Econ 711 – Midterm Exam, 4 November 2021

3 questions.
100 points total.
120 minutes.

No notes, no books, no devices, no calculators.

Please answer question 1 in a separate bluebook;
you can answer questions 2 and 3 in the same bluebook

Unless something is very wrong, the professor and TAs don't plan to answer any questions during the exam. If you think a problem is missing necessary information, state the assumptions you need to solve the problem and solve it as best you can.

Good luck!

Question 1. Consuming Something. **(40 points)**

Yumei has preferences over $\mathbb{R}^2_+$ represented by the utilty function

$$u(x) \quad = \quad \max\left\{x_1^{\frac{1}{3}}x_2^{\frac{2}{3}},\ \ x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}\right\}$$

Assume that prices and wealth are strictly positive.

(a) Are Yumei's preferences convex? Explain.

Since you can't use a calculator, feel free to use the fact that $2^{\frac{1}{3}}4^{\frac{2}{3}} \approx 3.17$ if that helps.

(b) Show that at any solution x^* to her consumer problem, if $p_1 \neq p_2$, Yumei demands strictly more of whichever good is cheaper.

(c) Solve the consumer problem and state $x^*(p, w)$.

(d) Holding p_1 fixed, describe how Yumei's demand for good 1 changes as p_2 increases. Is good 1 a gross complement or a gross substitute for good 2?

Now instead of just one consumer, suppose there were 20 consumers, each with preferences represented by this same utility function. From period to period, prices vary, as does each consumer's wealth level.

(e) If only prices and aggregate (total) demand were observed, would this data satisfy GARP? Why or why not?

Would the data be consistent with the choices of a single consumer with convex preferences? Explain.

Please start a new bluebook for the remaining questions.

Question 2. Producing Something. **(40 points)**

Hakeem runs a small farm that grows one kind of crop and can use either of two production technologies, one that is more dependent on labor and one that is more dependent on chemical fertilizer. Each technology uses its own technology-specific capital (specialized machinery). Hakeem's farm's production function is

$$f(k_1, k_2, f, \ell) \quad = \quad \max\left\{k_1^{\frac{1}{5}} f^{\frac{1}{5}} \ell^{\frac{2}{5}}, \;\; k_2^{\frac{1}{5}} f^{\frac{2}{5}} \ell^{\frac{1}{5}}\right\}$$

where k_i is technology-i-specific capital, f is fertilizer, and ℓ is labor. Suppose that k_1 and k_2 are fixed in the short term but can be changed in the long term, while f and ℓ can be freely adjusted in the short term. Let p be the output price, c the price of fertilizer, and w the price of labor. The two types of capital k_1 and k_2 have the same price r, but capital useful for one technology cannot be used for the other technology or swapped for the other type of capital in the short term.

(a) Show that the production function is not concave.

Is the production set $Y = \{(q, -z) : f(z) \geq q\}$ convex?

(b) Suppose that right now, $\max\{k_1, k_2\} > 0$ and $\min\{k_1, k_2\} = 0$ (Hakeem has one type of machinery and not the other). Is Hakeem's short-term profit maximization problem supermodular? What effect would a decrease in the price of fertilizer c have on his demand for labor in the short term?

(c) Suppose that right now, $k_1 = k_2 = 1$ (Hakeem has some of each type of machinery). Is his short-term problem supermodular? Explain. If yes, what effect would an increase in c have on his use of labor in the short run? If no, explain how an increase in c could lead to either an increase or a decrease in labor used in the short term.

Suppose that Hakeem is maximizing his profits given current prices, and is doing this by using technology 2, so $k_2 > 0 = k_1$. For parts (d) and (e), you don't need to give formal proofs, just a clear explanation of the intuition for the results.

(d) If c decreases, he'll continue to use technology 2 in the long run. (This is true, and you may use it without proving it.) Will the decrease in c lead to an increase or decrease in Hakeem's use of labor in the short run? In the long run? Will the long-run effect be larger or smaller than the short-run effect?

(e) Will an increase in c lead to an increase or decrease in his use of labor in the short run? Can you tell the effect it will have in the long run? Explain.

Exam continues on next page.

Question 3. Gambling. **(20 points)**

Lucky is an expected-utility-maximizing decisionmaker with Bernoulli utility function

$$u(x) \quad = \quad \frac{x}{1+x}$$

Note that throughout this problem, lotteries are described in terms of Lucky's final outcome, not as changes relative to some starting wealth level w.

(a) Is Lucky risk-averse, risk-neutral or risk-loving?

Does Lucky have increasing or decreasing absolute risk aversion?

Does Lucky have increasing or decreasing relative risk aversion?

Please answer parts (b), (c) and (d) **without plugging numbers into utility functions**. Lucky's preferences happen to make him indifferent between the following two lotteries:

lottery A		lottery B	
11	*with probability* 100%	9	*with probability* 50%
		14	*with probability* 50%

(b) Does Lucky prefer $A + 9$ or $B + 9$, i.e., a lottery giving 20 for sure or a lottery giving either 18 or 23 with equal probabilities? Explain.

Does Lucky prefer $10A$ or $10B$, i.e., a lottery giving 110 for sure or a lottery giving either 90 or 140 with equal probabilities? Explain.

(c) Lucky's friend Bucky is also an expected-utility maximizer, and has Bernoulli utility function $v(x) = 1 - e^{-2.5x}$. What is Bucky's coefficient of absolute risk aversion? Does Bucky prefer lottery A or B? Explain.

Consider the following three other lotteries C, D, and E:

lottery C			lottery D			lottery E		
8	*w.p.*	25%	9	*w.p.*	25%	9	*w.p.*	25%
10	*w.p.*	25%	11	*w.p.*	50%	12.5	*w.p.*	50%
14	*w.p.*	50%	14	*w.p.*	25%	14	*w.p.*	25%

(d) Does Lucky prefer lottery A or C? Explain.

Does Lucky prefer lottery A or D? Explain.

Does Lucky prefer lottery A or E? Explain.

Congratulations, you're done!