Econ 711 – Midterm Exam – October 23, 2019

No books, no notes, no calculators allowed

Three questions, 100 points total. Some parts of questions ask for more than one thing; **please read the questions carefully and be sure you're answering every part.**

Please use one bluebook for question 1, and a separate bluebook for questions 2 and 3, and be sure your name is on both bluebooks. (You may use additional bluebooks if needed.)

At the end of the exam, please put one bluebook inside the other and drop both in the cardboard box; you can keep the exam questions if you want.

In the interest of fairness, the professor/TAs/proctors will not answer any questions clarifying what type of answer we're looking for or what a question means. **If you think information is missing or you need to make further assumptions to solve a problem, note what you are assuming and do the best you can.**

Relax, and good luck!

Question 1. Sun-Dried Tomatoes **(5 parts, 50 points)**

I run a small business making fancy jars of sun-dried tomatoes. My production function is

$$f(z) \quad = \quad S^{0.3} z_1^{0.3} z_2^{0.4}$$

where z_1 and z_2 are tomato seeds and fertilizer – ordinary inputs whose levels I control – and S is the amount of sunlight my factory gets, which depends on the weather (and is out of my control). Let p denote the price of sun-dried tomatoes, and w_1 and w_2 the prices of seeds and fertilizer; assume I'm a price taker in all of these markets.

(a) State my profit maximization problem.

What happens to my use of seeds and fertilizer, and my output level, if the price of sun-dried tomatoes goes up? Explain.

What happens to my use of seeds and fertilizer, and my output level, if weather patterns change and my factory gets less sunlight? Explain.

(b) I learn of an opportunity to move my factory to a different location that's more expensive but has more exposure to sunlight. Am I more likely to do this if the price of fertilizer is high or low? Explain.

I choose to remain in my old location, and instead find some inexpensive heat lamps, which allow me to supplement natural sunlight with artificial light, changing my production function to

$$f(z) \quad = \quad (S + \sqrt{z_0})^{0.3} z_1^{0.3} z_2^{0.4}$$

where z_0 is electricity.

(c) Show that if the price w_0 of electricity falls, I use more of each input, and produce more.

(d) Rather than choosing z_0, we can think of my problem as choosing the total level of light, $L = S + \sqrt{z_0}$, and setting $z_0 = (L - S)^2$, making my problem[1]

$$\max_{L \geq S, z_1 \geq 0, z_2 \geq 0} \left\{ pL^{0.3} z_1^{0.3} z_2^{0.4} - w_0 (L - S)^2 - w_1 z_1 - w_2 z_2 \right\}$$

Show that an increase in S increases my profits, and increases my total use of light L and my use of seeds and fertilizer.

Question 1 continues on the next page

[1] Although the values over which you're choosing L depend on the parameter S, this turns out to "not be a problem" in this case and you can apply monotone comparative statics techniques as usual.

(e) Over two "periods," my business operates under two different combinations of sunlight and market prices, and generates the following data:

	S	w_0	w_1	w_2	p	z_0	z_1	z_2	output	profits
period 1	8	3	12	16	40	4	10	10	10	108
period 2	6	2	12	16	40	9	9	9	9	90

An econ PhD student observes market prices (w_0, w_1, w_2, p), my input purchases (z_0, z_1, z_2), and my output level for the two periods, but does not know my production function and does not realize that sunlight is one of my inputs. Can the student rationalize the data as profit-maximizing behavior for *some* production function, or will he or she (wrongly) conclude that I was running my business irrationally? Explain.

Please start a separate bluebook for the second half of the exam

Question 2. A Little Quasilinearity **(4 parts, 20 points)**

Consider a consumer with preferences represented by the utility function

$$u(x) \quad = \quad x_1 + U(x_2, x_3)$$

Suppose that U is increasing in both its arguments, continuous, and supermodular in (x_2, x_3). Assume the non-negativity constraint on x_1 does not bind (so you can ignore it), and the consumer problem has a unique solution.

(a) "Plug in" the budget constraint and write the consumer problem as an unconstrained maximization problem over x_2 and x_3. Are goods 2 and 3 normal or inferior? Is good 1 normal or inferior?

(b) Show that goods 2 and 3 are gross complements.

(c) Consider the consumer's expenditure minimization problem. Show that good 1 is a (Hicksian) substitute for the other two goods.

(d) Given your answer to the last part of (a), does (c) imply that good 1 must be a gross substitute for the other goods? Explain.

Question 3. A Little Separability **(5 parts, 30 points)**

A consumer has preferences over five goods – peanut butter, jelly, bacon, lettuce, and tomato – represented by the utility function

$$u(x) \quad = \quad \min\{x_1, x_2\}^{0.8} + \min\{x_3, x_4, x_5\}^{0.9}$$

Assume $p \gg 0$ and $w > 0$.

(a) Argue that the consumer will optimally consume all five goods.

(b) Are preferences homothetic? Do preferences over (x_3, x_4, x_5) depend on (x_1, x_2)? Do preferences over (x_1, x_2) depend on (x_3, x_4, x_5)?

(c) Define

$$v_1(p, m_1) \quad = \quad \max_{x_1, x_2 \geq 0} \left\{\min\{x_1, x_2\}^{0.8}\right\} \qquad \text{subject to} \quad p_1 x_1 + p_2 x_2 \quad \leq \quad m_1$$

$$v_2(p, m_2) \quad = \quad \max_{x_3, x_4, x_5 \geq 0} \left\{\min\{x_3, x_4, x_5\}^{0.9}\right\} \quad \text{subject to} \quad p_3 x_3 + p_4 x_4 + p_5 x_5 \quad \leq \quad m_2$$

as the "utility value," respectively, of spending m_1 optimally on goods 1 and 2 combined, or spending m_2 optimally on goods 3, 4 and 5 combined. Calculate the consumer's demand for goods 1 and 2, conditional on spending a total of m_1 on the two goods combined; calculate the demand for goods 3, 4 and 5 conditional on spending a total of m_2 on the three goods combined; and calculate $v_1(p, m_1)$ and $v_2(p, m_2)$.

(d) Consider the consumer's problem of allocating a budget w among the two sets of goods,

$$\max_{m_1, m_2 \geq 0} \{v_1(p, m_1) + v_2(p, m_2)\} \quad \text{subject to} \quad m_1 + m_2 \quad \leq \quad w$$

and let (m_1^*, m_2^*) be the solution. Show that m_1^* is decreasing in p_1 and p_2 and increasing in p_3, p_4 and p_5, and that m_1^* and m_2^* are both increasing in w. (You do not need to give closed-form solutions for m_1^* and m_2^*.)

(e) Use parts (c) and (d) to show that goods 1 and 2 are gross substitutes for good 5; that goods 3 and 4 are gross complements for good 5; and that all five goods are normal.

(If you failed to prove the results in part (d), you should still assume them here so you can answer part (e).)

You're done – congratulations!

Good luck the rest of the semester!