ECON 522 - SECTION 1 - MATH REVIEW AND EFFICIENCY DISCUSSION

I Derivatives

Recall the definition of a derivative for a function f defined on the real number line:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)=\frac{df(x)}{dx}$$

Which is just the slope of the function at the point x. Since we know that if a function is differentiable and has a maximum value, then at that maximum the slope must be zero. That is why we take "first order conditions" ($f'(x)=0$) when we look for maximum values.

I.1 Example: Tragedy of the Commons

Vairables:

h =Number of hours you fish
H =Total number of hours the entire village (twenty people including you) fish
$\underline{H}$ =Total number of hours other 19 villagers (not you) fish
$(260-H)$ = The rate you can catch fish (in fish per hour)
8 = Your disutility from fishing (in fish per hour)

Note: $\underline{H}+h=H$

So, how much will you decide to fish? You know your own utility: $u(h)=(260-H)h-8h$, so you simply take the derivative (remembering that $\underline{H}+h=H$), and set it equal to zero:

$$\begin{aligned}
&u(h)=(260-H)h-8h=(260-\underline{H}-h)-8h\\
\Rightarrow &u'(h)=260-\underline{H}-2h-8=0\\
\Rightarrow &2h=252-\underline{H}\\
\Rightarrow &h=126-\frac{\underline{H}}{2}
\end{aligned}$$

But we also know, since everyone in this village has the same utility, that $\underline{H}=19h$. Substituting this in we get:

$$\begin{aligned}
&h=126-\frac{19h}{2}\\
\Rightarrow &\frac{21h}{2}=126\\
\Rightarrow &h=12
\end{aligned}$$

The point is that this is not efficient: we end up overfishing. To see that the result is inefficient notice that the utility of each individual when $h=12$ is: $u(12)=(260-20\times 12)(12)-8\times 12=144$. However, if we force everyone to fish ten hours, then each villager's utility is: $u(10)=(260-20\times 10)(10)-8\times 10=600-80=520>144$. To figure out the optimal (efficient) number of hours to fish, we can solve for the level of h that maximizes the utility of the entire village by setting $H=20h$ in each individual's utility. This internalizes the externality that each villager imposes on the others. Previously we took the

derivative treating what everyone else did ($\underline{H}$) constant, but now lets act as a benevolent government that knows everyone is the same. So the government wants to maximize: $u(h) = (260 - 20h)h - 8h$. Which gives the FOC:

$$\begin{aligned} u'(h) &= 260 - 40h - 8 = 0 \\ \Rightarrow h &= \frac{252}{40} \\ &= 6.3 \\ &= 6hr\ 18m \end{aligned}$$

Now each villager has utility $u(6.3) = (260 - 20 \times 6.3)(6.3) - 8 \times 6.3 = 793.8 > 520 > 144$.

II Efficiency

Pareto Efficiency is one of the most important concepts in economics. Recall the definition: An allocation is **pareto efficient** if there does not exist any other allocation such that *everyone is at least as well off* as before, and *at least one person is strictly better off.*

Examples:

- I own all of the wealth in the world and you have nothing
- Everyone owns an equal amount of wealth, and nothing is being wasted.
- Any allocation of wealth in which nothing is being wasted. No money is "left on the table."

Kaldor-Hicks (K-H) Efficiency is very similar to Pareto efficiency, and as long as utility is transferable (for example via monetary transfers) then they are equivalent. The main difference between the two concepts is that a Pareto *imporvement* is not the same as a K-H *improvement*. Recall that a K-H improvement is any change in an allocation which can be turned into a Pareto improvement via monetary transfers. Thus, any time a situation is K-H efficient it is also Pareto efficient, and there are no Pareto or K-H improvements available. Also, any Pareto improvement is also a K-H improvement, but not every K-H improvement is a Pareto improvement. Take the example from class/section: You own a car which you value at \$4000. I have \$5000 and just happen to value your car at \$5000. It's a K-H improvement for the government to sieze your car and give it to me, but it is not a Pareto improvement, since you're worse off. However, once I have your car, the situation is both K-H and Pareto efficient, since there's no way to create more value in our little economy.