# ECON 522 - SECTION 1 - MATH REVIEW AND EFFICIENCY DISCUSSION

**I. Very Short Math Review: Derivatives**

Recall the definition of a derivative for a function $f$ defined on the real number line:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)=\frac{df(x)}{dx}$$

Which is just the slope of the function at the point $x$. Since we know that if a function is differentiable and has a maximum value, then at that maximum the slope must be zero. That is why we take "first order conditions" ($f'(x)=0$) when we look for maximum values.

**I.1 Example: Tragedy of the Commons**

Variables:

$h$ =Number of hours you fish
$H$ =Total number of hours the entire village (twenty people including you) fish
$\underline{H}$ =Total number of hours other 19 villagers (not you) fish
$(260-H)$ = The rate you can catch fish (in fish per hour)
8 = Your disutility from fishing (in fish per hour)

Note: $\underline{H}+h=H$

So, how much will you decide to fish? You know your own utility: $u(h)=(260-H)h-8h$, so you simply take the derivative (remembering that $\underline{H}+h=H$), and set it equal to zero:

$$\begin{aligned}
&u(h)=(260-H)h-8h=(260-\underline{H}-h)h-8h\\
\Rightarrow &u'(h)=260-\underline{H}-2h-8=0\\
\Rightarrow &2h=252-\underline{H}\\
\Rightarrow &h=126-\frac{\underline{H}}{2}
\end{aligned}$$

But we also know, since everyone in this village has the same utility, that $\underline{H}=19h$. Substituting this in we get:

$$\begin{aligned}
&h=126-\frac{19h}{2}\\
\Rightarrow &\frac{21h}{2}=126\\
\Rightarrow &h=12
\end{aligned}$$

The point is that this is not efficient: we end up overfishing. To see that the result is inefficient notice that the utility of each individual when $h=12$ is: $u(12)=(260-20\times 12)(12)-8\times 12=144$. However, if we force everyone to fish ten hours, then each villager's utility is: $u(10)=(260-20\times 10)(10)-8\times 10=600-80=520>144$. To figure out the optimal (efficient) number of hours to fish, we can solve for the level of $h$ that maximizes the utility of the entire village by setting $H=20h$ in each individual's utility. This internalizes the externality that each villager imposes on the others. Previously we took the derivative treating what everyone else did ($\underline{H}$) constant, but now lets act as a benevolent government that

knows everyone is the same. So the government wants to maximize: $u(h) = (260 - 20h)h - 8h$. Which gives the FOC:

$$\begin{aligned} u'(h) &= 260 - 40h - 8 = 0 \\ \Rightarrow h &= \frac{252}{40} \\ &= 6.3 \\ &= 6hr\ 18m \end{aligned}$$

Now each villager has utility $u(6.3) = (260 - 20 \times 6.3)(6.3) - 8 \times 6.3 = 793.8 > 520 > 144$.

**II. Efficiency**

**Pareto Efficiency** is one of the most important concepts in economics. Recall the definition: An allocation is **pareto efficient** if there does not exist any other allocation such that *everyone is at least as well off* as before, and *at least one person is strictly better off*.

Examples:

- I own all of the wealth in the world and you have nothing
- Everyone owns an equal amount of wealth, and nothing is being wasted.
- Any allocation of wealth in which nothing is being wasted. No money is "left on the table."

**Kaldor-Hicks (K-H) Efficiency** is very similar to Pareto efficiency, and as long as utility is transferable (for example via monetary transfers) then they are equivalent. The main difference between the two concepts is that a Pareto *imporvement* is not the same as a K-H *improvement*. Recall that a K-H improvement is any change in an allocation which can be turned into a Pareto improvement via monetary transfers. Thus, any time a situation is K-H efficient it is also Pareto efficient, and there are no Pareto or K-H improvements available. Also, any Pareto improvement is also a K-H improvement, but not every K-H improvement is a Pareto improvement. Take the example from class/section: You own a car which you value at \$4000, while I value your car at \$5000. It's a K-H improvement for the government to sieze your car and give it to me, but it is not a Pareto improvement, since you're worse off. However, once I have your car, the situation is both K-H and Pareto efficient, since there's no way to create more value in our little economy.

$$\begin{array}{ccc} \text{Kaldor-Hicks Efficient} & \begin{array}{c} \Rightarrow \\ \Leftarrow \text{ if utility is transferable} \end{array} & \text{Pareto Efficient} \\ \text{Kaldor Hicks Improvement} & \begin{array}{c} \not\Rightarrow \\ \Leftarrow \end{array} & \text{Pareto Improvement} \end{array}$$

***IMPORTANT*.** An allocation of property is Kaldor-Hicks efficient if and only if every object is owned by the person who values it the most.

**III. Next Week- Coase Theorem and Property Rights**

**Coase Theorem.** If property rights are well defined and tradable, and transaction costs are low, then the initial allocation of property rights does not affect efficiency.