-F(\hat{y}))^{n-1}(1-F(y)) + (1-F(y))^{n-1}(1-F(y-\alpha c)) > 0 \tag{18}$$

with the inequality because $\Pr(bid) \leq 1$, $(1-F(\hat{y}))^{n-1} < (1-F(y))^{n-1}$ (because $y < \hat{y}$), and $1-F(y) \leq 1-F(y-\alpha c)$ (because $y - \alpha c \leq y$). Since $\Delta(\widehat{y}) = 0$ and $\Delta'(y) > 0$ for all $y < \widehat{y}$ we conclude that $\Delta(y) < 0$ for all $y < \widehat{y}$, as was to be demonstrated.

Step 2: *In any PBE supporting the outcome of Proposition 10 for entry threshold $\hat{y}$ it is never a weak best response for any subcontractor of type $y \neq \hat{y}$ to enter and bid b.*

Consider first a subcontractor of cost $y > \hat{y}$ and suppose that entering and bidding b were a weak best response for him. In equilibrium y does not enter, so this deviation earns him a zero payoff. When bidding b a sub with cost $\hat{y}$ wins more often than y, and earns a higher margin whenever he does, so this deviation would earn $\hat{y}$ a strictly positive payoff. As the marginal entrant, a sub of cost $\hat{y}$ has an equilibrium payoff of zero, establishing that bidding b would be a strictly profitable deviation for $\hat{y}$, contradicting equilibrium.

Next, consider a sub with cost $y < \hat{y}$, who in equilibrium enters and earns positive surplus. Suppose that entering and bidding b were a weak best response for y, so that $\Delta(y) = 0$, where $\Delta(y)$ is given by (16). Then we must have $\Delta'(y) \leq 0$, for otherwise a slightly-higher-cost sub would find the bid b to be a strictly profitable deviation. But if $\Delta'(y) \leq 0$ it follows that if $y \geq y_m$ then (from (17))

$$\Pr(bid)(1-F(\hat{y}))^{n-1}(1-F(y_m)) \geq (1-F(y))^{n-1}(1-F(y-\alpha c)) > (1-F(\hat{y}))^{n-1}(1-F(\hat{y}-\alpha c)),$$

and as $b > b(\hat{y})$ and $y < \hat{y}$ this would make b a strictly profitable deviation for a sub with cost $\hat{y}$:

$$\Pr(bid)(1-F(\hat{y}))^{n-1}(1-F(y_m))(b-\hat{y}) - c > (1-F(\hat{y}))^{n-1}(1-F(\hat{y}-\alpha c))(\beta_{\hat{y}}(\hat{y})-\hat{y}) - c = U(\hat{y})$$

so b would be a strictly profitable deviation for type $\hat{y}$, contradicting equilibrium.

On the other hand, when $y < y_m$, $\Delta'(y) \leq 0$ would require (from (18)) that $\Pr(bid) > 1$, which is impossible; since $\Delta'(y) > 0$, we cannot have $\Delta(y) = 0$ for any $y < y_m$, or we would have $\Delta(y+\varepsilon) > 0$ for some small $\varepsilon > 0$, contradicting equilibrium.

We conclude that entering and bidding b is never a weak best response for any subcontractor of type $y \neq \hat{y}$.

Step 3: *No symmetric equilibrium with $\hat{y} < y^*$ survives NWBR.*

From the first two steps, the beliefs following receipt of a pre-auction subcontractor bid $b \in (\beta_{\hat{y}}(\hat{y}), b_{\max})$ must put all weight on $y = \hat{y}$. In the first step, we showed that these beliefs induce an expected cost of $C < r$ for the prime contractor that receives the bid b. Thus the prime contractor would bid for sure in the prime auction, giving a subcontractor of cost $\hat{y}$ who deviates and bids b an expected payoff of

$$(1-F(\hat{y}))^{n-1}(1-F(\hat{y}-\alpha c))(b-\hat{y}) - c \quad > \quad (1-F(\hat{y}))^{n-1}(1-F(\hat{y}-\alpha c))(\beta_{\hat{y}}(\hat{y})-\hat{y}) - c \quad = \quad 0,$$

contradicting equilibrium.

A.26 "More Aggressive" Bid Shopping (Multiple Outsiders)

Proposition 13 *Consider the model of bid shopping to outsiders, suppose bid shopping does not change the number of pre-auction subcontractors, and consider the limit $\alpha \to 0$. If the winning prime contractor will approach $K > 1$ new subcontractors for post-auction bids, then bid shopping* increases *the likelihood of successful procurement. If $r < v$ and the effect of bid shopping on the likelihood of successful procurement is small, bid shopping increases total surplus by* more than *the expected profit of the subcontractors bidding after the prime auction.*

With $\alpha \approx 0$, post-auction subcontractors will enter whenever their costs are below the incumbent's cost level, which we'll call y_{min}. So if all K post-auction subcontractors have costs above y_{min}, none of them enter and the price stays the same at $\beta(y_{min})$; if one has costs below y_{min}, he enters and the incumbent competes the price down to y_{min}; and if two or more new subcontractors have costs below y_{min}, all those with costs below y_{min} enter, and competition pushes the price paid by the prime contractor down to the second-lowest of their costs. The prime contractor's expected cost, given a pre-auction bid of b from a subcontractor with cost $y_{min} = y$, will therefore be

$$\begin{aligned} C(b,y) &= (1-F(y))^K b + K(1-F(y))^{K-1}F(y)y \\ &\quad + \int_0^y x\, d\left[1-(1-F(x))^K - K(1-F(x))^{K-1}F(x)\right] \end{aligned}$$

since $1-(1-F(x))^K - K(1-F(x))^{K-1}F(x)$ is the CDF of the second-lowest cost among the K post-auction subcontractors.[79] Integrating by parts and simplifying gives

$$C(b,y) \quad = \quad (1-F(y))^K b + \left(1-(1-F(y))^K\right) y - \int_0^y \left(1-(1-F(x))^K - K(1-F(x))^{K-1}F(x)\right) dx$$

Define

$$A(y) \quad \equiv \quad \int_0^y \left(1-(1-F(x))^K - K(1-F(x))^{K-1}F(x)\right) dx$$

[79] Since the subcontractors willing to enter are all those with costs below y_{min}, the second-lowest cost among those who enter is simply the second-lowest cost among those approached.

and note that this is positive (since the integrand is the probability two or more post-auction subcontractors have costs below x, and therefore non-negative). We can rearrange and find

$$C(b,y) - y + A(y) \quad = \quad (1-F(y))^K(b-y)$$

and note that the right-hand side is the incumbent subcontractor's post-auction expected payoff. Thus, by bidding enough to give the prime contractor expected costs of $C(b,y)$, a pre-auction subcontractor with cost y can now earn an expected payoff of $C(b,y) - y + A(y)$, contrasted with $C(b,y) - y$ when only a single post-auction subcontractor was approached. Since the marginal entrant induces a general contractor bid of r, the entry threshold must now satisfy

$$(r - y^{**} + A(y^{**}))\,(1-F(y^{**}))^{n-1} \quad = \quad c$$

so the entry threshold is now *higher* than without bid shopping.

Now, let

$$\beta(y) \quad = \quad y + \frac{c + \int_y^{y^{**}} (1-F(s))^{n-1}(1-F(s))^K ds}{(1-F(y))^{n-1}(1-F(y))^K}$$

We can verify that a prime contractor receiving a bid of $\beta(y^{**})$ from a subcontractor with costs y^{**} has expected cost

$$C(\beta(y^{**}), \beta^{**}) \quad = \quad y^{**} + \frac{c}{(1-F(y^{**}))^{n-1}} - A(y^{**}) \quad = \quad r$$

The rest of the proof that y^{**} and β define a symmetric equilibrium mirrors the proof for $K = 1$.

Since $y^{**} > y^*$ and bid shopping (by assumption) does not change the number of pre-auction subcontractors, it increases the likelihood at least one of them enters, which is the probability of successful procurement. Further, the combined payoff to everyone besides the post-auction subcontractors is equal to $W_{n,0}$ (with or without bid shopping), which is increasing in the entry level up until the entry threshold without bid shopping where $r = v$; so if $r < v$ and bid shopping does not change the entry threshold too much, it increases the combined payoffs of everyone but the post-auction subcontractors, increasing total surplus by more than the post-auction subs' expected surplus.

A.27 Bid Shopping with Added Costs

Return to the case of $K = 1$ (just one post-auction subcontractor will be approached), but now suppose that in addition to having sunk his participation costs, a subcontractor who submits the low pre-auction bid but gets replaced via post-auction bid shopping incurs an additional loss of $\varepsilon > 0$. (Perhaps due to capacity constraints, the subcontractor can't commit to another job until learning whether he'll be awarded this one; bid shopping delays learning whether his pre-auction sub-bid will be honored, imposing an added opportunity cost.)

If a pre-auction subcontractor with costs y bids b, this induces an expected cost

$$C(b,y) \quad = \quad (1-F(y-\alpha c))b + F(y-\alpha c)y$$

for the prime contractor as before, but an expected payoff of

$$(1-F(y-\alpha c))(b-y) + F(y-\alpha c)(-\varepsilon) \quad = \quad C(b,y) - y - F(y-\alpha c)\varepsilon$$

for the subcontractor in the event his is the lowest pre-auction subcontractor bid. That is, to induce an expected cost of $C(b,y)$ for the prime contractor, the subcontractor must now accept an expected payoff from winning the pre-auction competition of $C(b,y) - y - F(y-\alpha c)\varepsilon$, rather than $C(b,y) - y$ in the absence of this cost. Since the threshold type must still induce the prime contractor to bid the reserve price, the entry threshold y^{**} now must satisfy

$$(r - y^{**} - F(y^{**} - \alpha c)\varepsilon)(1-F(y^{**}))^{n-1} \quad = \quad c$$

so the entry threshold y^{**} must be lower than without bid shopping. We can verify that this entry threshold, along with the bid function

$$\beta(y) = y + \frac{c + \int_y^{y^{**}} (1-F(s))^{n-1}(1-F(s-\alpha c))ds}{(1-F(y))^{n-1}(1-F(y-\alpha c))} + \varepsilon \frac{F(y-\alpha c)}{1-F(y-\alpha c)}$$

constitutes a symmetric equilibrium. Since $y^{**} < y^*$, this means bid shopping lowers the likelihood of successful procurement, and (assuming $r \leq v$) lowers the combined surplus of the procurer, prime contractors, and pre-auction subcontractors. Depending on the magnitude of the costs, this lost surplus might be either more or less than the expected surplus of the post-auction subcontractor, so bid shopping might either increase or decrease total surplus.

If instead the added cost is borne by the winning prime contractor, the entry threshold instead satisfies

$$(r - y^{**} - \varepsilon)(1-F(y^{**}))^{n-1} = c$$

and the equilibrium bid function is

$$\beta(y) = y + \frac{c + \int_y^{y^{**}} (1-F(s))^{n-1}(1-F(s-\alpha c))^{n-1}ds}{(1-F(y))^{n-1}(1-F(y-\alpha c))}$$

but the result is otherwise the same:

Proposition 14 *Consider the model of bid shopping to outsiders, with the winning prime approaching one post-auction subcontractor. Assume $r \leq v$, and suppose bid shopping does not change the number of pre-auction subcontractors.*

If bid shopping involves added costs (incurred by either the winning prime contractor or the incumbent subcontractor), then it reduces the probability of successful procurement.

Depending on the magnitude of the new costs, bid shopping may increase or decrease total surplus relative to no bid shopping; if the former, total surplus increases by less than the expected profit of the post-auction subcontractors.

A.28 Proof of Proposition 12 (ex post regret)

As noted in the text, Rule 1 rules out subcontractor bids above r, since they cannot lead to a general contractor bid. The highest-cost subcontractor type to enter must therefore be willing to bid r with the risk of being bid-shopped; this leads to exactly the equilibrium with entry threshold y_*. Under rule 2, the entry cutoff remains y_*, and as noted in the text, subcontractor bids are still uniquely determined by the envelope theorem, and are the same as in the equilibrium with entry threshold y_* and no other constraint. The constraint requiring general contractors not to bid below their cost based on received sub-bids therefore simply inflates prime contractor bids (from their expected cost up to their received sub-bid), giving the winning prime contractor more surplus at the expense of the procurer.

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