ECON 522 - SECTION 1 - MATH REVIEW AND EFFICIENCY DISCUSSION

# I Derivatives

Recall the definition of a derivative for a function $f$ defined on the real number line:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \frac{df(x)}{dx}$$

Which is just the slope of the function at the point $x$. Since we know that if a function is differentiable and has a maximum value, then at that maximum the slope must be zero. That is why we take "first order conditions" ($f'(x) = 0$) when we look for maximum values.

## I.1 Example: Tragedy of the Commons

Vairables:

$h$ =Number of hours you fish
$H$ =Total number of hours the entire village (twenty people including you) fish
$\underline{H}$ =Total number of hours other 19 villagers (not you) fish
$(130 - H)$ = The rate you can catch fish (in fish per hour)
4 = Your disutility from fishing (in fish per hour)

Note: $\underline{H} + h = H$

So, how much will you decide to fish? You know your own utility: $u(h) = (130 - H)h - 4h$, so you simply take the derivative (remembering that $\underline{H} + h = H$), and set it equal to zero:

$$\begin{aligned} & u(h) = (130 - H)h - 4h = (130 - \underline{H} - h) - 4h \\ \Rightarrow & u'(h) = 130 - \underline{H} - 2h - 4 = 0 \\ \Rightarrow & 2h = 126 - \underline{H} \\ \Rightarrow & h = 63 - \frac{\underline{H}}{2} \end{aligned}$$

But we also know, since everyone in this village has the same utility, that $\underline{H} = 19h$. Substituting this in we get:

$$\begin{aligned} & h = 63 - \frac{19h}{2} \\ \Rightarrow & \frac{21h}{2} = 63 \\ \Rightarrow & h = 6 \end{aligned}$$

The point is that this is not efficient; we end up overfishing. To see that the result is inefficient notice that the utility of each individual when $h = 6$ is: $u(6) = (130 - 20 \times 6)(6) - 4 \times 6 = 60 - 24 = 36$. However, if we force everyone to fish three hours, then each villager's utility is: $u(3) = (130 - 20 \times 3)(3) - 4 \times 3 = 210 - 12 = 198 > 36$. To figure out the optimal (efficient) number of hours to fish, we can solve for the level of $h$ that maximizes the utility of the entire village by setting $H = 20h$ in each individual's utility. This internalizes the externality that each

villager imposes on the others. Previously we took the derivative treating what everyone else did ($\underline{H}$) constant, but now lets act as a benevolent government that knows everyone is the same. So the government wants to maximize: $u(h) = (130 - 20h)h - 4h$. Which gives the FOC:

$$\begin{aligned} u'(h) &= 130 - 40h - 4 = 0 \\ \Rightarrow h &= \frac{126}{40} \\ &= \frac{63}{20} \\ &= 3hr\ 9m \end{aligned}$$

# II Efficiency

**Pareto Efficiency** is one of the most important concepts in economics. Recall the definition: An allocation is **pareto efficient** if there does not exist any other allocation such that *everyone is at least as well off* as before, and *at least one person is strictly better off*.

Examples:

- I own al of the wealth in the world and you have nothing
- Everyone owns an equal amount of wealth, and nothing is being wasted.
- Any allocation of wealth in which nothing is being wasted. "No money is left on the table."

**Kaldor-Hicks (K-H) Efficiency** is just like Pareto efficiency, the difference between the two is in Pareto *imporvements* versus K-H *improvements*. Recall that a K-H improvement is any change in an allocation which can be turned into a Pareto improvement via monetary transfers. Thus, any time a situation is Pareto efficient it is also K-H efficient, and there are no Pareto or K-H improvements available. Also, any Pareto improvement is also a K-H improvement, but not every K-H improvement is a Pareto improvement. Take the example from class/section: You own a car which you value at \$4000. I have \$5000 and just happen to value your car at \$5000. It's a K-H improvement for the government to sieze your car and give it to me, but it is not a Pareto improvement, since you're worse off. However, once I have your car, the situation is both K-H and Pareto efficient, since there's no way to create more value in our little economy.