Search Models

Christopher Taber

Department of Economics
University of Wisconsin-Madison

November 7, 2015

This is a very large literature on the border of labor and macro

I am just going to scratch the surface

In particular I am really focused on the worker sides and on wages-not on the firm vacancy posting side or unemployment per se

Rasmus Lentz knows this much better than I do-if you are interested he should be teaching labor next year.

To start lets think about a very simple search model

I start with the paper “Search-Theoretic Models of the Labor Market: A Survey” by Rogerson, Shimer, and Wright, Journal of Economic Literature, 2005

I will use my own notation but follow them pretty closely otherwise for now

Discrete Time

Lets start with the discrete time model

The main features

- Agents infinitely lived
- Once they get a job they keep it forever
- They don't borrow and lend but consume their wage each period
    - Let $u(w)$ be the flow utility from wage w when working (strictly increasing in w)
    - u_0 be the flow utility when not working
    - $F(\cdot)$ be the distribution of offered wages
    - Arrival rate of an offer is λ (and only one per year)
    - Discount rate is β

When a worker gets a job they choose whether to take it or keep looking.

The value function of taking a job at wage w is

$$
\begin{aligned}
V(w) &= u(w) + \beta V(w) \\
&= \frac{u(w)}{1-\beta}
\end{aligned}
$$

Let V_0 be the value function of unemployment

So you take the job if $V(w) > V_0$

Reservation Wage

This gives a reservation wage property

- Define w^r by $V(w^r) = V_0$
- if $w > w^r$ you take the job
- if $w < w^r$ you keep searching

Note that all that is identified from the data is the job finding rate $\lambda\left[1 - F\left(w^r\right)\right]$ and the distribution of w when it is bigger than w^r

Lets solve for w^r

$$V_0 = u_0 + \beta \left[\lambda \int_{w^r}^{\infty} \frac{u(w)}{1-\beta} dF(w) + \left[1 - \lambda\left(1 - F(w^r)\right)\right] V_0 \right]$$
$$= u_0 + \beta \left[\lambda \int_{w^r}^{\infty} \frac{u(w)}{1-\beta} dF(w) + \left[1 - \lambda\left(1 - F(w^r)\right)\right] \frac{u(w^r)}{1-\beta} \right]$$
$$= \frac{u(w^r)}{1-\beta}$$

Which simplifies to

$$u(w^r) = u_0 + \frac{\beta}{1-\beta} \lambda \int_{w^r}^{\infty} \left[u(w) - u(w^r)\right] dF(w)$$

We can simplify further integrating by parts.

For some particular upper bound

$$\begin{aligned}\int_{w^r}^{\overline{w}} \left[u\left(w\right) - u(w^r)\right] dF\left(w\right) &= \left[u\left(\overline{w}\right) - u(w^r)\right] F\left(\overline{w}\right) - \int_{w^r}^{\overline{w}} u'\left(w\right) F\left(w\right) dw \\ &= \int_{w^r}^{\overline{w}} u'\left(w\right) \left[F\left(\overline{w}\right) - F\left(w\right)\right] dw\end{aligned}$$

Take limits as $\overline{w} \to \infty$ gives

$$u(w^r) = u_0 + \frac{\beta}{1-\beta}\lambda \int_{w^r}^{\infty} u'\left(w\right) \left[1 - F\left(w\right)\right] dw$$

Notice that the reservation wage will be increasing in u_0, λ, and β

Continuous Time

People who work on search like continuous time as it is more elegant

Also an ackward thing about discrete time is that its not clear why you would only get one offer at a time

I have never been completely comfortable with continuous time models and always need to start with the discrete time version and then send the time periods to zero

Luckily the Rogerson, Shimer, and Wright paper does that as well

We will start the same way as in the discrete time version, let Δ be the difference in time

Lets also add something else to the model now-assume that jobs can be destroyed as well at rate δ

$$\begin{aligned} V(w) =& \Delta u(w) + \frac{1}{1+r\Delta}\left[\Delta\delta V_0 + (1-\Delta\delta)V(w)\right] \\ V_0 =& \Delta u_0 \\ &+ \frac{1}{1+r\Delta}\left[\Delta\lambda\int_{w^r}^{\infty} V(w)\,dF(w) + \left[1-\Delta\lambda\left(1-F\left(w^r\right)\right)\right]V_0\right] \end{aligned}$$

or after some algebra

$$\begin{aligned} (r+\delta)V(w) &= (1+r\Delta)u(w) + \delta V_0 \\ rV_0 &= (1+r\Delta)u_0 + \lambda\int_{w^r}^{\infty}\left(V(w)-V_0\right)dF(w) \end{aligned}$$

Send $\Delta \to 0$

$$
\begin{aligned}
V(w) =& \frac{u(w) + \delta V_0}{r + \delta} \\
V_0 =& V(w^r) = \frac{u(w^r) + \delta V_0}{r + \delta} \\
rV_0 =& u(w^r) \\
=& u_0 + \lambda \int_{w^r}^{\infty} (V(w) - V_0)\, dF(w) \\
=& u_0 + \frac{\lambda}{r + \delta} \int_{w^r}^{\infty} (u(w) - u(w^r))\, dF(w) \\
=& u_0 + \frac{\lambda}{r + \delta} \int_{w^r}^{\infty} u'(w)\,[1 - F(w)]\, dw
\end{aligned}
$$

- increasing in u_0 and λ
- decreasing in r and δ

Firm Problem

But where does F come from?

The firm is a monopsonist at the point that you get a job

- No reason to pay more than w^r
- Seems like they should pay w at all firms so that $u(w) = u_0$
- We need to get away from this to understand world (and to make search frictions interesting)
  - Could allow for bargaining on job. As a worker I can abstract more surplus on the job for which I am more productive
  - On the Job Search

On the Job Search

On the job search can explain why similar workers can earn different wages

In the model so far there is full Monopsony power-no firm ever competes against others

With on the job search you sometimes have two firms competing over you so they no longer are pure monopsonists

We will look at two different versions of models that give competition

Burdett and Mortensen (1988)

In Burdett and Mortensen firms post wages and they are not renegotiated

Now we let

- λ_0 be the arrival rate of jobs when non-employed
- λ_1 the arrival rate when employed
- $V_1(w)$ is the value function for someone employed who currently earns wage w

First focus on the worker's decisions

With finite time Δ

$$
\begin{aligned}
V_0 =& \Delta u_0 + \frac{1}{1+r\Delta}\left(\lambda_0\Delta\int \max\{V_0, V_1(w)\}dF(w) + (1-\Delta\lambda_0)V_0\right)\\
V_1\left(w^*\right) =& \Delta u(w^*) + \frac{1}{1+r\Delta}\left(\left(1-\Delta\delta\right)\lambda_1\Delta\int \max\{V_1\left(w^*\right), V_1(w)\}dF(w) + \delta\Delta(1-\Delta\lambda_1)V_0\right.\\
&\left.+\Delta\delta\Delta\lambda_1\int \max\{V_0, V_1(w)\}dF(w) + \left(1-\Delta\lambda_1\right)\left(1-\Delta\delta\right)V_1\left(w^*\right)\right)
\end{aligned}
$$

or after some algebra

$$
\begin{aligned}
\left(r\Delta + \Delta\lambda_0\right)V_0 &= \left(1+r\Delta\right)\Delta u_0 + \left(\lambda_0\Delta\int \max\{V_0, V_1(w)\}dF(w)\right)\\
(r\Delta + \Delta\lambda_1 + \Delta\delta - \Delta\lambda_1\Delta\delta)V_1\left(w^*\right) &= \left(1+r\Delta\right)\Delta u(w^*)\\
&+ \left(1-\Delta\delta\right)\lambda_1\Delta\int \max\{V_1\left(w^*\right), V_1(w)\}dF(w) + \delta\Delta(1-\Delta\lambda_1)V_0\\
&+ \Delta\delta\Delta\lambda_1\int \max\{V_0, V_1(w)\}dF(w)
\end{aligned}
$$

cancelling terms, dividing by Δ and taking limits as $\Delta \to 0$ gives

$$
\begin{aligned}
(r+\lambda_0)\, V_0 =& u_0 + \lambda_0 \left[\int \max\left\{V_0, V_1(w)\right\} dF(w) \right] \\
(r+\lambda_1+\delta)\, V_1(w^*) =& u(w^*) + \lambda_1 \left[\int \max\{V_1\left(w^*\right), V_1(w)\} dF(w) \right] + \delta V_0
\end{aligned}
$$

Reservation Wage

Again for the reservation wage

$$V_1(w^r) = V_0$$

So plugging in $rV_0 = rV_1(w^r)$ into the two expressions above and solving gives

$$\begin{aligned} u(w^r) - u_0 &= (\lambda_0 - \lambda_1) \int_{w^r}^{\infty} [V_1(w) - V_0]\, dF(w) \\ &= (\lambda_0 - \lambda_1) \int_{w^r}^{\infty} \left[V_1'(w)(1 - F(w))\right] dw \\ &= (\lambda_0 - \lambda_1) \int_{w^r}^{\infty} \left[\frac{u'(w)(1 - F(w))}{r + \delta + \lambda_1 [1 - F(w^*)]}\right] dw \end{aligned}$$

They simplify by assuming r is small relative to λ_0 so we can ignore it and by defining

$$k_0 \equiv \frac{\lambda_0}{\delta}$$
$$k_1 \equiv \frac{\lambda_1}{\delta}$$

Then we can write

$$u(w^r) - u_0 = (k_0 - k_1) \int_{w^r}^{\infty} \left[\frac{u'(w)\left(1 - F\left((w)\right)\right)}{1 + k_1 \left[1 - F\left(w^*\right)\right]} \right] dw$$

Now we want to think about the behavior of firms.

To do this we need to think about steady state behavior of workers since that is what they will face.

Let u be the steady state rate of unemployment

Steady State Unemployment

In steady state, the number of workers losing jobs must be equal to the number of workers finding jobs

$$(1-u)\,\delta = u\lambda_0\,(1-F\,(w^r))$$

or

$$u = \frac{1}{1+k_0\,(1-F\,(w^r))}$$

Steady State Wage Distribution

Next note that the distribution of wages of employed workers will be different than the distribution of workers because people move up the wage ladder by getting outside offers.

Let the distribution of wages of workers in steady state be G.

Consider $G(w)$ for some w. This is the fraction of workers that are earning less than w.

In steady state the number of people moving into this state must be the same as the number moving out

$$u\lambda_0 \left[F(w) - F(w^r)\right] = (1-u)\, G(w) \left[\delta + \lambda_1 \left(1 - F\left(w\right)\right)\right]$$

So

$$
\begin{aligned}
G(w) =& \frac{u\lambda_0 \left[F(w) - F(w^r)\right]}{(1-u)\left[\delta + \lambda_1 \left(1 - F\left(w\right)\right)\right]} \\
=& \frac{\left[F(w) - F(w^r)\right]}{\left[1 - F\left(w^r\right)\right]\left[1 + k_1 \left(1 - F\left(w\right)\right)\right]}
\end{aligned}
$$

Note that

$$
G(w) < \frac{\left[F(w) - F(w^r)\right]}{\left[1 - F\left(w^r\right)\right]}
$$

which is the distribution you would get with no on the job search

Firm Size

In steady state

- $[G(w) - G(w-\varepsilon)](1-u)$ is the fraction of workers working at a firm that pays between $w-\varepsilon$ and w
- $[F(w) - F(w-\varepsilon)]$ is the fraction of firms that pay between $w-\varepsilon$ and w

so

$$\frac{G(w) - G(w-\varepsilon)}{F(w) - F(w-\varepsilon)}(1-u)$$

is the average size of these firms

Define

$$\ell(w; w^r, F) \equiv \lim_{\varepsilon \downarrow 0} \frac{G(w) - G(w-\varepsilon)}{F(w) - F(w-\varepsilon)}(1-u)$$
$$= \lim_{\varepsilon \downarrow 0} \frac{\frac{[F(w)-F(w^r)]}{[1+k_1(1-F(w))]} - \frac{[F(w-\varepsilon)-F(w^r)]}{[1+k_1(1-F(w-\varepsilon))]}}{F(w) - F(w-\varepsilon)} \frac{(1-u)}{1-F(w^r)}$$
$$= \frac{(1+k_1-F(w^r)k_1)}{[1+k_1(1-F(w))][1+k_1(1-F(w^-))]} \frac{k_0}{1+k_0(1-F(w^r))}$$

where

$$F\left(w^-\right) \equiv \lim_{\varepsilon \downarrow 0} F(w-\varepsilon)$$

Notice $\ell(w; w^r, F)$ is

- increasing in w
- continuous except where F has a mass point
- strictly increasing on the support of F
- constant on intervals where F is flat

Firm Behavior

OK time to consider firms

- They face worker behavior as we have described
- let p be the output per worker
- Thus their steady state profit at wage w is

$$(p - w)\,\ell\,(w; w^r, F)$$

- In equilibrium it must be
  - that for any wage that is offered

$$(p - w)\,\ell\,(w; w^r, F)$$

    must be the same
  - No other wage can make higher profits

Equilibrium

So what we can see with an equilibrium is

- No firm will offer a wage lower than w^r
- There won't be any mass points in equilibrium. If there were I could offer a wage of ε more and steal all workers from those firms when we are matched and make the same profit per worker but have more workers thus strictly higher profit
- There can not be a hole in the offered wage distribution. If there is an offered wage w^* but no offered wages in range $[w-\delta, w]$ a firm at w^* could receive larger rents by offering a wage in that range. They would make higher rents per worker and get as many workers.

This means that some firms will offer a wage of w^r

at w^r

$$\ell\left(w; w^r, F\right) = \frac{\left(1 + k_1 - F\left(w^r\right) k_1\right)}{\left[1 + k_1\left(1 - F\left(w^r\right)\right)\right]\left[1 + k_1\left(1 - F\left(w^r\right)\right)\right]} \frac{k_0}{1 + k_0\left(1 - F\left(w^r\right)\right)}$$

$$= \frac{k_0}{\left[1 + k_1\right]\left[1 + k_0\right]}$$

So

$$\pi = \left(p - w^r\right) \frac{k_0}{\left[1 + k_1\right]\left[1 + k_0\right]}$$

$$= \left(p - w\right) \frac{\left(1 + k_1\right)}{\left[1 + k_1\left(1 - F\left(w\right)\right)\right]\left[1 + k_1\left(1 - F\left(w\right)\right)\right]} \frac{k_0}{1 + k_0}$$

But notice that this gives us a closed form solution for F

$$F\left(w\right) = \left(\frac{1 + k_1}{k_1}\right)\left[1 - \sqrt{\frac{\left(p - w\right)}{\left(p - w^r\right)}}\right]$$

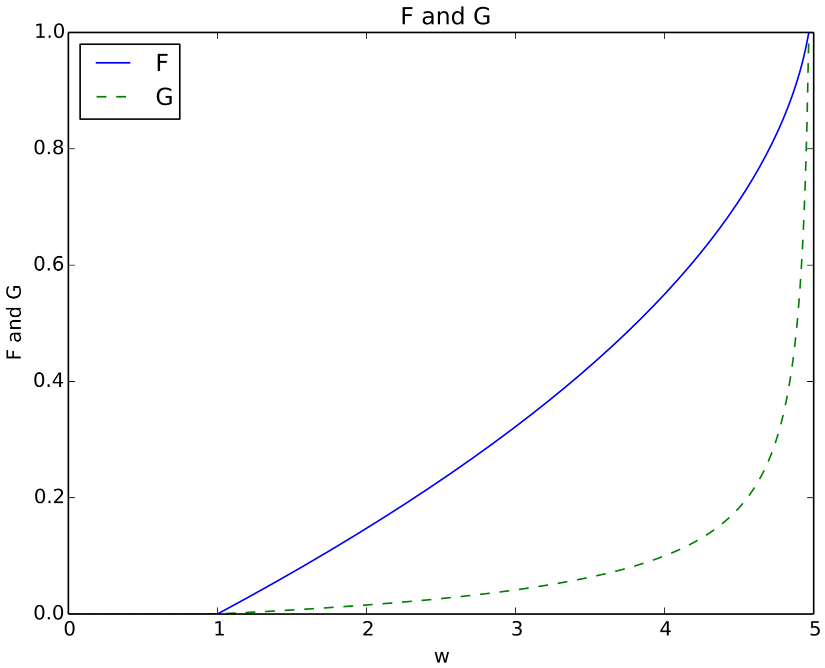
Given F we can also solve for G, w^r, and $\overline{w}$.

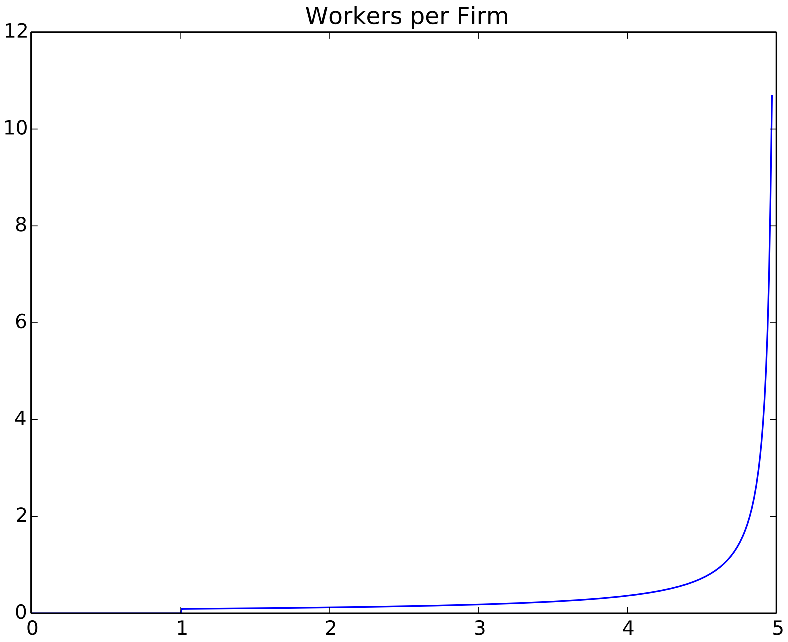
- We can get heterogeneity in wages despite no heterogeneity in firms or workers
- Can add worker heterogeneity or firm heterogeneity
    - With firm heterogeneity, higher productivity firm offer higher wages

Numerical Example

As an example take

- $u(w) = w$
- $\lambda_1 = \lambda_0 = 1$
- $\delta = 0.1$
- $u_0 = 1$
- $p = 5$


F and G
F
G
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
w
F and G


Workers per Firm
0
2
4
6
8
10
12
0
1
2
3
4
5

Postel-Vinay and Robin (2002)

This will be similar to the previous model in terms of notation, but the wage process will be very different.

In Burdett and Mortensen, when a worker gets an outside offer they just let them go

in Postel-Vinay and Robin a firm can respond to an outside offer

The main difference happen with the way the wage contracts work

- Firms can vary their wage offers according to the characteristics of the particular worker they meet.
- They can counter the offers received by their employees from competing firms.
- Firms make take-it-or-leave-it wage offers to workers. (the paper with Cahuc relaxes this)
- Wage contracts are long-term contracts that can be renegotiated by mutual agreement only.
- Complete Information.

Additions to Model

- Heterogeneity in productivity p of the firm which has distribution F
- Heterogeneity in worker ability ε so productivity at a firm p is

$$\varepsilon p$$

- Flow utility from non-employment is

$$u_0 = u(\varepsilon b)$$

- Death and birth at rate μ

Notation

- $V_0(\varepsilon)$ is value function for unemployed worker of type ε
- $V_1(\varepsilon, w, p)$ is value function for employed worker fo type ε currrently earning wage w at firm type p
- $\phi_0(\varepsilon, p)$ the wage offered to an ε type worker when hired from non-employment by a type p firm

Unemployed Workers

In equilibrium there is no reason for a firm with too low a productivity to make offers, so assume all offers are accepted

When a worker gets an offer, the firm will pay them a wage that makes them indifferent between working and staying unemployed

$$V_1(\varepsilon, \phi_0(\varepsilon, p), p) = V_0(\varepsilon)$$

Lets derive this thing

$$
\begin{aligned}
V_0(\varepsilon) =& \Delta u(\varepsilon b) + \frac{1}{1 + r\Delta}\left[\Delta\lambda_0 \int V_1(\varepsilon, \phi_0(\varepsilon, p), p) dF(p)\right. \\
& \left. + (1 - \Delta\lambda_0 - \Delta\mu) V_0(\varepsilon)\right] \\
=& \Delta u(\varepsilon b) + \frac{1}{1 + r\Delta}\left[\Delta\lambda_0 V_0(\varepsilon) + (1 - \Delta\lambda_0 - \Delta\mu) V_0(\varepsilon)\right]
\end{aligned}
$$

or

$$
(1 + r\Delta + \Delta\mu) V_0(\varepsilon) = (1 + r\Delta) \Delta u(\varepsilon b) + V_0(\varepsilon)
$$

taking limits as $\Delta \to 0$

$$
V_0(\varepsilon) = \frac{u(\varepsilon b)}{r + \mu}
$$

Search on the Job

Now our guy is working at the p firm at wage $\phi_0(\varepsilon, p)$ and suppose he gets an offer from a p' firm

- the maximum willingness to pay for the p firm is $p\varepsilon$
- the maximum willingness to pay for the p' firm is $p'\varepsilon$
- The firms will engage in Bertrand competition where
  - the firm with higher productivity will attract the worker
  - it needs to pay a wage high enough so that the other firm won't match it-but no higher

Therefore if $p' > p$ the worker will move to p' with wage $\phi(\varepsilon, p, p')$ such that

$$V\left(\varepsilon, \phi\left(\varepsilon, p, p'\right), p'\right) = V\left(\varepsilon, \varepsilon p, p\right)$$

What if $p' < p$?

This is symmetric now the current firm (the p firm) will offer the worker the wage to keep him

$$V\left(\varepsilon, \phi\left(\varepsilon, p', p\right), p\right) = V\left(\varepsilon, \varepsilon p', p'\right)$$

- Since an existing firm beats non-employment, $V(\varepsilon, \varepsilon p', p') > V_0(\varepsilon)$ so

  $$V\left(\varepsilon, \phi\left(\varepsilon, p', p\right), p\right) > V\left(\varepsilon, \phi_0\left(\varepsilon, p\right), p\right)$$

  and

  $$\phi\left(\varepsilon, p', p\right) > \phi_0(\varepsilon, p)$$

  that is this has to be an actual wage increase
- Both parties are willing to negotiate
  - The firm would lose the worker otherwise
  - The workers wage has gone up
- Thus we get firms to give raises to its employees in response to outside offers

Now Another Offer

Take the previous case with $p > p^{'}$ so our guy is still working at firm type p but now with wage $\phi\left(\varepsilon, p', p\right)$

Suppose now he gets an offer from a firm p^{*}.

There are three possibilities

$p* \leq p'$ In this case $V\left(\varepsilon, \varepsilon p^{*}, p^{*}\right) \leq V\left(\varepsilon, \varepsilon p', p'\right)$ so $\phi\left(\varepsilon, p^{*}, p\right) \leq \phi\left(\varepsilon, p', p\right)$. In this case the worker would have to take a wage cut. They do not agree to renogiate this wage so nothing happens.

$p' < p^* \leq p$ In this case

$$V(\varepsilon, \varepsilon p', p') < V(\varepsilon, \varepsilon p^*, p^*) \leq V(\varepsilon, \varepsilon p, p)$$

so $\phi(\varepsilon, p^*, p) > \phi(\varepsilon, p', p)$ where

$$V(\varepsilon, \phi(\varepsilon, p^*, p), p) = V(\varepsilon, \varepsilon p^*, p^*)$$

This is a wage increase. Both the firm and worker agree to raise the wage to $\phi(\varepsilon, p^*, p)$

$p^* > p$ This is the same as before. Worker will switch and be paid $\phi(\varepsilon, p, p^*)$ where

$$V(\varepsilon, \phi(\varepsilon, p, p^*), p^*) = V(\varepsilon, \varepsilon p, p)$$

Solving the Model

For a person of type ε earning wage w at a type p firm define $q(\varepsilon, w, p)$ such that

$$\phi(\varepsilon, q(\varepsilon, w, p), p) = w$$

That is q is the level of productivity of a firm that would give a current wage of w. What that means is that

- if I get an outside offer from a p' firm so that $p' > q(\varepsilon, w, p)$ then I will either renegotiate or leave
- If $p' < q(\varepsilon, w, p)$ renegotiating would lead to a lower wage, so the firm won't do that.

- We can write the discrete time Bellman equation as

$$
\begin{aligned}
V(\varepsilon, w, p) =& \Delta u(w) + \frac{1}{1+r\Delta}\left[\Delta\lambda_1 \int_{q(\varepsilon,w,p)}^{p} V\left(\varepsilon, \phi\left(\varepsilon, \varepsilon p^{'}, p\right), p\right) dF(p')\right. \\
& + \Delta\lambda_1 \int_{p}^{\infty} V\left(\varepsilon, \phi\left(\varepsilon, \varepsilon p, p^{'}\right), p^{'}\right) dF(p') + \Lambda\delta V_0(\varepsilon) \\
& + \left(1 - \Delta\delta - \Delta\lambda_1\left[1 - F(q(\varepsilon, w, p))\right] - \Delta\mu\right) V(\varepsilon, w, p)] \\
=& \Delta u(w) + \frac{1}{1+r\Delta}\left[\Delta\lambda_1 \int_{q(\varepsilon,w,p)}^{p} V(\varepsilon, \varepsilon p', p)\, dF(p')\right. \\
& + \Delta\lambda_1 (1 - F(p)) V(\varepsilon, \varepsilon p, p) + \delta V_0(\varepsilon) \\
& + \left(1 - \Delta\delta - \Delta\lambda_1\left[1 - F(q(\varepsilon, w, p))\right] - \Delta\mu\right) V(\varepsilon, w, p)]
\end{aligned}
$$

Some algebra and taking limits as $\Delta \to 0$

$$\left(r + \delta + \lambda_1 \left[1 - F\left(q(\varepsilon, w, p)\right)\right] + \mu\right) V\left(\varepsilon, w, p\right)$$
$$= u\left(w\right) + \left[\lambda_1 \int_{q(\varepsilon,w,p)}^{p} V\left(\varepsilon, \phi\left(\varepsilon, p^{'}, p\right), p\right) dF\left(p'\right)\right.$$
$$\left. + \lambda_1 \left(1 - F\left(p\right)\right) V\left(\varepsilon, \varepsilon p, p\right) + \delta V_0\left(\varepsilon\right)\right]$$

So

$$V(\varepsilon, \varepsilon p, p) = \frac{u(\varepsilon p) + \delta V_0(\varepsilon)}{r + \delta + \mu}$$

plugging this in, doing some algebra and integrating by parts gives

$$\begin{aligned} &(r + \delta + \mu) V(\varepsilon, w, p) \\ =& u(w) + \delta V_0(\varepsilon) + \frac{\lambda_1 \varepsilon}{r + \delta + \mu} \int_{q(\varepsilon, w, p)}^{p} u'(\varepsilon p') \left(1 - F(p')\right) dF(p') \end{aligned}$$

Now to figure out the wage for $p > p'$ we know that $V\left(\varepsilon, \phi\left(\varepsilon, p^{'}, p\right), p'\right) = V\left(\varepsilon, \varepsilon p, p\right)$ so

$$\begin{aligned} u\left(\varepsilon p'\right) + \delta V_0\left(\varepsilon\right) =& u\left(\phi\left(\varepsilon, p^{'}, p\right)\right) + \delta V_0\left(\varepsilon\right) \\ &+ \frac{\lambda_1 \varepsilon}{r+\delta+\mu} \int_{p'}^{p} u'\left(\varepsilon p^*\right)\left(1 - F\left(p^*\right)\right) dF\left(p^*\right) \end{aligned}$$

or

$$u\left(\phi\left(\varepsilon, p^{'}, p\right)\right) = u\left(\varepsilon p'\right) - \frac{\lambda_1 \varepsilon}{r+\delta+\mu} \int_{p'}^{p} u'\left(\varepsilon p^*\right)\left(1 - F\left(p^*\right)\right) dF\left(p^*\right)$$

a closed form solution which makes estimation very easy.

Notice as well that the second term is negative. This means if we moved from a p' firm to a p firm we could actually see wages fall at switches

Wages from Non-employment

Worrying about non-employment, we know that $V\left(\varepsilon,\phi_0\left(\varepsilon,p\right),p'\right)=V_0\left(\varepsilon\right)$ so

$$\begin{aligned}u\left(\varepsilon b\right)+\delta V_0 =& u\left(\phi_0\left(\varepsilon,p\right)\right)+\delta V_0\left(\varepsilon\right)\\ &+\frac{\lambda_1\varepsilon}{r+\delta+\mu}\int_b^p u'\left(\varepsilon p'\right)\left(1-F\left(p'\right)\right)dF\left(p'\right)\end{aligned}$$

so

$$u\left(\phi_0\left(\varepsilon,p\right)\right) = u\left(\varepsilon b\right)-\frac{\lambda_1\varepsilon}{r+\delta+\mu}\int_b^p u'\left(\varepsilon p'\right)\left(1-F\left(p'\right)\right)dF\left(p'\right)$$

Empirical Approach

They use the Declarations Annuelles des Donees Sociales which is a data set of matched employer-employee data from France

Use years 1996-1998

Divide by occupation

TABLE I

DESCRIPTIVE ANALYSIS OF WORKER MOBILITY

Occupation	Number of indiv. trajectories	Percentage with no recorded mobility (%)	Percentage whose first recorded mobility is from job… …to-job (%)	…to-out of sample (%)	Sample mean unemployment spell duration	Sample mean employment spell duration
Executives, managers, and engineers	22,757	46.2	23.4	30.4	0.96 yrs	2.09 yrs
Supervisors, administrative, and sales	14,977	48.1	19.3	32.5	1.16 yrs	2.11 yrs
Technical supervisors and technicians	7,448	55.5	16.0	28.6	1.07 yrs	2.28 yrs
Administrative support	14,903	54.3	8.2	37.5	1.30 yrs	2.23 yrs
Skilled manual workers	12,557	55.9	5.2	38.9	1.16 yrs	2.28 yrs
Sales and service workers	5,926	45.1	5.5	49.4	1.28 yrs	2.06 yrs
Unskilled manual workers	4,416	42.5	7.0	50.5	1.29 yrs	1.98 yrs

TABLE II

VARIATION IN REAL WAGE AFTER FIRST RECORDED JOB-TO-JOB MOBILITY (I.E. WITH LESS THAN 15 DAYS WORK INTERRUPTION) IN 96–98

Occupation	Nb. obs.	Median $\Delta \log$ wage (%)	% obs. such that $\Delta \log$ wage $\leq$				
			−0.10	−0.05	0	0.05	0.10
Executives, managers, and engineers	5,335	3.1	23.6	28.5	38.1	55.1	65.4
Supervisors, administrative, and sales	2,893	3.7	21.6	27.1	36.6	54.3	65.2
Technical supervisors and technicians	1,190	3.8	14.0	20.2	32.2	55.5	67.3
Administrative support	1,222	2.2	21.5	28.7	40.7	60.5	69.2
Skilled manual workers	657	0.5	33.2	37.7	49.2	62.3	72.0
Sales and service workers	326	1.4	31.3	37.7	45.1	58.0	67.5
Unskilled manual workers	310	−1.3	33.5	42.9	54.5	63.4	72.3

TABLE III

VARIATION IN REAL WAGE BETWEEN 01/01/96 AND 31/12/97 WHEN HOLDING THE SAME JOB OVER THIS PERIOD

Occupation	Nb. obs.	Median Δ log wage (%)	% obs. such that Δ log wage $\leq$				
			−0.10	−0.05	0	0.05	0.10
Executives, managers, and engineers	16,102	2.7	6.6	11.3	28.5	64.4	80.0
Supervisors, administrative, and sales	15,592	2.6	7.9	12.9	28.6	65.2	81.1
Technical supervisors and technicians	5,644	2.5	6.6	11.9	29.6	68.1	85.0
Administrative support	11,105	2.2	7.9	12.4	30.0	69.8	84.2
Skilled manual workers	9,747	1.9	7.9	15.0	34.9	69.5	85.1
Sales and service workers	4,192	2.5	7.4	12.8	31.4	64.5	79.1
Unskilled manual workers	2,847	2.2	7.7	14.6	32.9	66.4	81.9

For brevity I am going to skip the details of the estimation.

They estimate the model in three steps

1. Estimate transition parameters $(\delta, \mu, \lambda_0, \lambda_1)$ by maximizing the likelihood function
2. Estimate p_j from earnings by firm
3. Estimate rest of wage equation including distribution of ϕ and of ε

TABLE IV
Estimated Transition Parameters

Occupation	Parameter δ	μ	λ_0	λ_1	κ_1
Executives, managers, and engineers	0.0776	0.0070	2.104	0.643	7.61
	(0.0009)	(0.0005)	(0.063)	(0.009)	(0.14)
Supervisors, administrative, and sales	0.0859	0.0065	1.956	0.666	7.21
	(0.0014)	(0.0007)	(0.081)	(0.015)	(0.21)
Technical supervisors and technicians	0.0686	0.0042	2.055	0.646	8.87
	(0.0016)	(0.0008)	(0.137)	(0.021)	(0.37)
Administrative support	0.0932	0.0085	1.678	0.737	7.24
	(0.0020)	(0.0011)	(0.078)	(0.026)	(0.32)
Skilled manual workers	0.0886	0.0082	1.499	0.685	7.07
	(0.0020)	(0.0012)	(0.071)	(0.027)	(0.35)
Sales and service workers	0.1016	0.0045	1.486	0.716	6.75
	(0.0031)	(0.0016)	(0.097)	(0.038)	(0.44)
Unskilled manual workers	0.0989	0.0153	1.529	0.666	5.84
	(0.0036)	(0.0020)	(0.099)	(0.038)	(0.41)

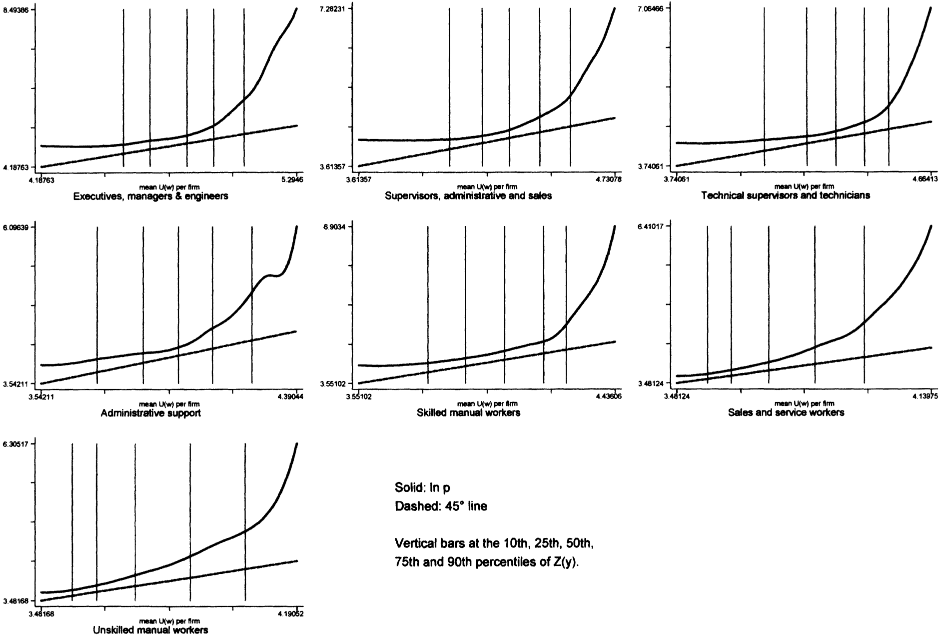


FIGURE 1.—Log marginal productivity and average log-wages (case $U(w) = \ln w$).

TABLE V

ESTIMATION OF THE REMAINING PARAMETERS

Occupation	Case	$U(b)$	$U(p_{\min})$	ρ	$V[U(\varepsilon)]$
Executives, managers, and engineers	$U(w) = \ln w$	4.62	4.74	0.128 (12% annual)	0.051 (0.0029)
	$U(w) = w$	97.1	112.9	0.353 (30% annual)	0.100 (0.0037)
Supervisors, administrative, and sales	$U(w) = \ln w$	3.99	4.21	0.320 (27% annual)	0.019 (0.0016)
	$U(w) = w$	53.6	67.2	0.471 (38% annual)	0.046 (0.0022)
Technical supervisors and technicians	$U(w) = \ln w$	4.07	4.22	0.240 (21% annual)	0.006 (0.0010)
	$U(w) = w$	56.8	66.5	0.361 (30% annual)	0.015 (0.0013)
Administrative support	$U(w) = \ln w$	3.69	3.84	0.678 (49% annual)	0.007 (0.0014)
	$U(w) = w$	40.0	46.5	0.678 (49% annual)	0.012 (0.0014)
Skilled manual workers	$U(w) = \ln w$	3.76	3.93	0.475 (38% annual)	−0.006 (0.0011)
	$U(w) = w$	43.3	50.3	0.443 (36% annual)	−0.001 (0.0013)
Sales and service workers	$U(w) = \ln w$	3.55	3.61	0.653 (48% annual)	0.003 (0.0011)
	$U(w) = w$	34.0	36.5	0.580 (44% annual)	0.004 (0.0013)
Unskilled manual workers	$U(w) = \ln w$	3.54	3.63	0.834 (57% annual)	−0.004 (0.0017)
	$U(w) = w$	33.9	37.1	0.796 (55% annual)	−0.006 (0.0017)

Variance Decomposition

They then use their model to decompose the variance of log wages First a trick. Let j be firms

$$
\begin{aligned}
V(Y) =& E\left[(Y-\mu_y)^2\right] \\
=& \sum_j E\left[(Y-\mu_y)^2 \mid j\right] p_j \\
=& \sum_j E\left[(Y - E(Y \mid j) + E(Y \mid j) - \mu_y)^2 \mid j\right] p_j \\
=& \sum_j E\left[(Y - E(Y \mid j))^2 \mid j\right] p_j \\
& + \sum_j E\left[(E(Y \mid j) - \mu_y)^2 \mid j\right] p_j \\
& + 2 \sum_j E\left[(Y - E(Y \mid j))(E(Y \mid j) - \mu_y) \mid j\right] p_j \\
=& \sum_j E\left[(Y - E(Y \mid j))^2 \mid j\right] p_j + \sum_j E\left[(E(Y \mid j) - \mu_y)^2 \mid j\right] p_j \\
=& E(Var(Y \mid j)) + Var(E(Y \mid j))
\end{aligned}
$$

because

$$E\left[\left(Y-E\left(Y\mid j\right)\right)\left(E\left(Y\mid j\right)-\mu_y\right)\mid j\right]=\left(E\left(Y\mid j\right)-\mu_y\right)E\left[Y-E\left(Y\mid j\right)\mid j\right] \\ =0$$

They get a third term

$$Var(\log(w)) = Var\left(\log\left(\varepsilon\right)\right)+E\left(Var\left(\log\left(\phi\left(1,q,p\right)\right)\mid p\right)\right) \\ +Var\left(E\left(\log\left(\phi\left(1,q,p\right)\right)\mid p\right)\right)$$

TABLE VII

LOG WAGE VARIANCE DECOMPOSITION

		Mean log wage:	Total log-wage variance/coeff. var.		Case	Firm effect: $VE(\ln w \mid p)$		Search friction effect: $EV(\ln w \mid p) - V \ln \varepsilon$		Person effect: $V \ln \varepsilon$	
Occupation	Nobs.	$E(\ln w)$	$V(\ln w)$	CV	$U(w) =$	Value	% of $V(\ln w)$	Value	% of $V(\ln w)$	Value	% of $V(\ln w)$
Executives, manager, and engineers	555,230	4.81	0.180	0.088	$\ln w$	0.035	19.3	0.082	45.5	0.063	35.2
					w	0.035	19.4	0.070	38.7	0.076	41.9
Supervisors, administrative and sales	447,974	4.28	0.125	0.083	$\ln w$	0.034	27.5	0.065	52.1	0.025	20.3
					w	0.034	27.9	0.069	55.1	0.022	17.8
Technical supervisors and technicians	209,078	4.31	0.077	0.064	$\ln w$	0.025	32.4	0.044	57.6	0.008	10.0
					w	0.025	32.8	0.047	60.6	0.005	6.6
Administrative support	440,045	4.00	0.082	0.072	$\ln w$	0.029	35.7	0.043	52.2	0.010	12.1
					w	0.028	34.6	0.045	55.7	0.008	9.7
Skilled manual workers	372,430	4.05	0.069	0.065	$\ln w$	0.029	42.9	0.039	57.1	0	0
					w	0.028	41.5	0.040	58.5	0	0
Sales and service workers	174,704	3.74	0.050	0.060	$\ln w$	0.020	40.8	0.029	58.7	0.0002	0.4
					w	0.019	37.1	0.029	57.9	0.0025	5.0
Unskilled manual workers	167,580	3.77	0.057	0.063	$\ln w$	0.027	48.3	0.029	51.7	0	0
					w	0.023	40.8	0.033	59.2	0	0