Econ 711 – Midterm Exam, 27 October 2020

Three questions.
100 points.
125 minutes minus whatever time you need to upload.

Instructions. Do your own work, don't discuss the problems with anyone, and be sure your answers are submitted on Canvas by 10:05 p.m. Submissions time-stamped after 10:05:59 will be penalized one point per minute. If Canvas gives you problems, email your answers to Dan (dquint@ssc.wisc.edu).

Srinivas, Cody, and Dan won't respond to questions during the exam. If you think a problem is missing necessary information, state the assumptions you need to solve the problem and solve it as best you can.

Good luck!

Question 1. A Consumer Problem. **(40 points)**

Consider the utility function

$$u(x) \quad = \quad \min\{x_1, x_2\}^{\alpha}(x_3 + x_4)^{1-\alpha}$$

with $\alpha \in (0, 1)$. Assume all prices are strictly positive, and ignore the cases where two or more prices are the same.

(a) Solve the consumer's problem, and calculate Marshallian demand $x(p, w)$ and the indirect utility function $v(p, w)$. Is good 1 normal or inferior?

(HINT: look for ways to simplify the problem before you solve it!)

(b) Use $v(p, w)$ to find the expenditure function $e(p, u)$, and use $e(p, u)$ to find $h_1(p, u)$, the Hicksian demand for the first good. Which goods are complements for good 1, and which goods are substitutes for it?

Now suppose that instead of a wealth endowment, the consumer has a positive endowment of goods 3 and 4, $e = (0, 0, e_3, e_4)$, which she can consume or sell in any quantity. We say the consumer is a *net buyer* of a good if she consumes more than her endowment of it, and a *net seller* if she consumes less than her endowment.

(HINT: if you think of the consumer as first selling her whole endowment and then deciding how to spend the proceeds, you shouldn't need to re-solve the consumer problem to answer parts (c) and (d).)

(c) If $p_3 > p_4$, is the consumer a net buyer or net seller of good 3? Is the demand for good 1 increasing or decreasing in p_3? Is this due to a substitution effect, a wealth effect, or both? Explain.

(d) If $p_3 < p_4$, when is the consumer a net buyer of good 3, and when is she a net seller? In each case, is the demand for good 1 increasing or decreasing in p_3? Is this due to a substitution effect, a wealth effect, or both? Explain.

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Question 2. Rationalizing Firm Behavior. **(30 points)**

Consider the following "dataset" of market prices and a firm's observed production:

prices p	production y
$p^1 = (1, 1, 1)$	$y^1 = (10, -3, -4)$
$p^2 = (2, 1, 1)$	$y^2 = (15, -6, -8)$
$p^3 = (1, 1, 2)$	$y^3 = (8, -5, -1)$

(a) Is the data consistent with the behavior of a profit-maximizing firm? Why or why not?

If yes, give a production set Y that rationalizes the data.

If no, give a different value for the observation y^3 which would make the data rationalizable, and give a production set Y that rationalizes the revised data.

(b) Is the original data consistent with a profit-maximizing firm whose production set Y is *convex*? Why or why not?

If yes, give a convex production set Y that rationalizes the data.

If no, give a different value for the observation y^3 that would make the data consistent with convex Y, and give a convex production set that would rationalize the revised data.

(c) Interpret each observed production plan as $y = (q, -z_1, -z_2)$, with $q = f(z_1, z_2)$, and suppose that each observed y^i was the firm's unique optimal plan at those prices. Is the original data consistent with a profit-maximizing single-output firm whose production function f is *supermodular*?

If yes, explain why.

If no, explain why not, and give a different value for the observation y^3 that would make the data consistent with supermodular f.

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Question 3. Lotteries and the Very Risk-Averse. **(30 points)**

Let X be a finite subset of $\mathbb{R}_+$. Given a lottery L over X, let L_* denote the worst-case outcome it allows, i.e., the lowest outcome that L puts positive probability on. Consider the *minmax preferences* where lotteries are evaluated purely based on their worst-case outcomes: $L \succsim_{mnx} L'$ if and only if $L_* \geq L'_*$.

(a) Is $\succsim_{mnx}$ complete? Is it transitive? Does it satisfy continuity? Does it satisfy independence? (For each, explain why or why not, don't just say "yes" or "no.")

Can $\succsim_{mnx}$ be represented by an expected utility function $U(L) = \sum_{i\,:\,x_i \in X} p_i u(x_i)$? Explain.

(b) Consider the CARA utility function $u(x) = 1 - e^{-cx}$. Show that for any two lotteries over X, if $L \succ_{mnx} L'$, then $U(L) > U(L')$ under CARA utility for c sufficiently high. (Thus, in a rough sense, CARA preferences approach minmax preferences as $c \to \infty$.)

(c) Consider the two lotteries

$$L \quad = \quad \frac{2}{5}(\$10) \oplus \frac{3}{5}(\$100) \qquad \text{and} \qquad L' \quad = \quad \frac{1}{5}(\$10) \oplus \frac{4}{5}(\$20)$$

Minmax preferences rank these equally ($L \sim_{mnx} L'$), but CARA utility does not. Which lottery is preferred based on CARA utility for c sufficiently large?

Describe, as completely as you can, the preferences over lotteries that correspond to the limit of CARA utility as $c \to \infty$.

Congratulations – you're done!
Good luck the rest of the semester!