Econ 711 – Midterm Exam – October 12, 2017

No books, no notes, no calculators allowed

Some parts of questions ask for more than one thing; **please read the questions carefully and be sure you're answering every part.**

Please use one bluebook for questions 1 and 2, and a separate bluebook for questions 3 and 4, and be sure your name is on both bluebooks.

At the end of the exam, please put one bluebook inside the other and drop both in the cardboard box; you can keep the exam questions if you want.

In the interest of fairness, the TAs will not answer any questions clarifying what type of answer we're looking for or what a question means. **If you think information is missing or you need to make further assumptions to solve a problem, note what you are assuming and do the best you can.**

Relax, and good luck!

1. **Aggregate Hicksian Demand (30 points)**

   Suppose there are $n \geq 2$ consumers, and consumer $i \in \{1, 2, \ldots, n\}$ has indirect utility

   $$v_i(p, w_i) \quad = \quad a_i(p) + \frac{w_i}{c(p)}$$

   This is the same formulation you've seen before, just with $c(p) = 1/b(p)$, because this will simplify algebra later in the problem. You've seen that this implies the combined Marshallian demand $\sum_i x^i(p, w_i)$ of the n consumers is the same as the demand of a single "representative consumer" with indirect utility

   $$V(p, W) \quad = \quad A(p) + \frac{W}{c(p)}$$

   where $A(p) = \sum_i a_i(p)$ and $W = \sum_i w_i$.

   (a) Calculate the expenditure function $e_i(p, u_i)$ for consumer i, and the expenditure function $E(p, U)$ for the representative consumer, and show that $\sum_i e_i(p, u_i) = E(p, \sum_i u_i)$.

   (b) Calculate consumer i's Hicksian demand $h_1^i(p, u_i)$ for good 1, and the representative consumer's $H_1(p, U)$, and show that $\sum_i h_1^i(p, u_i) = H_1(p, \sum_i u_i)$.

   (c) Show that if the price of good 1 falls from p_1^0 to p_1^1, the sum of the Compensating Variation of the n consumers is the same as the CV of the representative consumer.

2. **Zero Demand (20 points)**

   Suppose preferences are locally non-satiated. Show that if Marshallian demand for good i is 0 at prices p, it can't become positive when the price of good i rises.

   (Formally, if $p_i' > p_i$ and $p_j' = p_j$ for all $j \neq i$, show that if there's *any* $x \in x(p, w)$ such that $x_i = 0$, then at *every* $x' \in x(p', w)$, $x_i' = 0$.)

PLEASE START A NEW BLUEBOOK FOR QUESTIONS 3 AND 4.

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3. **The Slutsky Equation (20 points)**

   Suppose demand is single-valued and differentiable. Write the Slutsky equation for the change in demand for good 1 when the price of good 2 changes. For each of the following cases, explain whether you can predict the sign of $\frac{\partial x_1}{\partial p_2}$, and why or why not:

   (a) goods 1 and 2 are complements, good 1 is a normal good and good 2 is an inferior good

   (b) goods 1 and 2 are substitutes, and goods 1 and 2 are both normal goods

   (c) there are only two goods and good 2 is a Giffen good

4. **Intertemporal Choice (30 points)**

   Consider a simple model of intertemporal choice, within our static utility-maximization framework. The "goods" 1 through k represent consumption in each of k different time periods, and for $x \in \Re^k_+$,

$$u(x) = \sum_{i=1}^{k} \beta^{i-1} v(x_i)$$

   where $\beta \in (0,1)$ and $v : \Re_+ \to \Re$ is strictly increasing, strictly concave, differentiable, and satisfies an "Inada condition" $\lim_{z \to 0} v'(z) = +\infty$. The current "price" of good i is

$$p_i = \frac{1}{(1+r)^{i-1}}$$

   reflecting the fact that the consumption good costs the same in each period, but that money saved today grows at an interest rate r until it is used to purchase the consumption good in period i.

   (a) If $w > 0$, show that Marshallian demand for every good is strictly positive, $x(p,w) \gg 0$.

   (b) Show that if $\beta(1+r) > 1$, consumption increases from period to period ($x_{i+1} > x_i$), while if $\beta(1+r) < 1$, consumption decreases from period to period.

   (c) Now suppose one period has passed, and the consumer has consumed $x_1(p,w)$ according to plan. Show that his choices are *time-consistent*: that if he were to take his remaining budget $\bar{w} = (1+r)(w - p_1 x_1)$ and the current prices $p'_i = \frac{1}{(1+r)^{i-2}}$ and solve his forward-looking consumer problem

$$\max \sum_{i=2}^{k} \beta^{i-2} v(x_i) \qquad \text{subject to} \qquad \sum_{i=2}^{k} p'_i x_i \leq \bar{w}$$

   his new choices would match the original solution to his consumer problem.

You're done – congratulations!