Problem 1.

Take a lifecycle model in which people live forever. The first period they can either go to school or they can work.

If they go to school in the first period they earn \$100 in every period, if they don't they earn \$95. Let $\beta = 1/(1+r) = 0.95$. Assume everyone has initial assets of \$50 and pays tuition of \$10. Let the tastes for schooling have a normal distribution with mean zero and standard deviation 3.

a) People are not borrowing constrained what is the probability of attending college?

b) Suppose there are hard borrowing constraints so that you can't borrow at all when going to college. What is the probability of going to college?

c) Now suppose that tuition increases from \$10 to \$40. What happens to the probability of going to college for the two groups.

d) Now suppose that the wages as a high school graduate changes from \$95 to \$96. What happens to the probability of attending college for the two groups.

Problem 2

Take a four period discrete time Ben-Porath model with

$$H_{t+1} = A \left(H_t I_t \right)^{\alpha} + (1 - \sigma) H_t$$

where I_t is tiem investment in human capital, so that earnings in period t are $(1 - I_t)H_t$.

Take the parameter values

$$A = 2$$
$$\alpha = 0.6$$
$$\sigma = 0.05$$
$$r = 0.10$$
$$H_0 = 10$$

where r is the interest rate and people maximize the present discounted value of (after tax) income. Assume further that $0 \leq I_t \leq 1$.

a) First numerically solve the basic model without taxes. What are the values of I_t and H_t?

b) Now assume there are proportional taxes of 10%. Calculate the optimal values of I_t and H_t in this case. How does that compare to your answers to problem 1?

c) Now take progressive taxes. In particular, assume that for any earnings level w, the taxes that are collected are $0.01w^2$. Calculate the optimal values of I_t and H_t in this case. How does that compare to your answers to problems 1 and 2? Explain.